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A SIMPLE COMPONENT FIELD METHOD FOR SUSY EFFECTIVE POTENTIAL CALCULATIONS

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A B S T R A C T

S. Weinberg's tadpole method for the calculation of effective potentials of conventional field theories also provides an attractive means for calculating SUSY effective potentials. The essential trick is to leave the SUSY unconstrained and consider auxiliary field tadpoles. The method is illustrated for the Wess-Zumino model.

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1.-INTRODUCTION

In conventional field theory [by which I mean non-supersymmetric (SUSY) theories], one might say there are three popular ways of performing the calculation of effective potentials[1),2),3)]. There is the original method of Coleman and Weinberg [1)]. This method has a number of drawbacks. First of all, at any loop order there are an infinite number of Feynman graphs to consider. With these we have the problems of combinatorial factors, counting topologically distinct graphs, and then summation. On top of this, going beyond one-loop order in this fashion seems a formidable task[4)]. There is also the functional method of Jackiw[2)] which presents an improvement on the above. However, the most elegant method of the three mentioned is that due to S. Weinberg[3)]. Here the effective potential can be calculated by considering the scalar tadpole graphs of the translated theory. (A brief review will be given in §2.) The Coleman-Weinberg problems are avoided and further, one is considering the simplest possible loop graphs in the theory. Beyond a one-loop level, it provides <u>the</u> way to proceed [for example, see Refs. 4) and 5)].

When we consider SUSY theories there are a number of ways to proceed. One can either do a component field calculation[6),7),8)] or a superfield calculation[9),10)]. In principle, for either approach all three popular methods can be employed. Surprisingly enough, the tadpole method has not been pursued for SUSY calculations. For example, via component field methods, Huq[6)] pursues the functional method of Jackiw[2)]. O'Raifeartaigh and Parravicini[7)] reply upon the original Coleman-Weinberg paper[1)] and Lang and Fujikawa[8)] refer back to an earlier formulation of Feynman[11)]. As for the superfield approach, Huq applies the (non-graphical) method of Jackiw once more, and only recently have supergraph techniques been developed for such calculations [Grisaru, Riva and Zanon[10)]]. These authors tackle the problem via the Coleman-Weinberg approach (with necessary detours to simplify things).

In this letter I will show that the S. Weinberg tadpole method[3)], as in conventional field theory, provides the most attractive approach for the component field calculation. Application of the method to the Wess-Zumino (W-Z) model[12)] will show how the one-loop effective potential is derived in a very simple fashion. The principle ingredient which makes the calculation so simple is due to the fact that I will not constrain the theory, i.e., I will not eliminate auxiliary fields via their equations of motion. Consequently we can consider auxiliary field tadpole graphs. This approach will be particularly useful at higher loop order, since the auxiliary field does not couple directly to the spinor fields, hence there will be fewer graphs to evaluate than the approach where one uses spin zero component field tadpoles.

In section 2 I review the tadpole method. In section 3, the Wess-Zumino model is written down and propagators of the translated theory derived. In section 4, the auxiliary field tadpole method is displayed and finally, in section 5, I make some concluding remarks.

2.- DEFINING THE EFFECTIVE POTENTIAL AND THE TADPOLE METHOD

To arrive at the required results, it will be sufficient to consider a spin zero field B described by a Lagrangian $\mathcal{L}$. The effective action, the generating functional of one particle irreducible (1PI) amplitudes, will be denoted by Γ. This may be given a momentum space expansion in the classical field B_{cl},

$$\Gamma(B_{cl}) = \sum_n \int dp_1 \cdots dp_n \, (2\pi)^4 \delta(\sum_i p_i) \frac{1}{n!} \Gamma^{(n)}(p_1, \cdots p_n) \, B_{cl}(p_1) \cdots B_{cl}(p_n) \tag{2.1}$$

The effective potential for this theory is then defined by

$$V = \sum_n \frac{1}{n!} \left(- \Gamma^{(n)}(p_i = 0) \right) B_{cl}^n \tag{2.2}$$

We see

$$\left. \frac{dV}{dB_{cl}} \right|_{B_{cl}=0} = - \Gamma^{(1)}_{p_{extn}=0} \tag{2.3}$$

$\Gamma^{(1)}_{p_{extn}=0}$ is the 1PI tadpole of the theory with zero external momentum, calculated using $\mathcal{L}$. Now translate the theory, i.e. set

$$B_{cl} = B'_{cl} + b \tag{2.4}$$

where b is a constant. Expanding the theory about $B'_{cl} = 0$ we write $\mathcal{L}(B_{cl}) \equiv \mathcal{L}(B'_{cl} + b) = \mathcal{L}'(B'_{cl})$. Thus

$$\left. \frac{dV}{dB'_{cl}} \right|_{B'_{cl}=0} = - \Gamma^{(1)\prime}_{p_{extn}=0} \tag{2.5}$$

Here $\Gamma^{(1)'}_{p_{extn}=0}$ is the zero external momentum tadpole calculated using $\mathcal{L}'$. Equally, Eq. (2.5) may be written as

$$\left.\frac{dV(B_{cl})}{dB_{cl}}\right|_{B_{cl}=b} = -\Gamma^{(1)'}_{p_{ext\,n}=0} \tag{2.6}$$

Since $V(B_{cl})$ denotes V expanded in terms of the untranslated theory, it can have <u>no</u> b dependence; then (2.6) may be written as

$$\frac{dV(b)}{db} = -\Gamma^{(1)'}_{p_{ext\,n}=0} \tag{2.7}$$

This is the central equation of the tadpole method. It tells us to take the given theory, translate the fields, calculate the tadpole of the translated theory, substitute the result into (2.7) and then integrate with respect to b to obtain $V(b)$, finally to replace b by B_{cl} to obtain the effective potential expressed in terms of the untranslated theory. For greater detail, see Weinberg[3].

3.- THE WESS-ZUMINO MODEL AND THE PROPAGATORS OF THE TRANSLATED THEORY

The W-Z model in superfield notation[13] is given by

$$\mathcal{L} = [\bar{\phi}\phi]_{\theta^2\bar{\theta}^2} - [\ (\tfrac{m}{2}\phi\phi + \tfrac{\lambda}{3!}\phi\phi\phi)_{\theta^2} + H.C.] \tag{3.1}$$

ϕ, the chiral superfield, contains the component fields A (spin zero), ψ (spin one half) and the auxiliary field F. In terms of component fields, (3.1) may be written as[13]

$$\begin{aligned}\mathcal{L} = {} & i\partial_m\bar{\psi}\bar{\sigma}^m\psi + \bar{A}\Box A + \bar{F}F \\ & - [\ m(AF - \tfrac{1}{2}\psi\psi) + \tfrac{\lambda}{2}(AAF - \psi\psi A) + H.C.]\end{aligned} \tag{3.2}$$

[the metric signature $\eta = (-1,1,1,1)$ being employed throughout this paper].

Translating the Bose fields of this theory in the fashion

$$\begin{aligned} A &= A' + a \\ F &= F' + f \end{aligned} \tag{3.3}$$

we obtain

$$\mathcal{L}' = i\partial_m \bar{\psi}\bar{\sigma}^m \psi + \bar{A}'\Box A' + \bar{F}'F'$$
$$-[\chi(A'F' - \tfrac{1}{2}\psi\psi) + \tfrac{\lambda}{2}(A'A'F' - \psi\psi A') + \tfrac{\lambda}{2} f A'A'$$
$$+ F'(ma + \tfrac{\lambda a^2}{2} - \bar{f}) + A'\chi f + H.C.] \tag{3.4}$$

where

$$\chi = m + \lambda a \tag{3.5}$$

Before any perturbation theory can be done, we must know the propagators of this theory. The quadratic part of the (Bose) action is given by

$$S_0 = \int d^4x \; \tfrac{1}{2}\Phi^T \mathcal{A} \Phi + \Phi^T \mathcal{B} \tag{3.6}$$

where

$$\Phi^T = (A \quad \bar{A} \quad F \quad \bar{F})$$
$$\mathcal{B} = (J \quad \bar{J} \quad K \quad \bar{K}) \tag{3.7}$$

and

$$\mathcal{A} = \begin{pmatrix} \lambda f & \Box & \chi & 0 \\ \Box & \lambda \bar{f} & 0 & \bar{\chi} \\ \chi & 0 & 0 & 1 \\ 0 & \bar{\chi} & 1 & 0 \end{pmatrix} \tag{3.8}$$

J and K are the source terms for fields A and F respectively [note that linear terms in (3.4) can be absorbed into the definition of the sources; these terms will not affect the propagators]. The generating functional Z_0 is then given by

$$\operatorname{Ln} Z_0 = -\tfrac{1}{2}\int d^4x \; \mathcal{B}^T \mathcal{A}^{-1} \mathcal{B} \tag{3.9}$$

A is readily inverted to obtain

$$A^{-1} = \frac{1}{\Delta}\begin{pmatrix} -\lambda\bar{f} & \Box - \bar{\chi}\chi & -\bar{\chi}(\Box - \bar{\chi}\chi) & \lambda\bar{f}\chi \\ \Box - \bar{\chi}\chi & -\lambda f & \lambda f\bar{\chi} & -\chi(\Box - \bar{\chi}\chi) \\ -\bar{\chi}(\Box - \bar{\chi}\chi) & \lambda f\bar{\chi} & -\lambda f\bar{\chi}\bar{\chi} & -\lambda^2\bar{f}f + \Box(\Box - \bar{\chi}\chi) \\ \lambda\bar{f}\chi & -\chi(\Box - \bar{\chi}\chi) & -\lambda^2\bar{f}f + \Box(\Box - \bar{\chi}\chi) & -\lambda\bar{f}\chi\chi \end{pmatrix} \tag{3.10}$$

where

$$\Delta = (\Box - \bar{\chi}\chi)^2 - \lambda^2\bar{f}f \tag{3.11}$$

By looking at $\delta^2(\mathrm{Ln}Z_0)/\delta s_1\delta s_2|_{s=0}$ (where s denotes a source) the propagators of the theory are obtained directly. Although we will only need one of them later, for completeness they are all given in Figure 1.

4.- THE AUXILIARY FIELD TADPOLE METHOD

Now that the groundwork has been laid, I can show how simple the tadpole method becomes when applied to the W-Z model, unconstrained by auxiliary field equations of motion.

First the tree level result. At the tree level, the effective action is just the action evaluated at the classical field,

$$\Gamma_0 = S(\phi_{cl}) = \int \mathcal{L}' \, d^4x \tag{4.1}$$

From (3.4) we thus read off, the zero-loop, F tadpole of the translated theory (at zero external momentum)

$$\Gamma'_{F,0}\Big|_{P_{ext\underline{n}}=0} = -\left(ma + \frac{\lambda}{2}a^2 - \bar{f}\right) \tag{4.2}$$

Note that this term is not zero, as we have not applied the equation of motion for F. In other words, at this stage a and f are not related and we treat them as independent parameters.

Using the central equation in (2.7) we see

$$\frac{dV_0}{d\bar{f}} = \left(ma + \frac{\lambda}{2}a^2 - \bar{f}\right)$$

which when integrated gives

$$V_0 = \left(ma + \frac{\lambda a^2}{2}\right)f - \bar{f}f + H(\bar{f}, a, \bar{a}) \tag{4.3}$$

H is the constant of integration. It is easy to see that

$$H(\bar{f}, a, \bar{a}) = \left(m\bar{a} + \frac{\lambda\bar{a}^2}{2}\right)\bar{f} + I(\bar{a}, a) \tag{4.4}$$

either by symmetry or having integrated $dV_0/d\bar{f}$. Finally, using the fact that the effective potential must vanish at $f = 0$* we see

$$V_0 = \left(ma + \frac{\lambda a^2}{2}\right)f + \left(m\bar{a} + \frac{\lambda\bar{a}^2}{2}\right)\bar{f} - \bar{f}f \tag{4.5}$$

(having obtained the final answer we can now constrain the theory if we so wish; then $V_0 = \bar{f}f$ or $V_0 = \bar{F}_{cl}F_{cl}$ which is the familiar result).

Proceeding to the one-loop level, we see that Fig. 2 provides the only one-loop contribution to the F tadpole. Applying Feynman rules for the effective action (see Fig. 1 for the AA propagator), we see

$$\Gamma = \int \frac{dk}{(2\pi)^4}(2\pi)^4\delta(k) \int \frac{dp}{(2\pi)^4}(-\lambda)\cdot\frac{1}{2}\cdot\frac{(-\lambda\bar{f})}{(p^2+\bar{x}x)^2 - \bar{f}f\lambda^2} F'_{cl}(k) \tag{4.6}$$

+ (other terms of no interest) .

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* This is essentially the statement that if unbroken at the tree level, SUSY will never be broken by radiative corrections.

$-\lambda$ is the vertex factor read directly from (3.4) and 1/2 is the combinatorial loop factor. From this we see

$$\Gamma'_{F,1}\Big|_{p_{ext_n}=0} = \int \frac{dp}{(2\pi)^4} \, \frac{\lambda^2}{2} \, \frac{\bar{f}}{(p^2+\bar{\chi}\chi)^2 - \bar{f}f\lambda^2}$$

for the one-loop translated F tadpole at zero external momentum. Using (2.7) we obtain

$$\frac{dV_1}{df} = -\frac{\lambda^2}{2}\int \frac{dp}{(2\pi)^4} \, \frac{\bar{f}}{(p^2+\bar{\chi}\chi)^2 - \bar{f}f\lambda^2}$$

or

$$V_1 = \frac{\lambda^2}{2}\int \frac{dp}{(2\pi)^4} \, \mathrm{Ln}\left((p^2+\bar{\chi}\chi)^2 - \lambda^2\bar{f}f\right) + H(\bar{a},a) \tag{4.7}$$

the integration constant deduced (in the usual manner) to have no $\bar{f}$ dependence. Finally, as V_1 must vanish at $f = 0$ this fixes the integration constant and we obtain

$$V_1 = \frac{\lambda^2}{2}\int \frac{dp}{(2\pi)^4} \, \mathrm{Ln}\left(1 - \frac{\lambda^2\bar{f}f}{(p^2+\bar{\chi}\chi)^2}\right) \tag{4.8}$$

which is the familiar result obtained in the literature[9),10)] (recall that $f \to F_{cl}$ $a \to A_{cl}$ is understood). Although I have detailed the steps, it is clear that (4.8) from (4.6) is just a three-line derivation. This reflects the strength of the Weinberg tadpole method.

Equation (4.8) can also be derived using the A tadpole and integrating dV_1/da. Even at this level it is a more complicated approach, since the A field couples to the spinor field as well as to F. One thus sees that considering F tadpoles will always produce the easier calculation, particularly at higher loop order.

5.-CONCLUDING REMARKS

Hopefully I have convinced the reader that the S. Weinberg[3)] tadpole method of effective potential evaluation offers an attractive and simple means of calculating SUSY effective potentials. The trick is to maintain an unconstrained SUSY (i.e., do not eliminate auxiliary fields) and thus look at auxiliary field tadpoles.

I have discussed the method for self-interacting chiral superfields only. For SUSY gauge field theories one could apply the method by considering D and F tadpoles. In the introduction, I mentioned that supergraph techniques[10] have recently been developed for the calculation of SUSY effective potentials. The Coleman-Weinberg approach used by these authors faces difficulties at the gauge theory level. It is my view that a supergraph tadpole approach will ultimately provide the simplest technique for SUSY effective potential evaluations (this suggestion is not as insane as it first sounds, for one must remember that the tadpoles we evaluate are those of the translated theory which is manifestly non-SUSY, thus the tadpoles need no longer vanish). Work along these lines is in progress.

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FIGURE CAPTIONS

Fig. 1 Propagator Feynman rules for the translated W-Z model. Δ is defined in Eq. (3.11).

Fig. 2 The one-loop contribution to the 1PI F tadpole of the translated theory.

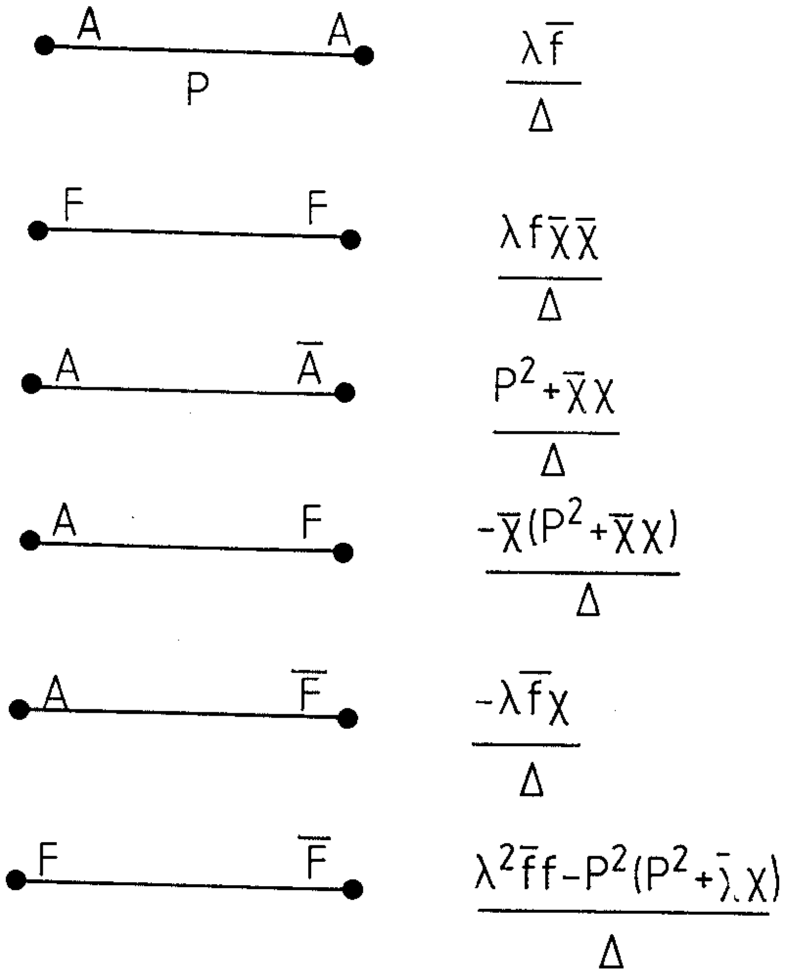
A
A
P
$\frac{\lambda\bar{f}}{\Delta}$
F
F
$\frac{\lambda f\bar{\chi}\bar{\chi}}{\Delta}$
A
$\bar{A}$
$\frac{P^2+\bar{\chi}\chi}{\Delta}$
A
F
$\frac{-\bar{\chi}(P^2+\bar{\chi}\chi)}{\Delta}$
A
$\bar{F}$
$\frac{-\lambda\bar{f}\chi}{\Delta}$
F
$\bar{F}$
$\frac{\lambda^2\bar{f}f-P^2(P^2+\bar{\chi}\chi)}{\Delta}$

FIG. 1

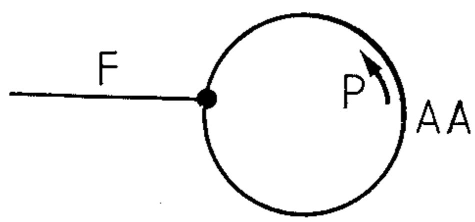
F
P
AA

FIG. 2