Intrinsic Multiscale Geometry

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Abstract

The geometric analysis of point cloud data as well as of triangle meshes is intimately tied to the notion of scale at which the data is examined. It is well known that different features of the data manifest at different scales and the discovery of scales at which prominent features appear in a stable form is a challenging research problem. This question has recently been addressed by both classical multiresolution analysis tools, as well as by persistent homology approaches. Many (though not all) of these tools, however, approach the data in an extrinsic form, requiring an explicit or implicit embedding of the shape or space from which the samples are taken and/or of any simplicial approximations obtained.

In this talk we focus on multiscale analysis in the intrinsic setting, an area that has been much less explored. Our aim is to study interesting properties of geometric data when only geodesic distances between the samples are given (and, if occasionally extrinsic information is needed as well, we aim to limit it to local embeddings of small neighborhoods of the shape). Specifically, we look at

- the reconstruction of shapes from samples using the witness complex, under mild sampling conditions – including the ability to have different reconstructions at different scales
- multiscale descriptors for shape neighborhoods obtained by examining heat diffusion on manifolds and restricting the heat kernel to the temporal domain – what we call heat kernel signatures (HKS)
- the analysis of scalar fields over shapes using persistent homology, exploiting recent advances in the stability of persistence diagrams

The approaches presented come with correctness guarantees and can be realized by algorithms with reasonable complexities. An advantage of the intrinsic approach is that the corresponding analysis can be used to compare different isometric embeddings of the same shape. It also allows us to consider the same questions in more general data analysis settings where the ambient dimension may be high or where no natural embedding of the data in a Euclidean space may be available.

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