DIGRAPH COVERING AND ITS APPLICATION TO TWO OPTIMIZATION PROBLEMS FOR DIGRAPHS

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Abstract

In this paper we define the notions of arc-disjoint and circuit coverings for directed graphs. Using these ideas we then show that certain diregular digraphs 'close' to Moore bound cannot exist.
Considering the relationship between degree, diameter and the number of vertices in a diregular digraph we define the following three optimization problems. The $N(d,k)$ problem: find the maximum possible number of vertices given degree d and diameter k. The $K(n,d)$ problem: find the minimum possible diameter given the number of vertices n and degree d. The $D(n,k)$ problem: find the minimum possible degree given the number of vertices n and diameter k. These three problems are related but as far as we know not equivalent.
In this paper we study the first two problems for $d = 2$. We introduce an efficient number-theoretic divisibility argument that shows $N(2,k) \leq 2^{k+1} - 4$ for many, but not all values of k. This new result also gives new values for the $K(n,2)$ problem when $n = 2^{k+1} - 3$.
The paper concludes with two tables, one giving a summary of our present knowledge of the $N(d,k)$ problem for $d = 2$; and the other giving the values of $K(n,d)$ for $d = 2$ and $n \leq 100$.

1. INTRODUCTION

A *directed graph*, or a *digraph*, $G = (V,E)$ where

(i) V is a nonempty set $V = \{v_1, v_2, \ldots, v_n, \ldots\}$ of distinct elements called *vertices*;

(ii) E is a bag $E = \{e_1, e_2, \ldots, e_m, \ldots\}$ of ordered pairs $(v_i \to v_j), v_i, v_j \in V$, called *arcs*.

In this paper we shall consider finite digraphs, that is, digraphs in which both V and E are finite. The number of vertices in the digraph is called the *order* of the digraph.

The *indegree* of a vertex $v \in G$ is the number of arcs of the form (u,v) in G. Similarly, the *outdegree* of a vertex $u \in G$ is the number of arcs of the form (u,v) in G. The *degree* of a vertex v is the sum of its indegree and outdegree. If in a digraph G every vertex has the same degree d then G is said to be a *regular digraph*, or a *regular digraph of degree d*.

However, if G is a regular digraph in which indegree=outdegree=d for every vertex in G then G is called a *diregular digraph of degree d.*

The *diameter* k of a digraph G is the maximum shortest distance between any two vertices of G.

A diregular digraph G of order n, degree d and diameter k, $G \in \mathcal{G}(n, d, k)$ consists of n vertices and nd arcs. For each vertex $v \in G$ there are d arcs whose initial vertex is v, and d arcs whose terminal vertex is v.

Given a digraph G, a set $l_1, l_2, \ldots, l_m$ of subdigraphs of G *covers* G if every arc and every vertex of G occurs in some l_i. and a collection $l_i, 1 \leq i \leq m$ of subdigraphs of G such that $\cup_{i=1}^{m} l_i = G$ is a *covering* of G. We introduce the following definitions.

Definition 1. A covering L of a digraph G is *arc-disjoint* if for all l_i and l_j in L, l_i and l_j have no arcs in common whenever $i \neq j$.

Similarly, we can define the concept of a *vertex-disjoint* covering.

Definition 2. A covering L of a digraph G is a *circuit covering* if for all $l_i \in L$, the subdigraph l_i is a circuit.

Furthermore, we call a circuit covering consisting of circuits of lengths $t_1, t_2, \ldots, t_s$ a $t_1, \ldots, t_s$*-circuit covering.* In particular, a $t-$*circuit covering* consists only of circuits of length t. Lastly, a $\leq t$*-circuit covering* consists of circuits of length t or less, and a $< t-$*circuit covering* consists of circuits of lengths less than t.

The three optimization problems $N(d, k)$, $K(n, d)$ and $D(n, k)$ for diregular digraphs $\mathcal{G}(n, d, k)$ are defined as follows.

1. The $N(d, k)$ problem : given d and k, find the maximum possible order $N(d, k)$. This problem has been also called the (d, k) problem.

2. The $K(n, d)$ problem : given n and d, find the minimum possible diameter $K(n, d)$.

3. The $D(n, k)$ problem : given n and k, find the minimum possible degree $D(n, k)$.

These three problems are related, but as far as we know, not equivalent. For further discussion on the relationships between these problems see [5].

The main result of this paper, Theorem 3, namely that for many values of k, $\mathcal{G}(n, 2, k)$ is empty, gives new results for the first two problems for $d = 2$. The proofs of all the theorems in this paper make use of circuit coverings.

Every diregular digraph $G \in \mathcal{G}(n, d, k)$ has a $\leq k + 1$-circuit covering since every vertex must reach itself in at most $k + 1$ steps (in at least d ways).

However, we shall be interested in those digraphs $G \in \mathcal{G}(n,d,k)$ which have $\leq k+1-$circuit covering that is arc-disjoint.

Not all diregular digraphs have such covering. For example, of the 5 nonisomorphic diregular digraphs in $\mathcal{G}(5,2,2)$ all except one (G_5) have an arc-disjoint $\leq 3-$circuit covering (Figure 1). The exception is the digraph with vertices i, $i = 0,1,2,3,4$ and arcs $(i, i+1 \ mod \ 5)$ and $(i, i+2 \ mod \ 5)$. Similarly, the digraph $G \in \mathcal{G}(7,2,3)$ with vertices $i, i = 0,1,2,3,4,5,6$ and arcs $(i, i+1 \ mod \ 7)$, $(i, i+2 \ mod \ 7)$ does not have an arc-disjoint $\leq 4-$circuit covering.

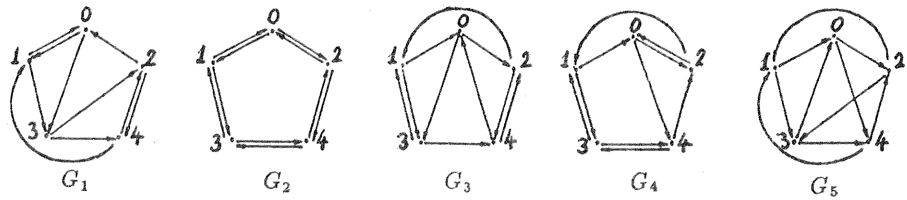

Figure 1.

If a diregular digraph $G \in \mathcal{G}(n,d,k)$ does have an arc-disjoint t-circuit covering L then the number of arcs in L is equal to nd and nd must be divisible by t. We shall exploit this fact to show the nonexistence of some digraphs.

The results of this paper are presented in the next section as Theorems 1, 2 and 3. Theorems 1 and 2 do not give new results.

A more general version of Theorem 1, namely that digraphs with degree d, diameter k and $1+d+...+d^k$ vertices do not exist for $d > 1$ and $k > 1$ was proved by Plesnik and Znam[6] in 1974, and in a simpler way by Bridges and Toueg[1] in 1980.

A more general version of Theorem 2, namely the nonexistence of diregular digraphs of degree 2, diameter k and $2+2^2+...+2^k (= 2^{k+1}-2)$ vertices for $k > 2$ was proved by Miller[4]. However, since the proofs of Theorems 1 and 2 are very simple, we include them here to illustrate the method used to prove Theorem 3 which is the main result of this paper.

Theorem 3 gives a new result, namely it proves the nonexistence of diregular digraphs of degree 2, diameter k and $2^{k+1}-3$ vertices for many values of k. This in turn gives new results for the $K(n,d)$ problem and it improves upon the bounds for the $N(d,k)$ problem.

2. RESULTS

We have already mentioned in the Introduction to this paper that for $d > 1$ and $k > 1$, $\mathcal{G}(1 + d + d^2 + \ldots + d^k, d, k)$ is empty[6][1]. We can prove this result for k such that $k+1$ does not divide $d(1+d+d^2+\ldots+d^k)$ in a much simpler way using digraph coverings.

Theorem 1. If $d > 1$, $k > 1$ and $k+1$ does not divide $d(1 + d + d^2 + \ldots + d^k)$ then $\mathcal{G}(1 + d + d^2 + \ldots + d^k, d, k)$ is empty.

Proof. If $G \in \mathcal{G}(1+d+d^2+\ldots+d^k, d, k)$ then every vertex $v \in G$ lies on d circuits, each consisting of $k+1$ distinct vertices.
These d circuits have only the vertex v in common. Thus there is an arc-disjoint $k+1$–circuit covering of G.
There are $d(1 + d + d^2 + \ldots + d^k)$ arcs in G so if $k+1$ does not divide $d(1 + d + d^2 + \ldots + d^k)$ then G does not exist. ∎

We proved (using brute force)[4] that for $k > 2$, $\mathcal{G}(2^{k+1}-2, 2, k)$ is empty. However, for k such that $k+1$ does not divide $2(2^{k+1}-2)$ we can prove this result in a much simpler way using digraph coverings.
The proof of the next theorem makes use of 'line digraphs'. The *line digraph* $L(G) \in \mathcal{G}(nd, d, k+1)$ of a digraph $G \in \mathcal{G}(n, d, k)$ is a digraph constructed from G as follows.

If $i \to j$ in G then there is a vertex (ij) in $L(G)$; and there is an arc (ij, mn) in $L(G)$ whenever $j = m$. The line digraph $L(G)$ of a digraph $G \in \mathcal{G}(n, d, k)$ has nd vertices, degree d and diameter $k+1$ (for more detailed explanation see for example [2]).

Theorem 2. If $k > 2$ and $k+1$ does not divide $2(2^{k+1}-2)$ then $\mathcal{G}(2^{k+1}-2, 2, k)$ is empty.

Proof. Suppose $k > 2$ and $G \in \mathcal{G}(2^{k+1}-2, 2, k)$ exists.
There are no circuits of length less than k in G. (Otherwise the diameter of G would be greater than k.)
Suppose we have a k-circuit in G. Then we can partially draw G as follows (Figure 2).

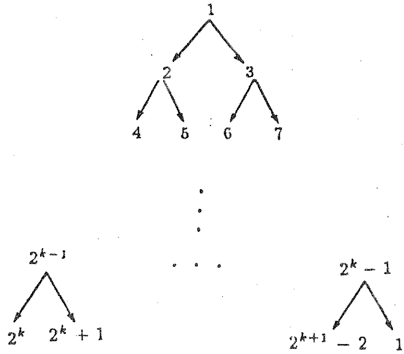

Figure 2.

Then to reach vertex 3 from vertex 2 in at most k steps, we need one of the vertices $2^k, \ldots, 3 \times 2^{k-1}$ to go to vertex 3, say $2^k \to 3$. For $k > 2$, to reach all vertices from 2^k we must have $2^k \to 2$.

Then every vertex lies on exactly one k-circuit and these circuits are disjoint. Furthermore, for all vertices $x \in G$, if $x \to x_1$, $x \to x_2$ and $y \to x_1$ then also $y \to x_2$, that is, G is a line digraph of $G^* \in \mathcal{G}(1 + 2 + 2^2 + \ldots + 2^{k-1}, 2, k-1)$.

But such G^* does not exist for $k - 1 > 1$, that is for $k > 2$ [1].

Hence G does not have a k-circuit.

Then each vertex of G must lie on (at least) two $k+1$-circuits. We will show that no vertex of G lies on more than two $k+1$-circuits which implies that these circuits are arc-disjoint.

Suppose there are three $k+1$-circuits in G.

Then we have (Figure 3).

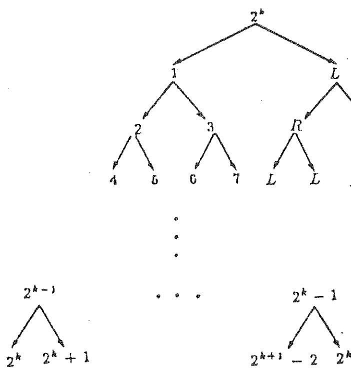

Figure 3.

where L denotes a vertex from $2^k+1, 2^k+2, \ldots, 3 \times 2^{k-1} - 1$ and R denotes a vertex from $3 \times 2^{k-1}, 3 \times 2^{k-1}+1, \ldots, 2^{k+1}-2$.
If $k > 2$ it is not possible to reach all of $2^k+1, \ldots, 2^{k+1}-2$ from 2^k in at most k steps.
Hence every vertex of G lies on exactly two $k+1$-circuits and any two such circuits are arc-disjoint, that is, there is an arc-disjoint $k+1$-circuit covering of G.
Hence if $k+1$ does not divide $2(2^{k+1}-2)$ then $\mathcal{G}(2^{k+1}-2, 2, k)$ is empty. ∎

To prove the main result of this paper, Theorem 3, we shall make use of the following two Lemmas.

Lemma 1. For $k > 3$ there are no $\leq k-$circuits in $G \in \mathcal{G}(2^{k+1}-3, 2, k)$.

Proof. Suppose 1,2,...,2^{k+1}-3 are the vertices of $G \in \mathcal{G}(2^{k+1}-3, 2, k)$.
Obviously, G cannot contain a circuit of length less than k since a vertex on a $< k-$circuit could not reach all the other vertices of G in at most k steps.
Assume there is a $k-$circuit in G.
Then we can partially draw G as in Figure 4.

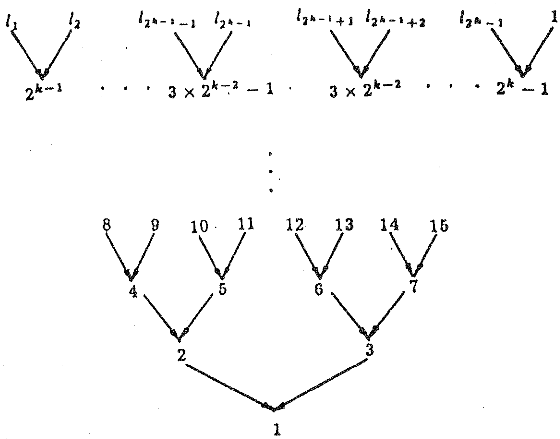

Figure 4.

To reach vertex 1 from all vertices in at most k steps, no vertex can reach 1 twice in less than k steps, so the vertices which are at most $k-1$ steps away from 1 must be all distinct.
Furthermore, to reach vertex 1 from all the $2^{k+1}-3$ vertices (including vertex 1 since we assume a $k-$circuit through vertex 1) in at most k steps, the vertices 1,2,...,2^k-1 and all except one of $l_1, \ldots, l_{2^k-1}$ must be distinct vertices.

Let $l_p \in \{l_1, \ldots, l_{2^k-1}\}$ be the vertex that reaches vertex 1 in at most k steps in two different ways. To reach vertex 2 from all vertices (in at most k steps) we must have the vertices in the set

$S_l = \{3, 6, 7, \ldots, 3 \times 2^{k-2}, \ldots, 2^k - 1, l_{2^{k-1}+1}, l_{2^{k-1}+2}, \ldots, l_{2^k-1}, 1\}$

(except l_p if $l_p \in \{l_{2^{k-1}+1}, \ldots, l_{2^k-1}\}$ and then possibly except x and/or y if $x \to l_p, y \to l_p$ and x and/or $y \in S_l$) go to the vertices $l_1, l_2, \ldots, l_{2^{k-1}}$.
Similarly, to reach vertex 3 from all vertices we must have the vertices in the set
$S_r = \{2, 4, 5, \ldots, 2^{k-1}, \ldots, 3 \times 2^{k-2} - 1, l_1, l_2, \ldots, l_{2^{k-1}}\}$
(except l_p if $l_p \in \{l_1, l_2, \ldots, l_{2^{k-1}}\}$ and then possibly also except x and/or y if $x \to l_p, y \to l_p$ and x and/or $y \in S_r$) go to the vertices $l_{2^{k-1}+1}, \ldots, l_{2^k-1}, 1$.
We have (Figure 5).

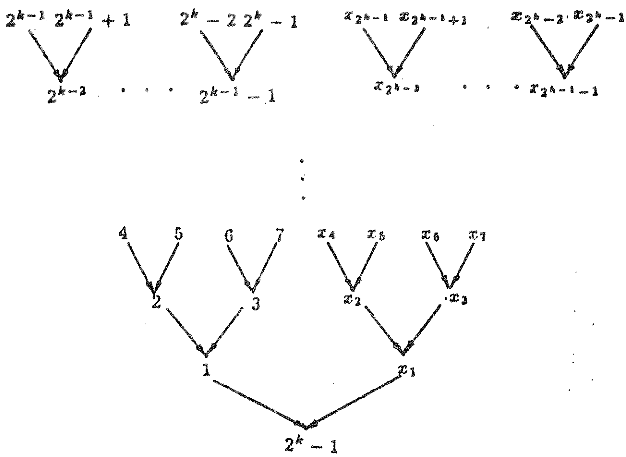

Figure 5.

where $x_1 = l_{2^k-1}$ and
$\{2^k, \ldots, 2^{k+1} - 3\} = \{l_1, l_2, \ldots, l_{2^k-1}\} - l_p = \{x_1, x_2, \ldots, x_{2^k-1}\} - x_q$ for some $x_q \in \{x_1, \ldots, x_{2^k-1}\}$.
If $x_1 \in \{2^{k-1}, \ldots, 2^k - 1\}$
or if $x_1 \in \{2^k, \ldots, 2^{k+1} - 3\}$ and $x_1 \neq l_p$
then we cannot have $l_i \to x_1, l_j \to x_1, l_i \neq l_j$
and $l_i \in \{l_1, \ldots, l_{2^{k-1}}\}, l_j \in \{l_{2^{k-1}+1}, \ldots, l_{2^k-1}\} - l_p$, or respectively
$l_i \in \{l_1, \ldots, l_{2^{k-1}}\} - l_p, l_j \in \{l_{2^{k-1}+1}, \ldots, l_{2^k-1}\}$.
Then, for $k > 3$, we cannot reach the vertex $2^k - 1$ from the 2^{k-1} (or respectively $2^{k-1} - 1$) vertices of $\{l_1, \ldots, l_{2^{k-1}}\}$ and from the $2^{k-1} - 2$ (or respectively $2^{k-1} - 1$) vertices of $\{l_{2^{k-1}+1}, \ldots, l_{2^k-1}\}$.

It remains to consider the case $x_1 = l_p \in \{2^k, \dots, 2^{k+1} - 3\}$.

Then to reach the vertex 2 from x_1 we must have $x_1 \in \{l_1, \dots, l_{2^{k-1}}\}$.

Let $x_1 = l_1 = 2^k$, $l_2 = 2^k + 1$.

We have (Figure 6).

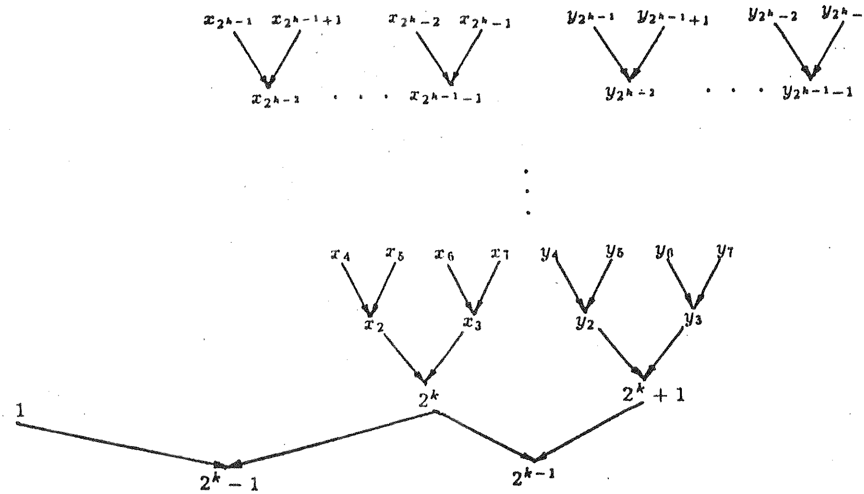

Figure 6.

Since all except at most one of $x_2, \dots, x_{2^k-1}$ are from $\{2^k + 1, \dots, 2^{k+1} - 3\}$ then all except at most one of $y_2, \dots, y_{2^k-1}$ must be from $\{1, 2, \dots, 2^k - 1\} - 2^{k-1}$. If $y_2 \neq 2$ or $y_3 \neq 3$ this is not possible.

Suppose $y_2 = 2$, $y_3 = 3$.

Then $x_q = 1$ ($= x_{2^k-1}$, say). But then we cannot reach both 2^k and $x_{2^{k-1}-1}$ from all vertices.

Hence there cannot be a $\leq k-$circuit in $G \in \mathcal{G}(2^{k+1} - 3, 2, k)$. ∎

Since the diameter of $G \in \mathcal{G}(2^{k+1} - 3, 2, k)$ is k and there are no $\leq k-$circuits in G, it follows that there is a $k+1-$circuit covering of G. We shall show that such a covering must be arc-disjoint.

Lemma 2. If $G \in \mathcal{G}(2^{k+1} - 3, 2, k)$ and $k > 3$ then there is an arc-disjoint $k+1-$circuit covering L of G.

Proof. Suppose $G \in \mathcal{G}(2^{k+1} - 3, 2, k)$, $k > 3$ and the following subdigraphs are in G (Figure 7).

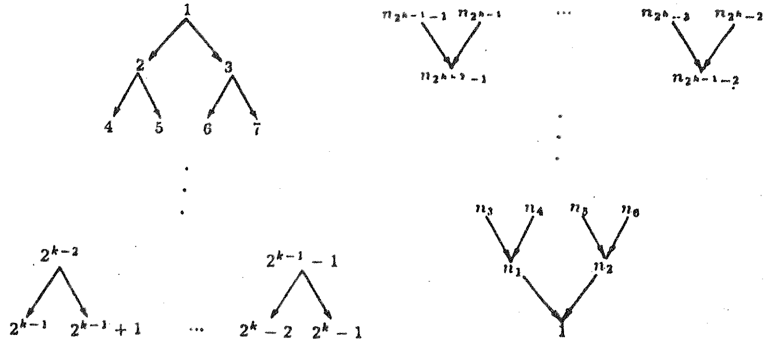

Figure 7.

To reach all the other $2^{k+1}-4$ vertices from vertex 1 in at most k steps the vertices $1, 2, \ldots, 2^k - 1$ must be all distinct.

To reach vertex 1 from all the other $2^{k+1}-4$ vertices in at most k steps, the vertices $1, n_1, n_2, \ldots, n_{2^k-2}$ must be all distinct.

Since by Lemma 1 there are no $\leq k-$circuits in G, we also have vertices $1, 2, \ldots, 2^k-1, n_1, n_2$ all distinct and vertices $1, 2, 3, n_1, \ldots, n_{2^k-2}$ all distinct.

To reach vertex 1 from vertex 2, one of the vertices $2^{k-1}, \ldots, 3 \times 2^{k-2} - 1$ must go to n_1 or n_2.

To reach vertex 1 from vertex 3, one of $3 \times 2^{k-2}, \ldots, 2^k - 1$ must go to n_1 or n_2.

To reach n_1 and n_2 from 1, one of $2^{k-1}, \ldots, 2^k - 1$ must go to n_1 and one of $2^{k-1}, \ldots, 2^k - 1$ must go to n_2.

Thus there must be two arc-disjoint $k+1-$circuits l_1 and l_2 going through vertex 1. Without loss of generality we can take $1 \rightarrow 2 \rightarrow 4 \rightarrow \ldots \rightarrow 2^{k-1} \rightarrow 2^k \rightarrow 1$ for l_1 and $1 \rightarrow 3 \rightarrow 7 \rightarrow \ldots \rightarrow 2^k - 1 \rightarrow 2^{k+1} - 3 \rightarrow 1$ for l_2.

Suppose there is another $k+1-$circuit going through vertex 1, say l_0 $(\neq l_1, l_2)$. Then l_0 must contain either $1 \rightarrow 3$ and $2^k \rightarrow 1$ (1)

or $1 \rightarrow 2$ and $2^{k+1} - 3 \rightarrow 1$ (2).

Note that l_0 cannot contain both the arcs $1 \rightarrow 3$ and $2^{k+1} - 3 \rightarrow 1$ because then there would be two different ways of reaching $2^{k+1} - 3$ from 3 in less than k steps and so we could not reach all vertices from 3.

Similarly, l_0 cannot contain both $1 \rightarrow 2$ and $2^k \rightarrow 1$.

Cases (1) and (2) being symmetric, let l_0 be the circuit

$$l_0 = 1 \rightarrow 3 \rightarrow 6 \rightarrow \ldots \rightarrow 3 \times 2^{k-2} \rightarrow 2^k \rightarrow 1.$$

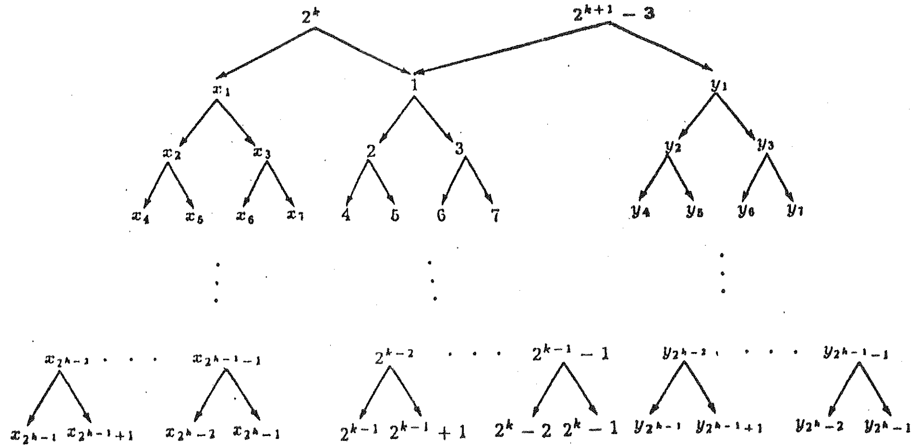

Figure 8.

Obviously, $x_1 \neq y_1$ since we must reach 2^k from y_1 in at most $k-1$ steps and if $x_1 = y_1$ then there would be a $k-$circuit containing the arcs $2^k \to x_1$ $(= y_1)$.

In general, if $a \to b, a \to c, d \to b,\ a, b, c, d \in G, d \neq a$, then $d \to c$ is not possible for otherwise there would be a $k-$circuit in G.

Consequently, if $x_1 \neq y_1$ then we cannot have both $x_2 = y_2$ and $x_3 = y_3$.

Hence there are (at least) two paths in G, p_1 and p_2 such that p_1 is (say) $2^k \to x_1 \to x_2 \to x_4 \to \ldots \to x_{2^{k-1}}$ and p_2 is $2^{k+1} - 3 \to y_1 \to y_2 \to y_4 \to \ldots \to y_{2^{k-1}}$.

If these are the only two such paths through 2^k and $2^{k+1} - 3$ respectively, i.e., if $x_i = y_i$ for all $x_i \notin p_1$ then all except at most two of $x_1, x_2, x_4, \ldots, x_{2^{k-1}}$ must be from $\{2^k + 1, \ldots, 2^{k+1} - 3\}$ (to reach all from 2^k) and all except at most two of $y_1, y_2, y_4, \ldots, y_{2^{k-1}}$ must be from $\{2^k, \ldots, 2^{k+1} - 4\}$.

One of $x_1, x_2, x_4, \ldots, x_{2^{k-1}}$ must be the vertex $2^{k+1} - 3$ (to reach $2^{k+1} - 3$ from 2^k), and one of $y_1, y_2, y_4, \ldots, y_{2^{k-1}}$ must be 2^k (to reach 2^k from $2^{k+1} - 3$).

Now for $k > 3$ there must be some $x_p \in \{2^k, \ldots, 2^{k+1} - 3\}$ and some $y_q \in \{2^k, \ldots, 2^{k+1} - 3\}$ such that $x_p = y_m \in p_2$ and $y_q = x_n \in p_1$ for some x_n and y_m.

But then there would be a $\leq k-$circuit through 2^k and $x_p(= y_m)$ or a $\leq k-$circuit through $2^{k+1} - 3$ and $y_m(= x_p)$.

Hence it is not possible to have $x_i = y_i$ for all $x_i \notin p_1$.

If $x_i \neq y_i$ for some $x_i \not\in p_1$ then there are in G vertices x_k, x_l, x_m such that $x_k \to x_m$, $x_l \to x_m$ and $x_k, x_l, x_m \in \{2^k, \ldots, 2^{k+1} - 3\}$.
This is not possible since we could not reach x_m from 1 in at most k steps.
Hence it is not possible for a vertex of G to lie on more than two $k+1-$circuits.
Every vertex lies on exactly two (arc-disjoint) $k+1-$circuits and so every arc lies on exactly one $k+1-$circuit. Thus there is an arc-disjoint $k+1-$circuit covering of G. ∎

Theorem 3. If $k > 2$ and $k+1$ does not divide $2(2^{k+1} - 3)$ then $\mathcal{G}(2^{k+1} - 3, 2, k)$ is empty.

Proof. If $k = 3$ then $\mathcal{G}(13, 2, 3)$ is empty [3].
If $k > 3$ then by Lemma 2, there is an arc-disjoint covering L of G such that $L = \{l : l \text{ is a } k+1-\text{circuit of } G\}$.
If $L = \{l_1, \ldots, l_m\}$ then L contains $m(k+1)$ distinct arcs.
However, G contains $2(2^{k+1} - 3)$ arcs and so if $k+1$ does not divide $2(2^{k+1} - 3)$ then $\mathcal{G}(2^{k+1} - 3, 2, k)$ is empty. ∎

3. CONCLUSION

Using Theorem 3 and the fact that for $k > 2$, $3 \times 2^{k-1} \leq n \leq 2^{k+1} - 2, K(n, 2)$ is either k or $k+1$ [3], it follows that if $k > 2$ and $k+1$ does not divide $2(2^{k+1} - 3)$ then $K(2^{k+1} - 3, 2) = k + 1$.
Furthermore, for such k's , $N(2, k) \leq 2^{k+1} - 4$ is an improvement upon the upper bound of $N(2, k)$.
Interestingly, we checked the divisibility of $2(2^{k+1} - 3)$ by $k+1$ for $3 \leq k \leq 10^7$ (using a computer) and we found that 274486 divides $2(2^{274486} - 3)$, 5035922 divides $2(2^{5035922} - 3)$ and $k+1$ does not divide $2(2^{k+1} - 3)$ for all the other values of k in this range.
Hence $K(2^{k+1} - 3, 2) = k + 1$ for all k, $3 \leq k \leq 10^7$ (except possibly when $k =$ 274485, $k = 5035921$).

We can now summarise our current state of knowledge of the $N(2, k)$ problem .

k	$N(2,k)$
0	1
1	3
2	6
3	12
4	$25 \le N(2,4) \le \mathbf{28}$
5	$50 \le N(2,5) \le \mathbf{60}$
6	$100 \le N(2,6) \le \mathbf{124}$
7	$200 \le N(2,7) \le \mathbf{252}$
8	$400 \le N(2,8) \le \mathbf{508}$
9	$800 \le N(2,9) \le \mathbf{1020}$
10	$1600 \le N(2,10) \le \mathbf{2044}$
$\vdots$	
t	$2^{t-4} \times 25 \le N(2,t) \le \mathbf{M_{2,t}}$
$\vdots$	

where $M_{2,t} = 2^{t+1} - 4$ if $t+1$ does not divide $2(2^{t+1} - 3)$
and $M_{2,t} = 2^{t+1} - 3$ otherwise.

Table 1.

Our current state of knowledge of the $K(n,2)$ problem is summarised in Table 2 (for $n \le 100$).

n	$K(n,2)$	n	$K(n,2)$	n	$K(n,2)$
1	0	34	5	67	6
2	1	35	5	68	6
3	1	36	5	69	6
4	2	37	5	70	6
5	2	38	5	71	6
6	2	39	5	72	6
7	3	40	5	73	6
8	3	41	5	74	6
9	3	42	5	75	6
10	3	43	5	76	6
11	3	44	5	77	6
12	3	45	5	78	6
13	4	46	5	79	6
14	4	47	5	80	6
15	4	48	5	81	6 or 7
16	4	49	5 or 6	82	6
17	4	50	5	83	6
18	4	51	5 or 6	84	6
19	4	52	5 or 6	85	6
20	4	53	5 or 6	86	6
21	4	54	5 or 6	87	6
22	4	55	5 or 6	88	6
23	4	56	5 or 6	89	6
24	4	57	5 or 6	90	6
25	4	58	5 or 6	91	6
26	4 or 5	59	5 or 6	92	6
27	4 or 5	60	5 or 6	93	6
28	4 or 5	61	6	94	6
29	5	62	6	95	6
30	5	63	6	96	6
31	5	64	6	97	6 or 7
32	5	65	6	98	6 or 7
33	5	66	6	99	6 or 7
				100	6

Table 2.

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