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Antimagicness of some families of generalized graphs

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Abstract

An *edge labeling* of a graph $G = (V, E)$ is a bijection from the set of edges to the set of integers $\{1, 2, \ldots, |E|\}$. The *weight* of a vertex v is the sum of the labels of all the edges incident with v. If the vertex weights are all distinct then we say that the labeling is *vertex-antimagic*, or simply, *antimagic*. A graph that admits an antimagic labeling is called an *antimagic graph*.

In this paper, we present a new general method of constructing families of graphs with antimagic labelings. In particular, our method allows us to prove that generalized web graphs and generalized flower graphs are antimagic.

1 Introduction

All graphs in this paper are finite, simple, undirected and connected, unless stated otherwise. An *edge labeling* of graph $G = (V, E)$ is a bijection $l : E \longrightarrow \{1, 2, \ldots, |E|\}$. The *weight* of a vertex v, $wt(v)$, is the sum of the labels of all edges incident with v.

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In 1990, Hartsfield and Ringel [7] introduced the concept of an antimagic labeling of graph. An *antimagic labeling* of a graph is an edge labeling in which the vertex weights are pairwise distinct. A graph is *antimagic* if it has an antimagic labeling.

Hartsfield and Ringel [7] showed that paths, stars, cycles, complete graphs K_m, wheels W_m and bipartite graphs $K_{2,m}$, $m \geq 3$, are antimagic. They conjectured that every connected graph, except K_2, is antimagic, a conjecture that has remained open for over two decades. Several families of graphs have been proved to be antimagic, for example, see [1, 2, 3, 4, 5]. Many other results concerning antimagic graphs are catalogued in the dynamic survey by Gallian [6]. Most recently, new families of antimagic graphs have been discovered by Phanalasy *et al.* [8] and Ryan *et al.* [9], using completely separating systems for the construction of the labelings.

In this paper we introduce a new approach which allows us to produce antimagic labelings of two new families of graphs called generalized web and generalized flower graphs. We also extend the results to more general cases, showing the antimagicness of the single apex multi-generalized web graphs and the single apex multi-(complete) generalized flower graphs. The antimagicness of paths and cycles that have been proved in [7] are special cases of our results.

2 Results

In this paper we extend definitions in [9]. Let G be a k-regular graph with p vertices. The *generalized pyramid graph* $P(G,1)$, is the graph obtained from the graph G by joining each vertex of G to a vertex called the *apex*; the graph G is called the *base*. Note that the wheel is a special case of the generalized pyramid graph $P(G,1)$ when $G = C_n$, $n \geq 3$. The *generalized pyramid graph* $P(G,2)$ is the graph obtained from the graph $P(G,1)$ by attaching a pendant vertex to each vertex of the base and then joining each pendant vertex to the corresponding vertex in a copy of G. By iterating the process of adding pendant vertices to the newest copy of G and joining them to form a new copy of G, we obtain the *generalized pyramid graph* $P(G,m)$, $m \geq 1$. Alternatively, to get $P(G,m)$, take the Cartesian product $G \times P_m$ and adjoin a vertex, the apex, to each vertex of one end copy of G.

The *generalized web graph* $WB(G,m,n)$ is the graph obtained from the generalized pyramid graph $P(G,m)$ by taking p copies of P_n ($n \geq 2$) and merging an end vertex of a different copy of P_n with each vertex of the furthermost copy of G from the apex. When G is a cycle, $WB(G,m,2)$ is simply called the *web graph*.

The *generalized flower graph with* p *petals* (or simply, *generalized flower graph*) $FL(G,m,n,p)$ is the graph obtained from the generalized web graph $WB(G,m,n)$ by connecting each of the p pendant vertices to the apex with an edge. A *petal* of $FL(G,m,n,p)$ is the subgraph C_{m+n} that contains the new edge. The *complete generalized flower graph with* $(m+n-2)p$ *petals* (or simply, *complete generalized flower*) $CFL(G,m,n,(m+n-2)p)$ is the graph obtained from the generalized web graph $WB(G,m,n)$ by adding $(m+n-2)p$ edges to ensure that the apex is adjacent to every other vertex. Petals in $CFL(G,m,n,(m+n-2)p)$ are defined as for the generalized flower graph and there are $m+n-2$ types of petals: $C_3, C_4, \ldots, C_{m+n}$. When G is a cycle, $CFL(G, m, n, (m+n-2)p)$ is simply called the *complete flower graph.*

An edge labeling l of a graph G will be described as an array L. Each row of L represents a vertex of G, and each entry of the array is an edge label and must appear in exactly two different rows. These two rows represent the two vertices incident to the edge with that label.

Hereafter we denote by T^t the transpose of the array T.

Theorem 1. *Let $G = (V, E)$ be a k-regular graph and $G \neq K_1$. Then the generalized flower graph $FL(G, m, n, p)$, $m \geq 1$, $n \geq 2$, is antimagic.*

Proof. Assume that G has p vertices and q edges. The construction of the generalized flower graph $FL(G, m, n, p)$ uses m copies of G. We choose any edge labeling of G. Let L_j, $1 \leq j \leq m$, be the array of edge labels of the j-th copy of graph G. Let v_i be the vertex represented by the i-th row of L_j. We arrange the rows of L_j so that $wt(v_i) \leq wt(v_{i+1})$, $1 \leq i \leq p-1$. Let T_h, $1 \leq h \leq m+n$, be the $(p \times 1)$-array of the edges e_i, $1 \leq i \leq p$, where e_i are the edges of $FL(G, m, n, p)$ that do not belong to any copy of G. We construct the array A of edge labels of the generalized flower graph $FL(G, m, n, p)$, $m \geq 1$, $n \geq 2$, in two cases as follows.

Case 1: $G = K_p$, $p \geq 2$.

(1) Label the edge e_i, $1 \leq i \leq p$, in row i of the array T_h, $1 \leq h \leq m+n$, with $i+(h-1)p$, for $1 \leq h \leq n+1$, and $i+(h-1)p+(h-n-1)q$, for $n+2 \leq h \leq m+n$;

(2) Replace the edge labels in the array L_j, $1 \leq j \leq m$, with new labels obtained by adding $(n+j)p+(j-1)q$ to each of the original edge labels;

(3) To form the array A,

If $m = 1$ and $n \geq 2$,

$$\begin{array}{ccc} & T_1 & T_2 \\ & T_2 & T_3 \\ & \vdots & \vdots \\ & T_{n-1} & T_n \\ & T_1^t & T_{n+1}^t \\ T_n & T_{n+1} & L_1 \end{array}$$

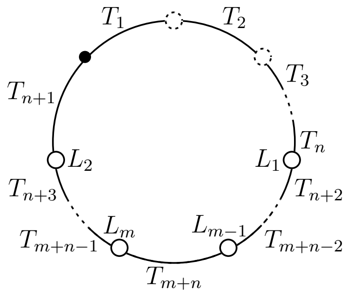

Figure 1: Illustration of the construction of the generalized flower graph $FL(K_p, m, n, p)$, for $p \geq 2$, $m \geq 2$, $n \geq 3$, and m is even

More generally, if $m \geq 1$ and $n \geq 2$,

$$\begin{array}{ccc}
 & T_1 & T_2 \\
 & T_2 & T_3 \\
 & T_3 & T_4 \\
 & \vdots & \vdots \\
 & T_{n-1} & T_n \\
 & T_1^t & T_{n+1}^t \\
T_n & L_1 & T_{n+2} \\
T_{n+1} & L_2 & T_{n+3} \\
\vdots & \vdots & \vdots \\
T_{m+n-2} & L_{m-1} & T_{m+n} \\
T_{m+n-1} & T_{m+n} & L_m
\end{array}$$

As an illustration, in Figure 1 we present the construction when m is even. The case of m being odd is similar and left to the reader.

By the construction of the array A, it is clear that the weight of each vertex (row) in the array is less than the weight of the vertex (row) below, except the weight of the row $T_1^t T_{n+1}^t$ and the weight of the first row of the subarray $T_n L_1 T_{n+2}$ need to be considered separately.

Let $r_{(n-1)p+1}$ and $r_{(n-1)p+2}$ be the row $T_1^t T_{n+1}^t$ and the first row in the subarray $T_n L_1 T_{n+2}$, respectively. Let $wt(r_{(n-1)p+1})$ and $wt(r_{(n-1)p+2})$ be the weights of $r_{(n-1)p+1}$ and $r_{(n-1)p+2}$, respectively. Since the least possible edge labels (that yield the least possible weight) of a vertex in the array L_1 are $1 + (n+1)p, 2 + (n+1)p, \ldots, (p-1) + (n+1)p$, it follows that $wt(r_{(n-1)p+2}) \geq (1 + (n-1)p) + (1 + (n+1)p) + \cdots + ((p-1) + (n+1)p) + (1 + (n+1)p + q) = 2 + (n-1)p + \frac{p(p-1)}{2} + (n+1)p^2 + q > (n+1)p^2 + p = 1 + \cdots + p + (1 + np) + \cdots + (p + np) = wt(r_{(n-1)p+1})$.

Case 2: $G \neq K_p$, $p \geq 2$.

(1) Label the edge e_i, $1 \le i \le p$, in the row i of the array T_h, $1 \le h \le m+n$, with $i+(h-1)p$, for $1 \le h \le n$, and $i+(h-1)p+(h-n)q$, for $n+1 \le h \le m+n$;

(2) Replace the edge labels in the array L_j, $1 \le j \le m$, with new labels obtained by adding $(n+j-1)p+(j-1)q$ to each of the original edge labels;

(3) To form the array A,

If $m = 1$ and $n \ge 2$,

$$\begin{array}{ccc} & T_1 & T_2 \\ & T_2 & T_3 \\ & \vdots & \vdots \\ & T_{n-1} & T_n \\ T_n & L_1 & T_{n+1} \\ & T_1^t & T_{n+1}^t \end{array}$$

If $m \ge 2$ and $n = 2$,

$$\begin{array}{ccc} & T_1 & T_2 \\ T_2 & L_1 & T_3 \\ T_3 & L_2 & T_4 \\ \vdots & \vdots & \vdots \\ T_{m+n-2} & L_{m-1} & T_{m+n-1} \\ T_{m+n-1} & L_m & T_{m+n} \\ & T_1^t & T_{m+n}^t \end{array}$$

and, in general, if $m \ge 1$ and $n \ge 2$,

$$\begin{array}{ccc} & T_1 & T_2 \\ & T_2 & T_3 \\ & \vdots & \vdots \\ & T_{n-1} & T_n \\ T_n & L_1 & T_{n+1} \\ T_{n+1} & L_2 & T_{n+2} \\ \vdots & \vdots & \vdots \\ T_{m+n-2} & L_{m-1} & T_{m+n-1} \\ T_{m+n-1} & L_m & T_{m+n} \\ & T_1^t & T_{m+n}^t \end{array}$$

The construction used here is similar to the illustration in Figure 1 and it is omitted.

By the construction of the array A, it is clear that the weight of each vertex (row) in the array is less than the weight of the vertex (row) below. □

Surprisingly, when the array L_j, $1 \le j \le m$, is removed from the construction in the proof of Theorem 1, when $n = 2$ we obtain an alternative proof of the following corollary. The reason this works is that K_1 has no edges, and hence the array of edge labels is empty. The cycle has been proved to be antimagic in [7].

Corollary 2. *The generalized flower graph* $FL(K_1, m, 2, 1) = C_{m+2}$, $m \geq 1$, *is antimagic.*

Corollary 3. *Let* $G = (V, E)$, *where* $G \neq K_1$ *is any* k*-regular graph. Then the generalized web graph* $WB(G, m, n)$, $m \geq 1$, $n \geq 2$, *is antimagic.*

Proof. In each case in the proof of Theorem 1, to construct an array which respects an antimagic labeling of the graph, remove each occurrence of T_1 and for $h \geq 2$, replace T_h by T_{h-1}. It is easy to check that in each case this yields an antimagic labeling of $WB(G, m, n)$, $m \geq 1$, $n \geq 2$.

□

When $n = 2$, removing the array L_j, $1 \leq j \leq m$ from the construction used in Corollary 3, results in an alternative proof of the following corollary. The reason this works is that K_1 has no edges, and hence the array of edge labels is empty. The path has been proved to be antimagic in [7].

Corollary 4. *The generalized web graph* $WB(K_1, m, 2) = P_{m+2}$, $m \geq 1$, *is antimagic.*

The antimagicness of complete generalized flower graphs is established by the following theorems.

Theorem 5. *Let* $G = (V, E)$ *be a* k*-regular graph and* $G \neq K_1$. *Then the complete generalized flower graph* $CFL(G, m, n, (m + n - 2)p)$, $m \geq 1$, $n \geq 2$, *is antimagic.*

Proof. Assume that G has p vertices and q edges. We divide the proof into three cases.

Case 1: $n = 2$ and $m = 1$.

The proof is the same as that of Theorem 1.

By definition, the construction of complete generalized flower graph $CFL(G, m, n, (m+n-2)p)$ uses m copies of G. We first choose any edge labeling of G. Let L_j, $1 \leq j \leq m$, be the array of edge labels of the j-th copy of graph G. Let v_i be the vertex represented by the i-th row of L_j. We arrange the rows of L_j so that $wt(v_i) \leq wt(v_{i+1})$, for $1 \leq i \leq p-1$. Let T_h, $1 \leq h \leq 2(m+n-2)+1$, be the $(p \times 1)$-array of edges e_i, $1 \leq i \leq p$, where e_i are the edges of $CFL(G, m, n, (m + n - 2)p)$ that do not belong to any copy of G. We construct the array A of edge labels of the generalized flower graph $CFL(G, m, n, (m + n - 2)p)$, $m \geq 1$, $n \geq 2$, as follows.

(1) Label the edge e_i, $1 \leq i \leq p$, in the row i of the array T_h, $1 \leq h \leq 2(m+n-2)+1$, with $i + (h - 1)p$, for $1 \leq h \leq m + 2n - 3$, and $i + (h - 1)p + (h - m - 2n + 3)q$, for $m + 2n - 3 < h \leq 2(m + n - 2) + 1$;

(2) Replace the edge labels in the array L_j, $1 \leq j \leq m$, with new labels obtained by adding $(m + 2n + j - 4)p + (j - 1)q$ to each of the original edge labels;

(3) To form the array A we consider two cases.

The array A is provided for all cases. The details of the proof are given only for the last, most general, subcase.

Case 2: $n = 2$.

Subcase 2.1: $m = 2$.

$$\begin{array}{cccc} & & T_1 & T_2 \\ & T_3 & L_1 & T_4 \\ T_2 & T_4 & L_2 & T_5 \\ & T_1^t & T_3^t & T_5^t \end{array}$$

Subcase 2.2: $m = 3$.

$$\begin{array}{cccc} & & T_1 & T_2 \\ & T_3 & T_4 & L_1 \\ T_2 & T_5 & L_2 & T_6 \\ T_4 & T_6 & L_3 & T_7 \\ T_1^t & T_3^t & T_5^t & T_7^t \end{array}$$

Subcase 2.3: $m \geq 4$.

$$\begin{array}{ccccccc} & & & & & T_1 & T_2 \\ & & & & T_3 & T_4 & L_1 \\ & & & T_2 & T_5 & T_6 & L_2 \\ & & & \vdots & \vdots & \vdots & \vdots \\ & & & T_{2m-6} & T_{2m-3} & T_{2m-2} & L_{m-2} \\ & & & T_{2m-4} & T_{2m-1} & L_{m-1} & T_{2m} \\ & & & T_{2m-2} & T_{2m} & L_m & T_{2m+1} \\ T_1^t & T_3^t & \dots & . & T_{2m-3}^t & T_{2m-1}^t & T_{2m+1}^t \end{array}$$

Case 3: $n \geq 3$.

Subcase 3.1: $m = 1$.

$$\begin{array}{cccccc} & & & & T_1 & T_2 \\ & & & T_2 & T_3 & T_4 \\ & & & \vdots & \vdots & \vdots \\ & & & T_{2n-4} & T_{2n-3} & T_{2n-2} \\ & & & T_{2n-2} & L_1 & T_{2n-1} \\ T_1^t & T_3^t & \dots & . & T_{2n-3}^t & T_{2n-1}^t \end{array}$$

Subcase 3.2: $m = 2$.

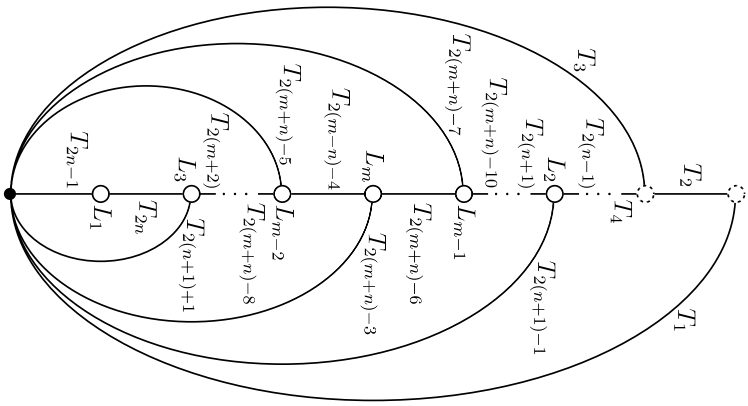

Figure 2: Illustration of the construction of the generalized flower $CFL(G, m, n, mp)$, for $p \geq 2$, $m \geq 3$, $n \geq 3$, and m is odd

$$
\begin{array}{ccccccc}
 & & & & & T_1 & T_2 \\
 & & & & T_2 & T_3 & T_4 \\
 & & & & \vdots & \vdots & \vdots \\
 & & & & T_{2n-4} & T_{2n-3} & T_{2n-2} \\
 & & & & T_{2n-1} & L_1 & T_{2n} \\
 & & & T_{2n-2} & T_{2n} & L_2 & T_{2n+1} \\
T_1^t & T_3^t & \ldots & . & T_{2n-3}^t & T_{2n-1}^t & T_{2n+1}^t
\end{array}
$$

Subcase 3.3: $m \geq 3$.

$$
\begin{array}{ccccccc}
 & & & & & T_1 & T_2 \\
 & & & & T_2 & T_3 & T_4 \\
 & & & & \vdots & \vdots & \vdots \\
 & & & & T_{2(n-2)} & T_{2(n-1)-1} & T_{2(n-1)} \\
 & & & & T_{2n-1} & T_{2n} & L_1 \\
 & & & T_{2(n-1)} & T_{2(n+1)-1} & T_{2(n+1)} & L_2 \\
 & & & \vdots & \vdots & \vdots & \vdots \\
 & & & T_{2(m+n-2)-6} & T_{2(m+n-2)-3} & T_{2(m+n-2)-2} & L_{m-2} \\
 & & & T_{2(m+n-2)-4} & T_{2(m+n-2)-1} & L_{m-1} & T_{2(m+n-2)} \\
 & & & T_{2(m+n-2)-2} & T_{2(m+n-2)} & L_m & T_{2(m+n-2)+1} \\
T_1^t & T_3^t & \ldots & . & T_{2(m+n-2)-3}^t & T_{2(m+n-2)-1}^t & T_{2(m+n-2)+1}^t
\end{array}
$$

The diagram in Figure 2 illustrates the construction used here when m is odd. When m is even, the construction is similar and left for the reader.

By the construction of the array A, it is clear that the weight of each vertex (row) in the array is less than the weight of the vertex (row) below, except the weight of the last vertex (row) of the subarray $T_{2(m+n-2)-2}T_{2(m+n-2)}$ $L_mT_{2(m+n-2)+1}$ and the row $T_1^tT_3^t\ldots T_{2(m+n-2)-1}^tT_{2(m+n-2)+1}^t$ that need to be verified.

Let r_f be the row f in the array A and $wt(r_f)$ be the weight of the row r_f. Let $e_{f,g}$ be the edge label in the row f and the column g in the array A.

Let $r_{(m+n-1)p}$ and $r_{(m+n-1)p+1}$ be the last row of the subarray $T_{2(m+n-2)-2}$ $T_{2(m+n-2)}L_mT_{2(m+n-2)+1}$ and the row $T_1^tT_3^t\ldots T_{2(m+n-2)-1}^tT_{2(m+n-2)+1}^t$, respectively. We have the edge labels of the rows when $G = K_p$, $p \geq 2$, as shown below.

$$\begin{array}{rcccccccc} r_{(m+n-1)p}: & & & & & . & 2lp+(m-1)q & \ldots & (2l+1)p+mq \\ r_{(m+n-1)p+1}: & 1 & \ldots & p & \ldots & . & (2l-1)p+(m-2)q & \ldots & (2l+1)p+mq \end{array}$$

where $l = m + n - 2$.

Since $\sum_{g=1}^{p} e_{(m+n-1)p+1,g} + e_{(m+n-1)p+1,(m+n-2)p} = \frac{p(p+1)}{2} + (2l-1)p + (m-2)q = 2lp+(m-1)q = e_{(m+n-1)p,(m+n-2)p}$ and $e_{(m+n-1)p+1,(m+n-2)p-1} > e_{(m+n-1)p,(m+n-2)p-1}$ and $e_{(m+n-1)p+1,g} > e_{(m+n-1)p,g}$ for $(m+n-2)p+1 \leq g \leq (m+n-1)p-1$, hence $wt(r_{(m+n-1)p+1}) > wt(r_{(m+n-1)p})$. This also warrants that $wt(r_{(m+n-1)p+1}) > wt(r_{(m+n-1)p})$, when $G \neq K_p$, $p \geq 2$. Therefore, for any k-regular graph $G \neq K_1$, $wt(r_{(m+n-1)p+1}) > wt(r_{(m+n-1)p})$. □

When the array L_j, $1 \leq j \leq m$, is removed from the construction in the proof of Cases 1 and 2 of Theorem 5, the following corollary emerges.

Corollary 6. *The complete generalized flower graph $CFL(K_1, m, 2, m)$, $m \geq 1$, is antimagic.*

3 Extension of Results

To extend our results in Section 2 to more general cases, we need the following definitions. The *single apex multi-generalized flower graph* (or *single apex multi-generalized web graph* or *single apex multi-complete generalized flower graph*) is the graph obtained from generalized flower graphs (or generalized web graphs or complete generalized flower graph) with a common apex.

Hereafter let $rG = \bigcup_r G$.

The proof of the following theorem is similar to a part of the proof of Theorem 1, and so it is omitted here.

Theorem 7. *Let H be a k-regular disconnected graph and $H \neq rK_1$. Then the single apex multi-generalized flower graph $MFL(H, m, n, p)$, $m \geq 1$, $n \geq 2$, is antimagic.*

When the array L_j, $1 \leq j \leq m$, is removed from the construction in the proof of Theorem 1, we have proved the following corollary.

Corollary 8. *The single apex multi-generalized flower graph $MFL(rK_1, m, 2, r)$, $m \geq 1$, is antimagic.*

The proof of the following corollary is omitted since it is similar to the proof of Corollary 3.

Corollary 9. *Let H be a k-regular disconnected graph and $H \neq rK_1$. Then the single apex multi-generalized web graph $MWB(H, m, n)$, $m \geq 1$, $n \geq 2$, is antimagic.*

When the array L_j, $1 \leq j \leq m$, is removed from the construction in the proof of Corollary 3, we have proved the following corollary. The reason this works is that rK_1 has no edge, and hence the array of edge labels is empty.

Corollary 10. *The single apex multi-generalized web graph $MWB(rK_1, m, 2)$, $m \geq 1$, is antimagic.*

Note that when $r = 2$, $MWB(2K_1, m, 2) = P_{2m+3}$, this is a special case of Corollary 4.

The proof of the following theorem is similar to the proof of Theorem 5, so it is omitted here.

Theorem 11. *Let H be a k-regular disconnected graph and $H \neq rK_1$. Then the single apex multi-complete generalized flower graph $MCFL(H, m, (m + n - 2)p)$, $m \geq 1$, $n \geq 2$, is antimagic.*

When the array L_j, $1 \leq j \leq m$, is removed from the construction in the proof of Theorem 5, we have proved the following corollary.

Corollary 12. *The single apex multi-complete generalized flower graph $MCFL(rK_1, m, 2, mr)$, $m \geq 1$, is antimagic.*

Example 1. *Using the construction of Subcase 2.1 of Theorem 5, we have the array A of edge labels of the single apex multi-complete generalized flower graph $MCFL(4K_2, 2, 2, 16)$*

												1	9
												2	10
												3	11
												4	12
												5	13
												6	14
												7	15
												8	16
											17	25	29
											18	25	30
											19	26	31
											20	26	32
											21	27	33
											22	27	34
											23	28	35
											24	28	36
										9	29	37	41
										10	30	37	42
										11	31	38	43
										12	32	38	44
										13	33	39	45
										14	34	39	46
										15	35	40	47
										16	36	40	48
1	…	8	17	…	24	41	42	43	44	45	46	47	48

and the corresponding antimagic single apex multi-complete generalized flower graph $MCFL(4K_2, 2, 2, 16)$ *as shown in Figure 3*

References

[1] N. Alon, G. Kaplan, A. Lev, Y. Roditty and R. Yuster, Dense graphs are antimagic, *J. Graph Theory* 47(4) (2004), 297–309.

[2] M. Bača, Y. Lin and A. Semaničová-Feňovčiková, Note on super antimagicness of disconnected graphs, *Int. J. Graphs Combin.* 6(1) (2009), 47–55.

[3] M. Bača and M. Miller, *Super Edge-Antimagic Graphs: A Wealth of Problems and Some Solutions*, Brown Walker Press, Boca Raton, Florida, USA, 2008.

[4] Y. Cheng, A new class of antimagic Cartesian product graphs, *Discrete Math.* 308(24) (2008), 6441–6448.

[5] D. W. Cranston, Regular bipartite graphs are antimagic, *J. Graph Theory* 60(3) (2009), 173–182.

[6] J. A. Gallian, A Dynamic Survey of Graph Labeling, *Electron. J. Combin.* 17(♯DS6), 2010.

[7] N. Hartsfield and G. Ringel. *Pearls in graph theory: A comprehensive introduction.* Academic Press Inc., Boston, MA, 1990.

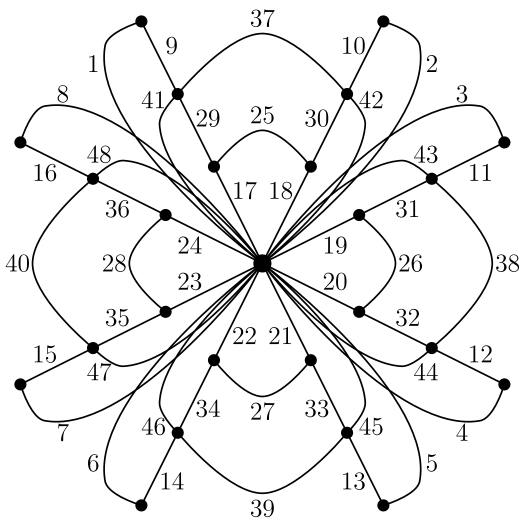

Figure 3: The single apex multi-complete generalized flower graph $MCFL(4K_2, 2, 2, 16)$ and its antimagic labeling.

[8] O. Phanalasy, M. Miller, L. J. Rylands and P. Lieby, On a Relationship between Completely Separating Systems and Antimagic Labeling of Regular Graphs, in *Proc. IWOCA10* (eds. C. S. Iliopoulos and W. F. Smyth), London, UK, July 2010. *Lec. Notes Comp. Sci.* 6460 (2011), 238–241.

[9] J. Ryan, O. Phanalasy, M. Miller and L. J. Rylands, On Antimagic Labeling for Generalized Web and Flower Graphs, in *Proc. IWOCA10* (eds. C. S. Iliopoulos and W. F. Smyth), London, UK, July 2010. *Lec. Notes Comp. Sci.* 6460 (2011), 303–313.

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