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Construction of 3-D Paper-made Objects from Crease Patterns

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Abstract

This paper proposes an approach to constructing a 3-D paper-made object from a crease pattern, a set of line segments (creases) on a sheet of paper which is usually produced when a paper-made object such as origami is unfolded. First, a coordinate transformation method from 2-D crease patterns to 3-D space is proposed. Since faces (formed by the creases) more than two are likely transformed onto the same plane in 3-D space, it is necessary to dispose these faces consistently. Therefore, second, a method for analyzing positional relationships of the overlapping faces is proposed. Finally, we show some examples of 3-D objects constructed by our method. The proposed method is useful for packaging and architectural modeling.

1 Introduction

Origami is a form of visual and sculptural representation that is defined primarily by the folding of the medium (usually paper). Traditional origami models have been presented by drill books, in which the folding processes consisting of several simple folding operations (e.g. mountain-folding) are intelligibly instructed step-by-step through a sequence of illustrations. Miyazaki et al. [4] have developed a virtual interactive manipulation system for each simple folding operation.

However, recently, we find that there are a lot of realistic, modern and complex origami models designed not by the traditional folding operations but by geometric origami design methods [3]. There is no specific process to fold these models. The origami creators draw line segments (creases) onto a sheet of paper to generate a crease pattern, the unfolded state of an origami model. In other words, they design a new model before they fold it. Designing such models consumes much time and energy of the creators because they have to imagine the completed model from a crease pattern without actually folding it.

In this paper, we propose an approach that represents 3-D models constructed from crease patterns to help origami creators easily see designed models as occasion demands. We notice the following three problems:

1. The crease patterns generated by geometric origami design methods do not include any information about folding process or folding operations.
2. The faces more than two formed by creases are likely overlapped on the same plane in 3-D space.
3. Some crease patterns allow several ways of folding.

To deal with these problems, we first propose a coordinate transformation method that is able to transform all faces in a 2-D crease pattern into the faces in 3-D space simultaneously, in section 3. Then, in section 4, we describe how to dispose the faces overlapped on the same plane in 3-D space so that the positional relations among them are consistent. Although there may be multiple solutions for ways of folding, we are trying to extract one of the feasible solutions.

As related work, Uchida et al. [6] have proposed an approach to deducing a folding process from a crease pattern of origami models, but it is premised that the origami models are designed by traditional folding operations. Eisenberg et al. [2, 1] have discussed a folding net problem, for the purpose of transforming 3-D virtual objects such as polyhedrons into a paper representation of the model as a form of hard copy from virtual environments, as the opposite of our approach. However, faces of the 3-D polygon models do not overlap each other. So, the second problem described above does not exist.

2 From Crease Patterns to 3-D Models

In order to construct 3-D virtual paper-made objects from crease patterns, the rotational transformation based on creases using the adjacent relationship among faces is needed. We generate a graph of the crease pattern in which nodes represent faces and edges represent creases is constituted, which comes to obtain the positional relationships among faces easily (Fig. 1). We call the graph *crease pattern graph* (*CP-graph* for short).

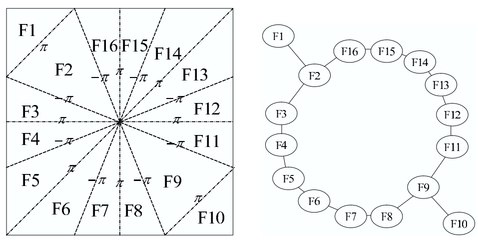


Figure 1. A crease pattern (left), and face-crease graph (right).

2.1 Crease Pattern

The crease pattern is given and satisfies the following preconditions;

- fold-able,
- has a set of faces divided by creases, and
- all the faces are polygons arranged on xy plane in the right-handed coordinate system, and on the obverse side

(the direction of normal vector is the positive z direction).

The crease pattern CP consists of vertices V, creases C and faces F. A crease $c_{i,j} \in C$ has information about its coordinates and the angle $\theta_{i,j}$ to be folded between two faces f_i, f_j. If $\theta_{i,j} = \pm\pi$ then the crease means valley/mountain folding. Moreover, in Fig. 2, when one of the right-handed-rotation vectors around the face f_i is defined as $\overrightarrow{c_{i,j}}$ which belongs to $c_{i,j}$, the unit vector of $\overrightarrow{c_{i,j}}$ is defined as $\widehat{c_{i,j}}$, and the vector from the origin to the starting point of $\overrightarrow{c_{i,j}}$ is defined as $t_{i,j}$.

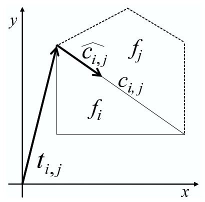


Figure 2. Vectors for a crease.

2.2 3-D Origami Model

Miyazaki et al. [4] have already proposed a data structure for origami which is able to represent the overlapping faces on the same plane. An example of this structure is shown in Fig. 3. The structure groups the faces on the same plane and hold the order of overlapping by a face list (in Fig. 3, that is $f_2 \to f_3$). Because the orders of overlapping faces are unknown when we try to transform a crease pattern into a 3-D origami model, we propose a method for consistently arranging the face lists from the given crease patterns.

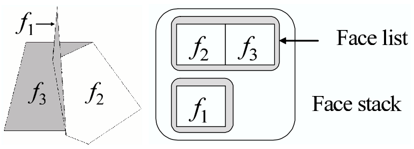


Figure 3. A structure for describing overlapping faces.

3 Coordinate Transformation

An arbitrarily face f_0 in a crease pattern is fixed on xy-plane and 3-D transformations of other faces are performed using the CP-graph. First, a path from f_0 to a face f_p to be transformed is searched. In order to decrease computational cost, the path is the shortest path (regard the weight of an edge as 1) is desired. When the obtained path is

$$f_0 \to \cdots \to f_q \to \cdots \to f_p,$$

the order of edges along this path is specified as

$$c_{0,1} \to \cdots \to c_{q-1,q} \to c_{q,q+1} \to \cdots \to c_{p-1,p}.$$

Next, the transformation matrices for each crease $c_{q-1,q}$ are calculated. The rotation matrix of an angle $\theta_{q-1,q}$ for the crease vector $\widehat{c_{q-1,q}}$ is defined as R_q, and the translation matrix for the vector $t_{q-1,q}$ is defined as T_q. Then, the total transformation matrix X_q for the crease $c_{q-1,q}$ is

$$X_q = T_q R_q T_q^{-1}.$$

Consequently, the 3-D (affine) transformation matrix Z_p for a face f_p based on the path from the fixed face f_0 is

$$Z_p(x, y, 0, 1) = X_1 ... X_q ... X_p(x, y, 0, 1)^t \quad \text{for } (x, y) \in f_p.$$

By calculating this Z_p for all faces in the crease pattern, it is possible to transform a crease pattern into a 3-D origami model (also other paper-made objects).

4 Arrangement of Face Lists

Firstly, the faces in 3-D space produced by the transformation method described in the previous section are divided into several face groups, each consists of the faces overlapping on the same plane.

Each face group is able to be defined as a CP-sub-graph and the sub-graph consists of the edges of only creases whose angles θ is $\pm\pi$, namely valley/mountain folding. Such a sub-graph can be painted with two colors to represent the front and the back of the paper, and two faces which have the same crease have different colors.

Secondly, by using the 2-colored sub-graph, a face list is consistently arranged so that the positional relationship between two faces in the list can be decided. The positional relationship between two faces is defined as follows.

Difinition 1 *When a face f_i should situate before a face f_j, $f_i > f_j$.*

Therefore, the face list is arranged so that all face pairs in the list satisfy $f_i > f_j$ for $i < j$, where the i-th face and the j-th face in the list are f_i and f_j, respectively. A method for decision of the relationship between two faces in a crease pattern is proposed in the following subsections. This method first judges the relationships between two neighboring faces in a crease pattern, and then, analyzes the relationships between all faces by using the obtained adjacent relationship.

4.1 Positional Relationship between Neighboring Faces

When there are two neighboring faces f_i, f_j have jointly a crease $c_{i,j}$, the condition for satisfying $f_i > f_j$ is:

- the side of f_i is the front and $\theta_{i,j}$ is $-\pi$, or
- the side of f_i is the back and $\theta_{i,j}$ is π.

Figure 4 shows an example of two neighboring faces. The side of the face f_1 is the front and the side of the face f_2 is the back. Moreover, the angle $\theta_{i,j}$ of the crease between the faces is $-\pi$, that is mountain-folding. Therefore, the positional relationship should be $f_1 > f_2$.

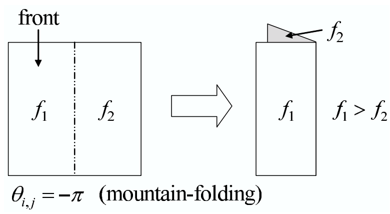


Figure 4. An example of determining positional relationship between neighboring faces.

4.2 Positional Relationship between Two Faces

By using the adjacent relationship, the positional relationship between two faces that are separate in a crease pattern is analyzed. We propose a method for analyzing the positional relationship based on *cross sections* of an origami model. Figure 5 shows an example of the cross section which is obtained by cutting an origami model. The obtained cross section in the crease pattern consists of a set of segments.

Two faces f_i, f_j whose positional relationship should be investigated are cut simultaneously and relationships between the faces are analyzed using the obtained cross sections. Moreover, the cross section between two faces needs to be connected. We have already proposed a method for generating the cross section by drawing line segments in the crease pattern [5]. This method generates the cross section using symmetry of the obtained segments. The obtained cross section S is defined as a set of line segments:

$$\{s_0, ..., s_i, ..., s_j, ..., s_n\} \in S$$

where S may be a simple path or a cycle, in the latter case $s_0 = s_n$.

In order to analyze the positional relationships among segments of the obtained cross section, each segment is arranged in 2-D plane as shown in Fig. 6. In this figure, there are segments $\{s_1, s_2, s_3, s_4, s_5\}$, the distance between terminal nodes p_i, p_{i+1} of the segments is represented as d_i. First, the segments are arranged parallel with y-axis in order at a regular interval, and y-coordinates of the terminal nodes correspond with the same nodes (Fig. 6(b)). Next, by using positional relationships between neighboring faces, the nodes of line segments are linked by vectors r_i if there are connectivity relations between corresponding neighboring faces (Fig. 6(c)). Moreover, if $f_i > f_{i+1}$, the direction of r_i is negative about x-axis. If $f_i < f_{i+1}$, the direction of r_i is positive.

The segments are rearranged about x-axis as shown in Fig. 7 and all arrangements (permutations) of the segments are analyzed. The segments should be arranged so that x-coordinates of segment s_i are larger than ones of s_j if $f_i > f_j$. The consistent permutation of the segments satisfies the following conditions;

1. All r_i are positive directions to the horizontal axis.
2. All r_i do not cross any segments.
3. When two vectors r_i, r_j are on the same straight line, if there is a terminal node of r_j between terminal nodes of r_i, there is also another node of r_j between terminal nodes of r_i.

Condition 1 means all the relationships between two neighboring faces are $f_i > f_j$ for $i > j$ (i, j are the values of x-coordinates). Permutation of Fig. 7(a) is infeasible because only r_2 is positive direction. Moreover, condition 2 means physical feasibility because it is impossible to fold the faces when a face collides with another face. Permutation of Fig. 7(b) is infeasible because r_3 crosses s_1. Furthermore, condition 3 represents a particular case of condition 2.

Figure 7(c) shows permutations which satisfy these conditions using the segments in Fig. 6. Only two permutations are consistent. Therefore, the positional relationship between two faces can be analyzed. For example, it is certain that $f_4 > f_2$, because s_4 is put more forward than s_2 on two permutations. However, the relationship between s_1 and s_4 is different at two permutations. This means that there may be multiple solutions. Although the positional relationships can not be solitarily determined in this case, only the feasible solutions are extracted by this method.

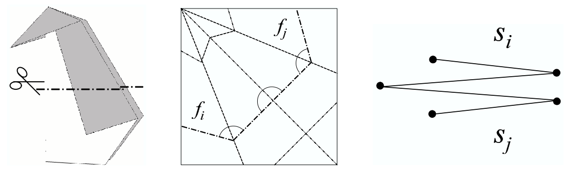
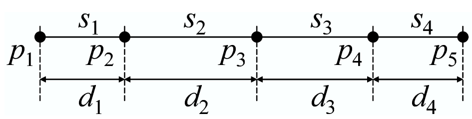
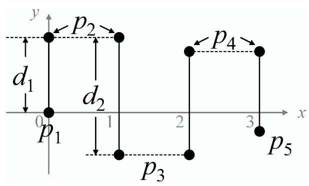


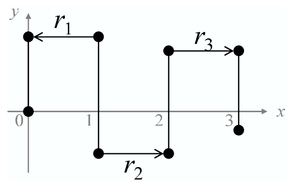
Figure 5. An example of generating a cross section.



(a) An cross section which consists of segments.



(b) Arrangement onto 2-D plane.



(c) The linked verctors.

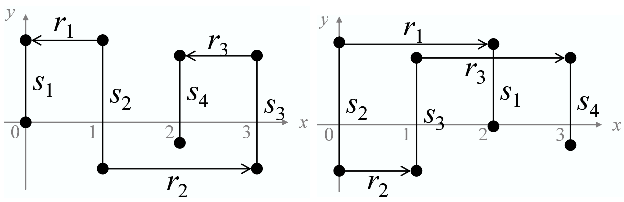
Figure 6. Arrangement of a cross section onto 2-D plane.

5 Experimental Results

We have implemented a prototype system based on the proposed method. Figure 8 shows the user interface of the system. A crease pattern given as input is represented on the left of the interface. When a crease pattern is input to the system, the system automatically constructs the 3-D paper-made object on the right. The implementation is tested with some fold-able crease patterns that are produced by hand and the resulting objects are represented in 3D virtual space, which is shown in Fig. 9. As a result, consistent paper-made objects can be constructed from crease patterns.

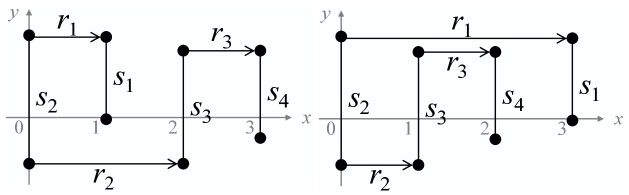
6 Conclusion

This paper presented an approach to constructing 3-D paper-made objects from crease patterns. The proposed coordinate transformation makes it possible to represent paper-made objects in 3-D virtual space. Furthermore, the proposed analyzing method of the positional relationships among faces arranges the faces overlapped on the same plane after the transformation in a consistent order. The experimental results have demonstrated that in our approach it is possible to construct consistent paper-made objects from crease patterns.



(a) There are vectors with different directions.

(b) A vector crosses a segment.



(c) Feasible permutations.

Figure 7. Permutations of segments in Fig. 6.

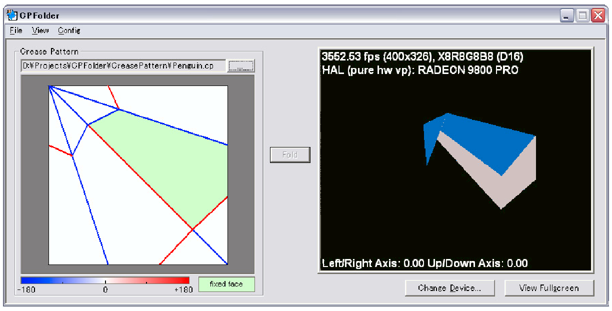


Figure 8. The user interface of the prototype system.

As our future work, it is necessary to deal with the problem how to find the "optimal" solution if multiple interpretations for one crease pattern exist. Moreover, in order to animate origami in 3-D virtual space, it is necessary to extract folding process or something close to it from given crease patterns.

Acknowledgement

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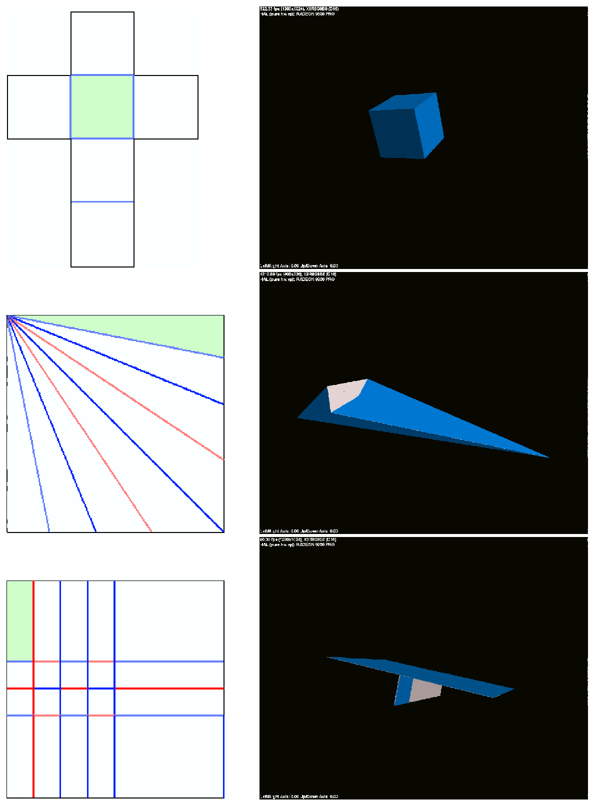

Figure 9. The experimental results.