Analysis of a plasma torch

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The physical behaviour of a plasma torch is described by means of a set of five equations with five variables, enabling calculation to be made of the characteristic quantities of the plasma flow, i. e. degree of ionisation, valocity, density, temperature, and enthalpy. In view of the practical application of this theory the equations are solved graphically. For purposes of comparison the values as found by a computer are also given.

A complete energy balance shows how the supplied electrical energy is distributed over the various components of the plasma and the cooling water. A relatively simple experiment to determine the speed of exhaust of the plasma illustrates the reliability of the calculations. A comprehensive discussion elucidates the merits of the theoretical analysis and the results for practical applications. Finally, it is pointed out how the results of this report can be made to apply to the practical spraying process.

1 INTRODUCTION

A large amount of literature on plasma torches has been published. Some authors[1-7] concentrate on the study of plasma torches as an individual subject, others[8-10] have made excellent contributions to handling the plasma torch as an instrument to be used in spraying processes, and Refs 11-15 mention studies concerning arc discharges and plasmas.

In studying systematically the plasma-spraying process, knowledge of the mass- and heat-transport properties in the plasma itself is indispensable. In the literature these quantities – viscosity, electrical and heat conductivity – are known as functions of the plasma temperature. Therefore, knowledge of plasma temperature is essential to make the handbook data accessible and with these the study of the spraying process.

However, the influence of the ionisation phenomena in the plasma on its temperature has, within the field of literature on spraying, not enjoyed the attention which, in the opinion of the present authors, it is worthy of. For example, the state of a plasma is often described by means of the ideal-gas law which, just because of the ionisation phenomena, is very incomplete.

The excellent work carried out in the field of temperature measurements has provided a clear insight into the temperature distribution of a streaming plasma. The apparatus involved is, however, rarely identical with the apparatus the research worker or industrial user has available in his own laboratory or workshop. Hasui, for instance, uses two different plasma torches for the work he has published,[7-10] which reduces the usefulness of the temperature measurements from Ref. 7 for the research purposes in Ref. 10.

The present analysis of a plasma torch makes it possible that every user of such an apparatus can carry out reliable calculations of the properties of the plasma under his working conditions, without having to perform supplementary measurements which are often time-consuming and require high investments. The energy balance gives a survey of the distribution

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of the energy supplied over cooling water and plasma components. This makes it possible to judge the merits of different arc gases and the construction of the equipment as regards efficiency. The analysis presented in this report is suitable for monatomic gases. The numerical calculations are limited to argon. It is the intention to extend the analysis to multi-atom gases and mixed gases. The reader is considered to have some general knowledge of the construction of plasma equipment. With this paper the authors hope to be offering a contribution to a more systematic study of the practical plasma-spraying process.

2 FORMULATION OF EQUATIONS

2.1 The degree of ionisation of a plasma

A plasma at atmospheric pressure and in the absence of strong electric fields satisfies in good approximation the condition of thermodynamic equilibrium. This condition underlies the Saha-Eggert equation[16,17] which, in turn, is based on Boltsmann's distribution:

$$\frac{n_{z+1} \cdot n_e}{n_z} = \frac{2 . W_{z+1}(T)}{W_z(T)} \cdot \frac{(2\pi m_e k)^{\frac{3}{2}}}{h^3} . T^{\frac{3}{2}} . \exp\left[-\frac{E_z - \Delta E_z}{kT}\right] \quad [1]$$

where
n_z = particle density of all z-fold ionised atoms (m^{-3}), e.g. z=0 for neutral particles, z=1 for one fold ionised particle
n_{z+1} = particle density of all z+1-fold ionised atoms (m^{-3})
n_e = electron density (m^{-3})
$W_z(T)$ = partition function of a z-fold ionised atom
$W_{zE1}(T)$ = partition function of a z+1-fold ionised atom
h = Planck's constant (=6.62 · 10^{-34} J. S)
k = Boltzmann's constant (=1.38 · 10^{-23} J kg^{-1})
m_e = mass of an electron (=9.1 · 10^{-31} kg)
E_z = ionisation energy for the ionisation process z z + 1
ΔE_z = reduction of ionisation energy due to the Coulomb interactions of charged particles in the plasma
T = absolute temperature (K)

In this equation some simplifications may be introduced which do not affect its value for practical application:

The reduction term ΔE_z is small. For a one-fold ionised argon plasma $(\Delta E_o)_{max.}$ 0.1 eV;[16,17] this is negligible with respect to the ionisation energy E_o = 15.68 eV

If z = 0, $\frac{W_{z+1}(T)}{W_z(T)} \cong 5.4$ if $T \leqslant 17\,000^oK$[17]

The Saha-Eggert equation for a one-fold ionised argon plasma is then reduced to:

$$\frac{n_{A^+} \cdot n_e}{n_A} = 2 \cdot 5.4 \frac{(2\pi m_e k)^{\frac{3}{2}}}{h^3} . T^{\frac{3}{2}} . \exp -\frac{E_o}{kT} \quad [2]$$

where n_{A^+} = particle density of one-fold ionised argon atoms (m^{-3})
n_A = particle density of all neutral argon atoms (m^{-3})
E_o = 15,68 (eV) = 25.1 · 10^{-19} (Nm)

In the literature the Saha-Eggert equation is also written as:

$$\frac{\zeta^2}{1-\zeta^2} = \frac{2 \cdot W_{z+1}(T)}{W_z(T)} \cdot \frac{(2\pi m_e)^{\frac{3}{2}}}{ph^3} . (kT)^{\frac{5}{2}} . \exp -\frac{E_z - \Delta E_z}{kT} \quad [3]$$

where $\zeta = \frac{n_z}{n}$, the degree of ionisation

n = total number of 'heavy' particles (particles with a nucleus) per unit of volume (m^{-3}); $n = n_o + n_1$ for a one-fold ionised plasma
p = gas pressure (Nm^{-2})

Equations 1 and 3 are equivalent as will be shown in the Appendix. For a one-fold ionised argon plasma, eq. 3 reduces to:

$$\frac{\zeta^2}{1-\zeta^2} = 2 \cdot 5.4. \frac{(2\pi m_e)^{\frac{3}{2}}}{ph^3} . (kT)^{\frac{5}{2}} . \exp\left[-\frac{E_o}{kT}\right] \quad [4]$$

Assume $\frac{(2\ m_e)^{\frac{3}{2}} . k^{\frac{5}{2}}}{h^3} = K_1$

$$\frac{E_o}{k} = \frac{eU_o}{k} = K_2$$

$$2 \cdot 5.4 = K_3$$

then eq. 4 may be written as:

$$\frac{\zeta^2}{2 - \zeta^2} = K_3 \cdot \frac{K_1}{p} \cdot T^{\frac{5}{2}} \cdot \exp\left[\frac{-K_2}{T}\right] \qquad [5]$$

From eq. 5 results the first of the five equations which were our aim to derive, i.e. the equation for the degree of ionisation:

$$\zeta = \sqrt{\frac{T^{\frac{5}{2}} \cdot \exp\left[\frac{-K_2}{T}\right]}{\frac{p}{K_3 \cdot K_1} + T^{\frac{5}{2}} \cdot \exp\left[\frac{-K_2}{T}\right]}} \qquad [6]$$

where, for argon, $K_1 = 3.35 \cdot 10^{-2}$
$K_2 = 18.2 \cdot 10^4$
$K_3 = 10.8$

In Fig. 1a, $\zeta = \zeta(T)$ represents for $p = 1$ (kgf cm^{-2}) $= 9.81 \cdot 10^4$ (Nm^{-2}).

2.2 Equation of state of a one-fold ionised monatomic gas

To simplify matters as an introduction the equation of state of an ideal gas is converted into a suitable form. All the following formulae hold for monatomic gases which are one-fold at most.

2.2.1 Ideal gas

For a kilomol ideal gas holds:

$$pV_m = R.T \qquad [a]$$

where p = gas pressure
V_m = molar volume = volume of 1 kilomol gas
R = gas constant = $8.135 \cdot 10^3$ (J. [kilomol K]$^{-1}$)
T = absolute temperature.

Division of eq. [a] by N = number of atoms per kilomol gas (Avogadro's number = $6.025 \cdot 10^{26}$ kilomol^{-1}) gives rise to:

$$p\frac{V_m}{N} = \frac{R}{N} \cdot T \qquad [b]$$

With $\frac{R}{N} = k$ = Boltzmann's constant, eq. [b] becomes:

$$p = \frac{H}{V_m} \cdot kT \qquad [c]$$

Now $\frac{N}{V_m} = n$ = number of particles per unit of volume (m^{-3}) and as a result eq. [c] may be written as:

$$p = n.k.T \qquad [7]$$

For one kg ideal gas holds:

$$p = \rho . R_{kg} . T \qquad [8]$$

where ρ = density (kgm^{-3})
R_{kg} = gas constant for 1 kg gas = $\frac{R}{at.weight}$;
for argon: $R_{kg} = 208$ ($J.kg^{-1} K^{-1}$)

2.2.2 One-fold ionised gas ~~Analagous to eq. 7~~ holds:

$$p = n_{tot} . k.T \qquad [d]$$

where $n_{tot} = n_0 + n_1 + n_e$, hence

$$p = (n_0 + n_1 + n_e)\, kT \qquad [9]$$

In addition the following equations hold:

$n = n_0 + n_1$ = number of particles with a nucleus per unit of volume
$n_1 = \zeta . n$
$n_0 = (1 - \zeta) . n$
$n_e = n_1 = \zeta . n$

Substitution in eq. 9 yields as the equation of state for a partially one-fold ionised gas:

$$p = (1 + \zeta) n\, k\, T \qquad [10]$$

and for 1 kg gas:

$$p = (1 + \zeta)\, \rho R_{kg}\, T \qquad [11]$$

2.2.3 Particle densities With the aid of the foregoing the particle densities in an argon plasma as a function of temperature can be calculated. Equation [d] leads to:

$$n_{tot} = \frac{p}{kT} = 7.108 \cdot 10^{27} \cdot \frac{1}{T}\ (m^{-3}) \qquad [e]$$

It follows from eq. 10:

$$n = \frac{p}{(1+\zeta)kT} \qquad [f]$$

Hence, the number, n, of particles with a nucleus can be calculated by substitution of $\zeta = \zeta(T)$ from eq. 6 in eq. [f]. It follows:

$$n_0 = n_{tot} - (n_1 + n_e) = n_{tot} = 2 \zeta n$$

$$n_1 = n - n_0$$

$$n_e = n_1$$

The results of the calculations are represented in Fig. 2.

2.3 Enthalpy

The enthalpy per kg of an ionised gas is defined as follows:

$$H = U + p V_{kg} + E \qquad [12]$$

where H = enthalpy
U = internal energy
V_{kg} = volume of one kg gas $= \frac{1}{\rho}$ (m^3)
E = ionisation energy.

2.3.1 The internal energy Under conditions of thermal equilibrium the average internal energy per particle for monatomic gases is:

$$U = \tfrac{1}{2} \overline{m u^2} = \frac{3}{2} kT \qquad [h]$$

where u = velocity of a particle.

Per unit of volume:

$$U = \frac{3}{2}(n_0 + n_1 + n_e) kT \qquad [i]$$

$$= \frac{3}{2}(1+\zeta) n k T$$

Per kg:

$$U = \frac{3}{2}(1+\zeta) n \frac{1}{p} kT \qquad [j]$$

2.3.2 The term $p V_{kg}$ becomes:

$$p V_{kg} = (1+\zeta) n . \frac{1}{} kT \qquad [k]$$

hence:

$$U + p V_{kg} = \frac{5}{2}(1+\zeta) n \frac{1}{} kT \qquad [l]$$

From eqs 10 and 11 it follows that:

$$nk = \rho R_{kg} \qquad (m)$$

Substituting eq. [m] in eq. [l] gives:

$$U + p V_{kg} = \frac{5}{2}(1+\zeta) R_{kg} T \qquad [n]$$

2.3.3 The ionisation energy To the ionisation energy per unit of volume applies:

$$E_{m^3} = n_1 . e U_0 \qquad [o]$$

where U_0 = ionisation potential for one-fold ionisation.

Using $n_1 = \zeta n$, eq. [o] becomes:

$$E_{m^3} = \zeta n \, e . U_0 \qquad [p]$$

For 1 kg gas this leads to:

$$E = \frac{1}{\rho} \zeta n \, e \, U_0 \qquad [q]$$

From eqs [q] and [m] it follows:

$$E = R_{kg} . \zeta . \frac{eU_0}{k} = R_{kg} \zeta K_2 \qquad [r]$$

The enthalpy of the ionised gas, H is equal to the sum of the terms [n] and [r]. This yields:

$$H = (1+\zeta)\frac{5}{2} R_{kg} T + \zeta R_{kg} K_2 \left(\frac{J}{kg}\right) \qquad [13]$$

2.4 The energy equation of the plasma flow at the exit plane

The energy balance of the plasma gun is:

$$W - q_{kw} = \Phi_m . H + \tfrac{1}{2} \Phi_m v^2 \qquad [14]$$

where W = electric power supplied by the rectifier (W)
q_{kw} = heat loss through cooling water in torch and power-carrying cables (W)
Φ_m = arc-gas flow (kg s^{-1})
v = average flow velocity of the plasma at the exit plane (ms^{-1}).

2.5 The continuity equation

For the arc gas holds:

$$\Phi_m = \rho . v . A \text{ (kg } s^{-1}) \qquad [15]$$

where: A = cross-section of the anode bore (m^2).

2.6 Survey of equations

Applying to 1 kg of partially one-fold ionised argon gas:

degree of ionisation:

$$\zeta = \sqrt{\frac{T^{\frac{5}{2}} \cdot \exp \frac{-K_2}{T}}{\frac{p}{K_3 \cdot K_1} + T^{\frac{5}{2}} \cdot \exp \frac{-K_2}{T}}} \qquad [6]$$

equation of state: $p = (1 + \zeta)\, \rho\, R_{kg}\, T$ [11]

enthalpy: $H = (1 + \zeta) \frac{5}{2} R_{kg} T + \zeta R_{kg} K_2$ [13]

energy equation: $W - q_{kw} = \Phi_m \cdot H + \frac{1}{2} \Phi_m v^2$ [14]

continuity equation: $\Phi_m = \rho \cdot v \cdot A$ [15]

where $K_1 = 3.35 \cdot 10^{-2}$; $K_2 = 18.2 \cdot 10^4$; $K_3 = 10.8$

The variables are: p, T, ρ, W, q_{kw}, Φ_m, v, H, ζ, and A, of which can be measured easily: p, W, q_{kw}, Φ_m, and A. Consequently, the following five unknown variables remain to be solved: T, ρ, v, H, and ζ. These can be solved by means of the given set of five equations.

3 SOLVING THE EQUATIONS

3.1 By computer

Using eqs 11, 13, and 15, the energy eq. 14 may be written:

$$W - q_{kw} = (1 + \zeta)\, \Phi_m \frac{5}{2} R_{kg} T$$

$$1 + \frac{2}{5} \frac{\zeta}{1+\zeta} K_2 + \frac{1}{5} (1 + \zeta) \frac{\Phi_m^2}{p^2 A^2} \cdot R_{kg} \cdot T \qquad [16]$$

In eq. 16, T is given as a function of measurable operating quantities. The choice of the gas determines R_{kg} and $K_2 = \frac{eU_0}{k} = \frac{E_0}{k}$; choice of the anode likewise determines A. W and Φ_m are still to be varied, while q_{kw} is determined through the construction of the plasma torch. The computer has solved the set of equations for eight discrete operating conditions. As control parameters of the torch, Φ_m (two values), and the current intensity, I (four values per arc gas flow), were used. Table 1 shows the values measured on the plasma apparatus; Table 2 gives the calculated values of T, ζ, H, ρ, v, and $\frac{H}{v}$.

3.2 Graphically

The set of equations can also be solved simply by using graphs after introducing a permissible simplification.

The right-hand side of eq. 16 comprises three terms. For temperatures below 15 000°K it may be assumed that only one-fold ionisation of argon occurs. The third term inside the square brackets may then be ignored with respect to the two others. Hence in very good approximation:

$$\frac{W - q_{kw}}{\Phi_m \cdot R_{kg}} \cong (1 + \zeta) \frac{5}{2} T + \zeta K_2 = \frac{H}{R_{kg}} \qquad [17]$$

The equations are now solved as follows. If the left-hand side of eq. 17 be L and the right-hand side be R, thus:

$$L = \frac{W - q_{kw}}{\Phi_m \cdot R_{kg}} \quad \text{and} \quad R = (1 + \zeta) \frac{5}{2} T + \zeta K_2$$

For a certain gas, L can be found from the chosen spraying conditions, because W, q_{kw}, and Φ_m are quantities that can be simply measured. Using Fig. 1a, R can be represented by a graph as a function of T independently of the spraying conditions. For example, from Fig. 1a it appears that, at T = 10 000°K, $\zeta = 2.1 \cdot 10^{-2}$, so R can be calculated as shown in Fig. 1b. Taking $L = R$ yields the temperature of the plasma under the chosen spraying conditions. Next, ζ at that temperature can be read from Fig. 1a. Using eq. 11, ρ can be found and, finally, v, using eq. 15.

4 APPLICATION OF THE GRAPHICAL METHOD OF SOLVING EQUATIONS TO AN EXISTING PLASMA TORCH

The graphical method has been used for the same eight discrete operating conditions that have been handled by the computer.

4.1 Determination of L

For determining L the arc voltage and current intensity were measured at the terminals of the rectifier. This fixes W. The temperatures of the ingoing and outgoing cooling water were so measured that the losses sustained in the water-cooled current carrying cables are included. (The measurements are obviously also necessary when the computer is being used.)

At the same time the pressure difference between the arc-gas chamber of the gun and the ambient air was measured in order to make possible an experimental check of the calculated plasma velocities. Table 1 gives the results and the calculation of L.

4.2 Determination of R

The value of R has been determined in the manner indicated in 3.1. Figure 1b shows R as a function of temperature.

From eqs 17 and 13 it follows that the enthalpy H can be calculated from R by multiplying by R_{kg}, hence $H = R_{kg} \cdot R \cdot (J \cdot kg^{-1})$. Curve H = H(T), which thus can beformed from Fig. 1b, is the enthalpy curve of argon, frequently cited in the spraying literature. Therefore, this curve can be calculated by the user of the plasma torch.

4.3 Equating L and R ($L=R$)

As an example, in Fig. 1 this equation has been worked out for $L = 27\,173$ (see Table 1); the plasma temperature, determined graphically, is then $T = 9800^{o}K$ and the degree of ionisation $\zeta = 1.8 \cdot 10^{-2}$.

5 RESULTS

5.1 The values of the graphically determined characteristic quantities of a plasma

These, for eight discrete operating conditions, are shown in Table 2, in which the computer values are also recorded.

To illustrate the characteristic differences resulting from the use of a great or a small arc-gas flow, the quantities T and ζ have been represented in Fig. 3; v and $\frac{H}{v}$ in Fig. 4; and Δp in the arc-gas chamber in Fig. 5, all related to the current intensity. The graphically represented values were calculated by computer.

5.2 The energy balance

Using the foregoing analysis, a detailed energy balance can be drawn up. Substitution of eq. 13 in eq. 14 yields:

$$W = \underbrace{\tfrac{1}{2}\,\Phi_m \cdot v^2}_{W_1} + \underbrace{\tfrac{5}{2}\,\Phi_m R_{kg} T}_{W_2} + \underbrace{\tfrac{5}{2}\,\Phi_m R_{kg}\,\zeta T}_{W_3} + \underbrace{\Phi_m R_{kg}\,\zeta \frac{eU_0}{k}}_{W_4} + \underbrace{q_{kw}}_{W_5}$$

where W_1 = kinetic energy
W_2 = enthalpy of the heavy particles (particles with a nucleus)
W_3 = enthalpy of electrons
W_4 = ionisation energy
W_5 = cooling-water losses

From the numerical evaluation of the energy balance given in Fig. 6, it appears that the cooling-water losses are considerable. It also appears that the ionisation phenomena are certainly not negligible with respect to the energy consumed.

Owing to the ionisation the gas absorbs $W_3 + W_4$, i.e. $\frac{W_3 + W_4}{W_1 + W_2 + W_3 + W_4} \cdot 100\%$ of the total plasma energy. From Fig. 6 it appears that this may be as high as 50%. The kinematic energy of the plasma flow is slight. However, a small absolute variation in the kinetic energy results in large differences in velocity. This point will be given further attention in the discussion. The total efficiency of the plasma equipment is moderate.

6 EXPERIMENTAL VERIFICATION OF THE CALCULATED SPEEDS OF EXHAUST OF THE PLASMA

The exhaust speed can be verified experimentally from the over-pressure Δp in the arc-gas chamber. According to Bernoulli's law for compressible gas flow it holds:[2,12,13]

$$\Delta p = p_i - p_0 = \tfrac{1}{2}\rho v^2_{exp} \cdot \left(1 + \frac{M^2}{4} + \ldots\right) \qquad [19]$$

where p_i = pressure in arc-gas chamber
p_0 = atmospheric pressure
v_{exp} = experimentally determined outflow velocity
M = Mach number at the temperature of the plasma

$$M = \frac{v}{C} \quad \text{and} \quad C = \sqrt{C_1 \frac{P_0}{\rho}}$$

where C = velocity of sound in the plasma
C_1 = a constant; $C_1 \cong \frac{5}{3}$

Using $\rho = \frac{\Phi_m}{vA}$, eq. 19 yields:

$$v_{exp} = \frac{2\ \ PA}{\Phi_m \ \frac{(1 + M^2}{4})} \qquad [20]$$

which expresses v_{exp} in simply measurable process parameters and a correction term still to be calculated. Table 3 gives the values of the exhaust speed, v, calculated from eqs 11 and 15, v_{exp}, and $\frac{v_{exp}}{v}$. Column $\frac{v_{exp}}{v}$ in particular gives an impression of the agreement between theory and experiment.

7 DISCUSSION OF THE RESULTS

7.1 In drafting the five equations, only one was based upon a universally acknowledged formula, i.e. the Saha-Eggert equation. The remaining equations were derived from simple physical laws; therefore, little misunderstanding can exist as regards the applicability of the set of equations.

7.2 The agreement between the approximate results obtained graphically and the results supplied by a computer is such that the use of a computer is not necessary, though it is convenient.

7.3 The energy balance shows clearly that the plasma torch which has been examined is only moderate efficient, which gives adequate grounds for searching for a better construction.

7.4 Column $\frac{v_{exp}}{v}$ of Table 3 shows a systematic deviation between theory and experiment. It appears that for the operating conditions considered $1.10 < \frac{v_{exp}}{v} < 1.8$ is the value of $\frac{v_{exp}}{v} = 1.30$ is disregarded. Moreover, $\frac{v_{exp}}{v}$ appears to be larger is more arc gas is used. Three reasons may be put forward why $\frac{v_{exp}}{v}$ must be larger than 1; to elucidate this, further consideration of the processes going on in the plasma gun is necessary.

Figure 7 is a diagrammatic representation of a plasma gun. A nearly ideal arc discharge II is shown between cathode and anode. The arc discharge is maintained by electron emission from the strongly heated cathode spot. The plasma is formed in the arc by thermal ionisation resulting from collisions between the particles. The gas traverses successively zones I, II, and III, in which it passes through the following stages:

Zone I: cold gas not ionised
Zone II: gas ionised thermically into plasma
Zone III: outflowing plasma
Zone II: is a current-carrying plasma, zone III a plasma that carries no current.

The plasma of zone III derives its high velocity from:

1 Very rapid increase in specific volume owing to heating up in the arc II. The density of the gas decreases from 1.78 (kg m^{-3}) to 0.05 (kg m^3) and less.

2 Increase in pressure and velocity in zone II owing to Lorentz forces in the current-carrying plasma at right angles to the lines of current. The lines of current j_1 and j_2 attract each other according to the Lorentz law.

$\vec{P}_L = \vec{j} \times \vec{B} = \text{grad } p + \rho v \text{ grad.}$[12,15]

The magnetically built-up pressure is highest where the current density is greatest, i.e. at the smallest cross-section of the arc. This magnetic pressure causes the plasma to pass to the areas of lower pressure, hence in the direction of the larger cross-sections of II. Owing to magnetic compression the pressure in I, II, and III increases, and the velocity in II and III increases as well. This is illustrated in Fig. 7, which is a diagram of a particle moving along path, i.

3 Contraction of the hot plasma stream III in the anode bore. The plasma cannot exist at the low temperature of the water-cooled wall, so that here a layer of a relatively cold and heavy gas separates the plasma stream from the wall. The outflowing plasma uses for its flow a smaller cross-section than the net cross-section A of the anode bore. This results in an increase in velocity, hence, also an increase in pressure in the arc-gas chamber.

The foregoing equations are based upon a model in which is considered only the increase in the specific volume as shown in curve (0) in Fig. 7. The computed velocity v_{exp} originating from the measured overpressure, Δp, in the arc-gas chamber is, therefore, already, for two reasons, higher than the theoretical outflow velocity, v.

There is a third pressure effect which was not considered in drawing up the energy balance. Equation 14 refers to the plasma in the form in which it leaves the gun, i.e. at the plane BB in Fig. 7. Between planes AA and BB energy is absorbed from the plasma by the cooling water. Hence, in plane AA the plasma will be richer in energy than in plane BB. This manifests itself in a higher enthalpy as well as in higher velocity at AA. This additional increase in velocity produces an additional pressure difference, Δp_3. Therefore, the measured overpressure in the arc-gas chamber is greater by the amount $\Delta p_1 + \Delta p_2 + \Delta p_3$ than the pressure on which the theoretical velocity, v, in Table 2 is based.

If it were possible to measure the overpressure $\Delta p' = \Delta p_0 + \Delta p_1 + \Delta p_2$ (hence, without Δp_3), the real outflow velocity v'_{exp} could be calculated from it. It will be clear that $1 < \frac{v'_{exp}}{v} < \frac{v_{exp}}{v}$, so that actually the figures in Table 3 give too unfavourable a picture of the discrepancy between theory and reality.

Finally, it should be pointed out that the kinetic energy of the outflowing plasma is very slight in respect of the enthalpy, and that a relatively great change in v only affects ρ but not H, ζ, or T. Consequently, the following procedure is suggested to make the theoretically calculated results practicable:

Calculate ρ, v, H, ζ, and T by solving the set of equations

Correct v by a factor 1.1 to $v' = 1.1 \cdot v$

Correct ρ likewise with this factor to $\rho' = \frac{\rho}{1.1}$.

Then consider the values of ρ', v', H, ζ, and T as the real solution.

8 PRACTICAL IMPLICATIONS

The present research was started with the object of stimulating the systematic study of the practical spraying process. Just a few possibilities will be mentioned here.

The determination of plasma properties as a function of the operating parameters renders the handbook values such as electrical conductivity, heat conductivity, and viscosity applicable to the experiments.

The choice of the arc gas can now be made starting from predictable properties, although development is still necessary to make the results of the present research applicable to diatomic gases and mixtures of gases.

Judgement of plasma torches as regards efficiency and construction is being investigated in our laboratory.

The study of the individual influence of the plasma properties such as v, T, and $\frac{H}{v}$ [18] on the properties of the sprayed product and its interaction with the substrate. $\frac{H}{v}$ is probably an important quantity which is codecisive for the behaviour of the spraying particle during the transport from the torch to the substrate.

In this connection the relative velocity of the spraying particle with respect to the average velocity, v, of the plasma plays a role. Ignoring for a moment the relative velocity, from eqs 13, 15, and 11 it follows that:

$$\frac{H}{v} = \frac{5}{2} \frac{p.A}{\Phi_m} \left[1 + \frac{2}{5} \zeta \frac{eU_0}{kT} \right] \quad [21]$$

and for argon:

$$\frac{H}{v} = \frac{5}{2} \cdot \frac{p.A}{\Phi_m} \left[1 + 0.73 \zeta \frac{10^5}{T} \right]$$

The ratio $\frac{H}{v}$ is, therefore, greater in the case of

1 a smaller degree of ionisation Φ_m
2 a higher degree of ionisation ζ.

If $T < 10^3$, ζ is zero and $\frac{H}{v} = \frac{5}{2} \cdot \frac{p.A}{\Phi_m}$ for $T \cong 10^4$ holds: $0.01 < \zeta < 0.15$ and $\frac{H}{v}$ increases with the gas temperature, T (Fig. 4). For $T \cong 10^5, \zeta \cong 1$ and $\frac{H}{v}$ reaches a maximum, decreasing again with increasing gas temperature. To what extent $\frac{H}{v}$ plays an important role is part of the continued research.

As an example, in the following problem the authors will apply the foregoing theory.

To calculate heat and impulse interactions between the plasma stream and the spraying particle, it is of importance to know whether the flow is laminar or turbulent. The plasma streams used in spraying work are very noisy and show a strong tendency towards mixing with the ambient atmosphere.[10] Apparently the flow outside the plasma torch is entirely or partly turbulent. Injection of the powder particles may take place in the plasma stream before the latter has left the gun. The plasma stream before the latter has left the gun. The plasma stream produced by the apparatus investigated is laminar within the anode bore, as will be shown below. The Reijnolds number for the plasma flow in the bore is $Re = \frac{\rho v d}{\mu}$

where d = anode bore = 7.4 mm
μ = dynamic viscosity

With the chosen operating parameters (see Table 2):

i.e. $m = 0.589 \cdot 10^{-3}$ (kg s^{-1}) $\hat{=}$ 21 (l min^{-1}), argon

$J = 510$ (A) = 20145 (W)

it appears $T = 12275$ (K)

$\rho = 0.03355$ (kg s^{-1}) and $\rho' = \frac{\rho}{1.1} =$ 0.0305 (kg s^{-1})

$v = 408$ (m s^{-1}) and $v' = 1.1\ v =$ 448.8 (m s^{-1})

From eq. 11 at $T = 12\,275^{0}$ (K) it follows $\mu = 36.10^{-5}$ (N s m^{-2}),

hence: $Re = 281 < R_{ckr} = 2300$,

where $R_{kr} = 2300$ indicates the limit of change from laminar to turbulent flow.

Whether a turbulent boundary layer has already formed at the exit plane can be calculated with eq. 19 (see Fig. 8):

$$\frac{\delta_t}{x} = 0.38\ Re_x^{-0.2}$$

$$Re_x = \frac{\rho x v}{\mu}$$

where δ_t = thickness of turbulent boundary layer

x = length of the anode channel = 13 (mm)

$\delta_t = x\ .\ 0.38\ (\frac{\rho x v}{\mu})^{-2} = 1.43$ (mm)

For $\delta_{t_{kr}} = \frac{d}{2} = 3.7$ (mm) the change to turbulent flow would take place. Consequently the change does not occur in the anode channel of the torch.

9 CONCLUSIONS

.1 It has been shown how to define the operation of a plasma torch for monatomic gases by means of a relatively simple set of equations. The ionisation phenomena play an important part in the process.

.2 The solution of the equations supplies the desired information on plasma properties such as ρ, v, T, H, ζ, and $\frac{H}{v}$

.3 From the quantities calculated the real mean values of these quantities can be derived by way of a simple correction.

10 APPENDIX

Equations 1 and 3 are equivalent, as shown below. Division of eq. 1 by eq. 3 yields:

$$\frac{n_{z+1}\ n_e}{n_z} \Big/ \frac{\zeta^2}{1-\zeta^2} = \frac{p}{kT} \qquad [s]$$

For one-fold ionisation [s] becomes:

$$\frac{n_1\ .\ n_e}{n_0} = \frac{p}{kT}\ .\ \frac{\zeta^2}{1-\zeta^2} \qquad [t]$$

Equations 1 and 3 are equivalent if eq. [t] is correct. The left-hand side of eq. [t] is written as a function of by means of the following expressions:

$n_1 = \zeta n$

$n_e = n_1 = \zeta n$

$n_0 = n - n_1 = n\ (1-\zeta)$

The equation of state for an ionised gas yields:

$n = \frac{p}{(1+\zeta)\ kT}$, hence,

$$n_1 = \frac{\zeta}{1+\zeta}\ .\ \frac{p}{kT}$$

$$n_e = \frac{\zeta}{1+\zeta}\ .\ \frac{p}{kT}$$

$n_0 = \frac{1-\zeta}{1+\zeta}\ .\ \frac{p}{kT}$, resulting in:

$$\frac{n_1\ .\ n_e}{n_0} = \frac{(\frac{\zeta}{1+\zeta})^2\ .\ (\frac{p}{kT})^2}{\frac{1-\zeta}{1+\zeta}\ .\ \frac{p}{kT}} = \frac{\zeta^2}{1-\zeta^2}\ .\ \frac{p}{kT} = QED$$

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Table 1 Measurements carried out on plasma apparatus and calculation of $L = \frac{W - q_{kw}}{\Phi_m \cdot R_{kg}}$

Arc gas[1]	Power supplied			Cooling-water losses		Δp^2	*L*
Φ_m, kg s^{-1}	I, A	V, V	W, W	q_{kw},	q_{kw}/W, %	mm H_2O^3	
$0.589 \cdot 10^{-3}$	205	30.2	6 191	3 340	53.9	202	23 271
	312	32.2	10 046	5 610	55.8	252	36 208
	411	36.0	14 796	8 700	58.7	298	49 758
	510	39.5	20 145	12 530	62.1	327	62 157
$0.897 \cdot 10^{-3}$	205	32.2	6 601	3 465	52.4	410	16 808
	310	35.0	10 850	5 780	53.2	527	27 173
	415	38.5	15 977	9 010	56.4	615	37 341
	518	42.2	21 859	12 800	58.5	715	14 553

Notes:

1 Arc gas = argon

$\Phi_m = 0.589 \cdot 10^{-3}$ (kg s^{-1}) ≙ 21 (lmin^{-1})

$\Phi_m = 0.897 \cdot 10^{-3}$ (kg s^{-1}) ≙ 32 (lmin^{-1})

2 Δp = overpressure in arc gas chamber with respect to atmosphere

3 1 mm H_2O ≙ 9.81 (Nm^{-2}).

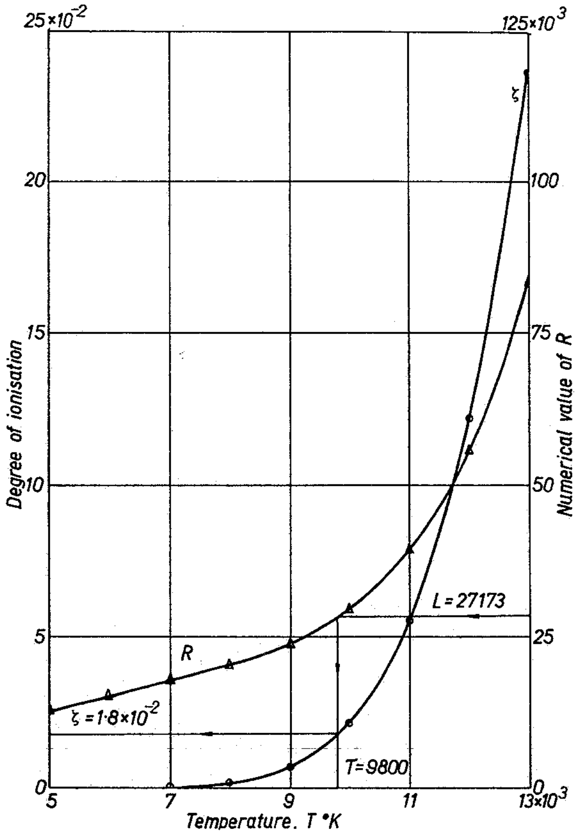
Table 2 Characteristic quantities of a plasma

Φ_m, kg s^{-1}	I, A	T, K	ζ, -	H, J kg^{-1}	ρ, kg m^{-3}	v, m s^{-1}	$\frac{H}{v}$, Js $kg^{-1}m^{-1}$
Determined graphically							
$0.589 \cdot 10^{-3}$	205	8 877	$0.5 \cdot 10^{-2}$	$4\,840 \cdot 10^{3}$	$5\,348 \cdot 10^{-5}$	256	$18.90 \cdot 10^{3}$
	312	10 750	$4.45 \cdot 10^{-2}$	$7\,531 \cdot 10^{3}$	$4\,200 \cdot 10^{-5}$	326	$23.10 \cdot 10^{3}$
	411	11 725	$9.6 \cdot 10^{-2}$	$10\,349 \cdot 10^{3}$	$3\,670 \cdot 10^{-5}$	373	$27.74 \cdot 10^{3}$
	510	12 275	$14.5 \cdot 10^{-2}$	$12\,928 \cdot 10^{3}$	$3\,355 \cdot 10^{-5}$	408	$31.68 \cdot 10^{3}$
$0.897 \cdot 10^{-3}$	205	6 625	-	$3\,496 \cdot 10^{3}$	$7\,119 \cdot 10^{-5}$	293	$11.93 \cdot 10^{3}$
	310	9 800	$1.8 \cdot 10^{-2}$	$5\,651 \cdot 10^{3}$	$4\,727 \cdot 10^{-5}$	442	$12.78 \cdot 10^{3}$
	415	10 850	$4.9 \cdot 10^{-2}$	$7\,766 \cdot 10^{3}$	$4\,143 \cdot 10^{-5}$	504	$15.40 \cdot 10^{3}$
	518	11 650	$9.1 \cdot 10^{-2}$	$10\,099 \cdot 10^{3}$	$3\,710 \cdot 10^{-5}$	563	$17.93 \cdot 10^{3}$
Calculated by computer							
$0.589 \cdot 10^{-3}$	205	8 808	$0.53 \cdot 10^{-2}$	$4\,807 \cdot 10^{3}$	$5\,325 \cdot 10^{-5}$	257	$18.68 \cdot 10^{3}$
	312	10 735	$4.36 \cdot 10^{-2}$	$7\,478 \cdot 10^{3}$	$4\,209 \cdot 10^{-5}$	326	$22.97 \cdot 10^{3}$
	411	11 678	$9.58 \cdot 10^{-2}$	$10\,281 \cdot 10^{3}$	$3\,684 \cdot 10^{-5}$	372	$27.64 \cdot 10^{3}$
	510	12 259	$14.64 \cdot 10^{-2}$	$12\,845 \cdot 10^{3}$	$3\,355 \cdot 10^{-5}$	408	$31.45 \cdot 10^{3}$
$0.897 \cdot 10^{-3}$	205	6 630	$0.012 \cdot 10^{-2}$	$3\,453 \cdot 10^{3}$	$7\,112 \cdot 10^{-5}$	293	$11.76 \cdot 10^{3}$
	310	9 575	$1.35 \cdot 10^{-2}$	$5\,560 \cdot 10^{3}$	$4\,859 \cdot 10^{-5}$	429	$12.94 \cdot 10^{3}$
	415	10 806	$4.65 \cdot 10^{-2}$	$7\,641 \cdot 10^{3}$	$4\,170 \cdot 10^{-5}$	500	$15.26 \cdot 10^{3}$
	518	11 587	$8.93 \cdot 10^{-2}$	$9\,943 \cdot 10^{3}$	$3\,736 \cdot 10^{-5}$	559	$17.80 \cdot 10^{3}$

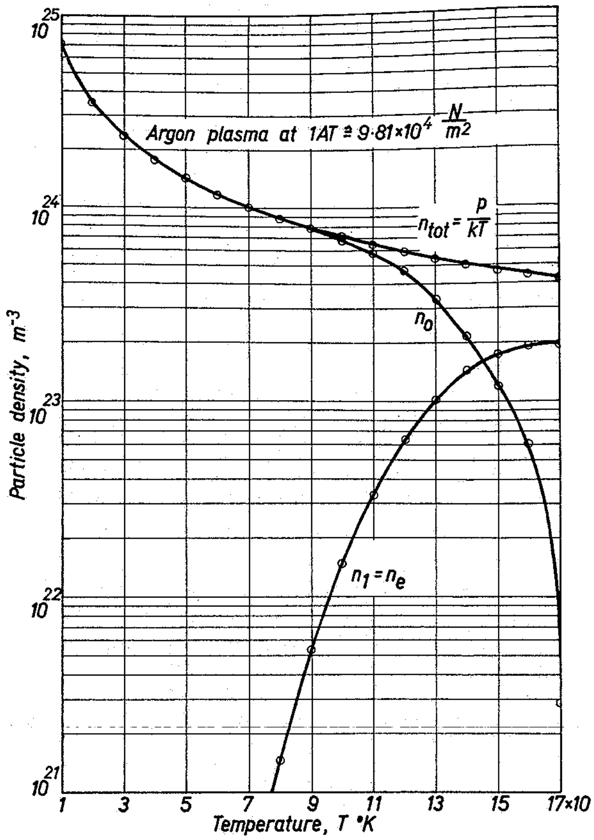
Table 3 Outflow velocity determined experimentally. Comparison with the theoretically calculated values

Φ_m, kg s^{-1}	I, A	v_{exp}, m s^{-1}	v, m s^{-1}	$\frac{v_{exp}}{v}$, -
$0.589 \cdot 10^{-3}$	205	287	257	1.116
	312	359	326	1.101
	411	424	372	1.139
	510	464	408	1.137
$0.897 \cdot 10^{-3}$	205	381	293	1.30*
	310	489	429	1.139
	415	569	500	1.138
	518	660	559	1.180

*Measuring error ?



1 (a) degree of ionisation of an argon plasma as a function of temperature, (b) right-hand side of eq.17 as a function of temperature



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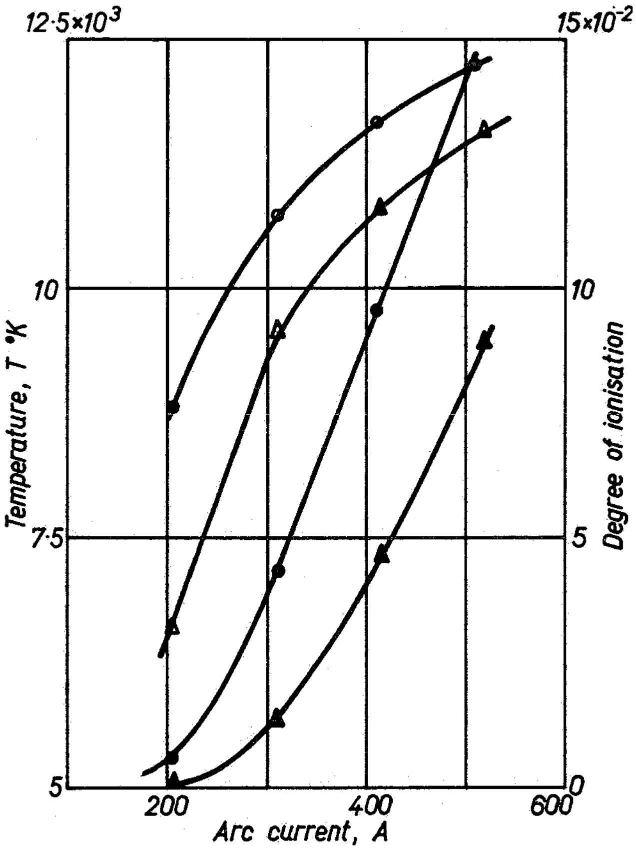
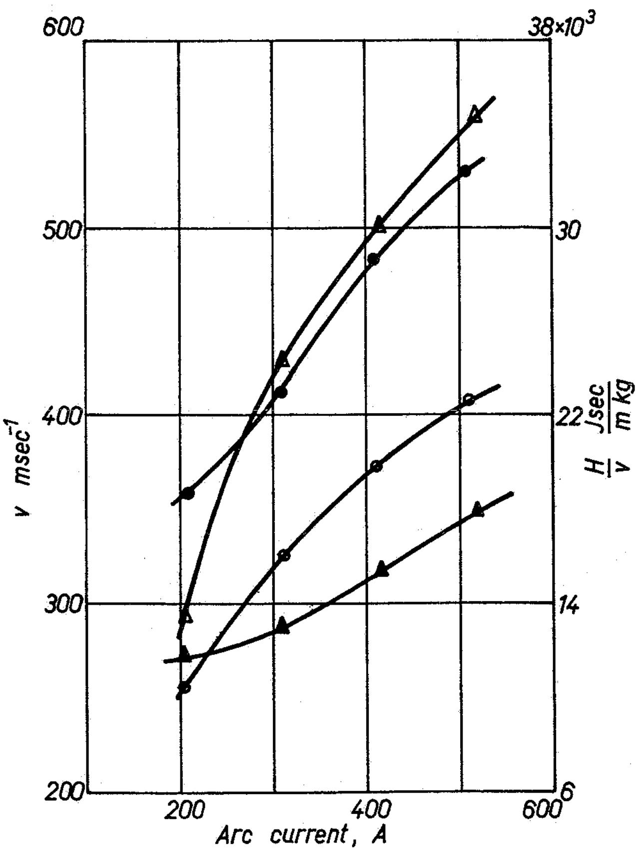
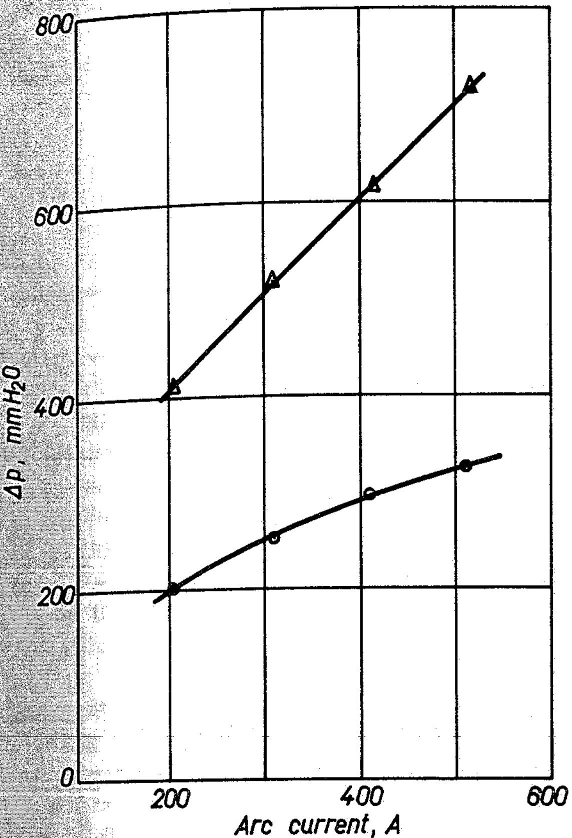


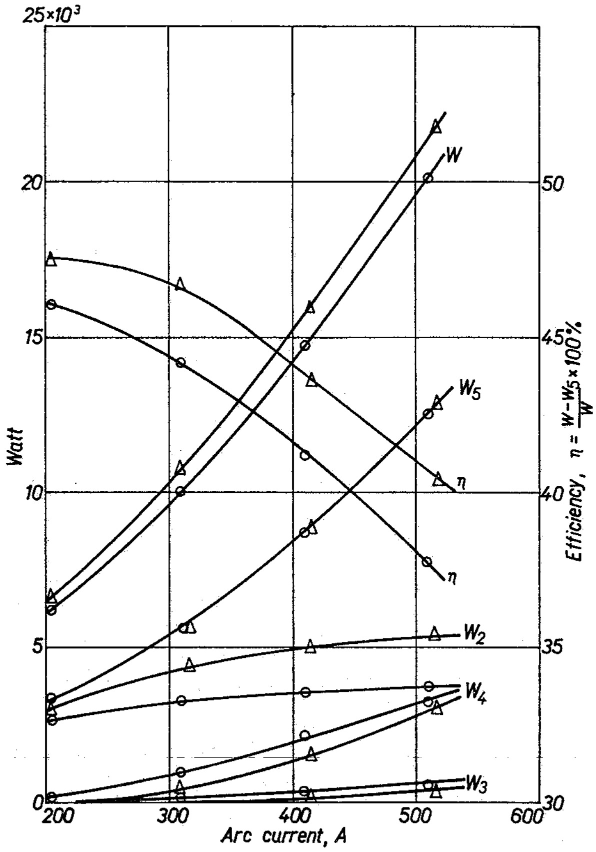
3 Temperature and degree of ionisation as a function of arc current. ○ – T at 21 l/min^{-1}; ● – ζ at 21 l/min^{-1}; △ – T at 32 l/min^{-1}; ▲ – ζ at 32 l/min^{-1}



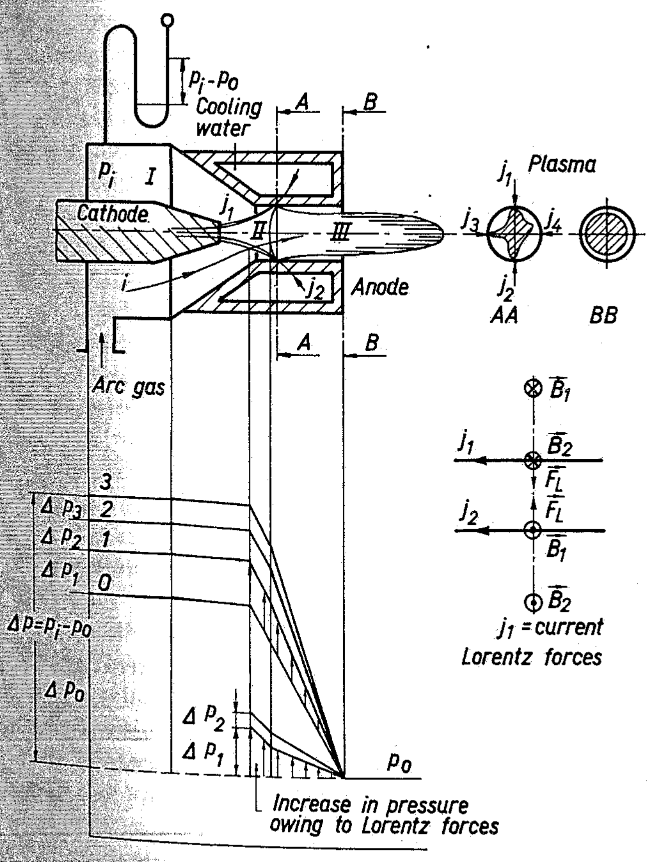
4 Plasma velocity and enthalpy/velocity ratio as a function of arc current. ○ – v at 21 l/min^{-1}; ● – $\frac{H}{v}$ at 21 l/min^{-1}; △ – v at 32 l/min^{-1}; ▲ – $\frac{H}{v}$ at 32 l/min^{-1}



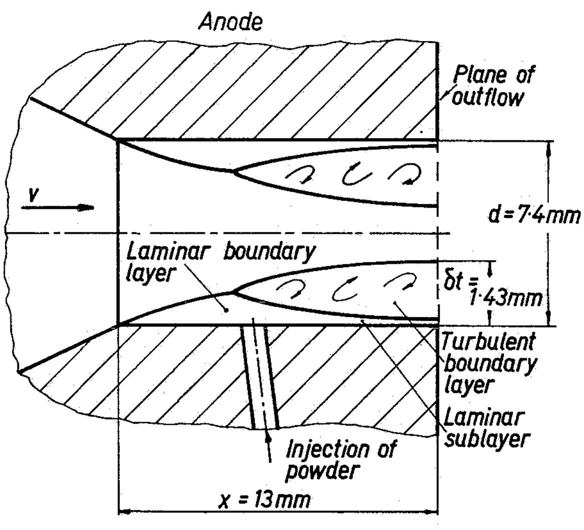
5 Pressure difference between arc-gas chamber and atmosphere as a function of arc current. ○ — at $21 l/min^{-1}$; △ — at $32 l/min^{-1}$



6 Distribution of supplied energy over cooling water and plasma components. ○ — $21 l/min^{-1}$; △ — $32 l/min^{-1}$; kinetic energy ($\frac{1}{2}\phi_m v^2$); $\phi_m \triangleq 21 l/min^{-1}$ $19 < \frac{1}{2}\phi_m v^2 < 49$; $\phi_m \triangleq 32 l/min^{-1}$ $39 < \frac{1}{2}\phi_m v^2 < 140$



7 Increase in pressure: Δp_1 — in arc-gas chamber owing to magnetic compression; Δp_2 — owing to contraction of plasma stream; Δp_3 — owing to loss of heat in anode bore



8