Adaptive Searching in One and Two Dimensions

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1 Introduction

Searching in a geometric space is an active area of research, predating computer technology. The applications are varied ranging from robotics, to search-and-rescue operations in the high seas [24, 23] as well as in land, such as in an avalanche [5] or an office space [12, 7, 13], to scheduling of heuristic algorithms for solvers searching an abstract solution space for a specific solution [16, 17, 22, 2, 19]. Within academia, the field has seen two marked boosts in activity. The first was motivated by the loss of weaponry off the coast of Spain in 1966 in what is known as the Palomares incident and of the USS Thresher and Scorpion submarines in 1963 and 1966 respectively [24, 26]. A second renewed thrust took place in the late 1980s when the applications for autonomous robots became apparent.

Geometric searching has proved a fertile ground within computational geometry for the design and analysis of search and recognition strategies under various initial conditions [14, 12, 6, 7, 8, 18, 20].

The basic search scenarios consist of exploring a one dimensional object, such as a path or office corridor, usually modeled as the real line, and of exploring a two dimensional scene, such as a room or a factory floor, usually modelled as a polygonal scene. However, in spite of numerous advances in the theoretical understanding of both of these scenarios, so far such solutions have generally had a limited impact in practice.

Over the years various efforts have been made to address this situation, both in terms of isolated research papers attempting to narrow the gap, as well as in organized efforts such as the Algorithmic Foundations of Robotics conference and the Dagstuhl seminars on on-line robotics which bring together theoreticians and practitioners. From these it is apparent that the cost model and hence the solutions obtained from theoretical analysis do not fully reflect real life constraints. Several efforts have been made to resolve this, such as including the turn cost, the scanning cost, and error in navigation and reckoning [9, 10, 15, 20, 18].

In this paper we address one more shortcoming of the standard model. Consider for example a vacuuming robot—such as Roomba(TM). Such a robot explores the environment using sophisticated motion planning algorithms with the goal of attaining complete coverage of the floor surface within a reasonable amount of time. It is not hard to devise worst case floor plans (such as complex mazes) which would not be covered very efficiently. In practice this is not a concern since (i) most rooms are relatively simple and (ii) if the robot ever encounters such a complex scene a drop in performance is only to be expected and users would not mind a severe degradation in performance. This naturally leads to the concept of adaptive algorithms, in which on simpler inputs the robot must perform more efficiently than on more complex ones.

In this paper we consider adaptive analysis of two basic geometric primitives: searching on the real line and looking around the corner.

Searching on the real line consists of finding a target t on the real line located at an unknown distance d (in either direction) from a search robot. The robot detects t upon contact. The optimal strategy visits the rays under a doubling strategy with competitive ratio of 9 [4, 11, 3, 21]. We refer the reader to the survey of Alpern and Gal [1] for a thorough discussion. However upon being presented by the optimal doubling strategy practitioners routinely report that they find the answer non-intuitive and generally "not optimal". This holds for the optimal strategy for either the average or the worst case. There are several non-mutually exclusive explanations for this disparity. In particular we incorporate the observation that in some settings, exploration is a valuable task in which case the goal is to simultaneously minimize the time to the target, and maximize the amount of information gained during the search. For this case we obtain an optimal strategy that is, subjectively, more pleasing to practitioners.

For the second case study we consider searching around a corner. Icking et al. [14] provided an algorithm with competitive ratio $c \approx 1.21218$ and proved that this is the best competitive ratio possible. We extend this result by applying adaptive analysis to this problem.

2 Searching on the Real Line

Without loss of generality, we assume the robot searches starting from the origin x_1 units to left, then it returns to the origin and moves past it x_2 units to the right. In general in the ith phase, it goes x_i units from origin to left or right (depending on the parity of i) and returns to the origin. The search ends when the robot finds the

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target. In the *doubling strategy* we have $x_i = 2^{i-1}$. In the standard cost model, we minimize the ratio of the distance travelled by the robot to the straight distance from the target to the origin, which is termed the competitive ratio. As stated before, the doubling method has competitive ratio 9, which is optimal.

In order to reflect the "waste" of robot when traversing a region that has already been explored, we propose a dual cost model. It costs one unit whenever the robot traverses one unit of distance of unknown territory, while it costs c units ($c \geq 1$) when the robot traverses a region that has already been explored.

In order to find the worst case for doubling method under the new cost model, assume that the target is located at distance $2^k + \varepsilon$ from the origin, for some integer k. Therefore robot will find the target at phase $k+3$. For $3 \leq i \leq k+2$, let $C(i)$ be the cost robot incurs at phase i. At phase i, the robot goes 2^{i-1} units away from the origin and then returns to the origin. Of the first 2^{i-1} units, 2^{i-3} units are already explored and $2^{i-1} - 2^{i-3} = 3 \times 2^{i-3}$ units are newly explored. All 2^{i-1} units on the robot's return to the origin are already explored. Therefore we have $C(i) = 2^{i-3}(5c+3)$. Thus the total cost of the first $k+2$ phases is $(1+c) + (2+2c) + \sum_{i=3}^{k+2} 2^{i-3}(5c+3) = (5 \times 2^k - 2)c + 3 \times 2^k$. In the last phase, the robot finds the target at distance $2^k + \varepsilon$, incurring cost $2^k c + \varepsilon$. Thus the competitive ratio of doubling is $\frac{(6 \times 2^k - 2)c + 3 \times 2^k + \varepsilon}{2^k + \varepsilon}$ which becomes arbitrarily close to $6c+3$ as k grows. Note that for $c = 1$ we get the standard competitive ratio of 9.

Observe that the doubling might no longer be the optimal strategy under the new model. As usual we consider the family of geometric search strategies A_r: we have $x_i = r^{i-1}$ for an arbitrary real number $r > 1$ (the doubling strategy corresponds to $\mathcal{A}_2$). Using arguments similar to the analysis of the doubling method, the cost of robot at phase $3 \leq i \leq k+2$ is $C(i) = r^{i-3}((r^2+1)c + (r^2-1))$ and the total cost of $\mathcal{A}_r$ is $(r + 1 + (r^2+1)(\frac{r^k-1}{r-1}) + r^k)c + (r^2-1)(\frac{r^k-1}{r-1}) + \varepsilon$. Thus the competitive ratio of A_r for this case is $CR(\mathcal{A}_r) = \frac{(r+1+(r^2+1)(\frac{r^k-1}{r-1})+r^k)c+(r+1)(r^k-1)+\varepsilon}{r^k+\varepsilon}$, which becomes arbitrarily close to $(\frac{r^2+r}{r-1})c + r + 1$ as k grows. Through symbolic manipulation, we find out that the competitive ratio is minimized for $r = 1 + \frac{\sqrt{2c+2c^2}}{c+1}$. As c goes to ∞, this optimal value of r goes to $1+\sqrt{2} = 2.414213\ldots$ with a search cost of $(3+2/\sqrt{2})c + 2 + \sqrt{2} \approx 5.83c + 3.41$. This improves over the $6c+3$ cost of doubling for large c.

Furthermore, this is optimal, as it can be shown using the Gal-Schuierer functional theorem [11, 25] as follows. For any given strategy, let $X = x_0, x_1, x_2, \ldots$ denote the (infinite) sorted sequence of turn points incurred by the strategy. Then using ideas similar to [22] we can lower bound the competitive ratio by $CR \geq cost(ALG)/cost(OPT)$, where $cost(ALG) = (x_0 + cx_0) + (x_1 + cx_1) + (x_2 - x_0 + cx_0 + cx_2) + \ldots + (x_{k+1} - x_{k-1} + cx_{k-1} + cx_{k+1}) + cx_k$, and $cost(OPT) = x_k$. Therefore, we have that

$$CR(X,k) \geq \frac{(c+1)\sum_{i=0}^{k+1} x_i + (c-1)\sum_{i=0}^{k-1} x_i + cx_k}{x_k} \quad (1)$$

Let $X^{+i} = (x_i, x_{i+1}, \ldots)$ denote the suffix of a sequence $X = (x_0, x_1, \ldots)$ starting at x_i.

Theorem 1 ([25]) *Let $X = (x_0, x_1, \ldots)$ be a sequence of positive numbers, r an integer, and $a = \limsup_{n\to\infty}(x_n)^{1/n}$, for $a \in \mathbb{R} \cup \{+\infty\}$. If F_k, $k \geq 0$, is a sequence of functionals which satisfy*
(1) $F_k(X)$ only depends on $x_0, x_1, \ldots, x_{k+r}$,
(2) $F_k(X)$ is continuous, $\forall x_i > 0$, with $0 \leq i \leq k+r$,
(3) $F_k(\alpha X) = F_k(X)$, $\forall \alpha > 0$,
(4) $F_k(X+Y) \leq \max(F_k(X), F_k(Y))$, and
(5) $F_{k+i}(X) \geq F_k(X^{+i})$, $\forall i \geq 1$,
then $\sup_{0\leq k<\infty} F_k(X) \geq \sup_{0\leq k<\infty} F_k(A_r)$.

It is not hard to verify that the hypothesis of the theorem holds for the modified cost model, and hence it suffices to consider x_i of the form r^{i-1} in the expression for $CR(X,k)$ above. Note that the left-hand side of inequality 1 above is precisely the expression we derived when upper-bounding the competitive ratio. Therefore, substituting r with $1+\sqrt{2}$ yields a lower bound on $CR(X,k)$ which is identical to the upper bound, which in turn implies that the geometric strategy with $r = 1+\sqrt{2}$ is in fact optimal.

We can extend our dual cost model to cases in which $c < 1$, i.e., revisiting is less expensive than discovering. As suggested by an anonymous reviewer, the case $c = 0$ can also be used to model two sequential communicating searchers. If $c < 0$, the robot can reduce its cost by revisiting the discovered territories forever and no optimal strategy exists. For $0 < c < 1$, we can use an analysis analogous to the case $c \geq 1$ to show that A_r with $r = 1 + \frac{\sqrt{2c+2c^2}}{c+1}$ is optimal. For $c = 0$, the optimal strategy is A_r with $r = 1+\varepsilon$ for a very small constant ε and this leads to the competitive ratio $2+\varepsilon$.

3 Looking Around a Corner

In this particular case we consider the setting in which the robot is exploring a man made setting in which there is a preferential occurrence for orthogonal and near orthogonal angles. We wish to explore the change in the nature of the solution when this assumption is made.

We follow the same approach as [14] and formulate the problem using a differential equation. Therefore we use similar terminology and notation and just highlight the differences between the methods; refer to [14] for omitted details. First we formally define the problem. Figure 1 shows a typical instance of the problem. The

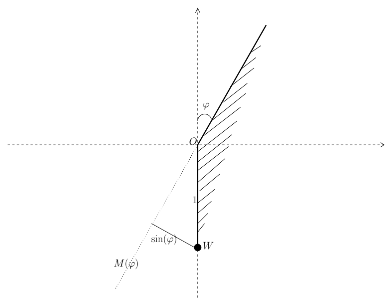

Figure 1: A typical instance of the corner problem.

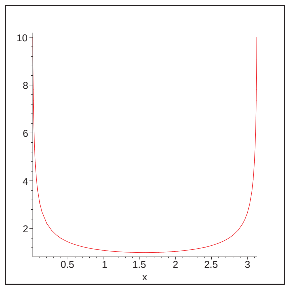

Figure 2: Plot of $d(\varphi) = \frac{1}{\sqrt{\sin\varphi}}$ for $0 < \varphi < \pi$.

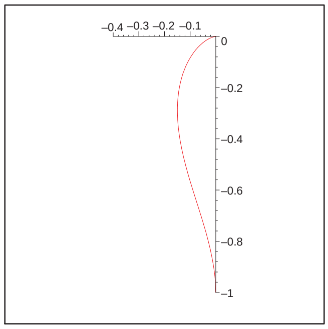

Figure 3: Robot's Optimal Path in the New Model.

corner is placed at the origin O and one of its halflines coincides with the negative y axis. The other halfline of the corner makes an angle $0 \le \varphi \le \pi$ with the positive y axis. A mobile robot is located at point $W = (0, -1)$ and is equipped with an on-board vision system facing O. When $\varphi > 0$, the robot cannot see the other halfline (wall) of the corner and his goal is to discover that (invisible) halfline by minimum movement. The robot sees the invisible line the first time it visits any point on the prolongation $M(\varphi)$ of the invisible line. Let $a(\varphi)$ be the distance between W and $M(\varphi)$. We have

$$a(\varphi) = \begin{cases} \sin\varphi & \text{if } 0 \le \varphi \le \pi/2 \\ 1 & \text{if } \pi/2 < \varphi \le \pi \end{cases} \tag{2}$$

If the robot knows φ then it can discover the invisible wall by the optimal movement $a(\varphi)$. However this is not the case and the robot should come up with a strategy S that works for all $0 \le \varphi \le \pi$. Let $A_S(\varphi)$ be the length of the path generated by S from W to the first point on $M(\varphi)$. Then the *competitive function* of S is defined as $f_S(\varphi) = \frac{A_S(\varphi)}{a(\varphi)}$ and the *competitive factor* of S is defined as $c_S = \sup_{\varphi\in(0,\pi]} f_S(\varphi)$.

In practical robot navigation most corners have angles close to $\pi/2$ and usually we do not have angles close to 0 or π. As a first attempt for applying adaptive analysis ideas we consider $d(\varphi) = 1/\sqrt{\sin\varphi}$ as difficulty measure. Figure 2 shows the behaviour of $d(\varphi)$ for $0 < \varphi < \pi$. We normalize the competitive function further by $d(\varphi)$ and the new competitive function is defined as $g_S(\varphi) = \frac{f_S(\varphi)}{d(\varphi)} = \begin{cases} \frac{A_S(\varphi)}{\sqrt{\sin\varphi}} & \text{if } 0 \le \varphi \le \pi/2 \\ A_S(\varphi)\sqrt{\sin\varphi} & \text{if } \pi/2 < \varphi \le \pi \end{cases}$

Icking et al. [14] describe the strategies by curves of form $S = (\varphi, s(\varphi))$ in polar coordinates about O that satisfy certain properties, e.g., $s(0) = 1$. They show that the optimal competitive strategy is given by the solution to

$$f_R(\varphi) = \frac{A_R(\varphi)}{\sin\varphi} = c,$$

for all $\varphi \in [0, \pi/2]$ and for some constant c (the smallest c if there are several solutions). For our cost model, the corresponding equation becomes $g_R(\varphi) = \frac{A_R(\varphi)}{\sqrt{\sin\varphi}} = c$. We have $A_R(\varphi) = c\sqrt{\sin\varphi} \Rightarrow \frac{c\cos\varphi}{2\sqrt{\sin\varphi}} = A_R'(\varphi) = \sqrt{r'^2(\varphi) + r^2(\varphi)} \Rightarrow r'(\varphi) = -\sqrt{\frac{c^2\cos^2\varphi}{4\sin\varphi} - r^2(\varphi)}$. We take the negative square root because in an optimal strategy the robot should always come closer to the corner. By replacing $r(\varphi)$ by $cu(\varphi)$ we get the differential equation

$$u'(\varphi) + \sqrt{\frac{\cos^2\varphi}{4\sin\varphi} - u^2(\varphi)} = 0, \tag{3}$$

with initial condition $u(0) = 1/c$. Therefore, our problem reduces to:

Problem Find the minimum $c > 1$, such that the ordinary differential equation (3) has a solution on some interval $[0, \sigma] \subseteq [0, \pi/2]$, subject to the following constraints:

$$u(0) = 1/c \qquad u(\varphi) > 0 \quad \text{for } \varphi \in [0, \sigma] \qquad u(\sigma) = 0$$

Since this type of differential equations generally do not have a closed form we use numerical methods to compute the solution $c \approx 1.08$. The strategy with this competitive factor is shown in Figure 3. We can prove the optimality of this strategy using arguments analogous to [14].

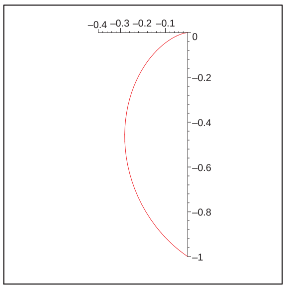

Figure 4: Robot's optimal path in the previous model.

The optimal strategy in the standard model is shown in Figure 4. It has competitive factor ≈ 1.21 [14]. Observe that since less weight is given to small angles the solution takes a shorter path to reach sightlines for angles around $\pi/4$.

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