

# 13—17 A Slanted Ellipse Detection by A Circle Detecting Hough Transform Using a Pair of Arcs.

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## Abstract

In this paper we propose a new method to detect a slanted ellipse using Hough transform with 5 parameters effectively. The procedure is as follows: First, a pair of arcs are detected as a part of the slanted ellipse using a circle detecting Hough transform with 3 parameters; the central point  $(x_c, y_c)$  and the slanted angle  $\theta$  can be obtained. Second, the slanted ellipse is transformed into a normal ellipse whose central point is  $(0, 0)$  and slanted angle is zero degree. Two parameters of a normal ellipse, the major axis and the minor axis ( $a$  and  $b$ ) can easily be obtained by 2 parameters Hough transform. Thus, the slanted ellipse can be detected with less memory and less data processing.

## 1 Introduction

Employing a Hough transform is an effective method to detect an object represented by geometric equations with stability[1]. In order to recognize an object by a Hough transform, a certain number of dimensional parameter space (voting space), which is equivalent to the number of object parameters, is required.

For example, a line ( $y = ax + b$ ) has 2 parameters,  $a$  and  $b$ . Therefore, a 2 dimensional parameter space is necessary. A circle consists of 3 parameters; the central point  $(x_c, y_c)$  and the radius  $r$ , thus requires a 3 dimensional parameter space. Moreover, a slanted ellipse requires a 5 dimensional parameter space since it is a 5 parameters object; the central point  $(x_c, y_c)$  (the cross point of the major axes and the minor axes), the slanted angle  $\theta$  and the major/minor axes ( $a, b$ ). That means traditional appli-

cation of a Hough transform to detect a slanted ellipse is unpractical. Here, we propose a new method to detect a slanted ellipse without complex 5 dimensional data processing [2].

## 2 Detecting Method

A traditional Hough transform to detect a slanted ellipse requires a 5 dimensional parameter space. The proposed method here enables the down-sizing of parameter space by dividing it into 3 dimensional (circle detection) array and 2 dimensional (normal ellipse detection) array data processing.

### 2.1 Estimation of the Central Point and the Slanted Angle of the ellipse

Fig. 1 shows that the central point of a slanted ellipse can be obtained as a midpoint of the line segment, which is made by connecting central points of correspondent arcs (circles). The slanted angle can subsequently be detected as the angle which is made when the line segment meets the  $x$  coordinate. A method to detect circles (arcs) which are used for approximating a slanted ellipse is stated in ref. [3] and [4].

### 2.2 Estimation of the Major/Minor Axes of the ellipse

A slanted ellipse whose central point and slanted angle are known can be transformed into a normal ellipse in the following way; move the central point of the slanted ellipse parallel to the origin, then rotate it so that the major/minor axes lap over the coordinates. To detect a normal ellipse, two parameters of the major/minor axes ( $a, b$ ) are required. Thus a 2 dimensional Hough transform is applied. The major/minor axes can also be obtained as the cross points of the coordinates and a normal ellipse.

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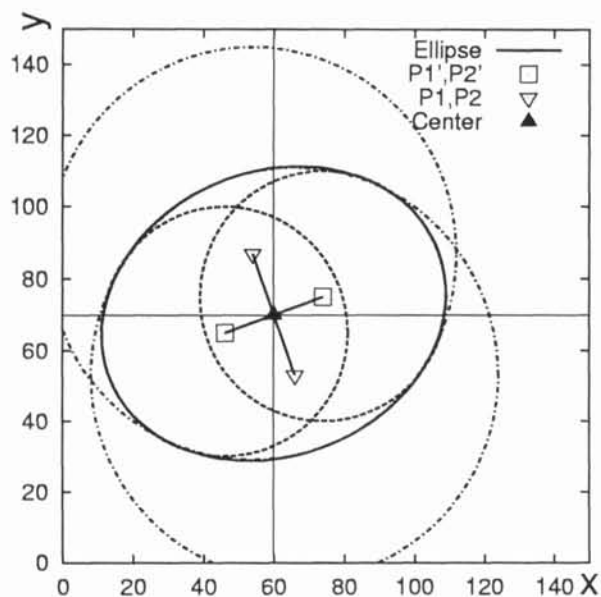


Figure 1: An example of a slanted ellipse and its circles for detection

### 3 Ellipse Detection Experiment

First, a detecting experiment of a slanted ellipse using a constructed image was made. Then detection using two kind of photo images were done.

#### 3.1 Detection Experiment using a Constructed Image

To examine our method by detecting a slanted ellipse from a constructed image, we made several rotated ellipse as the origin image.

To evaluate the detecting accuracy to the slanted angle, 10 different slanted angles were used as a major parameter.

Table 1: Constructed parameters of slanted ellipses

|                     |                              |
|---------------------|------------------------------|
| slanted angle(deg.) | 0,10,20,30,40,50,60,70,80,90 |
| central point       | (60,70)                      |
| major/minor axes    | 50, 40                       |
| image size          | 256x256                      |

The circle detecting Hough transform uses a 3 dimensional (central point  $(x_c, y_c)$ , radius  $r$ ) parameter space. The array size for the central point  $(x_c, y_c)$  was set to 255, which corresponds to the image size. For radius  $r$ , the size was set to 399, which corresponds to the length of the diagonal line of the image. We set the element of the array as 1 byte. So the total size of array became 25MB ( $256 * 256 * 400$ ).

As a result of the experiment, the parameters of the slanted ellipse (the central point, the major/minor axes) were obtained precisely. Errors of the slanted angle were within  $\pm 1.70$  degree. (We defined an error here as the difference between the

detected and actual angle.) The method is therefore, effective in detecting a slanted ellipse with sufficient accuracy.

#### 3.2 Detection Experiment using Photo Images

We made detecting experiments from 2 photo images. Fig. 2 is a photographed plate. Fig. 5 is a photographed lemon. The size of each image is 640x480 pixels. Fig. 3 and 6 are the binarized images. The size of Fig. 3 and 6 is 256x256 pixels for convenience' sake. Fig. 4 and 7 show 2 pairs of arcs detected by the first stage of Hough transform application and a slanted ellipse made by one of 2 pairs of arcs and an original image.

Table 2: Detected parameters about Fig. 4; a plate

|                            |   |
|----------------------------|---|
| circle 1                   | central point:(123,198), radius:98  |
| circle 2                   | central point:(130, 62), radius:98  |
| a detected slanted ellipse | central point:(127,130), major axis:61, minor axis:30, slanted angle:4 deg. |

Table 3: Detected parameters about Fig. 7; a lemon

|                            |   |
|----------------------------|---|
| circle 1                   | central point:(97,121), radius:62   |
| circle 2                   | central point:(95,133), radius:62   |
| a detected slanted ellipse | central point:(96,127), major axis:62, minor axis:56, slanted angle:9.46 deg. |

A slanted ellipse appeared on the outline of a plate was detected as we expected.(Fig. 4)

We aimed to detect a slanted ellipse which lap over the outline of the lemon most. In this experiment we got a satisfactory result. Since lemon does not have a precise ellipse outline, many other ellipses could be detected.

### 4 Memory Estimation for the Traditional Hough Transform

We also made an experiment by the traditional Hough transform for comparison with our method. We used the same constructed image as we used for our method. The 5 dimensional Hough transform to detect a slanted ellipse requires an array of 440GB ( $45 * 256 * 256 * 400 * 400$ ). Since it is impossible to work on a program with this huge amount of memory, we designed an experiment requiring less memory under the following restricting conditions: (a) slanted angle  $\theta$  is 0, (b) range of the central point and the major/minor axes is limited ( $x_c : 0 - 63, y_c : 0 - 90, a, b : 0 - 63$ ). Under these conditions, the amount of required memory was 23.5MB ( $64 * 91 * 64 * 64$ ).

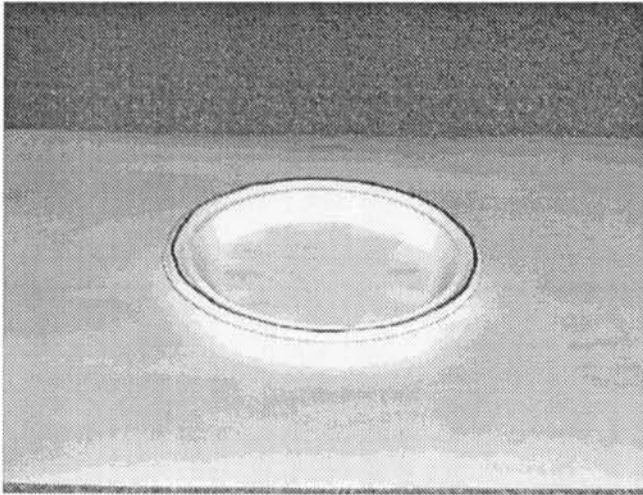


Figure 2: Original Image:plate

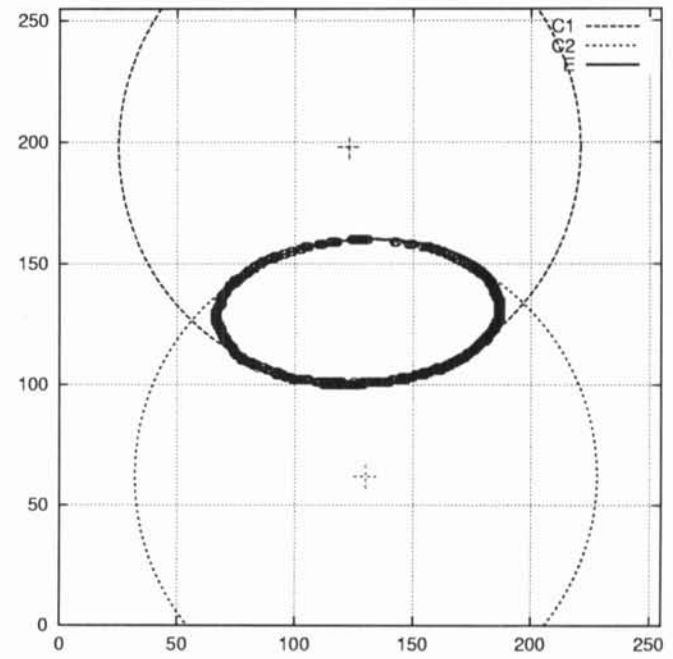


Figure 4: Detected Result:plate

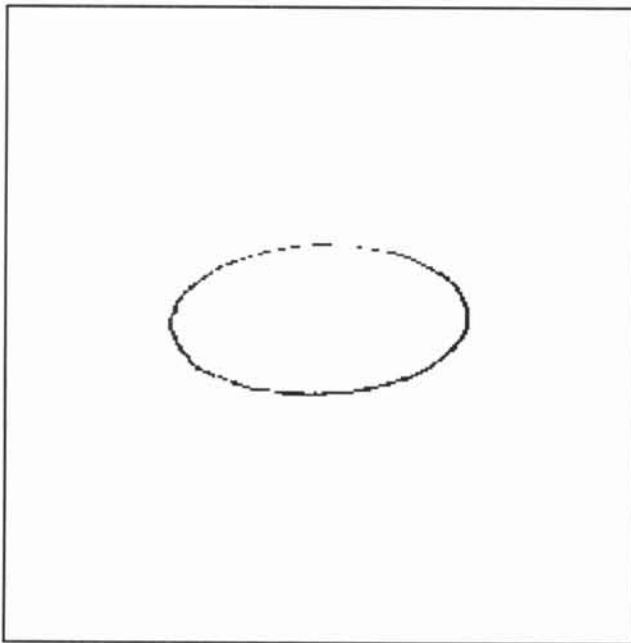


Figure 3: Binarized Image:plate

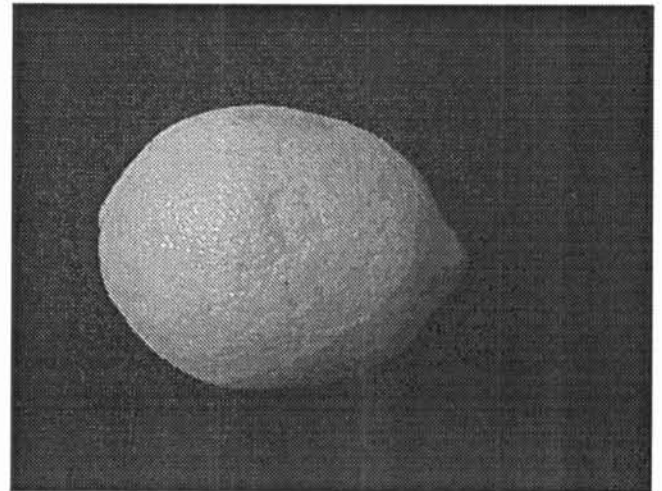


Figure 5: Original Image:lemon

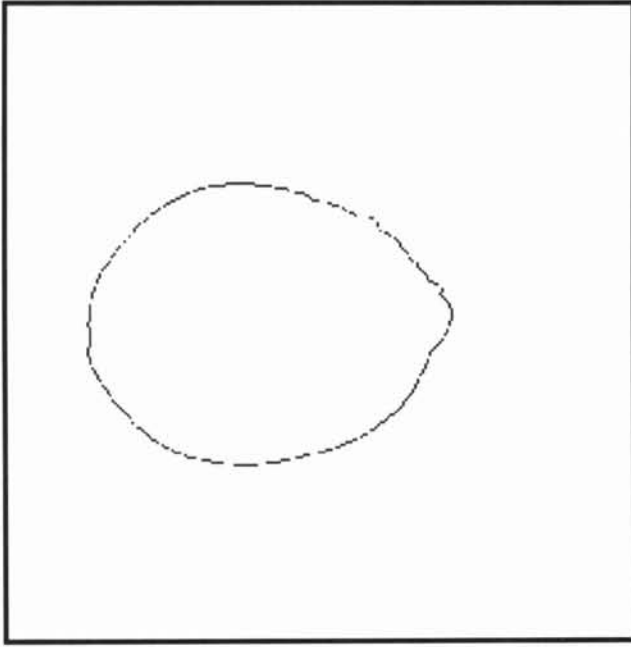


Figure 6: Binarized Image:lemon

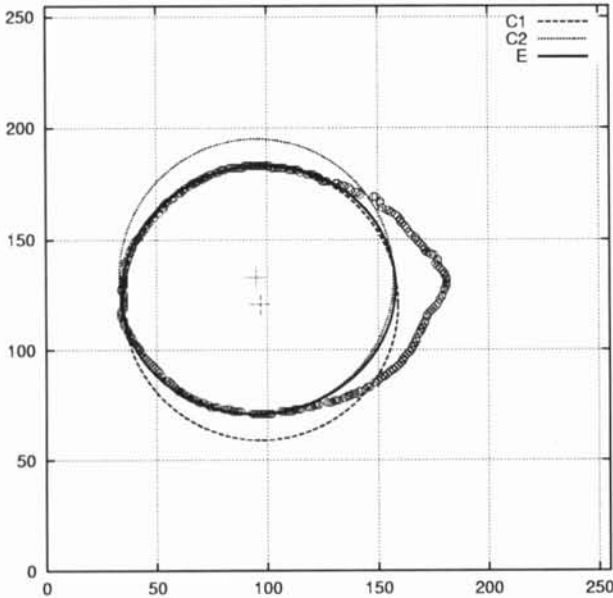


Figure 7: Detected Result:lemon

However, the system which is available under such severely restricted conditions is far from practical use. In addition, it does not suit our goal of detecting a slanted ellipse, since the conditions demand ellipses with no slanted angle. Thus we did not make a precise comparison experiment with our proposed method.

## 5 Conclusion

We proposed a new method to detect a slanted ellipse using a Hough transform without requiring a huge amount of memory. The proposed method consists of two stages of Hough transform application: First, the central point and the slanted angle are obtained by approximation with a pair of arcs. Second, the slanted ellipse is transformed into a normal ellipse to detect the major/minor axes. In our experiments, we could successfully detect a target object with sufficient accuracy and significantly reduced memory for practical use.

## References

- [1] Hough P.V.C.: "Method and Means for Recognizing Complex Patterns", U.S. Patent 3069654 (Dec. 1962).
- [2] Yasusuke ASAYAMA and Mitsuru SHIONO: "An Experiment on a Slanted Ellipse Detecting by Circle Detecting Hough Transform using of Arcs", MIRU98, pp.II-113 - II-117(1998).
- [3] Mitsuru SHIONO: "High Speed Drawing of a Slanted Ellipse Using Circle Drawing Command and Its Error Analysis", IEICE,D-II Vol.J75-D-II No.6 pp.1048-1057, (1992).
- [4] Mitsuru SHIONO: "An Experiment on Arc Detection by Hough Transform Using Random Sampling of Black Points and the Central of Gravity", IEICE,D-II Vol.J75-D-II No.7 pp.1195-1201, (1992).