Proc. IEEE 1975 Int. Conf. Cybernet. Society. New York. N.Y. pp. 163-165

PAPER WeAM 4-5

THE PHASE CORRELATION IMAGE ALIGNMENT METHOD

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Abstract

A new correlation method based upon the inverse Fourier transform of the phase difference between two images is described. The result is a highly accurate alignment technique which exhibits an extremely narrow correlation peak, is relatively scene-independent, and is insensitive to narrow bandwidth noise and convolutional image degradations. Through the use of phase weighting functions, the method may be generalized in order to provide immunity to various types of noise or image distortions. An efficient implementation can be devised which involves no greater complexity than that required for circular cross correlation.

Introduction

The accurate registration of displaced images is an important problem in many areas of current technology such as target tracking, platform stabilization, velocity sensors, map matching, scene change detection, and multispectral image correlation. The most commonly used image matching techniques employ the cross correlation algorithm or closely related variations. Insufficient attention has been devoted to the development of alternative correlation methods which are optimal for image degradations which cannot be approximated by additive white noise.

The phase correlation algorithm is based upon the fact that the information pertaining to the displacement of two images resides in the phase of the cross power spectrum. This concept has been generally overlooked in previous treatments of the image matching problem. The application of the phase correlation technique results in a sharp peak at the point of registration on the order of one resolution element in width. The method is relatively scene-independent and ambiguities which arise through the use of circular cross correlation are avoided. It is particularly useful for aligning images taken with different sensors or under varying conditions of illumination. Large relative displacements can be accurately determined without requiring that one image be contained within the other.

Phase Correlation Algorithm

The phase correlation algorithm proceeds as follows. The two-dimensional discrete Fourier transforms G_1 and G_2 of a pair of sampled images g_1 and g_2 are computed, and the phase difference $e^{j(\phi_1 - \phi_2)}$ is obtained for each spatial frequency $\vec{f}$, where $G_i(\vec{f}) = |G_i(\vec{f})|e^{j\phi_i(\vec{f})}$, $(i = 1, 2)$. If either G_1 or G_2 is 0 at some frequency, the corresponding phase factor is ambiguous and is therefore replaced by zero. The phase correlation function is given by:

$$d = F^{-1}\{e^{j\phi}\} \qquad (1)$$

where $\phi = \phi_1 - \phi_2$ and $F^{-1}\{\ \}$ denotes the inverse Fourier transform. The function $e^{j\phi}$ thus represents the phase of the cross power spectrum, $G_1G_2^*$.

For infinite or cyclically shifted images such that $g_2(\vec{r}) = g_1(\vec{r} + \vec{L})$, the Fourier shift theorem[1] gives

$$G_2(\vec{f}) = G_1(\vec{f})e^{j2\pi\vec{f}\cdot\vec{L}} \qquad (2)$$

so that $\phi = -2\pi\vec{f}\cdot\vec{L}$. The correlation function (1) is then a delta function located at the point of registration, $\delta(\vec{r} - \vec{L})$. The discrete form of the phase correlation function for sampled, cyclically shifted images is given by the product of shifted sinc-functions of the rectangular coordinates. This result holds remarkably well for finite images taken from a continuous scene, even when the common area (overlap) between the images is small, or when a substantial amount of noise is present.

Some related studies have been performed by Lo and Parikh[2] who attempted to estimate the shift vector in the transform domain by using differences between successive phase angles. The ensemble of shift vector determinations derived in this manner was then averaged in order to obtain a best estimate for the displacement. The results were considerably less accurate than those obtained using phase correlation because of the large dispersion that is usually present in the shift vector data. This problem is neatly eliminated in the phase correlation method by taking the inverse Fourier transform of the phase difference matrix, $e^{j\phi}$.

The phase correlation algorithm may be generalized by introducing an arbitrary weighting function $H(\vec{f})$ in the spatial frequency domain giving

$$d_H = F^{-1}\{He^{j\phi}\} \ . \qquad (3)$$

For instance, if the weighting function is of the form $|G_1G_2^*|^\alpha$, the resulting family of correlation algorithms include both phase correlation ($\alpha = 0$) and conventional cyclic cross correlation ($\alpha = 1$). Immunity to various types of noise or image distortions may be provided by the proper choice of weighting function.

Performance Characteristics

The phase correlation signal/rms noise ratio (SNR) can be expressed quite simply as a function of the peak amplitude and the square root of the total number of sample points.

Consider first the case of two NxN element images having no common features. The phase angle difference ϕ can be considered to be a random variable uniformly distributed over 2π radians. The N^2 d-matrix amplitudes (normalized to unit total power) can then be approximately represented by gaussian random variables having zero mean and standard deviation equal to N^{-1}.

In the case of two images which have some degree of congruence, the power is divided between a coherent peak located at the point of image registration (signal) and the incoherent peaks (noise) resulting from the random component of the phases. The amplitude of the coherent peak is a direct measure of the degree of image congruence. If the peak amplitude is denoted by A, where $0 \le A \le 1$, then the signal power is A^2 and the noise power is $1 - A^2$. The standard deviation of the noise is thus:

$$\sigma = \frac{1}{N}(1 - A^2)^{1/2} \qquad (4)$$

and the phase correlation SNR is

$$\psi = \frac{A}{\sigma} \qquad (5)$$

$$= \frac{NA}{(1 - A^2)^{1/2}} \ . \qquad (6)$$

Using the gaussian model for the distribution of noise peaks, the probability that a noise peak will be $\ge$ some threshold T is given by:

$$P_T = \frac{N^2}{(2\pi)^{1/2}\sigma} \int_T^1 e^{-\frac{1}{2}(x/\sigma)^2} dx \quad . \qquad (7)$$

Using the maximum standard deviation, $\sigma = N^{-1}$, in Eq. (7) and assuming that $NT \gg 1$, the error probability can be written as

$$P_T \leq \frac{N}{(2\pi)^{1/2}} \frac{e^{-\frac{1}{2}(NT)^2}}{T} \quad . \qquad (8)$$

The normalization procedure used to obtain the phase matrix effectively "whitens" each image with respect to itself so that the resulting correlation measure is relatively scene-independent. As an example, the Fourier phase factor and hence the phase correlation function are invariant with respect to either a scaling or level shift of an image brightness function. In addition, the results are fairly insensitive to convolutional image degradations which can be represented by a transfer function T. This can be understood from the fact that for noise free images subject to the same degrading process, the degraded image cross power spectrum is related to the undegraded spectrum by a factor $|T|^2$, so that the phase-difference matrix remains unchanged.

Phase correlation is clearly a preferred correlation algorithm in the presence of narrow band noise of unknown spectral content, since all spectral phase terms are treated on an equal basis. The technique is particularly useful for aligning images obtained under differing conditions of illumination, since illumination functions are normally slowly varying and therefore concentrated at low spatial frequencies. The cross correlation function, on the other hand, is dominated by the largest spectral components and is optimal with respect to white noise.

The position of a correlation surface peak is in general a continuous function of image displacement. Given a sufficiently high SNR, it is therefore possible to measure non-integer displacements thru the use of interpolation. It is particularly advantageous to use the phase correlation surface for interpolation since it is characterized by a very sharp, symmetric peak of known functional form. Computer simulations have verified that highly accurate displacement computations can be made by using interpolation with only a few data points around the maximum value.

Implementation

A hardware implementation of the phase correlation method can be made involving no greater complexity than that required for circular cross correlation. The computation consists of the following steps:

1. The input consists of two sampled images g_1 and g_2 which have the same dimensions (say NxN);
2. The two-dimensional Fast Fourier transform[3] (FFT) is taken of each image resulting in two complex NxN element arrays, G_1 and G_2;
3. The phase difference matrix is derived by forming the cross power spectrum, $G_1G_2^*$, and dividing by its modulus;
4. The phase correlation function, d, is then obtained as a real NxN element array by taking the inverse FFT of the phase difference array.

The computation can be further simplified by replacing step 3 with a phase quantization algorithm. With this approach, the phase difference matrix $e^{j(\phi 1 - \phi 2)}$ is approximated by quantizing each phase to eight levels using three binary decisions involving the real and imaginary components of the spectral elements, followed by a 3-bit subtraction. This process is clearly easier to implement than multiplying two complex quantities and dividing by the modulus of the product. In order to test the phase quantization concept, a series of computer simulations was made using the inverse chirp z-transform[4] in order to obtain a high resolution sampling of the phase correlation function. It was found that for 64x64 element images, the use of an eight level quantization introduced a rms displacement error of only 1/80 of a resolution element.

Computer Simulation

An image registration experiment was performed using two successive frames of imagery taken from a moving platform. The film was scanned and digitized to 8 bits with a resolution of 256x256 pixels. The digitized images are reproduced in Figure 1 using 64 gray shades. The image overlap region, which represents only 44% of the image area, has been outlined in white. The common region in the two frames reveals differences due to change in illumination and perspective in addition to film grain noise.

The correlation surfaces obtained using phase correlation and cyclic cross correlation are shown in Figure 2 a, b. In both cases, the positive amplitude variation has been scaled to cover the range (0, 1). Cross sections along the row and column which intersect at the peak are shown in Figure 2 c, d. The phase correlation surface is characterized by a very sharp peak at the correct point of registration plus very low amplitude peaks at other locations. The cross correlation surface has a lower, broader peak centered at the registration point which is only a local maximum. Higher amplitudes are obtained over other portions of the cross correlation surface due to illumination differences between the images.

References

1. J. W. Goodman, Introduction to Fourier Optics, New York: McGraw-Hill, 1968, p. 9.
2. R. C. Lo and J. A. Parikh, "A Study of the Application of Fourier Transforms to Cloud Movement Estimation from Satellite Photographs," University of Maryland Computer Science Center Technical Report TR-242, May 1973.
3. J. W. Cooley and J. W. Tukey, "An Algorithm for the Machine Calculation of Complex Fourier Series," Math. Computation, Vol. 19, pp. 297-301, April 1965.
4. L. R. Rabiner, R. W. Schafer, and C. M. Radar, "The Chirp Z-Transform Algorithm and Its Applications," Bell Syst. Tech. Journal, Vol. 48, pp. 1249-1292, May 1969.

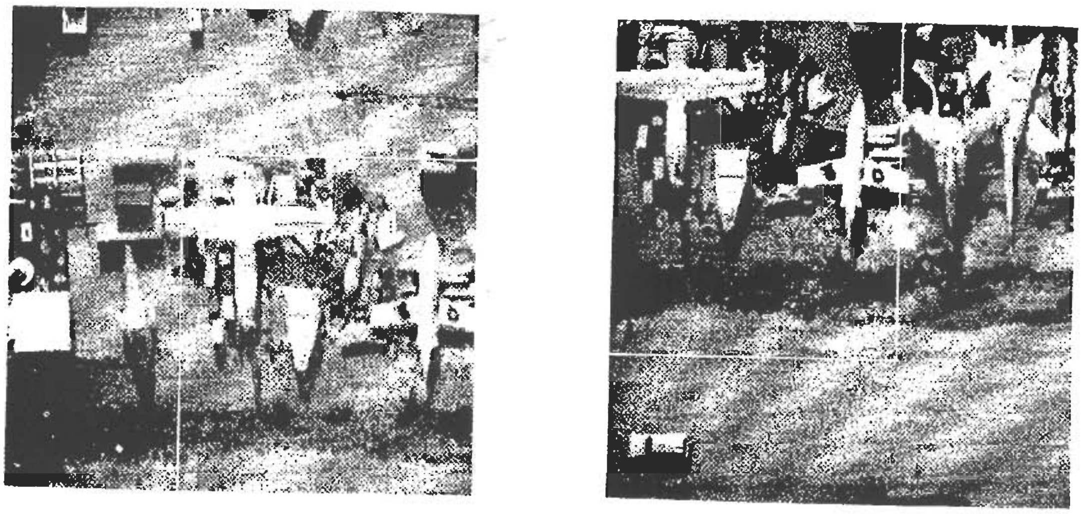

Fig. 1 Displaced Aerial Photos.

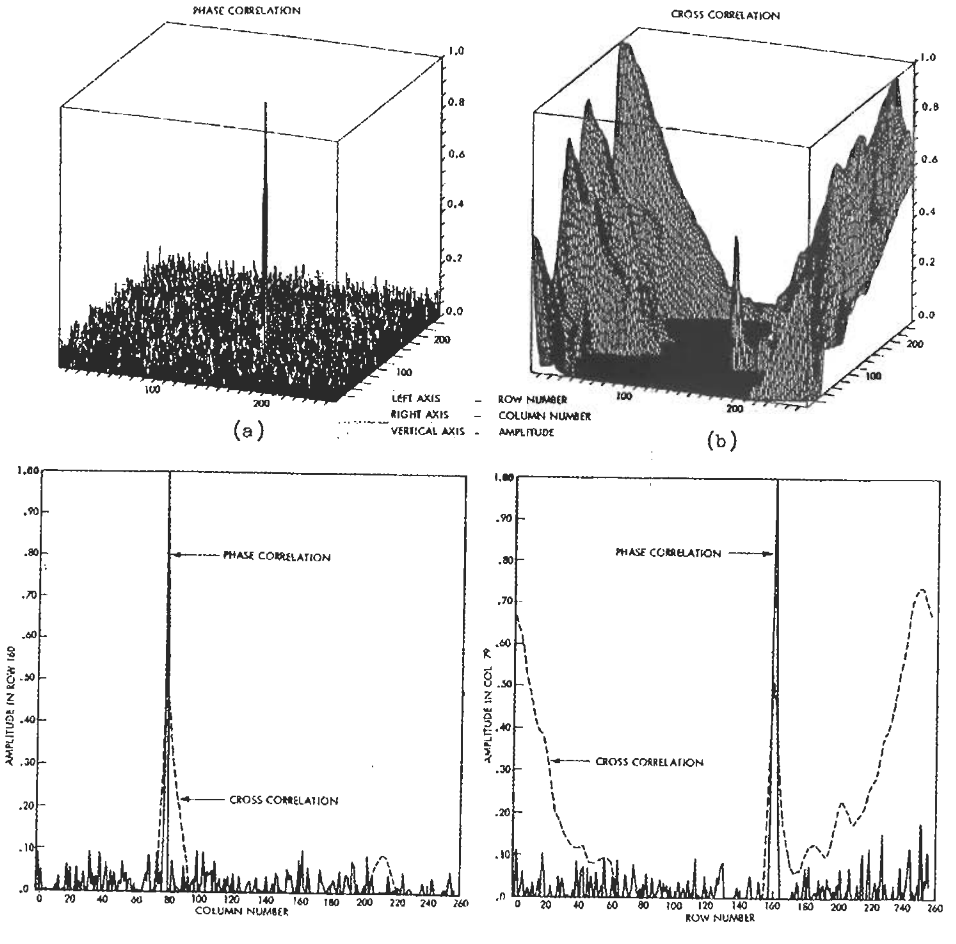

Fig. 2 Correlation Surfaces Obtained Using Displaced Aerial Photos. Phase correlation and cross correlation results are shown in (a) and (b). Row and column cross sections through the peak are shown in (c) and (d).