Routing with Guaranteed Delivery on Virtual Coordinates

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Abstract

We propose four simple algorithms for routing on planar graphs using virtual coordinates. These algorithms are superior to existing algorithms in that they are oblivious, work also for non-triangular graphs, and their virtual coordinates are easy to construct.

1 Introduction

Local geographic routing methods for networks use only information about the location of the current node, its neighbors and the destination to route a packet. The main challenge is to guarantee that the packet will actually arrive at the intended destination. If this is guaranteed – we say that the algorithm *delivers*. A routing algorithm is called *oblivious* if it does not require any extra information to be added to the routed packet, apart from information on the destination node. It is called *competitive* if the ratio of the routing path length (in hops) to the shortest path between the source and the destination is bounded by some constant. Bose et al. [5] proved that there is no oblivious and competitive geographic routing algorithm which delivers for *all* convex planar tilings.

An alternative to geographic routing is to assign carefully chosen *virtual* coordinates to the nodes [4,14], and apply routing methods based on these new "locations".

In this paper we provide four constructions of virtual coordinates and accompanying oblivious routing algorithms. For the sake of brevity, we omit the (non-trivial) proofs of the theorems that the routing algorithms deliver. These, involving extensive use of duality and convex embeddings, will appear in a future full-length version of this paper. The algorithms may be applied to any 3-connected planar network connectivity. We experimentally show that the average performance of these algorithms is comparable to existing alternatives.

2 Previous Work

Important algorithms for oblivious geographic routing on planar graphs, with an analysis of their performance, were provided by Bose and Morin and co-workers [5,6,7]. The main difference between the algorithms is the forwarding rule. Among a node's neighbors there is always one (D) which is closest in Euclidean distance to the destination, one which is closest in clockwise angular distance to the line connecting the node to the destination (CW), and one which is closest in counterclockwise angular distance to the line connecting the node to the destination (CCW). *Greedy* routing forwards a packet from a node to D. *Compass* routing forwards to the closest (in angular distance) among CW and CCW. *Greedy compass* routing forwards to the closest in Euclidean distance to the destination among CW and CCW. *Random compass* routing forwards to a random choice among CW and CCW.

Bose et al. [5,6] show that *all* these routing algorithms work on Delaunay triangulations. Greedy compass routing works on arbitrary triangulations, and compass routing works on a subclass of triangulations known as *regular* triangulations (these are similar to the rubber band embeddings defined in Section 3.1.1). None of the deterministic algorithms work for non-triangular planar tilings. This is a severe limitation, as it is not always possible to find a triangulation subgraph of an arbitrary graph. For example, if the graph is the communication graph of a sensor network, there may be regions called "gaps", or "communication voids", which do not contain sensors at all. Such gaps are usually also the cause of failure for greedy routing algorithms, as the algorithm may get stuck there in a local minimum. Random compass routing works on any convex planar tiling, but due to its indeterminism, generates quite long paths in practice.

A common remedy to a routing algorithm getting stuck in a local minimum on a convex planar tiling is to continue from that point with so-called *face routing*. Examples of these are GFG [3], GPSR [12], and GOAFR+ [11], but they are not oblivious.

Virtual coordinates for routing were first introduced by Morin [4]. Papadimitriou and Ratajczak [13] conjecture that every 3-connected planar graph has an embedding in the plane such that greedy routing works. They propose an embedding of such graphs as a special convex 3D polyhedron edges-tangent to a sphere [16], and a routing algorithm that exploits angles between vectors. While ingenious, these embeddings are quite difficult to construct for large graphs [2], thus have limited practical value.

3 The Routing Algorithms

We describe now our virtual coordinate constructions and accompanying routing algorithms. All the constructions apply to 3-connected planar graphs.

Definitions:

Let G(V,E) be a 3-connected planar graph, with an outer face B. An *embedding* of G in R^d is a function f: $V \rightarrow R^d$ that assigns to each vertex of G coordinates in R^d. The edges of G are embedded as straight lines between neighboring vertices. A *planar embedding* is an embedding in R^2, such that the faces of G occupy disjoint regions of the plane. A *convex tiling* is a planar embedding whose faces are all convex.

An *oblivious routing scheme* on an embedding is a function R:$V\times V \rightarrow V$. Given two vertices (v,t) the function designates one of v's neighbors, u, to be the *next* vertex. This choice depends only on the coordinates of v – the current vertex, t – the destination vertex, and the neighbors of v. An *oblivious routing path* from s to t is generated by applying R successively, with its first argument starting at s, continuing with the vertex specified by R. Its second argument is always t. We say that a routing scheme *works* for an embedding, if for any two vertices (s,t) in the embedding there exists a finite oblivious routing path from s to t generated by the routing scheme.

In the next subsections, we describe four oblivious routing schemes that work on wide sub-classes of convex tilings, using virtual coordinates which are relatively easy to construct.

3.1 Left Compass Routing

3.1.1 The Virtual Coordinates

An easy way to generate a convex tiling from a 3-connected planar graph is by using the well-known convex-combination embedding of Tutte [17]. A face is first identified as the outer face, or *boundary*, of the graph. The boundary vertices are constrained to form a strictly convex polygon, and the location of each interior vertex to be some strictly convex combination of the locations of all its neighbors. The latter involves solving a linear set of equations. A *rubber band embedding* is such an embedding, where the convex combination weights are edge-symmetric, i.e. $w_{ij} = w_{ji}$. A rubber band embedding has a physical interpretation as the equilibrium of a constrained spring system with zero lengths at rest. Note that rubber band embeddings are a **strict** subset of convex tilings. In practice we use the largest face in the graph as the boundary, embedded to equally spaced points along the unit circle, and unit weights.

3.1.2 The Routing Algorithm

For the routing algorithm we use the Left Compass algorithm [4], which always forwards the packet to the CCW vertex. Morin [4] showed that Left Compass delivers on a special embedding of a class of graphs that includes 3-connected planar graphs. However, their embedding is more complex than the rubber band embedding, and is harder to construct locally.

Theorem 1: Left compass routing works for any rubber band embedding of a 3-connected planar graph. ♦

3.2 Visibility Dual Face Walking

3.2.1 The Virtual Coordinates

While extremely simple to implement, left compass routing sometimes generates long routing paths winding through the graph. It seems more natural that the path follows the edges closest to a straight line in the plane between the source and destination. Our next routing algorithm is based on this idea, but is a little more complicated. The virtual "coordinates" of the graph vertices are derived from the faces of a rubber band embedding of the *dual* graph.

It is not obvious which face of the dual should be considered the outer face, and, even if it was obvious, the dual mapping causes a primal vertex to vanish as it is mapped to this outer face. To overcome this difficulty, we *stellate* the outer primal face by augmenting the primal with a *dummy* vertex connected it to all the vertices on this face. Then we compute the combinatorial dual of the stellated graph and identify its boundary vertices as the primal faces incident on the dummy vertex. Thus, the primal vertex that corresponds to the outer face of the dual (and vanishes) is the dummy vertex.

After creating a rubber band embedding of the dual, each primal vertex stores the locations of the vertices dual to its incident faces. This allows the primal graph to simulate a face walking algorithm on the dual graph.

3.2.2 The Routing Algorithm

We use the visibility face walking algorithm [8] on the dual graph. This was originally designed as a point location algorithm on a convex planar tiling. When at a face f, visibility walk proceeds to a neighboring face g if the line supporting the common edge between f and g separates the destination t from f. We will run a visibility face walking algorithm on the dual graph, by simulating it on the vertices of the primal graph. Given a destination vertex t, our dual destination point t' will be the barycenter of the face dual to t. When at vertex v, we forward to neighboring vertex u if the edge e' dual to the primal edge $e = (v,u)$ separates the destination t' from the face dual to v. Note that e' is an edge of this dual face, and the computation may be facilitated by examining its barycenter.

Theorem 2: Visibility dual face walking works for any rubber band embedding of the dual of a 3-connected planar graph. ♦

3.3 Three Dimensional Hill Climbing

3.3.1 The Virtual Coordinates

Steinitz's theorem states that every 3-connected planar graph has an embedding as a strictly convex polyhedron in R^3. This embedding is not unique, and a number of such constructions exist. The simplest is probably the Maxwell-Cremona *lifting* of the Tutte rubber band embedding with a triangular boundary [15]. Thus an arbitrary triangle is chosen to be this boundary. In the rare case that the 3-connected planar graph does not contain a triangle that may be used as the boundary, the dual graph, its lifting and polar dual may be used instead, since either the primal or its dual must contain a triangle [15].

For the virtual coordinates will use the 3D coordinates of the vertices of this polyhedron, with the following twist: the coordinates of the target vertex will be the unit normal of a *supporting plane* of that vertex – a plane through the vertex such that the polyhedron is entirely on one side of it. The existence of such a plane is guaranteed by the convexity of the polyhedron, and may be computed by linear programming at each node, involving just the coordinates of the node and its neighbors.

3.3.2 The Routing Algorithm

At vertex *v*, we forward the packet to the neighbor of *v* which is closest to the destination *t*'s supporting plane. This is implemented by forwarding to the neighbor *u* such that

$$u = \arg\max_{u \in N(v)} (n_t \cdot u)$$

where n_t is the unit normal of that plane. The algorithm delivers since, as Papadimitriou and Ratajczak [13] observed, in a strictly convex polyhedron every vertex has a neighbor which is strictly closer to another vertex' supporting plane. Indeed, for the special case of a polyhedron which is edge tangent to a sphere, our algorithm is equivalent to Papadimitriou and Ratajczak's *polyhedral routing* algorithm.

Theorem 3: Three dimensional hill climbing works for any 3D lifting of a 3-connected planar graph. ♦

3.4 Greedy Power Routing

Our final algorithm has more theoretical than practical value. Although it is well known [6] that greedy routing works for Delaunay triangulations, there is no known generalization to non-triangulated planar graphs. Moreover, not every planar graph whose faces are all triangles is Delaunay-realizable [9]. We present a generalization of greedy routing to general 3-connected planar graphs. The embedding is planar, but the routing requires the use of an extra scalar value for each vertex.

3.4.1 The Virtual Coordinates

An *orthogonal dual* of a convex tiling is a planar embedding of the graph dual to the tiling, such that primal-dual edge pairs lie on orthogonal lines. We consider the setting in which the faces dual to boundary vertices are unbounded, and the vertex dual to the outer face is not embedded.

For a 3-connected planar graph, there may exist many orthogonal primal/dual embedding pairs. Define a *contained embedding* of a 3-connected planar graph to be an orthogonal primal/dual embedding pair, such that each primal vertex is strictly contained in its dual face.

A *power diagram* on a set of sites v_1 ,.., v_n having coordinates (x_i,y_i) in the plane and associated power radii r_i, is the partition of the plane to convex regions such that the all points x in the region R_i associated with v_i are closer to v_i than to any other site using the distance function $pow(x,v_i) = d^2(x,v_i) - r_i^2$, where $d(x,v_i)$ is the Euclidean distance between x and v_i, and r_i is the power radius associated with v_i. The famous Voronoi diagram is the special case when all r_i are identical (not necessarily zero). Power diagrams are sometimes called *weighted Voronoi* diagrams.

Lemma 1: Any 3-connected planar graph and its dual have a *contained embedding*.

Proof: Follows from the "kissing disks" embedding theorem of Koebe and Andre'ev [16], which is by definition a contained embedding. ♦

Note that a contained embedding is not necessarily unique. For example, if the graph happens to be a Delaunay-realizable triangulation, then any Delaunay realization and its dual Voronoi diagram are also a contained embedding.

A theorem of Aurenhammer [1] states that all orthogonal duals are power diagrams of the primal vertices (with some radii). In the special case of the Koebe-Andreev contained embedding, it is easy to show that the radii of the orthogonal dual power diagram are also the radii of the inscribed circles of the dual faces (whose centers are at the primal vertices).

For a given 3-connected planar graph, the virtual coordinates that we propose are the locations of the primal vertices in a contained embedding, and the accompanying power radii of the dual.

3.4.2 The Routing Algorithm

The routing algorithm is greedy using the *pow* distance function associated with the power radii. Namely, to route to destination *t* when at vertex *v*, forward to the neighboring vertex *u* such that

$$u = \arg\min_{u \in N(v)} pow(t,u)$$

Theorem 4: Greedy power routing works for any contained embedding of a 3-connected planar graph, and its associated power radii. ♦

4 Experimental Validation

The competitiveness of routing algorithms is an important feature. Although none of our algorithms is competitive, it is still useful to know how well they perform in practice, namely, what the average routing path lengths are. In our experiments, we used uniformly-distributed random 3-connected planar graphs [10], and applied the methods described in this paper to them. Following Bose et al [6], we define the *competitive ratio* for a pair of vertices (s,t) to be *routing path length*(s,t)/*shortest path length*(s,t). Path lengths are measured in hops (and not in Euclidean distance). For each graph and each routing algorithm we computed the average competitive ratio for all vertex pairs in the graph. We repeated this experiment with 10 graphs per graph size (averaging over all 10 graphs), and compared the results with those of state-of-the-art algorithms which are guaranteed to deliver on general convex tilings: GPSR (which is very similar to the GFG algorithm), random compass and polyhedral routing. GPSR and random compass were run on the simplest possible rubber band embeddings of the graphs – where the largest face is embedded as a circular boundary and all weights are unit.

Figure 1 shows our results. As expected, random compass performs significantly worse than the others, whose average competitive ratios are below 2.0. The two 3D algorithms are the most efficient and perform comparably. Construction of the lifting for hill climbing required dealing with some minor numerical precision problems, as the dihedral angle between adjacent faces is sometimes quite small. Other numerical problems were also encountered in the routine [2] for computing the edge-tangent polyhedron required for polyhedral routing. Of the 2D algorithms, our power routing seems to be the most efficient, but GPSR is only slightly worse. The performance of these algorithms is closely followed by that of dual face walking and left compass routing.

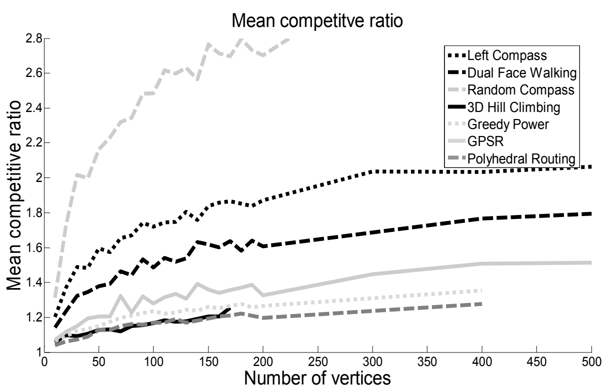

Figure 1: Mean competitive ratios for some routing algorithms.

5 Conclusions and Future Work

Although many local geographic routing algorithms exist, there is no deterministic oblivious algorithm which guarantees delivery on a general 3-connected plane graph. Thus the only hope is that a *specific* embedding may be computed, which does work. We proposed four such embeddings and associated routing algorithms, and showed that their performance is good in practice, and one of them is superior to all the existing ones. In real-world applications the virtual coordinates must be constructed in a distributed manner. Our first three constructions are simple enough to facilitate this.

Choice of a routing algorithm involves a performance/implementation tradeoff. Left compass routing is extremely simple to implement, but it may generate long routing paths. Dual face routing generates better paths, but it requires a more sophisticated construction of the virtual coordinates. Hill climbing requires 3-dimensional virtual coordinates, but these coordinates are significantly easier to construct than those of polyhedral routing. Moreover, this algorithm has an additional advantage over our first two algorithms - it can be applied to a non-planar graph once the virtual coordinates of a planar subgraph are found, without compromising the guaranteed delivery property.

Despite its good performance, greedy power routing is the least practical algorithm among those we propose here, because of the complicated construction involved. But it is not the least interesting. The only method at our disposal today to generate a contained embedding of a 3-connected planar graph is the implementation of Bobenko [2] of the "kissing disk" embeddings [16], which is much stronger than what we need. If an easier way of generating a contained embedding is found, this algorithm may be of practical value, beyond its theoretical value as a generalization of greedy routing on Delaunay triangulations.

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