



DIST: Rendering Deep Implicit Signed Distance Function with Differentiable Sphere Tracing

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ETH zürich

Google

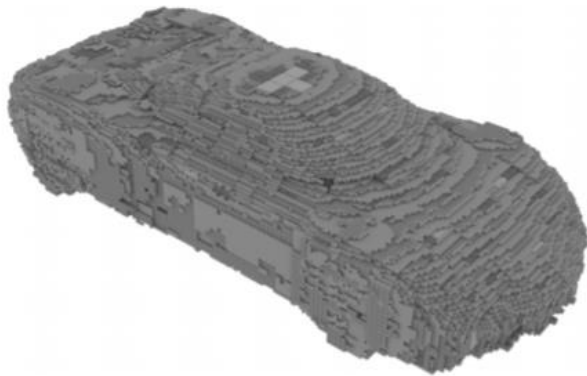


 Microsoft

Deep Implicit Signed Distance Functions (DeepSDF)

- Infinite-Resolution
- Lightweight

Effective compared to
voxels, point clouds and triangular meshes



Voxel-based Representation
[Tatarchenko *et al.*]

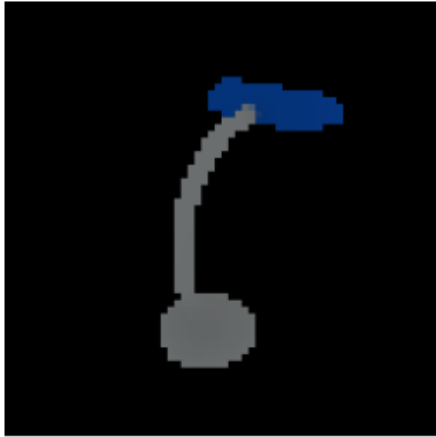


DeepSDF Representation
[Park *et al.*]

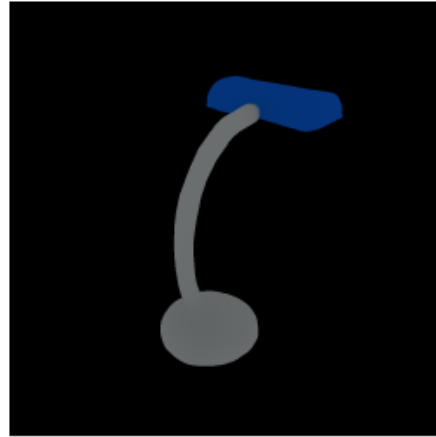
☹️ **No differentiable renderer for DeepSDF!**

Feedforward Rendering Results

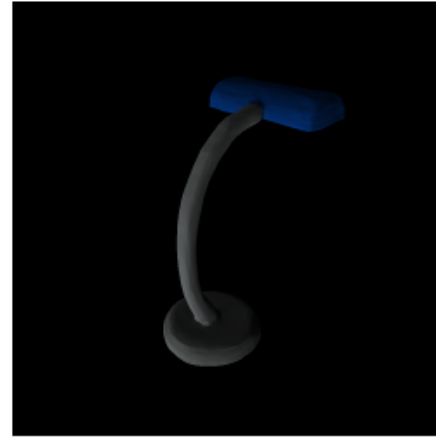
LR texture



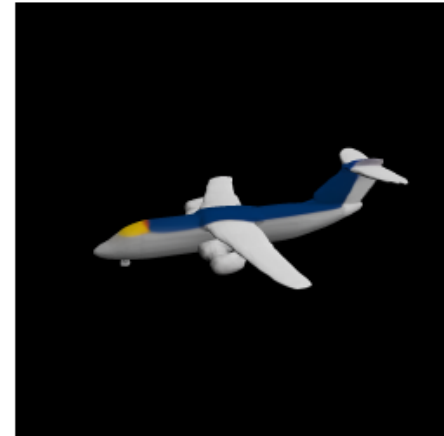
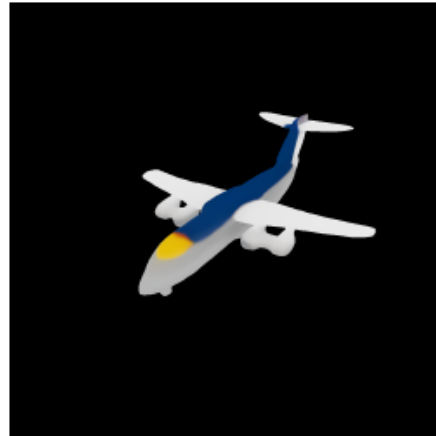
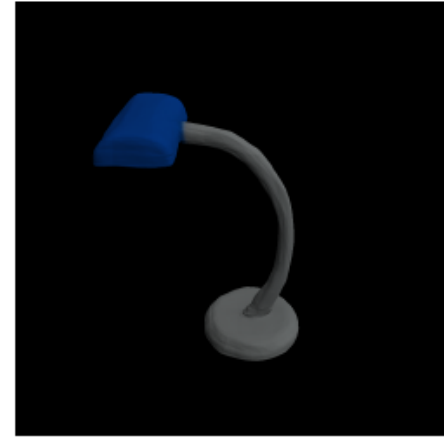
32x HR texture



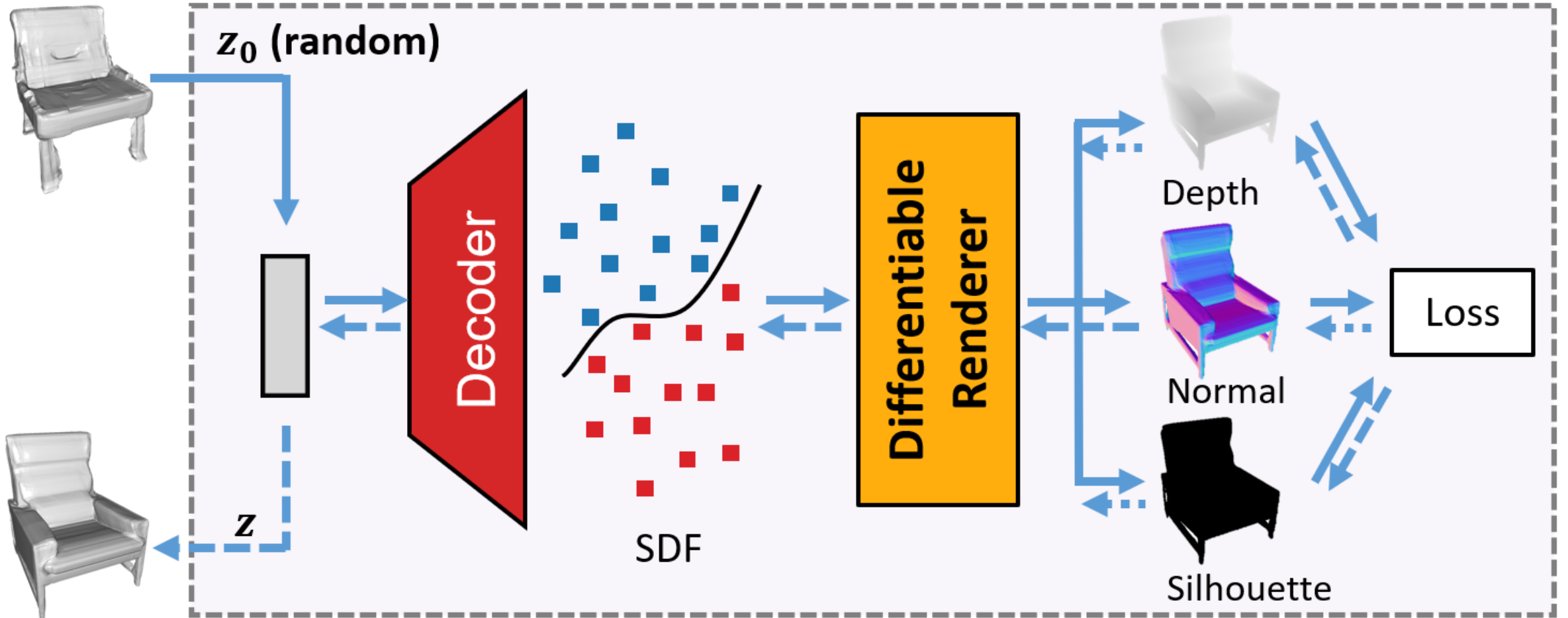
HR Relighting



HR 2nd View

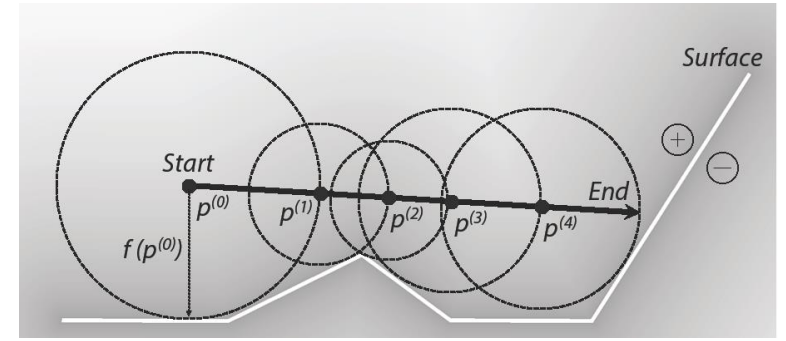


Optimization over Shape Code



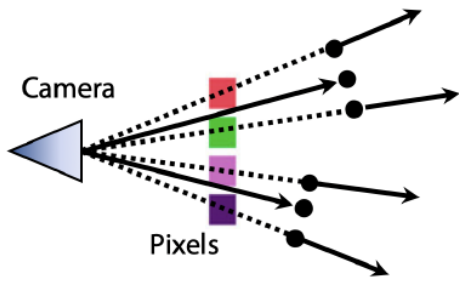
DIST - Differentiable Sphere Tracing

How to make feedforward efficient?

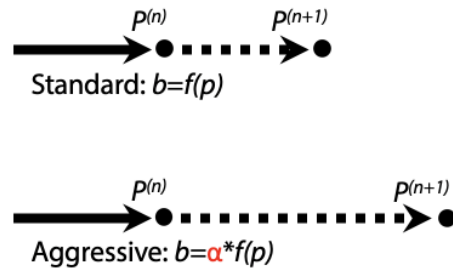


What ray convergence criteria is the best setup?

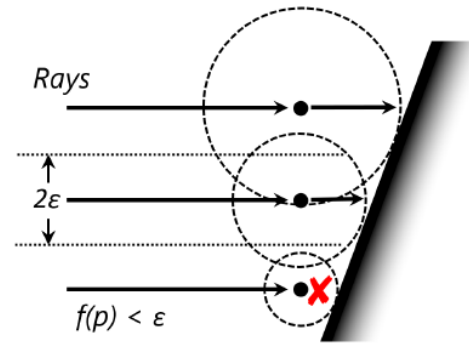
Efficient feedforward of sphere tracing algorithm



(a) Coarse-to-fine Strategy

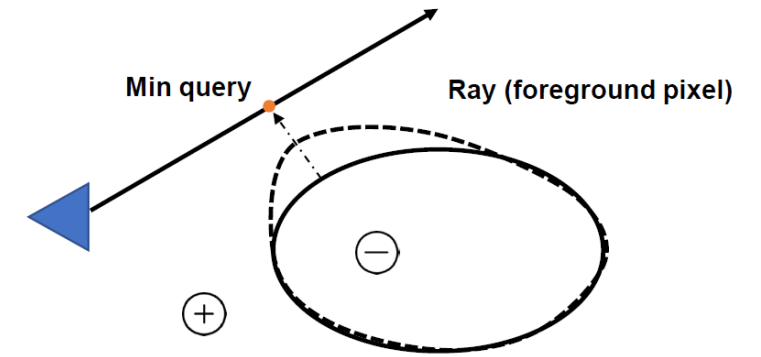


(b) Aggressive Marching



(c) Convergence Criteria

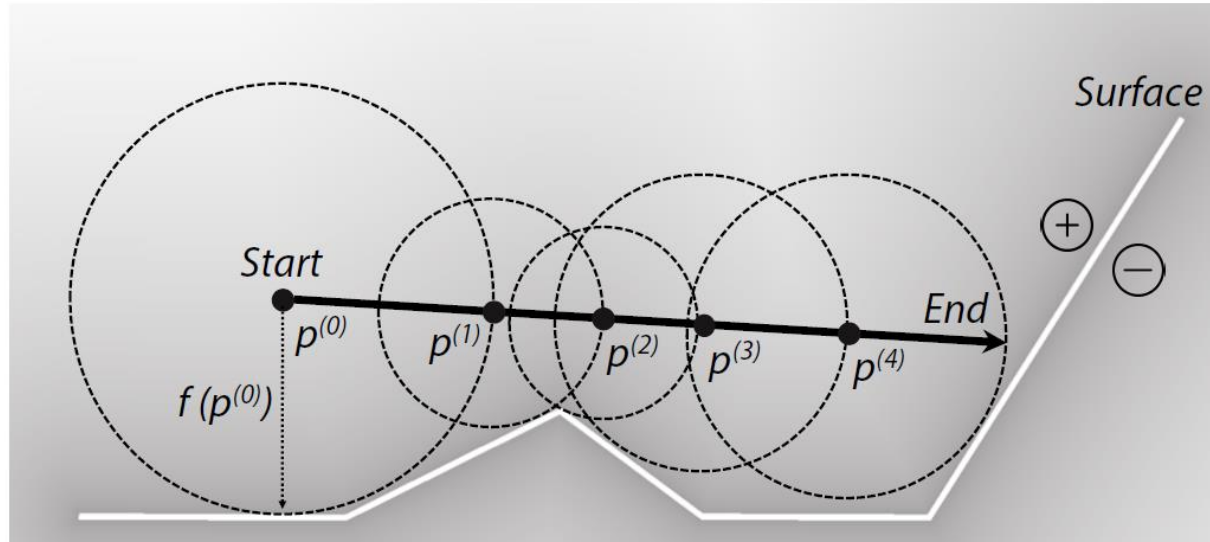
Gradient computation on the rendered silhouette



What can we do to resolve the GPU memory burden?

How to deal with non-differentiable rendered silhouette?

DIST Feedforward – Naive Sphere Tracing

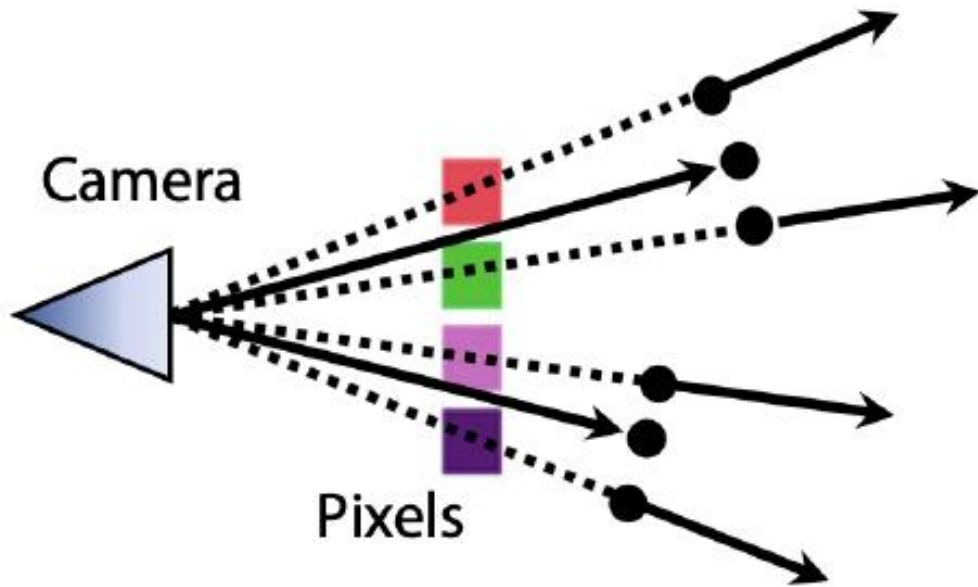


Algorithm 1 Naive sphere tracing algorithm for a camera ray $L : \mathbf{c} + d\tilde{\mathbf{v}}$ over a signed distance fields $f : \mathbb{N}^3 \rightarrow \mathbb{R}$.

- 1: Initialize $n = 0$, $d^{(0)} = 0$, $\mathbf{p}^{(0)} = \mathbf{c}$.
 - 2: **while** not converged **do**:
 - 3: Take the corresponding SDF value $b^{(n)} = f(\mathbf{p}^{(n)})$ of the location $\mathbf{p}^{(n)}$ and make update: $d^{(n+1)} = d^{(n)} + b^{(n)}$.
 - 4: $\mathbf{p}^{(n+1)} = \mathbf{c} + d^{(n+1)}\tilde{\mathbf{v}}$, $n = n + 1$.
 - 5: Check convergence.
 - 6: **end while**
-

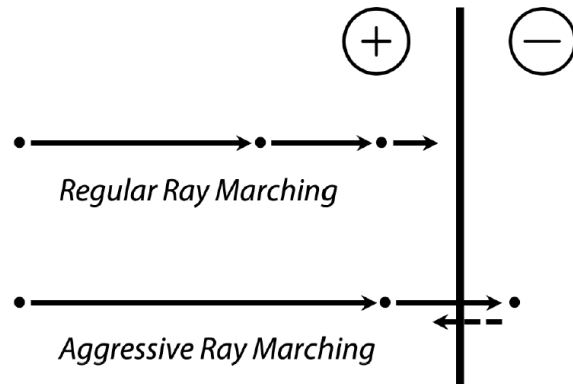
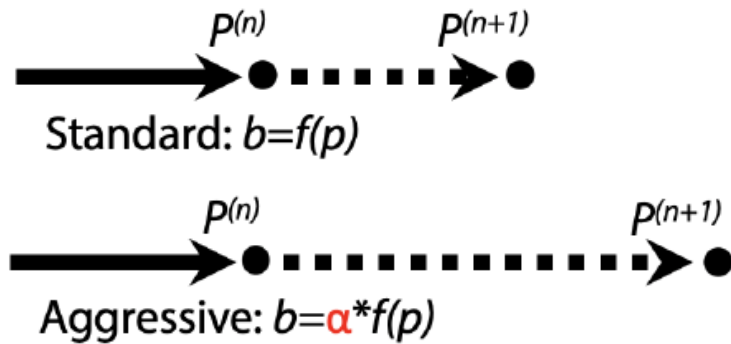
For each camera ray, march along the ray direction at each step with the queried SDF value until convergence.

DIST Feedforward - Coarse-to-Fine Strategy

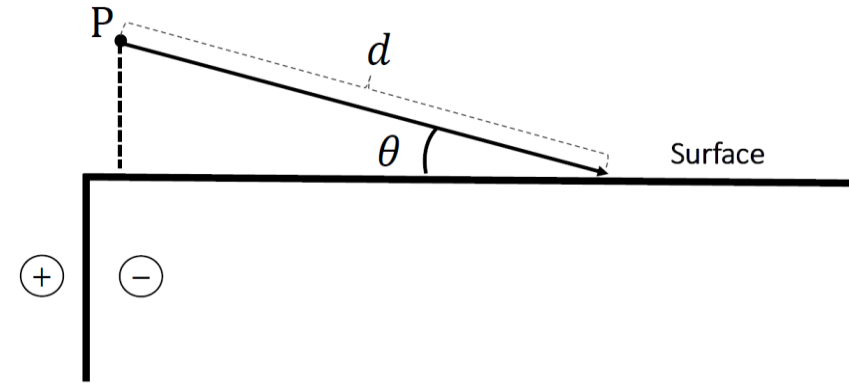


We start the sphere tracing over an image with $\frac{1}{4}$ resolution, and split each ray twice during the marching process, which saves computation at the early stage.

DIST Feedforward – Aggressive Marching



Setting step size $\alpha > 1$ incurs bouncing between both sides.

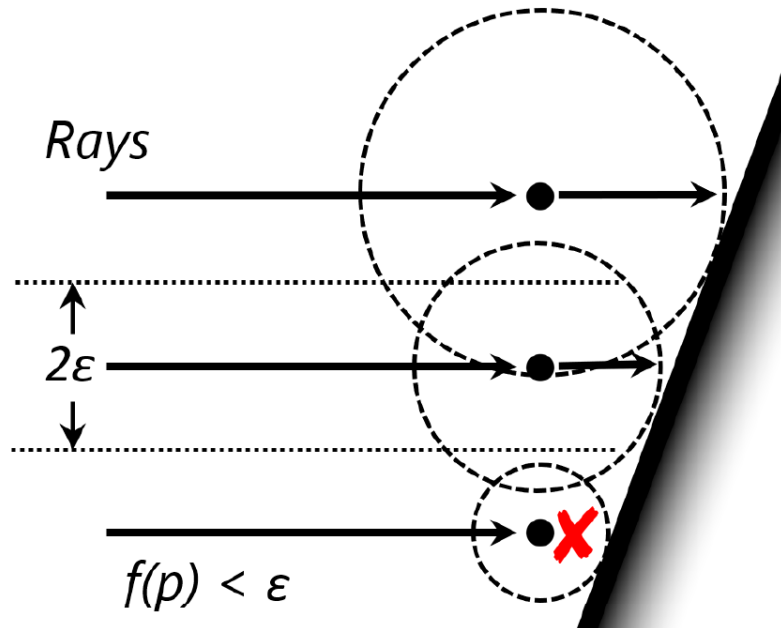


$$|d(1 - \alpha \sin \theta)^k| < \epsilon$$

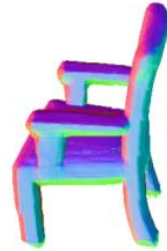
$$k > k_{min} = \frac{\log \epsilon - \log d}{\log |1 - \alpha \sin \theta|}$$

Setting step size $\alpha > 1$ speeds up convergence.

DIST Feedforward – Convergence Criteria



$$\epsilon = 5 \times 10^{-2}$$



$$\epsilon = 5 \times 10^{-4}$$



$$\epsilon = 5 \times 10^{-6}$$



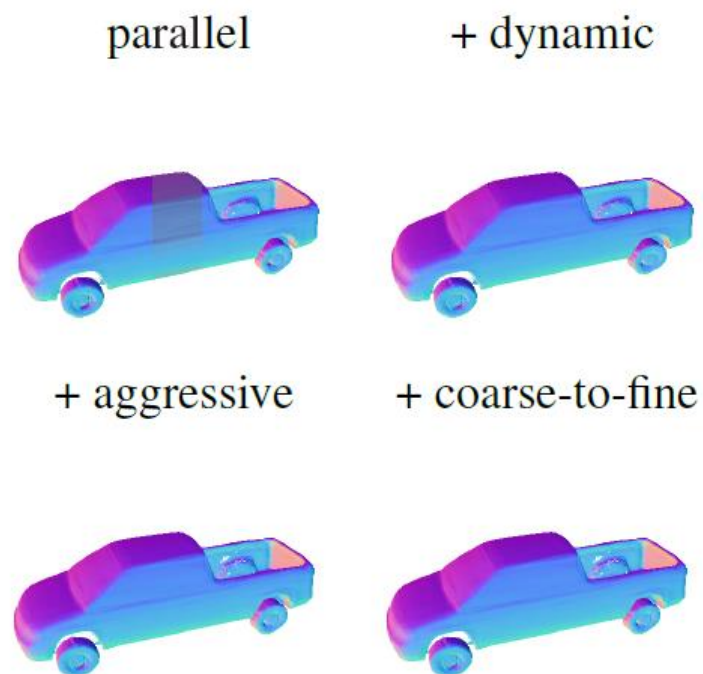
$$\epsilon = 5 \times 10^{-8}$$



We stop the marching once the SDF value is smaller than $1/2 \epsilon$.

A large threshold ϵ causes dilation, while a small threshold leads to erosion.

DIST Feedforward – Results



Method	size	#step	#query	time
Naive sphere tracing	512^2	50	N/A	N/A
+ practical grad.	512^2	50	6.06M	1.6h
+ parallel	512^2	50	6.06M	3.39s
+ dynamic	512^2	50	1.99M	1.23s
+ aggressive	512^2	50	1.43M	1.08s
+ coarse-to-fine	512^2	50	887K	0.99s
+ coarse-to-fine	512^2	100	898K	1.24s

The computation becomes affordable while the results remain almost unchanged.

DIST Backward – Recursive Gradients

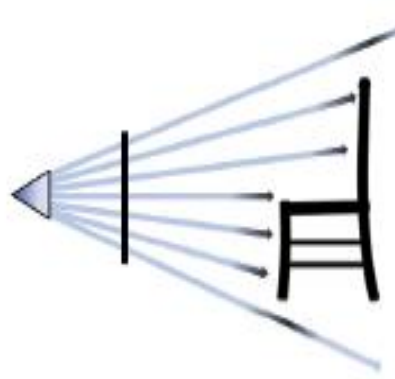
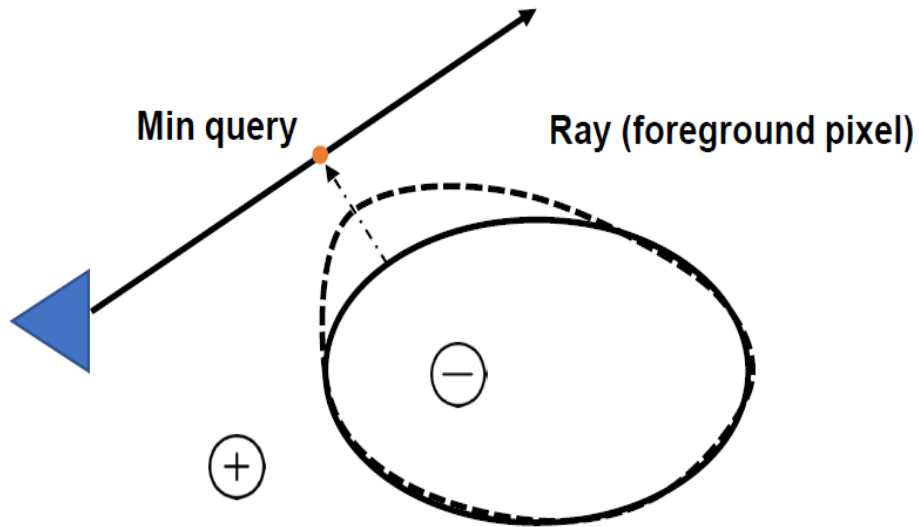
Each query location depends on the previous ones, incurring recursive gradients.

$$d = \alpha \sum_{n=0}^{N-1} f(\mathbf{p}^{(n)}) + (1 - \alpha)f(\mathbf{p}^{(N-1)}) = d' + e$$

$$\begin{aligned} \frac{\partial d'}{\partial \mathbf{z}} \Big|_{\mathbf{z}_0} &= \alpha \sum_{i=0}^{N-1} \frac{\partial f_{\theta}(\mathbf{p}^{(i)}(\mathbf{z}), \mathbf{z})}{\partial \mathbf{z}} \Big|_{\mathbf{z}_0} \\ &= \alpha \sum_{i=0}^{N-1} \left(\frac{\partial f_{\theta}(\mathbf{p}^{(i)}(\mathbf{z}_0), \mathbf{z})}{\partial \mathbf{z}} + \boxed{\frac{\partial f_{\theta}(\mathbf{p}^{(i)}(\mathbf{z}), \mathbf{z}_0)}{\partial \mathbf{p}^{(i)}(\mathbf{z})} \frac{\partial \mathbf{p}^{(i)}(\mathbf{z}_0)}{\partial \mathbf{z}}} \right) \end{aligned}$$

This term is omitted as it empirically has less impact on the optimization process.

DIST Backward – Differentiable Silhouette



Sphere Tracing



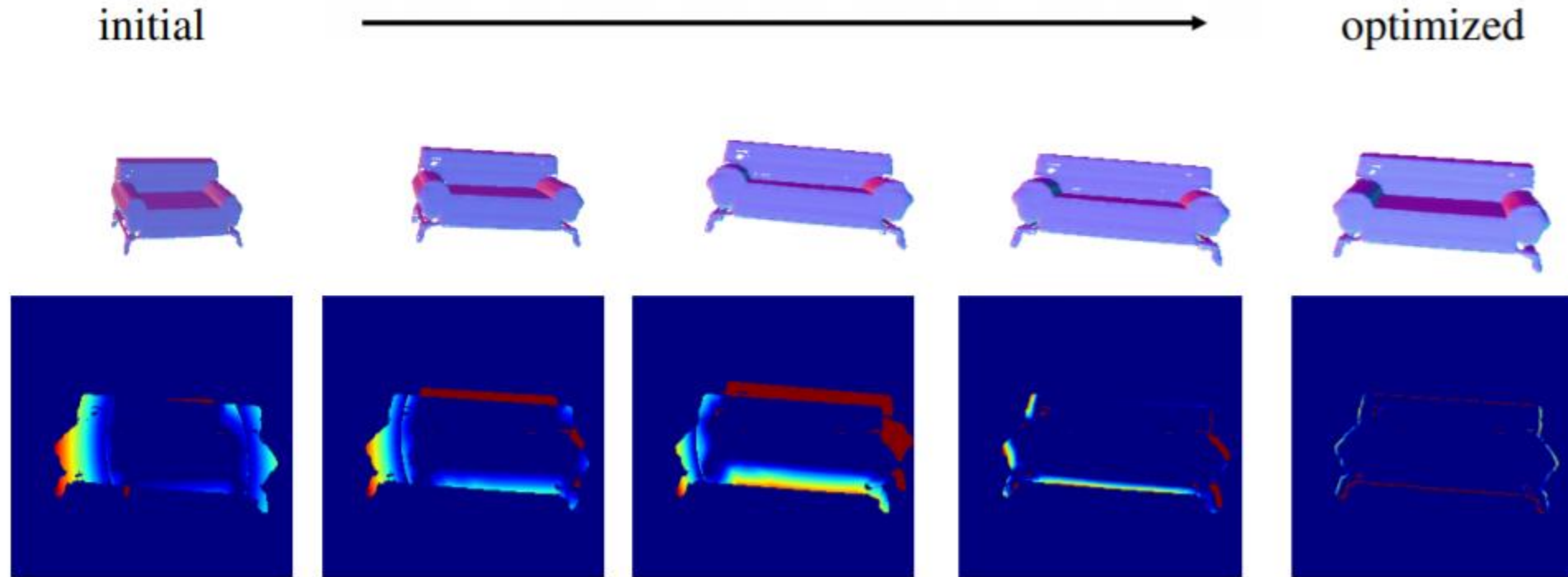
$\min(\text{abs}(\text{SDF}))$



$\min(\text{abs}(\text{SDF})) < \epsilon$

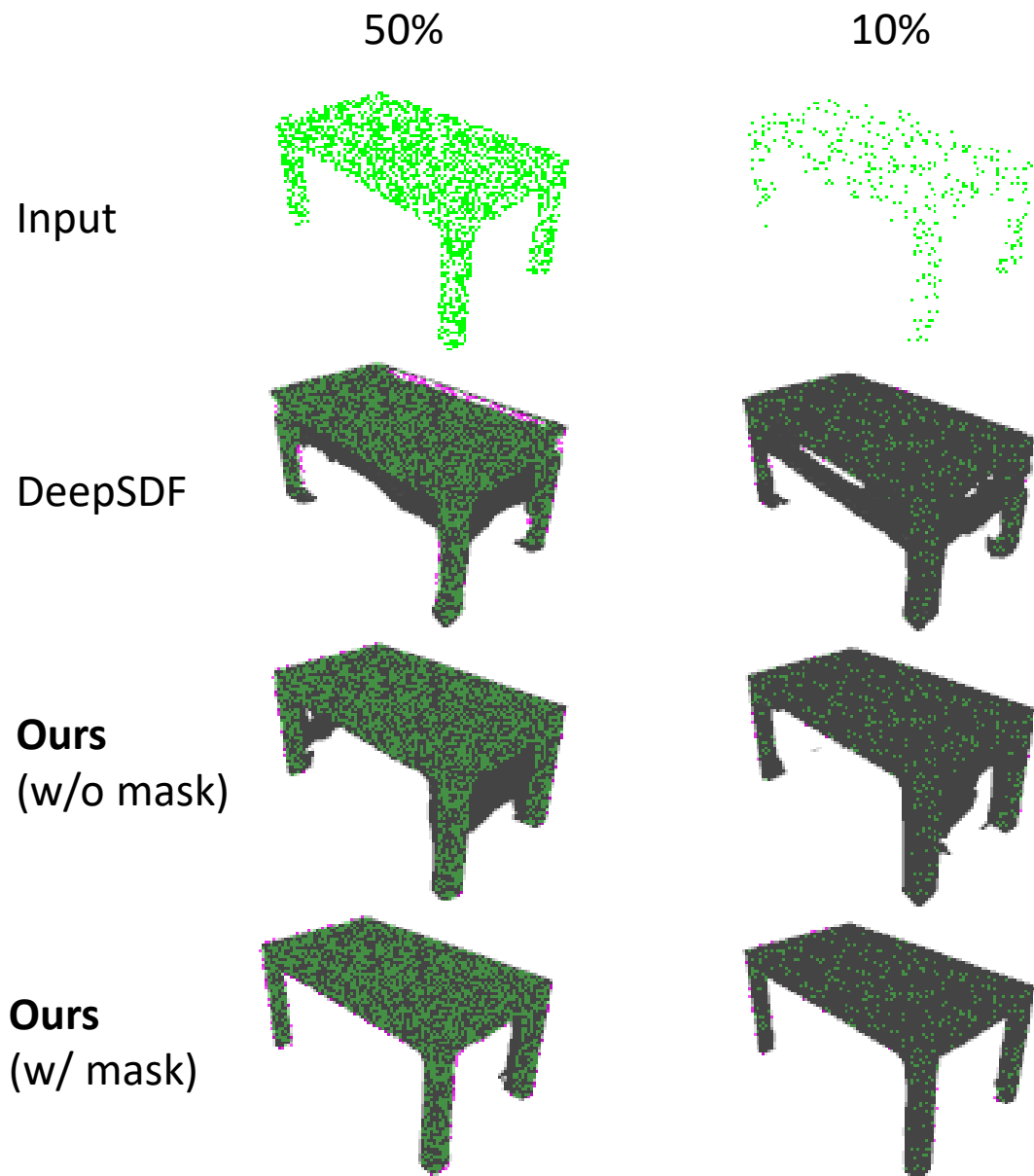
We make use of the nice property of signed distance function to optimize the nearest surface geometry.

Optimization over Camera Parameters



Given a fixed shape, our differentiable renderer can successfully backpropagate gradients to the camera parameters with respect to 2D observations.

Results - Reconstruction from Sparse Depth Images



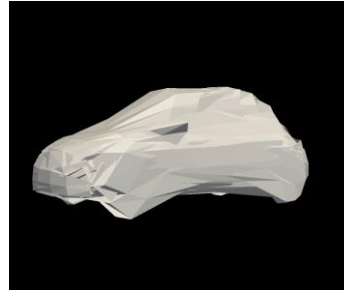
	dense	50%	10%	100pts	50pts	20pts
sofa						
DeepSDF	5.37	5.56	5.50	5.93	6.03	7.63
Ours	4.12	5.75	5.49	5.72	5.57	6.95
Ours (mask)	4.12	3.98	4.31	3.98	4.30	4.94
plane						
DeepSDF	3.71	3.73	4.29	4.44	4.40	5.39
Ours	2.18	4.08	4.81	4.44	4.51	5.30
Ours (mask)	2.18	2.08	2.62	2.26	2.55	3.60
table						
DeepSDF	12.93	12.78	11.67	12.87	13.76	15.77
Ours	5.37	12.05	11.42	11.70	13.76	15.83
Ours (mask)	5.37	5.15	5.16	5.26	6.33	7.62

Results - Reconstruction from Video Sequences

Synthetic #1



Lin *et al.*



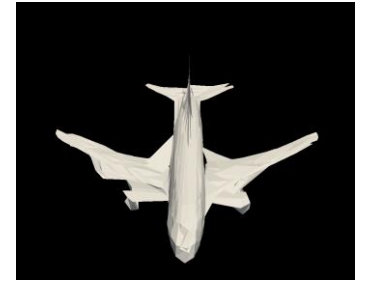
Ours



Synthetic #2



Lin *et al.*



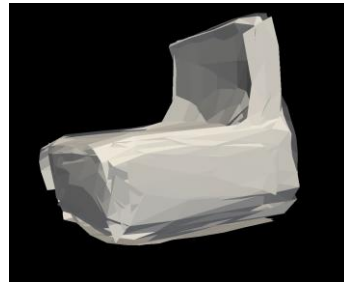
Ours



Real-world #1



Lin *et al.*



Ours



Real-world #2



Lin *et al.*

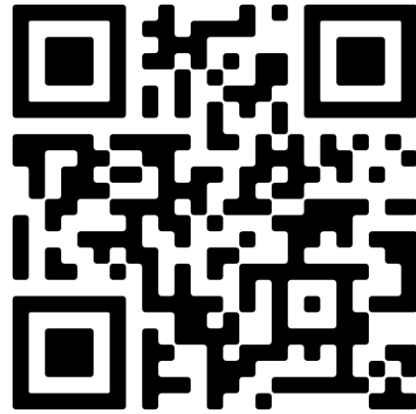


Ours





Code and Demo are available here



<http://b1ueber2y.me/projects/DIST-Renderer/>



Microsoft

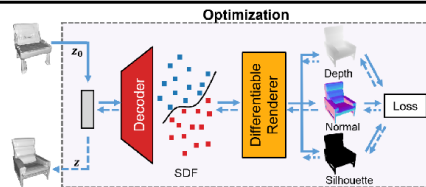
Motivation & Pipeline

[Project Page](#)



The recently proposed deep implicit signed distance function [1] is effective on representing 3D shapes. Advantages: infinite resolution, lightweight, etc.

⊗ **No differentiable renderer exists** for this representation, making it infeasible to be optimized over 2D observations.



Feedforward Rendering

Method	#query	time
Naive sphere tracing + practical grad.	N/A	N/A
+ parallel	6.06M	3.39s
+ dynamic	1.99M	1.23s
+ aggressive	1.43M	1.08s
+ coarse-to-fine	887K	0.99s

Image size = 512 x 512
marching step = 50

Reconstruction from Video Sequences

Results on synthetic data

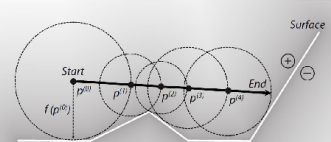
Input video	PMO [2] random init.	PMO [2]	Ours random init.

Results on real data

Input video	PMO [2]	Ours

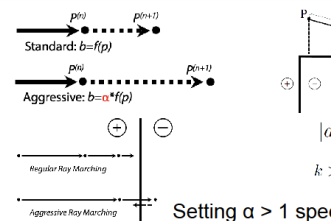
DIST - Feedforward

Naive Sphere Tracing

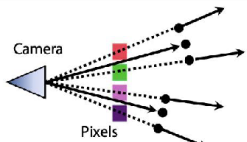


For each camera ray, march at each step with the queried SDF value until convergence.

Aggressive Marching



Coarse-to-fine Strategy



We start the sphere tracing over an image with 1/4 resolution, and split each ray twice during the marching process, which saves computation at the early stage.

Convergence Criteria

$$\frac{S/R \cdot \cos(\theta)}{f/\cos(\theta)} = 2\epsilon \quad \epsilon = \frac{d_{min} \cdot S \cdot \cos^2(\theta)}{2 \cdot f \cdot R}$$

Take focal length $f = 60\text{mm}$, sensor size $S = 32\text{mm}$, resolution $R = 512$, minimum depth $d_{min} = 10\text{cm}$. We can get $\epsilon = 5 \times 10^{-2}$, $\epsilon = 5 \times 10^{-4}$, $\epsilon = 5 \times 10^{-6}$, $\epsilon = 5 \times 10^{-8}$.

A large threshold causes dilation, while a small threshold leads to erosion.

Optimization over Camera Parameters

Reconstruction from Sparse Depths

	dense	50%	10%	100pts	50pts	20pts
sofa						
DeepSDF	5.37	5.56	5.50	5.93	6.03	7.63
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Ours (mask)	5.37	5.15	5.16	5.26	6.33	7.62

Density 50% 10%

Input

DeepSDF [1]

Ours (w/o mask)

Ours (w/ mask)

DIST - Backward

Memory issue caused by Recursive Gradients

$$d = \alpha \sum_{n=0}^{N-1} f(p^{(n)}) + (1-\alpha)f(p^{(N-1)}) = d' + \epsilon$$

$$\frac{\partial d'}{\partial z_0} = \alpha \sum_{i=0}^{N-1} \frac{\partial f_{\theta}(p^{(i)}(z_0), z_0)}{\partial z_0}$$

$$= \alpha \sum_{i=0}^{N-1} \left(\frac{\partial f_{\theta}(p^{(i)}(z_0), z_0)}{\partial z_0} + \frac{\partial f_{\theta}(p^{(i)}(z_0), z_0)}{\partial p^{(i)}(z_0)} \frac{\partial p^{(i)}(z_0)}{\partial z_0} \right)$$

Each query location depends on the previous one, incurring recursive gradients. We make approximations over sphere tracing by omitting high-order gradients.

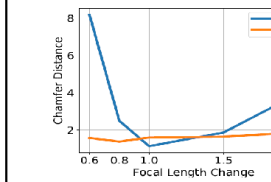
Differentiable Silhouette

Min query

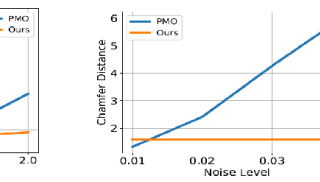
Ray (foreground pixel)

We make use of the nice property of signed distance function to optimize the nearest surface geometry.

Generalization across different focal lengths



Generalization across different noise levels



References:
[1] Park et al. "DeepSDF: Learning Continuous Signed Distance Functions for Shape Representation", CVPR '19.
[2] Lin et al. "Photometric Mesh Optimization for Video-Aligned 3D Object Reconstruction", CVPR '19.