

## Calibration of HyperOmni Vision based on Conic Curve

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### Abstract

*In this paper, we propose a calibration method for catadioptric camera systems consisting of a rotation symmetry mirror, like HyperOmni Vision, and an affine camera. The proposed method is based on conventional camera calibration and mirror posture estimation. Many methods for camera calibration have previously been proposed. In the last decade, methods for catadioptric camera calibration have also been proposed. The main problem with catadioptric camera calibration is mirror posture estimation because the degree of freedom of a mirror posture is limited and the accuracy of the estimated parameters is inadequate, owing to nonlinear optimization. Our method can estimate the six degrees of freedom of mirror posture and can be free from the volatility of nonlinear optimization. Our method uses a conic curve in an image, the borderline between mirror and non-mirror and estimate up to for mirror posture. This method is an application of extrinsic parameter calibration based on conic fitting. The mirror posture estimated analytically is not unique, so we propose a selection method for finding the best one. Because of the conic-base analytical method, our method can avoid the initial value problem arising from nonlinear optimization. We conduct experiments on synthesized images and real images to test the performance of our method, and discuss its accuracy.*

### 1 Introduction

Catadioptric camera systems are used for various applications such as security systems, environment recognition, and robot navigation. Many kinds of catadioptric camera are designed for various applications according to their use.

The disadvantage of catadioptric cameras is that alignment of the mirror and camera must be exact. If there is misalignment, the camera cannot maintain desired optical characteristics such as a single viewpoint. Misalignment problem causes various errors in systems using catadioptric cameras. The method used to correct misalignment depends on the purpose for which the camera is used. In measurement applications, it is not necessary for cameras to maintain their designed optical characteristics, but the geometric arrangement between pixels, viewpoints, and rays

should be correct. In other words, the position of the camera and mirror should be calibrated exactly. In visualization applications, accurate alignment of the camera and mirror is not required and, when errors in viewpoint position and ray direction are within acceptable levels, the viewer does not have any sense of incongruity. In both cases, the catadioptric camera calibration is an important problem.

The design of catadioptric camera systems is the combination of mirror shapes and camera models. In these systems, catadioptric camera calibration is as important as it is in regular camera calibration, but is more intricate, because it includes regular camera calibration and mirror posture estimation, and the methods for catadioptric calibration differ according to the kinds of mirrors, reflection, and camera models that are used. In this paper, we propose a calibration method for a catadioptric camera system consisting of a rotation symmetry mirror, HyperOmni Vision [1], and an affine camera model.

As mentioned above, the catadioptric camera calibration can be divided into regular camera calibration and mirror posture estimation. Many camera calibration methods have already been proposed, for example, Tsai's calibration [2], conic based methods [3, 4], and so on. Therefore, we do not discuss the regular camera calibration but focus on the mirror posture estimation.

#### 1.1 Related Work

Many catadioptric camera systems have been proposed over the last decade. Typical catadioptric camera systems have been proposed by Nayar [5] and Yamazawa et al. [1]. The first uses an orthogonal camera model and a parabolic mirror. Yamazawa et al. uses a perspective camera model and a hyperboloidal mirror. Catadioptric cameras that have non-single viewpoints have also been proposed [6, 7].

Much work has been done on developing methods to calibrate catadioptric cameras [8, 9]. Geyer and Daniilidis [8] proposed a method of calibration to estimate intrinsic parameters of a catadioptric camera system that consists of a paraboloid mirror and an orthographic lens. Strelow et al. [9] proposed a model for relation between the mirror and camera with 6 degrees of freedom (translation and rotation). They determined 6 parameters through nonlinear optimiza-

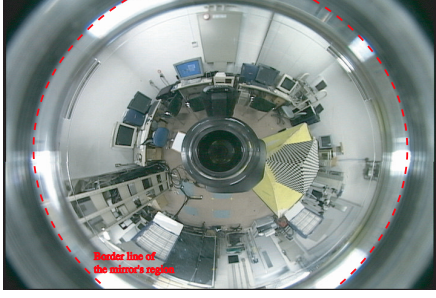


Figure 1: Omnidirectional image and the borderline between mirror and non-mirror.

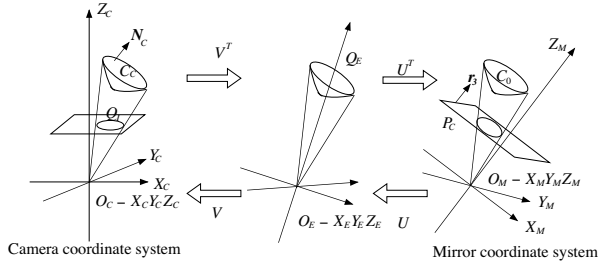


Figure 2: Relationships among coordinate systems and matrices.

tion. This has the advantage that translation and rotation parameters are simultaneously determined, but the disadvantage is that the accuracy of the estimated parameters is worse and depends on the initial values because of nonlinear optimization.

The main problem with the catadioptric camera calibration method is mirror posture estimation because the degree of freedom of a mirror posture is limited and the accuracy of the estimated parameters is inadequate owing to nonlinear optimization. Our method estimates the six degrees of freedom of mirror posture and is free from the volatility of nonlinear optimization such as the local minimum problem, the initial value problem, and the computation complexity problem. Our method uses a conic curve in an image, the borderline between mirror and non-mirror region, and is based on the extrinsic parameter calibration using a circular pattern[3]. Because of the conic-base analytical method, our method avoids the initial value and local minimum problem arising from nonlinear optimization.

## 2 Catadioptric Camera Calibration

In this section, we present a calibration method for catadioptric camera systems consisting of a rotation symmetry mirror and an affine camera model. The catadioptric camera calibration can be divided into four steps: normal camera calibration, mirror posture estimation, ray tracing, and calculation of mirror reflection.

Our method uses a conic curve in an image, the borderline between mirror and non-mirror regions, and is an application of extrinsic parameter calibration based

on conic fitting. We actually applied Wu's method [3], based on conic fitting, to calibrate extrinsic parameters.

In order to estimate a mirror posture, we assume the following conditions: the camera is calibrated, the rank of camera intrinsic matrix,  $K$ , is three (full rank), the borderline between mirror and non-mirror regions (Figure 1) is within an input image, and its radius,  $r$ , is known.

The borderline mentioned above is projected to an omnidirectional images as a ellipse (conic) curve, and its equation is  $ax^2 + by^2 + 2fx + 2gy + 2hxy + c = 0$ , where  $(x, y)$  is in the image coordinate system. The quadratic form of that equation is

$$\tilde{\mathbf{x}}^T Q_I \tilde{\mathbf{x}} = 0, \quad (1)$$

where

$$Q_I = \begin{bmatrix} a & h & f \\ h & b & g \\ f & g & c \end{bmatrix} \quad (2)$$

and  $\tilde{\mathbf{x}} = (x, y, 1)^T$  is the augment vector of a point in the image coordinate system. The relation between a point in the image coordinate system and one in the camera coordinate system is expressed by the following equation:

$$\tilde{\mathbf{x}} = sK\mathbf{X}_C, \quad (3)$$

where  $K$  is the camera intrinsic matrix and  $\mathbf{X}_C$  is the point in the camera coordinate system. By substituting Eq. 3 for Eq. 1, we obtain

$$s^2 \mathbf{X}^T Q_e \mathbf{X} = 0, \quad (4)$$

where  $Q_e = K^T Q_I K$ . By eigenvalue decomposition,  $Q_e = V\Lambda V^T$ .

We consider a circle centered at  $(x_0, y_0, z_0)$  on  $Z = z_0$  plane with radius  $r$ . According to Wu's method [3], the circle can be written in a quadratic form:

$$Q_C = \begin{bmatrix} 1 & 0 & \frac{-x_0}{z_0} \\ 0 & 1 & \frac{-y_0}{z_0} \\ \frac{-x_0}{z_0} & \frac{-y_0}{z_0} & \frac{x_0^2 + y_0^2 - r^2}{z_0^2} \end{bmatrix}, \quad (5)$$

$$\mathbf{X}^T Q_C \mathbf{X} = 0. \quad (6)$$

We consider the rotation from the coordinate system,  $O_E - X_E Y_E Z_E$  (See Figure 2), to the mirror coordinate system,  $O_M - X_M Y_M Z_M$ . The Z-axis of the mirror coordinate system is parallel to the normal vector,  $N_C$ , of the cross-section surface,  $P_C$ . To express that rotation, we consider the rotation matrix  $U$  and the relation can be expressed as follows.

$$U^T \Lambda U = kQ_C. \quad (7)$$

To solve the above equation, we obtain  $U$  by using Wu's method [3]. By substituting  $U$  and  $r$  for Eq.

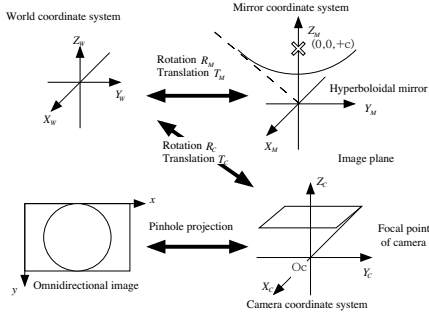


Figure 3: Coordinate systems

7, we can compute  $C_0 = [x_0, y_0, z_0]$ , the center of the circle. The rotation matrix,  $R$ , from the mirror coordinate system to the camera coordinate system is obtained by  $R = VU$ . Figure 2 shows the relationships among each coordinate system and rotation matrices. The center of the circle in the camera coordinate system,  $C_C$ , is obtained by the equation:  $C_C = RC_0$ . Since  $R$  is a rotation matrix, it can be represented by three orthogonal unit vectors:  $[\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3]$ . Specifically,  $\mathbf{r}_3$  is the normal vector of the circle, the aspect of the the mirror, in the camera coordinate system.

Ray tracing is implemented using coordinate transformation. Ray tracing uses four coordinate systems (image, camera, mirror, and world), and traces incident rays from the camera through coordinate transformation (Figure 3). Finally a viewpoint,  $\mathbf{P}_M$ , and a direction of reflected ray,  $\mathbf{V}_{M_{out}}$ , are calculated from each pixel.

The mirror posture is not unique and has 4 solutions by using Wu’s method. Here, we propose a method for selecting the best one of them. In this selection method, we use the rays from pixels projecting a line far from camera.

The condition that the rays,  $\mathbf{P}_i + k_i \mathbf{V}_i$ , are on a plane,  $(\mathbf{X}, \mathbf{N}) = S$ , is expressed as  $(\mathbf{P}_i + k_i \mathbf{V}_i, \mathbf{N}) = S$ , where  $(\cdot)$  expresses the inner product and  $S$  is the distance from the origin. In other words, a normal vector  $\mathbf{N}$  exists and satisfies the equations:  $(\mathbf{V}_i, \mathbf{N}) = 0$  and  $(\mathbf{P}_i, \mathbf{N}) = S$ .

The rays from a catadioptric camera intersecting with a line in a 3D-space are expressed as  $\mathbf{P}_{Mi} + k_i \mathbf{V}_{M_{out}i}$ . In the case of an aligned single viewpoint catadioptric camera, a normal vector,  $\mathbf{N}$ , exists and satisfies the following equation:

$$(\mathbf{P}_{Mi} + k_i \mathbf{V}_{M_{out}i}, \mathbf{N}) = S, \quad (8)$$

because the catadioptric camera can be assumed as a normal camera. In the case of misalignment,  $\mathbf{N}$  does not exist. However, if the line is very far from camera (i.e.  $k_i \rightarrow \infty$ ), we can assume that equation 8 as  $(\mathbf{V}_{M_{out}i}, \mathbf{N}) \rightarrow 0$ , and this can be regarded as the case of aligned mirror. We apply this assumption to select mirror posture. If the posture is correct,  $\mathbf{N}$  exists and satisfies  $(\mathbf{V}_{M_{out}i}, \mathbf{N}) \rightarrow 0$ , otherwise  $\mathbf{N}$  does not satisfy the condition because the rays

don’t intersect the line. We estimate  $\mathbf{N}$  by minimizing  $\Sigma(\mathbf{V}_{M_{out}i}, \mathbf{N})^2$ . Such  $\mathbf{N}$  is the solution of the equation,  $\nabla \Sigma(\mathbf{V}_{M_{out}i}, \mathbf{N})^2 = 0$ . And  $\mathbf{N}$  is the eigenvector that has the minimum eigenvalue of  $\nabla \Sigma(\mathbf{V}_{M_{out}i}, \mathbf{N})^2$ . The minimum eigenvalue can be regarded as an evaluation value. The mirror posture that has the minimum evaluation value is the correct posture.

## 3 Experiments

### 3.1 Mirror Posture Estimation

We conduct experiments on synthesized images to evaluate the accuracy and performance of our method. At first, we make an ellipse image by projecting the misaligned mirror. Next, we estimate ellipse parameters by least square error estimation. Finally, the mirror posture is estimated by our method.

Table 1 shows the experimental result. In table 1, the translation of the ground truth is the position of the circle’s center, the rotation of ground truth is the normal vector of the plane including the circle, the borderline between mirror and non-mirror regions, the translation error is Euclid distance between the ground truth and the estimated mirror position,  $C_C$ , and the rotation error is the angle, inner product between the ground truth and estimated normal vector  $\mathbf{r}_3$ . We can see that our method can be used to accurately estimate the mirror posture.

The factor of the estimation error is only quantization error of projection and the error of the ellipse estimation because the experiments are simulation. It is impossible to completely eliminate the quantization error. The accuracy of our method depends on the accuracy of camera calibration and ellipse estimation. To apply our method to a catadioptric camera system and real images, it is important to correctly estimate the ellipse parameters and the intrinsic parameters.

### 3.2 View Reconstruction

The advantage of our method is shown in image transformation. If an omnidirectional image is transformed into a perspective or panorama image assuming that the mirror is aligned while the mirror is misaligned, the transformed image has distortion and/or skew. If the mirror is misaligned, we can not transform omnidirectional image into perspective image because HyperOmni Vision does not keep single viewpoint. But we can transform omnidirectional image into approximate perspective image by assuming an arbitrary point to the viewpoint. We decide the viewpoint by the error, mean squared distance between the viewpoint to each ray.

Figure 4 and 5 shows that a perspective image transformed from the omnidirectional image (Figure 1). Figure 4 is transformed image without using calibration data, and Figure 5 is transformed image by using calibration data. We can see the distortion and skew in Figure 4, on the other hand we can see little distortion and skew in Figure 5. From these experimental results, our method is effective for omnidirectional camera calibration.

Table 1: Accuracy of mirror posture estimation

Ground truth						Error	
Translation [mm]			Rotation			Translation [mm]	Rotation [deg]
X	Y	Z	X	Y	Z		
0	0	-0.5	0	0	1	0.0701	0.464
0	0	3.5	0	0	1	0.120	0.828
-4.5	0	0	0	0	1	0.108	0.800
0	-2.19	-0.0229	0	-0.0209	0.999	0.172	1.13
-4.38	2.19	-0.115	-0.0419	0.0209	0.999	0.0506	0.333



Figure 4: Uncalibrated perspective view.

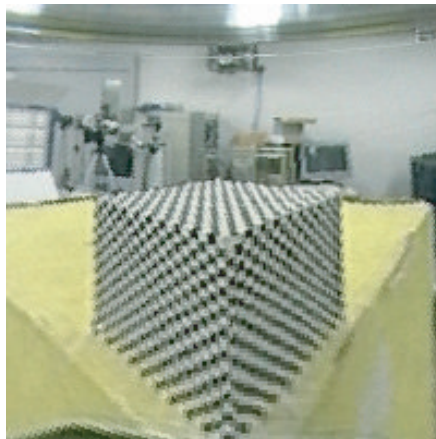


Figure 5: Calibrated perspective view.

#### 4 Conclusion

In this paper, we propose a calibration method for a catadioptric camera system consisting of a rotation symmetry mirror, such as HyperOmni Vision, and an affine camera. The proposed method is based on camera calibration and mirror posture estimation. Our method can estimate the six degrees of freedom of mirror posture and can be free from the volatility of nonlinear optimization. Our method uses a conic curve in an image, the borderline between mirror and non-mirror and is an application of extrinsic parameter calibration based on conic fitting. Because of the conic-base analytical method, our method can avoid the initial value problem arising from nonlinear optimization. We also proposed a method for mirror posture selection because Wu’s method has 4 solutions of mirror posture.

We conducted experiments on synthesized images and real images to test the performance of our method, and discussed its accuracy. In future work, we will evaluate the accuracy of our method by 3D reconstruction with real images.

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