A Time Complexity Analysis

As defined before, T is the number of time points, and p is the number of variables. The time complexity of computing each single hierarchical clustering on p entities at time t_i is $O(p^3)$, and calculating the entries of W_i would cost $O(p^2)$ time. Thus, the total additional cost to compute $\{W_i\}$ is $O(p^3T)$.

Section 3.4 illustrates that at each iteration, the optimization consists of three steps. Step (13) and (15) are same as in SINGLE, and their cost are dominated by eigendecomposition $O(p^3)$ and matrix additions $O(p^2)$ for each time point, thus total $O(p^3T)$ [Monti et al., 2014].

For step (14), there are total $O(p^2)$ number of equations (3.4) to solve. Each (3.4) is split into a set of Fused Lasso problem of length l_d which has $O(l_d \log l_d)$ cost to solve at each iteration [Hoefling, 2010]. Considering that $\sum l_d = T$ and disgarding the number of iterations, we can estimate the upperbound of step (14) as $O(p^2T \log T)$.

To sum up, the cost of CFGL update per iteration is $O(p^2T \log T + p^3T)$. Since a one time calculation cost $O(p^3T)$ for $\{W_i\}$ can be ignored, amortized for different runs and hyper-parameter settings, CFGL has same time complexity as SINGLE [Monti et al., 2014].

In addition, similar to SINGLE [Monti et al., 2014], the update steps of $\{\Theta_i\}$ (13) can be parallelized with respect to time index i, and the update steps of $\{Z_i\}$ (14) can be parallelized with respect to matrix index k, l. Thus the optimization algorithm achieves very good scalability.

B Graph Evolution in Simulated Experiments

In this section, we provide additional plots to show how the CFGL estimated graphs change at the switching points. The following tables of figures list randomly selected experiments from each signal group, with focus on the time points when structures changes happen (including both switchings of static state to event state and event state to static state). The graph structures inside each state are omitted since they are almost the same as the beginning and ending time's structures.

It can be seen that in most cases, CFGL estimated structures can reflect the true structure changes, even large changes happen in two consecutive time points.

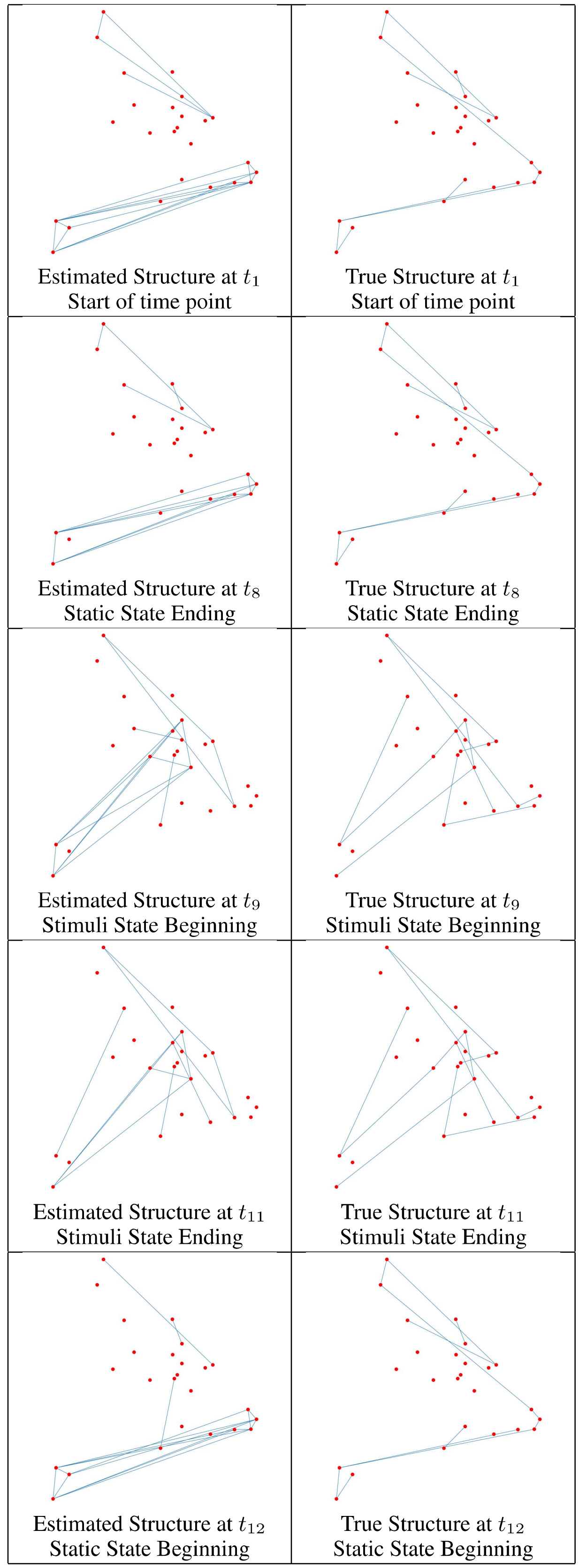

Figure 7: Estimated Graph of Signal Group 1 (Part1)

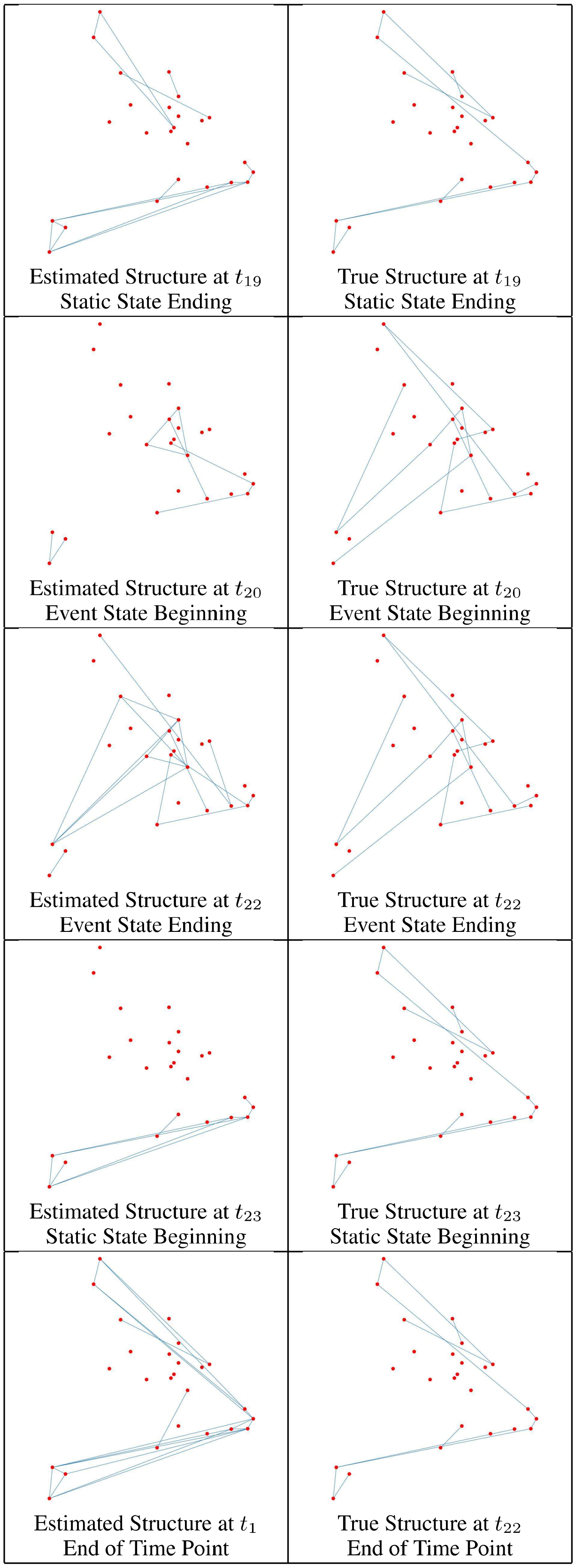

Figure 8: Estimated Graph of Signal Group 1 (Part2)

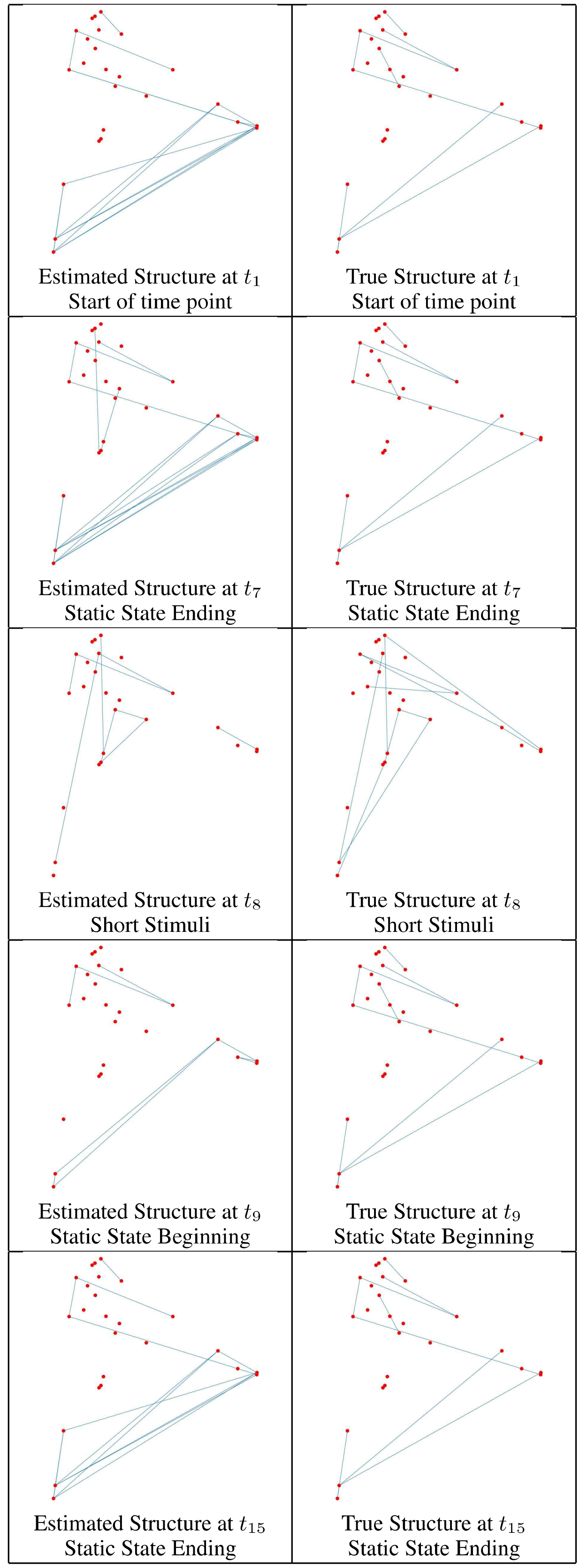

Figure 9: Estimated Graph of Signal Group 2 (Part1)

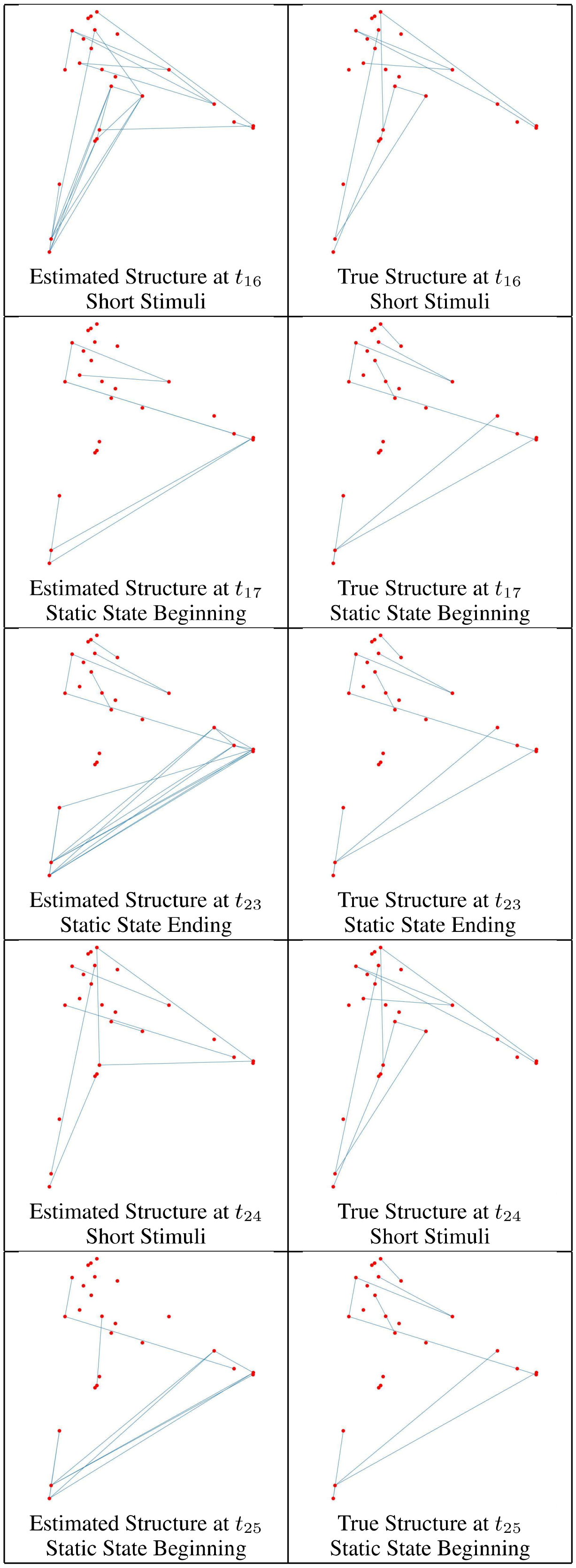

Figure 10: Estimated Graph of Signal Group 2 (Part2)

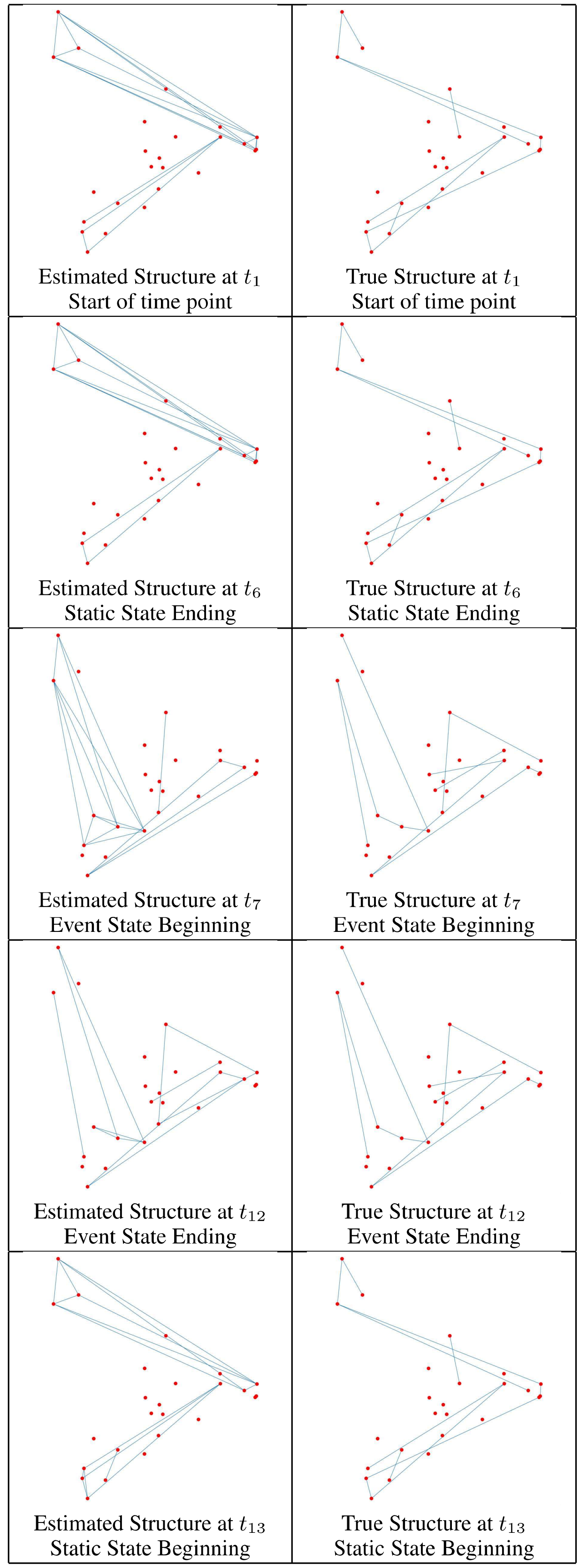

Figure 11: Estimated Graph of Signal Group 3 (Part1)

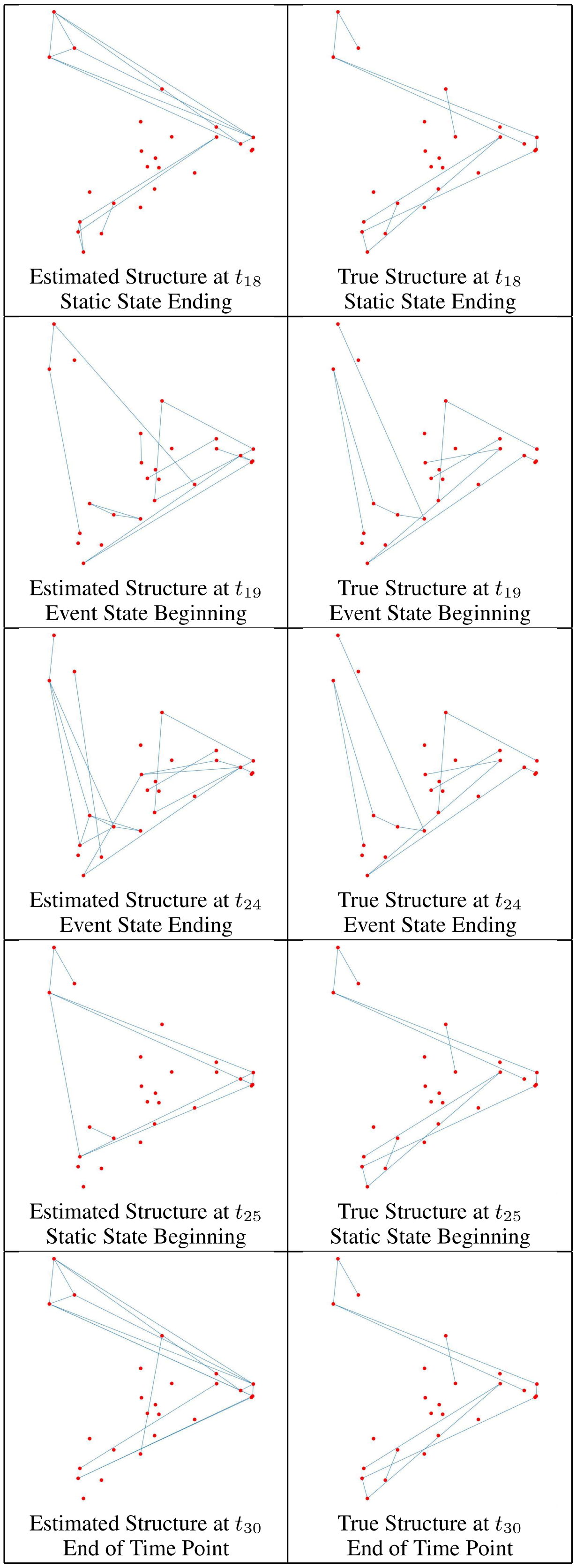

Figure 12: Estimated Graph of Signal Group 3 (Part2)