Cannons at Sparrows*

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The story told in this lecture starts with an innocuous little geometry problem (one that Erdős would have liked), posed in a September 2006 blog entry by R. Nandakumar, an engineer from Calcutta, India: "Can you cut every polygon into a prescribed number of convex pieces that have equal area and equal perimeter?" This little problem is a "sparrow", tantalizing, not as easy as one could perhaps expect, and *Recreational Mathematics*: of no practical use.

I will sketch, however, how this little problem connects to very serious mathematics, including Computational Geometry: For the modelling of this problem we employ insights from a key area of *Applied Mathematics*, the Theory of Optimal Transportation, which leads to weighted Voronoi diagrams with prescribed areas. This will set up the stage for application of a major tool from *Very Pure Mathematics*, known as Equivariant Obstruction Theory. This is a "cannon", and we'll have fun with shooting it at the sparrow.

On the way to a solution, combinatorial properties of the permutahedron turn out to be essential. These will, at the end of the story, lead us back to India, with some time travel 100 years into the past: For the last step in our (partial) solution of the sparrows problem we need a simple divisibility property for the numbers in Pascal?s triangle, which was first observed by Balak Ram, in Madras 1909.

But even if the existence problem is solved, the Computational Geometry problem is not: If the solution exists, how do you find one? This problem will be left to you. Instead, I will comment on the strained relationship between cannons and sparrows, and to this avail quote a poem by Hans Magnus Enzensberger.

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