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GOLDSTONE MECHANISM FROM GLUON DYNAMICS

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ABSTRACT

A single assumption about pure gluon (Yang-Mills) theory is shown to imply the Nambu-Goldstone realization of chiral symmetry in QCD as well as the resolution of its U(1) problem.

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$$\langle D_i\, Q \rangle\big|_{\ell=1} = -2 \langle Q\, Q \rangle\big|_{Y.M.} \tag{1b}$$

$$\langle D_i\, D_j \rangle\big|_{b=\ell=2} = 4 \langle Q\, Q \rangle\big|_{Y.M.} \tag{1c}$$

$$i, j = 1 \ldots L$$

where L is the number of light flavours and, in standard notation[4),9)]:

$$\begin{aligned} \Delta_i &= -2 m_i \bar{q}_i q_i \\ \partial_\mu J^{(i)}_{\mu,5} &\equiv \partial_\mu (\bar{q}_i \gamma_\mu \gamma_5 q_i) = D_i + 2Q \\ D_i &= 2 i m_i \bar{q}_i \gamma_5 q_i \\ Q &= \frac{g^2}{64\pi^2} \varepsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma} \end{aligned} \tag{2}$$

The notation $\langle\cdot\cdot\rangle|_{\cdot\cdot}$ stands for Fourier transform at zero momentum with some topological qualification, i.e.,

i) all diagrams considered have planar topology, hence no handles (h = 0 in the notation of Ref. 11));

ii) diagrams have no superfluous quark loops (w = 0 in the notation of Ref. 11));

iii) because of ii) the number b of boundaries (distinct quark loops on which an insertion is made[11)]) equals the total number ℓ of quark loops;

iv) gluonic correlations such as $\langle QQ \rangle$ are computed in pure Yang-Mills (YM), i.e., in the absence of quarks.

The usual procedure[7),9)] consists of assuming chiral symmetry breaking in the form

$$\langle \bar{q}_i q_i \rangle\big|_{\ell=1} = O(1) \;, \text{ i.e. } \langle \Delta_i \rangle\big|_{\ell=1} = O(m_i) \tag{3}$$

Then, since the left-hand side of Eq. (1a) is superficially of $O(m_i m_j)$, one needs L^2 NG bosons of squared mass O(m) in the planar (b = 1, h = 0) approximation. Next, one assumes[7),9)] that the right-hand side of Eqs (1b) and (1c) is non-zero:

$$\langle Q\, Q \rangle \Big|_{Y.M.} = i A \quad ; \quad A = c \Lambda^4 \tag{4}$$

with c, in principle, a calculable number of O(1) and $\Lambda \simeq 0.5$ GeV the scale of QCD. Noticing that the left-hand side of Eq. (1c) has double NG poles (because of b = 2), one checks that such an equation is indeed satisfied provided that there is a non-zero gluonic contribution to the pseudoscalar mass matrix:

$$\langle \pi_i | \Sigma_{gluon} | \pi_j \rangle = 2A/F_\pi^2 \neq 0 \; ; \; i,j = 1 \ldots L \tag{5}$$

where π_i is the planar level $q_i\bar{q}_i$ "pion"[10] and $F_\pi \neq 0$ is defined by:

$$\langle 0 | J^{(i)}_{\mu,5} | \pi_j \rangle = -i \delta_{ij} \sqrt{2} F_\pi q_\mu \; ; \; F_\pi = O(\sqrt{N_c}\,\Lambda) \; ; \quad F_\pi^{exp.} \simeq 95 \text{ Mev} \tag{6}$$

Equation (5) solves the U(1) problem[7),9),10)].

Our simple observation here is that assumption (4), with $A \neq 0$, implies both Eq. (5) and Eq. (6), hence SCSB.

In order to prove this, consider again Eq. (1c). If $A \neq 0$, such an equation requires light "pions" π_i ($m^2_{\pi_i} \sim O(m_i)$) coupled to $J_{\mu,5}$ as in Eq. (6) and satisfying:

$$2 F_\pi^2 \langle \pi_i | \Sigma_{gluon} | \pi_j \rangle = 4A \neq 0 \; ; \; i,j = 1 \ldots L \tag{7}$$

Notice that only the $O(N_c^{1/2})$ non-anomalous contribution to $\langle 0|D|\pi\rangle$ is picked up.

Since neither F_π nor Σ_g can be infinite (we assume[9)] no massless glueballs in pure YM theory) neither one can be zero, thus proving our assertion. By inserting the above information into Eq. (1a), it is also simple to show that

$$\langle \bar{q}_i\, q_i \rangle \Big|_{\ell=1} = O(m^2_{\pi_i}/m_i) = O(1) \quad \text{as } m_i \to 0$$

in clear violation of chiral symmetry[*)].

In view of the well-known[6)] selection rules relating chirality and topological charge the above conclusion is hardly a surprise. The only new and interesting feature here is that we were able to connect two properties of QCD, chiral symmetry breaking and the absence of a U(1) NG particle, to a property of the (a priori academic) Yang-Mills theory of gluons!

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*) The necessity of NG bosons can also be proved directly in the chiral limit ($m_i = 0$) and even if all quarks had a definite helicity. I thank D. Amati for a useful discussion on this point.

Yet, this is not the first example of this type of connection. The famous Wilson criterion for confinement[12)] is supposed to apply to YM theory, but still to imply confinement of quarks.

The above observation suggests that we should attempt to compute A by techniques of the type successfully used to test Wilson's criterion and to establish the phase structure of lattice QCD: we refer to the strong coupling expansion[13)] and to Monte Carlo methods[14)]. It is possible that one can get an evaluation of A in terms of the string tension by these methods or, at least, that one can understand the relation, if any, between a non-vanishing string tension (confinement) and a non-vanishing value for A (chiral symmetry breaking without a U(1) Goldstone boson). Following Ref. 15), one indeed expects $A \neq 0$ in lattice QCD.

An alternative, somewhat more formal approach could follow recent arguments by Kugo[16)] according to which the same (modified) Kogut-Susskind mechanism provides the solution of the U(1) problem and automatic quark confinement. If this daring connection could be made rigorous and extended to the large N expansion framework, one could then argue that colour confinement in large N_c YM theory ensures SCSB in QCD, in agreement with the claims made in Refs 1) and 3).

In any event, our connection between properties of a pure gauge theory ($A \neq 0$) and chiral symmetry breaking when (massless) fermions are added in, could provide useful constraints on candidate (non-QCD-like) models for composite quarks and leptons.

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FIGURE CAPTION

(a) Non-anomalous Ward identity in the one-loop planar approximation.

(b)(c) Anomalous Ward identities with minimal quark loop number. Shaded blobs represent the sum of all diagrams of a given topology.

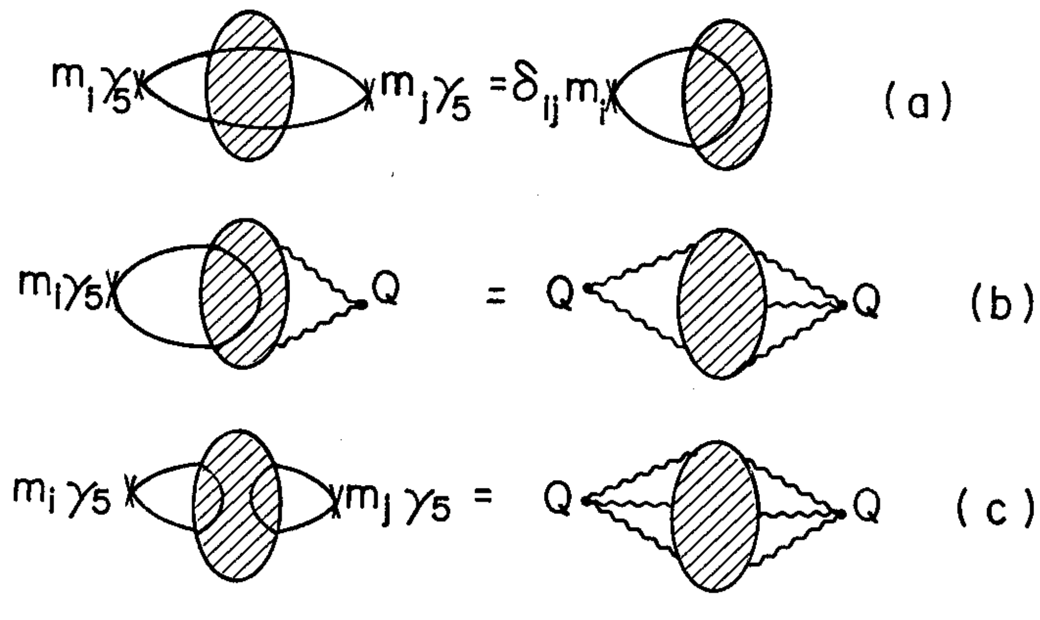

fig. 1