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Natural Surface Characterization by Multifractals

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ABSTRACT

This paper deals with multifractal theory applied to image analysis. Multifractals show many interesting peculiarities in the characterization of textured surfaces. In the paper, a method for the computation of multifractal parameters is proposed, together with some application results. Moreover, results are shown that depict the exploitation of such features in a data-fusion environment for SAR image analysis and recognition.

INTRODUCTION

In the field of image processing and recognition, texture is a fundamental but difficult to be measured feature. Many algorithms have been proposed in the literature, based on either statistical or syntactical approaches.

In recent years new prospects have been offered by the development of the fractal geometry, a new branch of mathematics that deals with shapes and phenomena close to the natural ones [1]. In particular, it is a powerful tool with which interesting results have been obtained in the characterization of natural surfaces. Such surfaces are well characterized by their fractal dimensions, which can be deduced from the degree of surface roughness [2]. For example, by using fractals one can descriminate among different soil conditions in an image acquired by a Synthetic Aperture Radar (SAR) system, nevertheless the signal is heavily corrupted by speckle noise. It has been proved that the fractal disparity between two different surfaces is not changed by the presence of speckle [3].

However, even if in analysing remote-sensed images of the Earth the roughness parameter is very important, it is not exaustive.

Other characteristics do not appear, such as arrangement, spatial distribution of grey levels and so on. There exist many types of textures that, even though characterized by the same fractal dimension, are in fact very different.

To avoid such a drawback, multifractal theory has been developed. In the following, this new kind of approach to texture characterization is described more in depth, together with some textural analysis results.

Such results (i.e. the fractal characterization of each pixel of the scene) are used as a *virtual* sensor acquiring the scene (it is not a physical sensor, of course, but a numerical transformation).

Other kinds of virtual data can be collected, and then used to recognise the scene by using a data-fusion approach.

In our case, we have used the fractal sensor and the physical one (i.e. the original SAR image) in order to extract a region map of the scene by following a hybrid clustering region growing approach.

The segmentation process was notably improved by the use of multifractal information, as one can notice in the results section.

INADEQUACY OF SINGLE FRACTAL DIMENSION

The single fractal dimension is not sufficient to fully characterize a line or a surface. Concepts such as roughness or spatial frequencies, can be associeted with fractal dimension, but it cannot, however, ollow one to evaluate the organization and distribution of pixels.

Fig.1 shows two surfaces, that look clearly different to the human eye, but are characterized by the same fractal dimension. One is a Takagi fractal surface [4], the other is a fractal surface generated by the Fractional Brownian Motion (FBM) algorithm [5]. The Takagi fractal surface is very regular and its texture is even: it could be an artificial object; the surface generated by the FBM algorithm could be a natural surface. As they have the same fractal dimension, we cannot differentiate them by using the single fractal dimension.

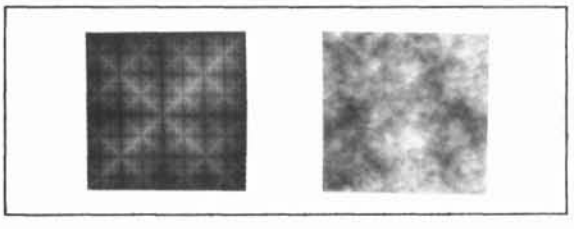

Fig.1: Takagi and FBM surfaces

Another problem of single fractal dimension is the impossibility of distinguishing between subsets with different fractal dimensions. For example: let us consider a fractal curve generated by the union of two curves with

different fractal dimension(e.g. two Koch's curves).
The fractal dimension of this set is:

$$D = - \lim_{\delta \to 0} \frac{\log\left(N_1(\delta) + N_2(\delta) \right)}{\log \delta}$$

Knowing that $N_1(\delta) = \delta^{-D1}$ and $N_2(\delta) = \delta^{-D2}$, it results:

$$D = - \lim_{\delta \to 0} \frac{\log(\delta^{-D1} + \delta^{-D2})}{\log \delta} = \max \left\{ D_1 , D_2 \right\}$$

So the fractal dimension of the curve equals the greater of the fractal dimensions of the two sets.
So, we can try to examine not only the whole fractal set but also its subsets; using a mathematical terminology, this means to estimate the fractal dimensions of the subsets with homogeneous features. To this end, multifractal theory will be used.

MULTIFRACTALS

The most widely used method for fractal analysis of sets is the *box-counting* [6] method. According to this method, an observed set is partitioned into cubes of side δ, and one has to count the number $N(\delta)$ of cubes that contain at least one point of the set. In this way, however, one loses the information about the distribution of the set points. To overcome this drawback a solution is to take into account the number of points contained in each box.

If $\mu_i = \Re_i / \Re$, where $\Re$ is the total number of points and $\Re_i$ the number of points inside the i-th box, the set $M=\{\mu_i, i=0..N\}$ (where N is the number of boxes necessary to cover the set) contains the whole amount of information about the set-points distribution.

The value $D = - \lim_{\delta \to 0} \frac{\log N}{\log \delta}$
is the box-dimension of the observed set.

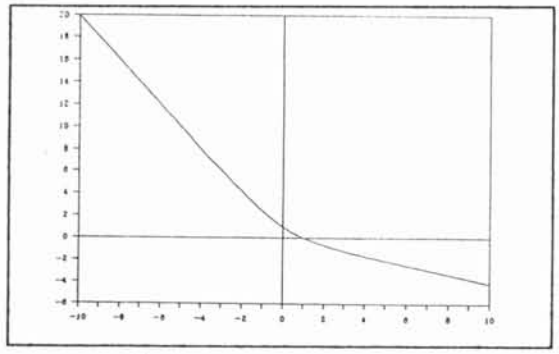

Fig.2: Typical $\tau(q)$ curve

The following formula

$$M_d(q,\delta) = \sum_{i=0}^{N(\delta)} \mu_i^q \delta^d = N(q,\delta)\delta^d$$

is a kind of measure that takes into account the point distribution by raising the masses μ_i to the q power. Low values of μ_i prevail when $q \to -\infty$, whereas high values prevail when $q \to +\infty$.

The parameter $\tau(q)$ represents the value for which the limit $\delta \to 0$ of $M_d(q,\delta)$ is finite and not equal to zero:

$$\tau(q) = - \lim_{\delta \to 0} \frac{\log(N(q,\delta))}{\log \delta}$$

THE SPECTRUM OF FRACTAL DIMENSIONS D(q)

The function D(q), introduced by Grassberg, Hentschel and Procaccia [7][8][9], can be defined in terms of $\tau(q)$ but has a peculiar characteristic: D(q) is constant for sets of constant density in the E-dimensional space, and it equals the fractal dimension of the set.

$$D(q) = \begin{cases} \dfrac{\tau(q)}{1 - q} & \text{if } q \neq 1 \\ \lim\limits_{\delta \to 0} \dfrac{\sum_i \mu_i \log \mu_i}{\log \delta} & \text{if } q = 1 \end{cases}$$

It can be demonstrated that this function decreases and that

$$\lim_{q \to +\infty} D(q) = \lim_{q \to +\infty} \frac{\log \mu_{min}}{\log \delta} = \alpha_{min}$$

$$\lim_{q \to -\infty} D(q) = \lim_{q \to -\infty} \frac{\log \mu_{max}}{\log \delta} = \alpha_{max}$$

It should be pointed out that both $\tau(0)$ and D(0) equal the fractal dimension of the whole set.

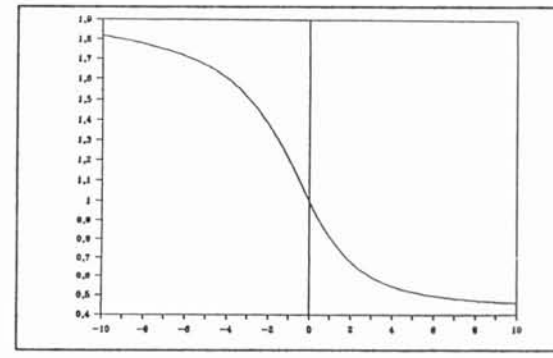

Fig.3: Typical D(q) curve

Using this function, one can overcome the drawback due to the use of the single fractal dimension (as mentioned in the second section). As a matter of fact, in the case of the union of two uniform fractal sets (sets that have uniform D(q) that equals the fractal dimension of the whole set) as is well-known the D (fractal dimension) of the resulting set is equal to the greater of D1 and D2 (fractal dimensions of the two subsets). However, the D(q) function is sensitive to the union (if D2>D1):

$$D(q) = \begin{cases} D_2 & \text{if } q < 1 \\ \frac{D1M1+D2M2}{M1+M2} & \text{if } q = 1 \\ D_1 & \text{if } q > 1 \end{cases}$$

D(q) COMPUTATION

Let us consider a digital image representation, that is a discrete set of points. We propose an approximate D(q) computation, as it is not possible to reach an arbitrarily small δ. The algorithm is composed of two steps:

1. The image is divided, without loss of generality, into boxes of size $\delta=2,4,8,..,n_{max}$, then we count the number $\mathcal{N}_i(\delta)$ of pixels contained in the i-th box.
2. The interpolation of multiple δ values is required to estimate the right D(q) value.

Starting from

$$D_q(\delta) = - \frac{\log(N(q,\delta)}{\log \delta}$$

if δ is sufficiently small, we obtain

$$N(q,\delta) = k(q)\cdot\delta^{-Dq}$$

and

$$D_q(\delta) = D_q - \frac{\log\ k(q)}{\log\ \delta}$$

So we can estimate D_q through a linear interpolation.

IMAGE ANALYSIS USING MULTIFRACTALS: AN ADAPTIVE APPROACH

In order to analyze texture, which is a basic feature of a region made up of connected pixels, we use a multifractal approach. Fractal dimensions are not related to pixel characteristics, such as gray levels, but are particular properties of a region. The goal of adaptive methods is to assign to each pixel the fractal dimensions of the region it belongs to.

To this end, we propose a new adaptive approach that chooses from a certain number of regions that contain the examined pixel, the one with the best uniformity in fractal dimensions. Using multifractals we have another tool for verifying the region uniformity: the D(q) curve points out the fractal evness of the examined set. The parameter H1, defined below,has proved very useful to obtain this goal.

$$H_1 = \left.\frac{\partial Dq}{\partial q}\right|_{q\to 1} = \lim_{\delta\to 0} \frac{1}{2\log\delta} \left[\sum_i \mu_i \log^2\mu_i - \left(\sum_i \mu_i \log\mu_i\right)^2\right]$$

Fig.4 shows how the H1 value is very high for the masks covering area with two different fractal dimensions. Consequently one chooses the mask(containing the pixel) with the lowest H1 value as it covers only one region.

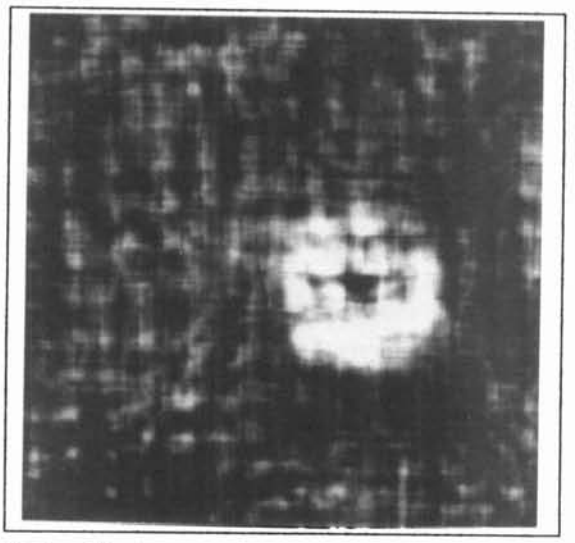

Fig.4: H1 image of an area with different fractal dimension

Thus, we can retain the details of the original image; instead the details are generally lost when using non adaptive methods as an image should be divided into boxes to compute its fractal dimensions.

RESULTS

One can sees in fig.5 an original SAR image of a ground area of Algeria where two lakes are present. The image is heavily corrupted by speckle noise. The segmentation based only on the original filtered image (edge preserving filter) has a great number of regions.

Fig.5: SAR image of a ground area of Ageria

Fig.6 and Fig.7 show the results of multifractal analysis (D(0) and D(1)) of fig.5. We use masks of size 16×16 pixels in different positions and directions. One can notice how the details are retained even if the masks have considerable dimensions, while different soil conditions are well

distinguished through their multifractal disparity

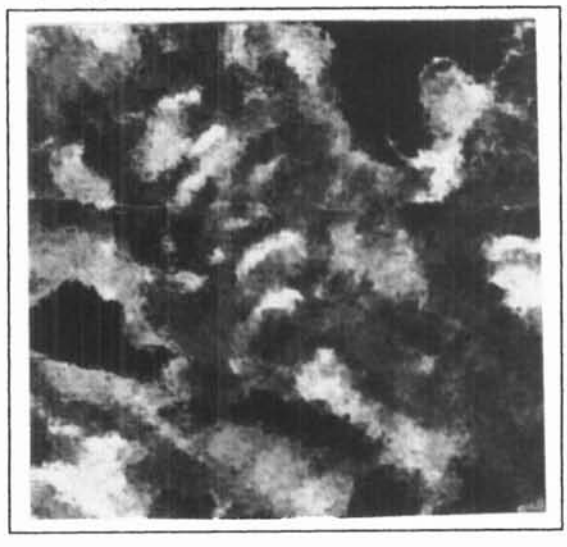

Fig.6: D(0)

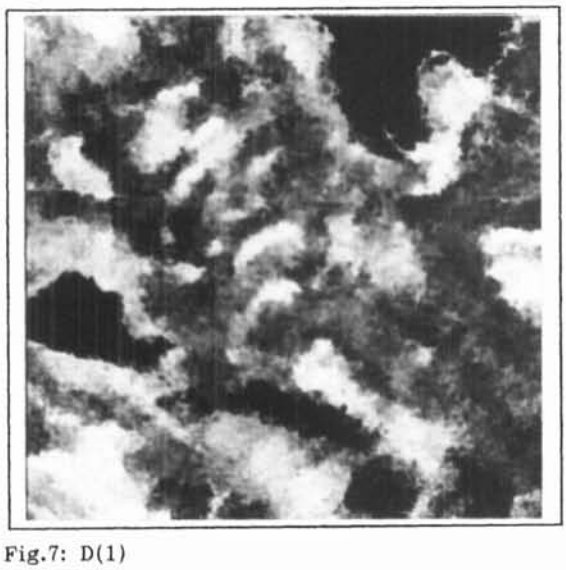

Fig.7: D(1)

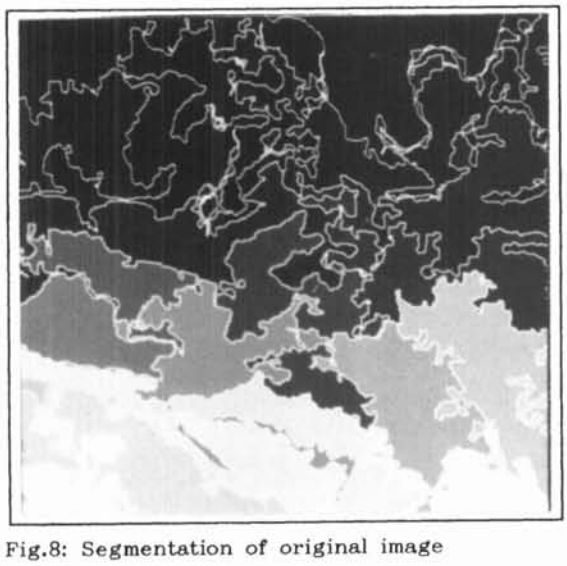

Fig.8: Segmentation of original image

In order to achieve the image segmentation we use a hybrid approach that take into account the multifractal data. As a matter of fact, the segmentation system uses, in order to compare the current pixel with its neighbour, some local parameter extracted from the imput images [10].

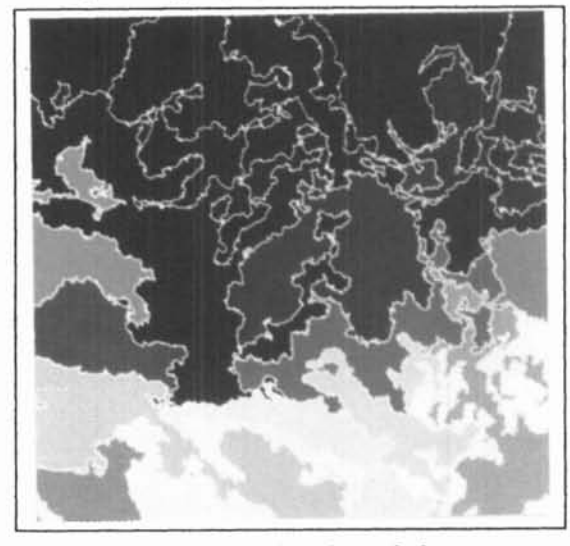

Fig.9: Segmentation using fractal data

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