

On the paper
“Some constraints on the missing Moore graph”

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Unfortunately, my paper “Some constraints on the missing Moore graph”, *Australas. J. Combin.* 77(3) (2020), 373–382, contains a computational error in the proof of Theorem 3.1, thereby rendering it invalid. Fortunately, the primary result, Corollary 3.1 remains true. As many of the remaining theorems rely upon Theorem 3.1, their statements must be revised. (Specifically, their hypotheses must be strengthened by adding the requirement that the biregular bipartitions have equal size parts.)

The corrected version of Theorem 3.1 should read as follows (with the same notation and assumptions as before).

Theorem 3.1. *If $k = 57$, then the only possible biregular bipartition of Γ_k having equal size parts has bidegree $(32, 32)$.*

Proof. The beginning of the proof goes through as before. The equation preceding (12) should now read

$$\alpha^2 + \alpha + (k - \delta)^2 \left(\frac{v_2}{v_1} \right) = (k - 1) + v_1.$$

(This is where the error occurred in the original. The factor (v_2/v_1) was inadvertently omitted.) Equation (12) itself is correct, but the two equations following it are wrong. The equation following (12) should have been

$$\alpha^2 + \alpha + (k - \alpha)^2 \left(\frac{v_1}{v_2} \right) = (k - 1) + v_1. \quad (*)$$

When $k = 57$, Equation (*) becomes

$$(65\alpha - v_1 - 455)(50\alpha - v_1 + 400) = 0,$$

which admits viable integral solutions for all $1 \leq \alpha \leq 56$. But with the stronger hypothesis of equal size parts, we have $v_1 = v_2 = v/2$ (and $\alpha = \delta$). Substituting this constraint into (*) gives

$$(k - (2\alpha + 1))^2 = 2\alpha. \quad (**)$$

In particular, the right hand side of (**) must be a perfect square. There are only five possibilities for $1 < \alpha < 57$, namely $\alpha \in \{2, 8, 18, 32, 50\}$. The case $\alpha = 2$ yields $k = 3$ or $k = 7$, which correspond to the Petersen and Hoffman-Singleton graphs, respectively. The cases $\alpha \in \{8, 18, 50\}$ require $k \in \{13, 21, 31, 43, 91, 111\}$, all of which are impossible. The case $\alpha = 32$ yields $k = 57$ (the missing Moore graph case) and $k = 73$ (which is impossible). \square

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