

# Appearance Manifold with Embedded Covariance Matrix for Robust 3D Object Recognition

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## Abstract

*We propose use of an appearance manifold with embedded covariance matrix as a technique for recognizing 3D objects from images that are influenced by geometric and quality-degraded effects. Our strategy covers the construction of this appearance manifold by giving consideration to pose changes. In the proposed method, the correspondence of each learning pose is not based on the eigenpoint but directly from the covariance matrix. Thus, we eliminate the dependency on eigenpoint-to-eigenpoint correspondence, which is the main cause of misclassification due to the phenomenon of the eigenpoint's shifting position. Experimental results show that our approach achieves higher recognition accuracies than using a simple appearance manifold. Consequently, it can provide a more efficient way of developing a robust 3D object recognition system.*

## 1. Introduction

Visual learning of 3D objects has been one of the most challenging problems in vision systems. 3D objects are visually complex and highly dependent on environmental conditions. Therefore, it is necessary to figure an object in such a way that can fully represent the characteristics of the object. In general, capturing the characteristics of a 3D object can be done by using several combinations of 2D images or by constructing a high-cost 3D shape model. Here, we focus on an appearance-based approach that uses combinations of images to capture the appearance variability of a 3D object.

Over the past decade, there has been a growing trend to use appearance-based approaches for 3D object recognition. Appearance-based approaches often start with the concept of Principal Component Analysis (PCA). This concept enables a method to efficiently present a series of sample images in a low-dimensional feature description, called the eigenspace. For years, the eigenspace has provided an efficient and easy way to solve many recognition problems. Some of the earlier works in this domain include the eigenpictures of Kirby and Sirovich [1] for characterizing the human face, the eigenfaces of Turk and Pentland [2], Moghaddam's [3] proposal of probabilistic PCA, and the Parametric Eigenspace of Murase and Nayar [4].

Just as the appearance of an object highly depends on the image conditions, the image's position in eigenspace relies on the object's appearance. For handling changes caused by pose and illumination variability, Murase and Nayar's Parametric Eigenspace method could give more satisfactory results than the traditional eigenspace method. Unfortunately, this method tends to fail when there are significant variations in scale, orientation, noise and degradation in the input image. This failure is mainly caused by the eigenpoint's shifting position in the input image, which is influenced by various degradation effects, from the learning images.

To overcome this limitation, we propose the construction of an appearance manifold with embedded covariance matrix. The basic idea is to eliminate the eigenpoint-to-eigenpoint correspondences of each learning pose class and then to construct the correspondences from the covariance matrices directly. By using this model, the appearance manifold helps the system to analyze image conditions such as pose changes, while the embedded view-dependent covariance matrix defines the scope of an eigenpoint's shifting positions in eigenspace.

Our paper is organized as follows: we describe the process of constructing the appearance manifold with an embedded covariance matrix in Section 2. Next, Section 3 covers the development of a 3D object recognition system. Finally, our conclusions are presented in Section 4.

## 2. Appearance Manifold with Embedded Covariance Matrix in Eigenspace

This section describes the detailed process of constructing the appearance manifold with an embedded covariance matrix, which consists of the development of the eigenspace using PCA, the construction of various techniques of appearance manifold in eigenspace, and the recognition of input images using the Mahalanobis distance measurement.

### 2.1. Eigenspace representation

Appearance-based approaches usually deal with a set of learning images in various poses. These images are represented in a very-high-dimensional space, and thus they cannot be applied directly due to efficiency reasons. Here, PCA provides a technique to efficiently represent a collection of images by reducing their dimensionality.

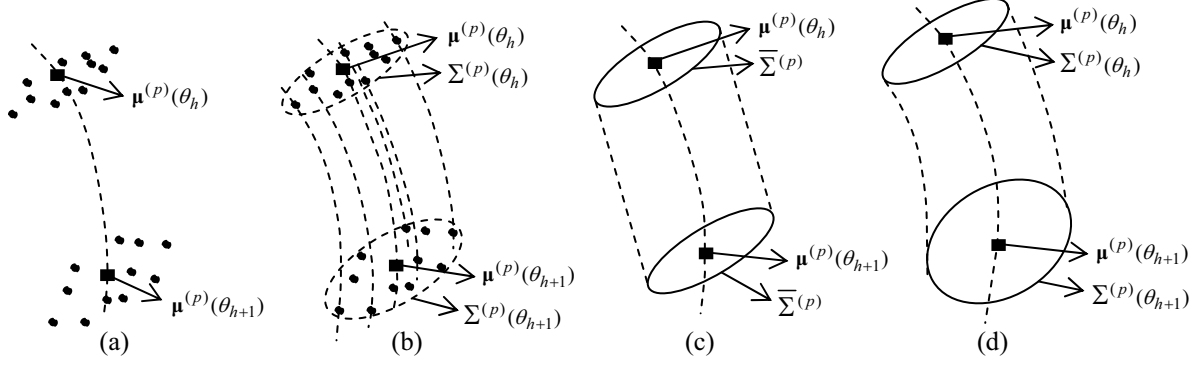


Figure 1. Construction models of appearance manifold (a) Parametric Eigenspace (PE), (b) points interpolation (AMPI), (c) constant covariance matrix (AMCC), (d) view-dependent covariance matrix (AMVC).

Generally, the captured images should be normalized in brightness and scaled in order to be invariant to image magnification and illumination intensity. These normalized images can be written as a vector  $\mathbf{x}$  by reading the number of pixels ( $N$ ) in an image:

$$\mathbf{x} = [x_1, x_2, \dots, x_N]^T. \quad (1)$$

Let  $M$  be the number of images in a learning set. By subtracting the average image  $c$  of all images, the learning set  $\mathbf{Y}$  will be obtained:

$$\mathbf{Y} = [\mathbf{x}_1 - c, \mathbf{x}_2 - c, \dots, \mathbf{x}_M - c]. \quad (2)$$

Next, define the auto-correlation matrix by

$$\mathbf{Q} = \mathbf{Y}\mathbf{Y}^T \quad (3)$$

and determine the eigenvalues  $\lambda_i$  with their corresponding eigenvectors  $\mathbf{e}_i$  by solving the following eigenvector decomposition problem:

$$\lambda_i \mathbf{e}_i = \mathbf{Q}\mathbf{e}_i. \quad (4)$$

To reduce the dimension, simply ignore small eigenvalues and use only  $k$  corresponding eigenvectors by using a  $T$  threshold value:

$$\frac{\sum_{i=1}^k \lambda_i}{\sum_{i=1}^N \lambda_i} \geq T, \quad (5)$$

where  $k \ll N$ .

The first  $k$  eigenvectors will be used to project  $S$  learning samples of  $P$  objects with  $H$  poses. Project  $\mathbf{x}_s^{(p)}(\theta_h)$  as the  $s$  sample image of object  $p$  with horizontal viewpoint  $\theta_h$  into the eigenspace:

$$\mathbf{g}_s^{(p)}(\theta_h) = [\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_k]^T (\mathbf{x}_s^{(p)}(\theta_h) - \mathbf{c}). \quad (6)$$

By projecting all of the learning samples into the eigenspace, learning features are represented efficiently as a set of discrete points in a low-dimensional space.

## 2.2. Construction of Appearance Manifold with Embedded Covariance Matrix

In this section, we present various techniques to construct the appearance manifold. Figure 1 shows the four types of construction models for the appearance manifold:

the simple manifold used in the Parametric Eigenspace (PE) method, the appearance manifold using the point interpolation (AMPI) method, the appearance manifold with constant covariance matrix (AMCC) method, and the appearance manifold with view-dependent covariance matrix (AMVC) method.

Although each method uses a different type of construction technique for the appearance manifold, in general they use the same basic steps. First, after transforming learning images to the eigenspace, calculate the mean vector  $\boldsymbol{\mu}^{(p)}(\theta_h)$  and the covariance matrix  $\Sigma^{(p)}(\theta_h)$  for each object  $p$  for horizontal viewpoint  $\theta_h$ .

The mean vector is typically estimated using

$$\boldsymbol{\mu}^{(p)}(\theta_h) = \frac{1}{S} \sum_{s=1}^S \mathbf{g}_s^{(p)}(\theta_h), \quad (7)$$

where  $s$  is the number of learning samples from each class, and  $\mathbf{g}_s^{(p)}(\theta_h)$  is the image sample  $s$  from class viewpoint  $\theta_h$  and object  $p$ . On the other hand, the covariance matrix is typically estimated by

$$\Sigma^{(p)}(\theta_h) = \frac{1}{S-1} \sum_{s=1}^S (\mathbf{g}_s^{(p)}(\theta_h) - \boldsymbol{\mu}^{(p)}(\theta_h))(\mathbf{g}_s^{(p)}(\theta_h) - \boldsymbol{\mu}^{(p)}(\theta_h))^T \quad (8)$$

Next, create  $\tilde{\boldsymbol{\mu}}^{(p)}(\theta)$  as a continuous manifold of the mean vector and  $\tilde{\Sigma}^{(p)}(\theta)$  for the covariance matrix. The processes of creating manifolds  $\tilde{\boldsymbol{\mu}}^{(p)}(\theta)$  and  $\tilde{\Sigma}^{(p)}(\theta)$  might be different from one method to another.

The PE method uses a simple manifold obtained from the interpolation of the mean vector of the eigenpoints in two consecutive poses. However, for the covariance matrices, the PE method simply applies the values of the identity matrix. The construction model of the appearance manifold in the PE method is depicted in Fig. 1(a).

Next, Fig. 1(b) shows the appearance manifold with the point interpolation (AMPI) method. It obtains the appearance manifold by interpolating every eigenpoint in each pose class to other eigenpoints in the consecutive pose classes that have similar characteristics, such as degradation effects. After creating those manifolds for each eigenpoint, generate the new eigenpoints for every in-between class pose, and then calculate their mean vectors and covariance matrices for every pose class.

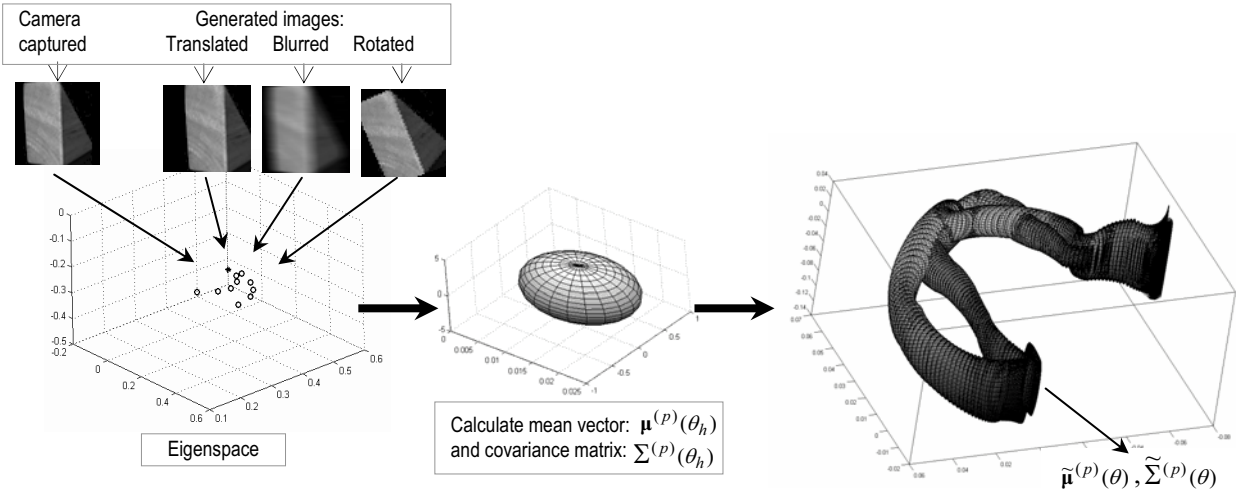


Figure 2. Scheme of AMVC method for 3D object recognition.

Figure 1(c) shows the tube appearance manifold with a constant covariance matrix (AMCC). After calculating the mean vectors and covariance matrix values for each learning pose in (7) and (8), apply an interpolation method for the mean vector of two consecutive learning poses to obtain the manifold of mean vector  $\tilde{\mu}^{(p)}(\theta)$ . On the other hand, the manifold of covariance matrix  $\tilde{\Sigma}^{(p)}(\theta)$  contains the same value for every viewpoint  $\theta_h$  by applying the average covariance matrix

$$\bar{\Sigma}^{(p)} = \frac{1}{H} \sum_{h=1}^H \Sigma^{(p)}(\theta_h) \quad (9)$$

with  $H$  number of viewpoint classes for each object.

Next, Fig. 1(d) shows another type of appearance manifold method, called the appearance manifold with view-dependent covariance matrix (AMVC). This method uses the appearance manifold embedded with view-dependent covariance matrix that changes along with the function of viewpoints. The manifold  $\tilde{\mu}^{(p)}(\theta)$  could be obtained by applying an interpolation method between two consecutive mean vectors  $\mu^{(p)}(\theta_h)$  and  $\mu^{(p)}(\theta_{h+1})$ . Then, the manifold  $\tilde{\Sigma}^{(p)}(\theta)$  could be obtained by interpolating the covariance matrices  $\Sigma^{(p)}(\theta_h)$  and  $\Sigma^{(p)}(\theta_{h+1})$ , respectively. Here, since we use only the horizontal pose parameter  $\theta_h$ , the surface of the appearance manifold in the AMVC method will be a tube. Figure 2 shows the scheme of the AMVC method with its tube surface.

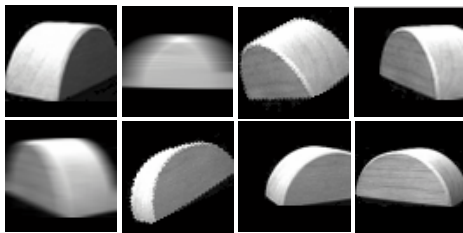


Figure 3. Sample images of 3D objects with various translation, rotation, and motion blur effects.

## 2.3. Classification using distance measurement

In order to recognize an input image  $\mathbf{u}$ , first project it into the eigenspace

$$\mathbf{z} = [\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_k]^T (\mathbf{u} - \mathbf{c}) \quad (10)$$

and then calculate distance  $d$  between the projected-image in the eigenspace  $\mathbf{z}$  and the manifold.

Since we have the parameter of mean vector and covariance matrix in the appearance manifold, the sufficient distance measurement to classify the input image is the Mahalanobis distance, defined in this formula:

$$d^{(p)}(\mathbf{z}) = \min_{\theta} (\mathbf{z} - \tilde{\mu}^{(p)}(\theta))^T (\tilde{\Sigma}^{(p)}(\theta))^{-1} (\mathbf{z} - \tilde{\mu}^{(p)}(\theta)) \quad (11)$$

The Mahalanobis metric provides a sufficient way to classify images based on their related characteristics and likelihood in each pose class.

## 3. Application in 3D Object Recognition

To evaluate the performance of our proposed methods, explained in section 2.2, we developed an application in 3D object recognition. The developed system was used to recognize seven objects with various horizontal pose positions and influenced by geometric and quality-degradation effects, such as translation, rotation, and motion blur. Samples of 3D objects with various translation, rotation, and motion blur effects are shown in Fig. 3.

In the learning stage, the images were first normalized into 32 x 32-pixel grayscale images. Then, the system was learned with a total of 6,552 images. Each object consists of 36 poses with 10-degree intervals of horizontal positions ( $0^\circ, 10^\circ, 20^\circ \dots 350^\circ$ ), and each pose consists of 26 learning images with an original camera-captured image and 25 generated images with various degradation effects. Those generated images were obtained by composing artificial noises with the MATLAB program, such as left and right translations (3, 6, 9, 12, 15 pixels), clockwise and counter-clockwise rotations ( $5^\circ, 10^\circ, 15^\circ, 20^\circ, 25^\circ$ ), and motion blur (5%, 10%, 15%, 20%, 25%).

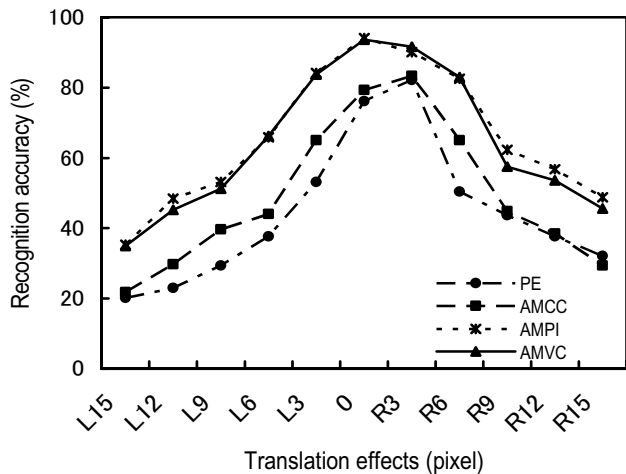


Figure 4. Recognition accuracies of images with left (L) and right (R) translation effects.

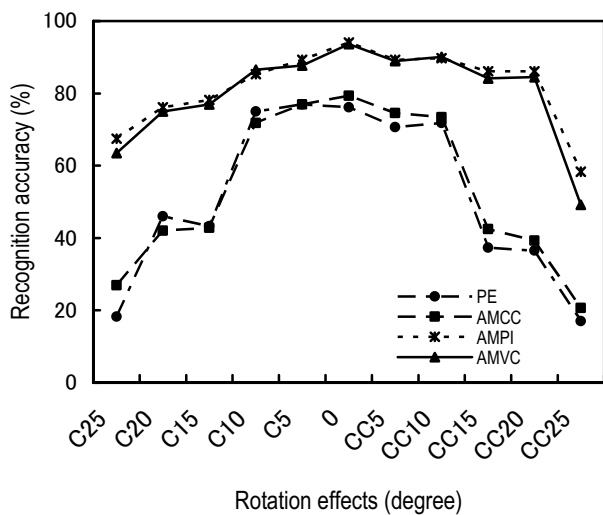


Figure 5. Recognition accuracies of images with clockwise (C) and counter-clockwise (CC) rotation effects.

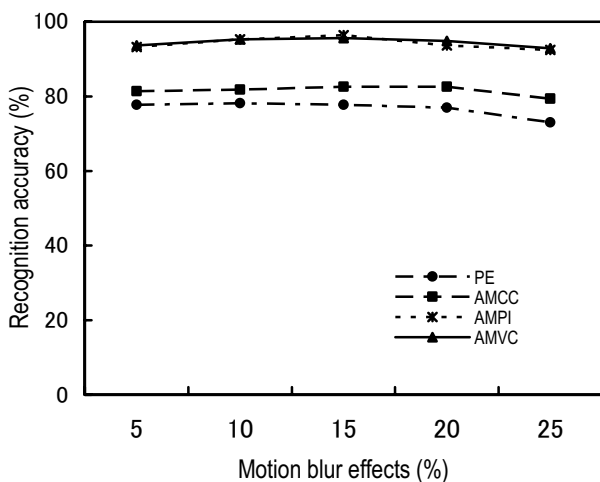


Figure 6. Recognition accuracies of images with motion blur effects.

Next, those images were projected into the eigenspace, and the appearance manifolds were created based on each construction method, as explained in section 2.2. We used spline interpolation technique to interpolate the mean vectors and linear interpolation technique to interpolate the covariance matrices.

Finally, we tested the system with input images that were different from the learning images ( $5^\circ, 15^\circ, 25^\circ \dots 355^\circ$ ) in horizontal poses and influenced by various types of degradation effects. For classification, we applied the Mahalanobis distance, as explained in section 2.3.

Figures 4, 5, and 6 show a series of the recognition accuracies of four methods in recognizing images influenced with translation effects, rotation effects, and motion blur effects, respectively. All figures indicate that the AMPI method and AMVC method, with their view-dependent covariance matrices, always achieved higher recognition accuracies than the PE method or AMCC method. For recognizing non-degraded images, the AMPI method achieved 94.05%, while the AMVC method achieved 93.65% recognition accuracy. When recognizing images with 3 pixels of right translation effects, the AMPI method achieved 90.08%, while the AMVC method achieved 91.67%. Furthermore, the AMPI method achieved 89.68% while the AMVC method achieved 90.08% when recognizing images with 10-degree counter-clockwise rotation effects.

#### 4. Conclusion and Future Works

In this paper, we presented the use of an appearance manifold with an embedded covariance matrix as a technique to recognize 3D objects from images that are influenced by geometric and quality-degraded effects. Our proposed appearance manifold with view-dependent covariance matrix method could outperform the accuracy of the simple appearance manifold method. Moreover, performing direct covariance matrix interpolation for approximation in the AMVC method by some means worked effectively and efficiently for a relatively small interval of learning pose.

Our future works include recognizing 3D objects from images that are influenced by other types of degradation effects, as well as developing a recognition system that uses fewer learning image samples by implementing a larger interval of viewpoint orientations.

#### References

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