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A TADPOLE SUPERGRAPH METHOD FOR THE EVALUATION OF SUSY EFFECTIVE POTENTIALS

Robert D. C. Miller
CERN - Geneva

A B S T R A C T

The S. Weinberg tadpole method for the evaluation of effective potentials of conventional field theory is here extended to the supergraph level, providing a new means for the evaluation of SUSY effective potentials. The method is illustrated for the Wess-Zumino model.

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1. INTRODUCTION

There are a number of ways in which one can calculate effective potentials in conventional [i.e., non-supersymmetric (SUSY) theories] field theories. Three popular methods are those of Coleman and Weinberg [1] (C-W), Jackiw [2] and S. Weinberg [3]. For SUSY theories we also have the options of either a component field or a superfield formalism [see Wess and Bagger for a recent review [4]].

As for component field methods, O'Raifeartaigh and Parravicini [5] have employed the C-W approach and Huq [6] has employed the functional method of Jackiw. Only recently has the S. Weinberg tadpole method been investigated for SUSY theories [7].

The one-loop effective potential of any gauge theory is essentially known if we combine the works of the original C-W paper with the general evaluation of SUSY mass matrices given by Barbieri et al. [8]. However, there are two set-backs with this approach. Firstly, the C-W results were derived for a covariant Landau gauge ($\alpha = 0$). This gauge choice is non-SUSY. Secondly the C-W method is virtually impossible to implement [9],[10] beyond the one-loop level. SUSY effective potentials beyond a one-loop level are not just of abstract theoretical interest but could prove useful in SUSY GUT models of the type in Ref. 11).

The methods of Jackiw [2] and S. Weinberg [3] both allow practical means to higher loop order calculations as was shown in conventional field theory. [See Kang [12] for two-loop Jackiw and both Lee and Sciacciluga [9] and Mahanthappa and Sher [10] for two-loop Weinberg in conventional field theories.] The gauge question, already raised, suggests it would be beneficial to work with a manifestly SUSY calculational method. This is also suggested by common sense, namely, if a theory possesses a certain symmetry, why not exploit it. Thus it appears what we would ideally like is either a superfield formalism of Jackiw's method or a supergraph analogue of Weinberg's tadpole method. Huq [13] has already considered Jackiw's method in such a fashion. However, it is the present author's opinion that even in conventional field theory, the tadpole method is intrinsically simpler than the Jackiw method. It is thus the purpose of this paper to develop the basic theory of a tadpole supergraph method and illustrate its use for a simple model, the Wess-Zumino (W-Z) model. It is worth while noting the recent paper by Grisaru et al.[14] which investigates a C-W supergraph method (which reveals that all three popular methods have now been persued to the superfield level).

Let me first begin by responding to a series of natural objections which usually arise with the mention of a tadpole supergraph method. For example is it not well known that tadpole supergraphs vanish ? The answer here is yes, but only for SUSY theories. The tadpoles evaluated in the S. Weinberg method are associated with a translated (hence broken) SUSY theory (they need not vanish). Another immediate objection follows; how can one possibly talk of supergraphs for a broken SUSY theory ? The answer here is that the prerequisite for supergraphs is a superfield formulation of one's theory <u>not</u> that it has to be SUSY. Of course SUSY theories will have simpler supergraph Feynman rules. A final objection might be that one cannot understand how a superfield of a broken SUSY theory, containing component fields of different mass, could possibly have a single expression for its propagators. The answer here is that superfield propagators are differential operators in superspace and this intuitive objection fails to materialize.

To summarize then, I will show that supergraph Feynamn rules are possible for a broken SUSY theory. Their application to my formulation of the tadpole supergraph method confirms it all works.

Since this paper deals with a calculational technique, I have presented the work on a slightly pedagogical level, particularly with the worked example. In Section 2, the basic methods are formulated. In Section 3, tree-level implementation provides a simple example of how it works. In Section 4 implementation at the one-loop level is persued, beginning with a derivation of the necessary (formal) supergraph Feynman rules of the broken SUSY theory. In 4.2 these are utilized to obtain a simple expression for the one-loop effective potential. In 4.3 we must go beyond the formal Feynman rules and derive an explicit expression for a specific operator inverse. (This point poses the greatest difficulty for the method, hence requires a separate discussion.) It concludes by showing how the one-loop-effective potential is finally arrived at. Section 5 provides a summary.

2. DEVELOPING THE METHOD

In this Section, I will show how the supergraph tadpole method may be formulated in such a way that we arrive at a simple recipe for its implementation.

Let ϕ denote a superfield described by a supersymmetric Lagrangian. Γ will denote the effective action of this theory. Since our theory is SUSY, we may apply the familiar supergraph techniques of Grisaru, Siegel and

and Roček [15] to obtain a loop expansion for Γ. Γ can always be reduced to a single $d^4\theta$ integration [15] (generally denoted by $\int d\theta$ alone), thus in general we may write,

$$\Gamma(\phi) = \int d\theta \int d\tilde{p}\, X(p,\theta)\,\phi(p,\theta) \ + \text{other terms} \tag{2.1}$$

[$\int d\tilde{p}$ will denote $(dp/(2\pi)^4)(2\pi)^4\delta(p)$ throughout this paper]. The effective potential is then defined by

$$V = -\int X(\theta)\,\phi(\theta) + \cdots\cdots d\theta \tag{2.2}$$

where it is understood in (2.2) and (2.3) that ϕ denotes the classical superfield, ϕ_{cl}, also that $X(\theta)$ means $X(0,\theta)$, i.e., X evaluated at zero external momenta. The loop expansion for V is obtained through the loop expansion for X (and, of course, the other terms not explicitly presented) which is directly calculable via supergraph techniques [and would usually be extracted from (2.1)].

Let us define $\mathcal{V}$ by

$$V = \int d\theta\, \mathcal{V} \tag{2.3}$$

Then

$$\mathcal{V} = -X(\theta)\,\phi(\theta) + \cdots\cdots \tag{2.4}$$

in particular,

$$\left.\frac{d\mathcal{V}}{d\phi}\right|_{\phi=0} = -X(\theta) \tag{2.5}$$

Suppose we now expand our theory about $\phi' = 0$, where

$$\phi = \phi' + \sigma \tag{2.6}$$

(σ denotes a momentum space constant superfield), then

$$\left.\frac{d\mathcal{V}}{d\phi'}\right|_{\phi'=0} = -X'(\theta) \tag{2.7}$$

$X'(\theta)$ is to be calculated as in (2.1) but using the translated theory, $\mathcal{L}'(\phi') = \mathcal{L}(\phi'+\sigma)$. [The reader will note that it is no longer obvious that

$X'(\theta)$ can be calculated via supergraph techniques, since $\mathcal{L}'(\phi')$ will be manifestly non-supersymmetric so standard supergraph Feynman rules no longer apply. This issue will be discussed later.]

With (2.6), (2.7) may be written as

$$\left.\frac{d\mathcal{V}}{d\phi}\right|_{\phi=\sigma} = -X'(\theta) \tag{2.8}$$

Now $\mathcal{V}(\theta)$ is $\mathcal{V}$ calculated via $\mathcal{L}$, thus $\mathcal{V}$ has no σ dependence, hence we may write (2.8) as

$$\frac{d\mathcal{V}(\sigma)}{d\sigma} = -X'(\theta) \tag{2.9}$$

with (2.3) we obtain the results

$$V = -\int d\theta \left[\int d\sigma \; X'(\theta) \right] \tag{2.10}$$

Observe that the introduction of $\mathcal{V}$ has avoided functional derivatives, also that the order of integration in (2.10) is strictly fixed because σ is θ dependent.

At this stage we have our recipe for the calculation of V.

1. We choose $\mathcal{L}$.
2. With $\phi = \phi' + \sigma$ construct $\mathcal{L}' = \mathcal{L}(\phi' + \sigma)$.
3. With $\mathcal{L}'$ and supergraph Feynman rules calculate $X'(\theta)$ appearing in

$$\Gamma = \int d\theta \, d\tilde{p} \;\; X'(p,\theta) \;\phi'(p,\theta) + \cdots\cdots$$

4. V is then given by (2.10).
5. If ϕ is a chiral superfield then σ is chiral and will have the form

$$\sigma = a + \theta^2 f$$

if ϕ is a vector superfield σ must have the form (general gauge)

$$\sigma = c + \frac{i}{2}\theta^2(m+in) - \frac{i}{2}\bar{\theta}^2(m-in) + \frac{1}{2}\theta^2\bar{\theta}^2 d$$

where a, f, c, m, n and d are constants. The effective potential in terms of classical fields is then given by replacing $a \to A_{cl}$, $f \to F_{cl}$ ... $d \to D_{cl}$ where A_{cl} and F_{cl} are the spin zero and auxiliary fields of ϕ_{cl} (ϕ_{cl} chiral) and C_{cl}, M_{cl}, N_{cl}, D_{cl} are the auxiliary field of ϕ_{cl} (ϕ_{cl} vector).

6. In the case of chiral superfields a special subtlety should be noted. Integration of (2.10) will lead to a result such as

$$V = -\int d\theta \left[R_2(\sigma,\bar\sigma) + R_1(\sigma) + \rho(\bar\sigma) \right]$$

where $\rho(\bar\sigma)$ is the "constant" of integration. It is then easy to see *)

$$V = -\int d\theta \left[R_2(\sigma,\bar\sigma) + R_1(\sigma) + R_1(\bar\sigma) \right]$$

In other words, do not forget to symmetrize your result in σ and $\bar\sigma$ otherwise you could throw away potentially important terms !

It is clear that the method is well defined, what remains to be seen is how it is implemented. This we now investigate for the Wess-Zumino model (its extension to the general chiral superfield theory is trivial).

3. IMPLEMENTATION AT THE TREE LEVEL

Our choice of $\mathcal{L}$ is the Wess-Zumino model

$$\mathcal{L} = \bar\phi\phi\Big|_{\theta^2\bar\theta^2} - \left[\left(\frac{m}{2}\phi\phi + \frac{\lambda}{3!}\phi\phi\phi \right)_{\theta^2} + \text{H.C.} \right] \tag{3.1}$$

We perform the field translation $\phi \to \phi+\sigma$ to obtain

$$\mathcal{L}' = \bar\phi\phi\Big|_{\theta^2\bar\theta^2} - \Big[\Big((m\sigma + \frac{\lambda\sigma^2}{2} - \bar f)\phi + \frac{(m+\lambda\sigma)}{2}\phi\phi + \frac{\lambda}{3!}\phi\phi\phi \Big)_{\theta^2} + \text{H.C.} \Big] \tag{3.2}$$

where σ is a constant chiral superfield, hence must have the form

$$\sigma = a + \theta^2 f \tag{3.3}$$

At the present stage we do not need supergraph Feynman rules since we know that the tree level effective action Γ_0 is simply given by the action evaluated at the classical field.

Thus

$$\Gamma_0 = \int d\theta\, d\check{p}\, \left(-(m\sigma + \frac{\lambda\sigma^2}{2} - \bar f)\, \delta^{(2)}(\bar\theta) \right)\phi + \cdots \tag{3.4}$$

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*) E.g., by integrating $dV/d\bar\sigma$ and using linear independence arguments.

[where we have used the fact that $K(\theta,\bar{\theta})|_{\theta^2} = \int d\theta K(\theta,\bar{\theta})\delta^{(2)}(\bar{\theta})$]. According to rule 3, we read off the zero loop $X'(\theta)$,

$$X'(\theta) = -\left(m\sigma + \frac{\lambda}{2}\sigma^2 - \bar{f}\right)\delta^{(2)}(\bar{\theta}) \tag{3.5}$$

Rule 4 and (2.10) directly give

$$V_0 = \int d\theta \left[\int d\sigma \left(m\sigma + \frac{\lambda\sigma^2}{2} - \bar{f}\right)\delta^{(2)}(\bar{\theta})\right]$$

integrating and symmetrizing (see rule 6) gives

$$V_0 = \int d\theta \left[-\bar{\sigma}\sigma + \left\{ \delta^{(2)}(\bar{\theta})\left(\frac{m\sigma^2}{2} + \frac{\lambda}{3!}\sigma^3\right) + \text{H.c.} \right\}\right]$$

[where we observe $\int d\theta\,(-)\,\sigma\bar{f}\delta^{(2)}(\bar{\theta}) = \int d\theta\,(-)\,\bar{\sigma}\sigma$].

Finally, utilizing (3.3), we obtain

$$V_0 = -\bar{f}f + \left(maf + \frac{\lambda}{2}a^2 f\right) + \left(m\bar{a}\bar{f} + \frac{\lambda}{2}\bar{a}^2\bar{f}\right) \tag{3.6}$$

Using rule 5 and the equations of motion for $\bar{F}_{cl}$, $\bar{F}_{cl} = mA_{cl} + (\lambda/2)A_{cl}^2$ we get the familiar result

$$V_0 = F_{cl}\bar{F}_{cl} \tag{3.7}$$

This at least provides a simple example of how the method works (in particular rule 6 plays a crucial rôle here).

4. IMPLEMENTATION AT THE ONE-LOOP LEVEL

4.1 Supergraph Feynman rules for the broken SUSY theory, $\mathcal{L}'$

The one-loop contribution to Γ, of interest to us (see rule 3), comes from the one-loop ϕ tadpole as illustrated in Fig. 1 *). Anyone familiar with supergraph Feynman rules will recall that such a graph normally would vanish, due to the structure of a $\phi\phi$ propagator. However, recall that this supergraph is to be evaluated using $\mathcal{L}'$, and since f [see (3.3)] is a priori arbitrary, $\mathcal{L}'$ represents a broken SUSY theory. The tadpole need not vanish.

To evaluate Fig. 1 we thus need supergraph Feynman rules for the broken SUSY theory, $\mathcal{L}'$. That such Feynman rules exist is not obvious. Intuitively one would believe that the possibility of defining supergraph

*) At this stage ignore the $\bar{D}^2$ and θ indexing.

Feynman rules is a consequence of having a SUSY Lagrangian. In this Section I will show that this is not correct, supergraph Feynman rules for $\mathcal{L}'$ do exist.

In fact a simple intuitive argument can be given to support such a conclusion. My premise is that all one needs to define supergraphs and their Feynman rules is a superfield formulation of the theory dealt with, $\mathcal{L}'$ in (3.2) is such a theory. The reasoning is that if the action can be written totally in terms of superfields then it is clearly intuitive that the effective action should be expressible in terms of classical superfields. However, given the effective action in terms of classical superfields, we can almost directly extract the supergraph Feynman rules of the theory.

My intention is to develop necessary supergraph Feynman rules for $\mathcal{L}'$ along the lines of the treatment given by Grisaru, Siegel and Roček [15] for SUSY theories. A glance at (3.2) shows that we expect three propagators $\phi\phi$, $\bar{\phi}\bar{\phi}$ and $\phi\bar{\phi}$, a cubic vertex ϕ^3 (and $\bar{\phi}^3$) and also a $\phi(\bar{\phi})$ tadpole vertex. In the present paper, since we are only considering one-particle irreducible (IPI) diagrams (Γ is their generator) we may forget about the tadpole vertex. Secondly, (3.2) shows that the ϕ^3 term has been unaltered by the field translation, thus we expect its vertex factor to be the same as that in an unbroken SUSY theory. This is not immediately guaranteed because we recall that in the SUSY theory the vertex factor and propagators are intimately related in that $\bar{\phi}^3(\phi^3)$ vertices provide $-D^2/4$ $(-\bar{D}^2/4)$ factors which act on propagators. That vertex factors can be defined in this way relies upon the structure of the propagators themselves.

In essence then this is our only problem, to derive the propagators of $\mathcal{L}'$ (in the process hopefully establishing the standard vertex factor). We commence in a standard manner, by writing down the quadratic part of the action with a chiral source term, J. From (3.2)

$$S_o = \int dz\; \bar{\phi}\phi - \left\{ \left(\int d\rho\; \frac{\mathcal{M}}{2}\phi\phi + J\phi \right) + H.c. \right\} \qquad (4.1)$$

I adopt the notation convention of Grisaru, Siegel and Roček [15] where $dz = d^4x d^4\theta$ and $d\rho = d^4x d^2\theta$ (however, my spinor index conventions will follow that of Wess and Bagger [4),*)]). Here I have defined a mass $\mathcal{M}$ given by

$$\mathcal{M} = m + \lambda\sigma \qquad (4.2)$$

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*) The reader with any doubts as to the meaning of my notation should refer to these references.

We recall that m and σ are co-ordinate space constants. However, σ is chiral [$\bar{D}_{\dot{\alpha}}\sigma = 0$, see (3.3)] hence so is $\mathcal{M}$. $\mathcal{M}$ we may refer to as a constant chiral superfield (constant referring to co-ordinate space dependence).

Using (3.3) we may write

$$\mathcal{M} = \chi + \lambda f \theta^2 \tag{4.3}$$

where

$$\chi = m + \lambda a$$

at this stage one can comprehend the possible difficulties we face. The mass of our chiral superfield ϕ <u>is itself a chiral superfield</u> ! Thus superfield ϕ has a mass which depends upon its position in superspace, this is exactly what one expects in a broken SUSY theory. We can also give a different interpretation to this effect, namely that according to ϕ superspace is not uniform. [One can imagine the analogous effect in co-ordinate space should one break Poincaré invariance by introducing a co-ordinate space dependent VEV. A particle's mass and the strength of its coupling would then depend upon "where it is" and "when it is" in space time.]

Returning to the problem at hand, suppose ξ and η are two chiral superfields, then it follows [13]

$$\int d\varsigma \; \xi \eta = \int dz \; \xi \frac{D^2}{-4\Box} \eta \tag{4.4}$$

Since J is a chiral superfield, as is the product $\mathcal{M}\phi$ (since $\mathcal{M}$ and ϕ are individually chiral), then we may write

$$S_0 = \int dz \; \tfrac{1}{2} \Psi^T A \Psi + \Psi^T B \tag{4.5}$$

with

$$B^T = \left(-\frac{P_+}{\Box^{1/2}} J \quad -\frac{P_-}{\Box^{1/2}} \bar{J} \right) \tag{4.6}$$

and

$$A = \begin{pmatrix} \frac{\mathcal{M}}{\Box^{1/2}} P_+ & 1 \\ 1 & \frac{\bar{\mathcal{M}}}{\Box^{1/2}} P_- \end{pmatrix} \tag{4.7}$$

where I employ the p operators and follow the notation of Wess and Bagger [4]

$$P_+ = D^2/4\Box^{1/2} \; , \; P_- = \bar{D}^2/4\Box^{1/2}$$

$$P_1 = D^2\bar{D}^2/16\Box \; , \; P_2 = \bar{D}^2 D^2/16\Box \; , \; P_3 = -D\bar{D}^2 D/8\Box \qquad (4.8)$$

[whose multiplication table may be found in Ref. 4)].

It is then not difficult to show that the standard expression for the generating functional Z_0 results

$$\mathrm{Ln}\, Z_0 = -\frac{1}{2}\int dz\; B^T A^{-1} B \qquad (4.9)$$

In the present case, we are only interested in the $\phi\phi$ propagator so let us concentrate upon this (the other propagators then follow in a straightforward manner).

The big task is to invert A in (4.7). This is a rather straightforward exercise in p operator algebra when m is just a number. However here m is a constant chiral superfield and does not commute with the p operator algebra ! This precludes the possibility of using p operator methods since it is not difficult to convince oneself that the commutators take one outside the closed p algebra. For the moment, however, we can make significant progress with a formal inverse.

If we denote $(A^{-1})_{11}$ by x then from $A^{-1}A = 1$ we find

$$x\left(1 - \frac{m}{\Box^{1/2}} P_+ \frac{\bar{m}}{\Box^{1/2}} P_-\right) = -\frac{\bar{m}P_-}{\Box^{1/2}}$$

Since $[p_+, \bar{m}] = 0$ we obtain the formal expression

$$x = -\frac{\bar{m}}{\Box^{1/2}} P_- \left(1 - \frac{m^2}{\Box} P_1\right)^{-1} \qquad (4.10)$$

where $m^2 = \bar{m}m$. With (4.10) in (4.9) we obtain the relevant term,

$$\mathrm{Ln}\, Z_0 = -\frac{1}{2}\int dz_0 \left(-\frac{P_+^{(0)}}{\Box^{1/2}} J\right)(-)\frac{\bar{m}_{(0)}}{\Box^{1/2}} P_-^{(0)}\left(1 - \frac{m_{(0)}^2}{\Box} P_1^{(0)}\right)^{-1}\left(-\frac{P_+^{(0)}}{\Box^{1/2}} J\right) \qquad (4.11)$$

+ other terms of no interest

The sub- and super-scripts (0) denote a labelling on $\theta(\bar{\theta})$, i.e., $\theta_{(0)}$ $(\bar{\theta}_{(0)})$. Since

$$\frac{\delta J_1}{\delta J_2} = -\Box^{1/2}\, P_-^{(2)}\, \delta(z_{12}) \tag{4.12}$$

(generally t_{12} will denote $t_1 - t_2$), then it is straightforward to show

$$\left.\frac{\delta^2 \operatorname{Ln} Z_0}{\delta J_2\, \delta J_1}\right|_{J=0} = P_-^{(2)} P_-^{(1)} \left\{ P_+^{(2)} \left[\frac{\bar{m}_{(2)}}{\Box^{1/2}} P_-^{(2)} \mathcal{O}_{(2)}^{-1} P_+^{(2)} \delta(\theta_{12}) \right] \right\} \delta(P_{12}) \tag{4.13}$$

where we go into momentum space and still retain the $\Box$ notation for "$-p^2$". Here I have introduced the notation

$$\mathcal{O} = \left(1 - \frac{m^2}{\Box} P_1 \right) \tag{4.14}$$

At this state we see we recover the standard vertex factor since $-\Box^{\frac{1}{2}} p_-^{(2)}$ and $-\Box^{\frac{1}{2}} p_-^{(1)}$ may be associated with the vertices at either end of the propagator $(-\Box^{\frac{1}{2}} p_- = -\bar{D}^2/4)$, as shown diagrammatically in Fig. 2.

Finally, from (4.13) we read off directly the formal $\phi\phi$ propagator Feynman rule, it is given in Fig. 3. Note that the p operators will not commute through $\mathcal{O}^{-1}$, further that the issue of an explicit form for $\mathcal{O}^{-1}$ will be dealt with later.

4.2 The one-loop contribution to V

We can now return to the evaluation of Fig. 1. Following the standard supergraph Feynman rules [15] with our modified propagator we obtain,

$$\Gamma_1 = \int d\tilde{k} \int d\mathcal{P} \int d\theta_0\, (-\lambda) \cdot \frac{1}{2} \cdot \left[-\frac{\bar{D}_{(1)}^2}{4} \cdot \frac{1}{\Box} \cdot \frac{\bar{m}_{(2)}}{\Box^{1/2}} P_1^{(2)} \mathcal{O}_{(2)}^{-1} P_+^{(2)} \delta(\theta_{12}) \right] \cdot \phi(k, \theta_0, \bar{\theta}_0) \Big|_{\theta_0 = \theta_1 = \theta_2} \tag{4.15}$$

+ other terms of no interest.

where $\frac{1}{2}$ is the closed loop combinatorial factor and $-\bar{D}_{(1)}^2/4$ comes from the vertex [we recall [15] that, as we have one external chiral superfield at the vertex, only one such term arises]. The $d\tilde{k}$ notation was introduced in (2.1) and $d\mathcal{P}$ denotes $dp/(2\pi)^4$.

We immediately deduce from (2.10)

$$V_1 = -\int d\theta_0 \int d\sigma \int d\rho \, \frac{\lambda}{2} \left[\frac{\bar{m}_{(2)}}{\Box} P_1^{(2)} \mathcal{O}^{-1}_{(2)} P_+^{(2)} P_-^{(1)} \delta(\theta_{12}) \right]_{\theta_0=\theta_1=\theta_2}$$

Since

$$\mathcal{O}^{-1}\mathcal{O} = 1 \quad , \quad \mathcal{O} \equiv \mathcal{O}\left(\frac{m^2}{\Box} P_1^{(2)} \right)$$

then it follows

$$\left[\mathcal{O}^{-1}, \frac{m^2}{\Box} P_1^{(2)} \right] = 0$$

finally utilizing the fact that [15)]

$$P_+^{(2)} P_-^{(1)} \delta(\theta_{12}) \rightarrow P_+^{(2)} P_-^{(2)} \delta(\theta_{12}) = P_1^{(2)} \delta(\theta_{12})$$

we obtain

$$V_1 = -\frac{\lambda}{2} \int d\theta_0 \, d\rho \, d\sigma \, m^{-1}_{(0)} \mathcal{O}^{-1}_{(0)} \frac{m^2_{(0)}}{\Box} P_1^{(0)} \delta(\theta_{10}) \Big|_{\theta_0=\theta_1}$$

observing that $X/(1-x) = -1+1/(1-x)$

$$V_1 = -\frac{\lambda}{2} \int d\theta_0 \, d\rho \, d\sigma \, m^{-1}_{(0)} \mathcal{O}^{-1}_{(0)} \delta(\theta_{10}) \Big|_{\theta_0=\theta_1} \qquad (4.16)$$

This can be further simplified by changing variables from σ to f using (3.3), thus giving

$$\int d\theta_0 \, d\sigma \, m^{-1}_{(0)} = \int d\theta_0 \int df \, \theta_0^2 \, m^{-1}_{(0)} = \chi^{-1} \int d\theta_0 \, df \, \theta_0^2$$

which in (4.16) gives,

$$\frac{\partial V_1}{\partial f} = -\frac{\lambda}{2} \chi^{-1} \int d\rho \int d\theta_0 \, \theta_0^2 \, \mathcal{O}^{-1}_{(0)} \delta(\theta_{10}) \Big|_{\theta_1=\theta_0} \qquad (4.17)$$

At this stage the present supergraph method has been relatively simple to implement. The most difficult aspect is associated with obtaining an explicit expression for $\mathcal{O}^{-1}$. My approach to its evaluation is discussed in the following Sub-Section.

4.3 Inversion of $\mathcal{O}$ and the final result

I mentioned earlier that p operator methods are useless for the inversion of $\mathcal{O}$

$$\mathcal{O} = \left(1 - \frac{m^2}{\Box} P_1\right)$$
$$\equiv \left(1 + \frac{m^2}{16\Box^2} \varepsilon^{\alpha\beta} \varepsilon^{\dot\alpha\dot\beta} D_\alpha D_\beta \bar{D}_{\dot\alpha} \bar{D}_{\dot\beta}\right) \tag{4.18}$$

due to the fact that m is a constant chiral superfield and it does not commute with the p operator algebra, i.e., the commutators $[p, m^2]$ generate terms outside the p algebra. We must then seek alternative means for the inversion of $\mathcal{O}$. The approach I have developed is rather straightforward, although it requires a deal of algebra. Here I will outline the method, the Appendix will provide greater detail for the interested reader.

We begin by observing that $\mathcal{O}^{-1}$ must be a function of covariant derivatives. Its most general form must be

$$\mathcal{O}^{-1} = C_i(\theta, \bar\theta)\, \Omega_i \tag{4.19}$$

where $\{\Omega_i\}$ is a basis of nine linearly independent covariant tensors, as listed in the Table. One observes that C_i are in general $\theta(\bar\theta)$ dependent and they do not commute with Ω_i. (It should be pointed out that dealing with a tensor basis is much simpler than dealing with a scalar, i.e., contracted basis, which would increase the amount of algebra to be performed.) With (4.19) in (4.17) we see that life simplifies even further, since $\Omega_i \delta(\theta_{10})|_{\theta_1=\theta_0} = 0$ for $i = 0, \ldots, 7$, and $\Omega_{8,\alpha\beta\dot\alpha\dot\beta} \delta(\theta_{10})|_{\theta_1=\theta_0} = -4\varepsilon_{\dot\alpha\dot\beta}\varepsilon_{\alpha\beta}$ hence

$$\frac{\partial V_1}{\partial f} = -\frac{\lambda}{2} \chi^{-1} \int d\rho \left(-4 \varepsilon_{\alpha\beta} \varepsilon_{\dot\alpha\dot\beta}\right) \partial_{\bar\theta^2} \left[C_8^{\alpha\beta\dot\alpha\dot\beta}(\theta=0, \bar\theta)\right]_{\bar\theta=0} \tag{4.20}$$

so we see we may set $\theta = 0$ in deriving $\mathcal{O}^{-1}$.

Our requirement is that $\mathcal{O}^{-1}\mathcal{O}=1$. By analyzing this equation and using the linear independence of $\{\Omega_i\}$ (see Appendix for details), one finds (after much algebra!) the following solutions,

$$
\begin{array}{ll}
C_0 = 1 & C_5^{\dot\alpha\dot\beta} = \varepsilon^{\dot\alpha\dot\beta}[-4\lambda f \bar{m} m^2/16\Box\Delta] \\
C_1^{\alpha} = 0 & C_6^{\alpha\beta\dot\alpha} = 0 \\
C_2^{\dot\alpha} = 0 & C_7^{\alpha\dot\alpha\dot\beta} = \varepsilon^{\dot\alpha\dot\beta}\,\bar{p}^{\dot\gamma\alpha}\bar{\theta}_{\dot\gamma}[4\lambda^2\bar{f}f m^2/16\Box\Delta(\Box-m^2)] \\
C_3^{\alpha\beta} = 0 & C_8^{\alpha\beta\dot\alpha\dot\beta} = \varepsilon^{\alpha\beta}\varepsilon^{\dot\alpha\dot\beta}[-m^2(\Box-m^2)/16\Box\Delta] \\
C_4^{\alpha\dot\alpha} = 0 &
\end{array}
\qquad (4.21)
$$

where $\theta=0$ is understood (e.g., $m=\chi$). Here

$$
\Delta = (\Box-m^2)^2 - \lambda^2\bar{f}f\left(1-\frac{2\lambda\bar{f}m\bar{\theta}^2}{\Box-m^2}\right) \qquad (4.22)
$$

and $\bar{p}^{\dot\alpha\alpha} = p^m\bar{\sigma}_m^{\dot\alpha\alpha}$ [Wess-Bagger [4] notation].

Note that the non-zero C_7 solution confirms my earlier remark that p operator methods cannot be used to invert $\mathcal{O}$ (for Ω_7 cannot be expressed in terms of the p algebra). The final step is to substitute the C_8 solution into (4.20) and perform the differentiation, the result is,

$$
\frac{\partial V_1}{\partial f} = -\frac{\lambda^2}{2}\bar{f}\int d\rho\,\frac{1}{(\Box-\bar{\chi}\chi)^2-\lambda^2\bar{f}f} \qquad (4.23)
$$

which is, of course, the correct result [a few simple steps take it through, see Ref. 7].

5. CONCLUDING REMARKS

In conventional field theory, S. Weinberg's tadpole method [3] of effective potential evaluation provides (in this author's opinion) the simplest and most elegant approach amongst the other popular methods [1),2)]. Recently, I have shown [7] how this appraisal can also be applied to SUSY theories analyzed via the (component) auxiliary field tadpole method.

Recently [14] supergraph methods have been applied to the calculation of SUSY effective potentials. These authors employ the Coleman-Weinberg [1] approach principally because it requires no modification to standard supergraph Feynman rules [15].

From what has been said, it is an attractive idea to investigate what the tadpole supergraph method might have to offer in terms of simplifying the calculation of SUSY effective potentials, in addition to providing a practical means of going beyond the one-loop level and also a way of performing manifestly SUSY gauge fixing. In this paper I have developed such a formalism and illustrated its implementation for the Wess-Zumino model. The formalism is quite compact and was summarized in recipe form. Implementation is also reasonably simple up to the point where an operator denoted by $\mathcal{O}$ (associated with the chiral superfield propagators) must be inverted. This is the only difficult point faced in the tadpole supergraph method. Inversion of $\mathcal{O}$ cannot proceed via standard p operator methods. I have shown how $\mathcal{O}$ may be inverted, but the procedure is not as simple as one might hope for. Simplification of this procedure, I believe, would place the Weinberg tadpole method as the more attractive supergraph approach. We should keep in mind after all that explicit superfield propagator Feynman rules (for the broken SUSY theory) need only be derived once !

Thus to conclude, the tadpole supergraph method has been shown to work. Its extension to SUSY gauge field theories is an interesting prospect and should be the next step in its development. At the same time, a simplified method for operator inversion, similar to the p operator methods, would be welcome.

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APPENDIX

FURTHER DETAILS ON THE INVERSION OF $\mathcal{O}$

From (4.18) and (4.19) $\mathcal{O}^{-1}\mathcal{O} = 1$ implies

$$1 = c_0\left(1 - \frac{m^2}{\Box} P_1\right) + \sum_{i=1}^{8}\left\{ c_i \Omega_i + \frac{\varepsilon^{\alpha\beta}\varepsilon^{\dot\alpha\dot\beta}}{16\Box^2} c_i K_i \right\} \tag{A.1}$$

where

$$K_i = \Omega_i \left(m^2 D_\alpha D_\beta \bar D_{\dot\alpha} \bar D_{\dot\beta} \right) \tag{A.2}$$

(spinor indexing will be given only when necessary).

Linear independence arguments on the set $\{\Omega_i\}$ are to be used, thus K_i must be expressed in the form $K_i = \kappa_j \Omega_j$, which requires Ω_i to be taken through m^2 (or the $m^2 D_\alpha D_\beta$ when appropriate). Note that this is necessary since we <u>cannot</u> take the Ω_i through to the left for they will not commute with the C_i, and as yet the C_i are unknown. The following (anti-) commutators are found to be useful in this respect, all derivable from the basic superalgebra;

$$\{ D_\alpha , \bar D_{\dot\alpha} \} = -2 P_{\alpha\dot\alpha} \tag{A.3}$$

where $p_{\alpha\dot\alpha} = p_n \sigma^n_{\alpha\dot\alpha}$ (p_n the momentum operator when in co-ordinate space) and my spinor notation is that of Ref. 4).

$$\{ D_\alpha , \theta_\beta \} = -\varepsilon_{\alpha\beta} \qquad \{ \bar D_{\dot\alpha} , \bar\theta_{\dot\beta} \} = \varepsilon_{\dot\alpha\dot\beta}$$

$$[D_\alpha , \theta^2] = 2\theta_\alpha \qquad [\bar D_{\dot\alpha} , \bar\theta^2] = 2\bar\theta_{\dot\alpha}$$

$$[D_\alpha , m^2] = 2\bar m \lambda f \theta_\alpha \qquad [\bar D_{\dot\alpha} , m^2] = 2 m \lambda \bar f \bar\theta_{\dot\alpha}$$

$$\begin{aligned} [D_\alpha , \bar D_{\dot\alpha} \bar D_{\dot\beta}] &= 2 [P_{\alpha\dot\beta} \bar D_{\dot\alpha} - P_{\alpha\dot\alpha} \bar D_{\dot\beta}] \\ [\bar D_{\dot\alpha} , D_\alpha D_\beta] &= 2 [P_{\beta\dot\alpha} D_\alpha - P_{\alpha\dot\alpha} D_\beta] \end{aligned} \tag{A.4}$$

The general results

$$\begin{aligned} \{a, bc\} &= \{a, b\}c - b[a, c] \\ &= [a, b]c + b\{a, c\} \\ [a, bc] &= [a, b]c + b[a, c] \\ &= \{a, b\}c - b\{a, c\} \end{aligned} \tag{A.5}$$

also prove useful. Having expanded K_i in Ω_j, substituted into (A.1), we then compare coefficients of the Ω_j (linear independence). The result of this is

$$\begin{aligned} C_0 &= 1 \\ C_i &= 0 \qquad i = 1, 2, 3, 4, 6 \end{aligned} \tag{A.6}$$

while C_5, C_7 and C_8 are the solutions of a system of simultaneous equations

$$0 = C_5^{\dot\alpha\dot\beta} + \frac{\varepsilon^{\alpha\beta}\varepsilon^{\dot\alpha\dot\beta}}{16\Box^2} 8\bar{m} P_{\alpha\dot\gamma_2} P_{\beta\dot\gamma_1}\left[m C_5^{\dot\gamma_1\dot\gamma_2} - 2\lambda f \varepsilon_{\gamma_1\gamma_2} C_8^{\gamma_1\gamma_2\dot\gamma_1\dot\gamma_2}\right] \tag{A.7}$$

$$\begin{aligned} 0 = C_7^{\alpha\dot\alpha\dot\beta} &+ \frac{\varepsilon^{\alpha\beta}\varepsilon^{\dot\alpha\dot\beta}}{16\Box^2}\left\{16 m\lambda\bar{f} P_{\beta\dot\gamma_2}\bar\theta_{\dot\gamma_1} C_5^{\dot\gamma_1\dot\gamma_2} + C_8^{\gamma_1\gamma_2\dot\gamma_1\dot\gamma_2} 32\lambda^2\bar{f}f\varepsilon_{\gamma_1\gamma_2} P_{\beta\dot\gamma_1}\bar\theta_{\dot\gamma_2}\right\} \\ &+ \frac{\varepsilon^{\gamma_1\beta}\varepsilon^{\dot\alpha\dot\beta}}{16\Box^2}\left\{C_7^{\alpha\dot\gamma_1\dot\gamma_2} 8m^2 P_{\gamma_1\dot\gamma_2} P_{\beta\dot\gamma_1}\right\} \end{aligned} \tag{A.8}$$

and finally

$$\begin{aligned} 0 = C_8^{\alpha\beta\dot\alpha\dot\beta} + \frac{1}{16\Box^2}\Big[&\varepsilon^{\alpha\beta}\varepsilon^{\dot\alpha\dot\beta}\{(m^2 + 2m\lambda\bar{f}\varepsilon_{\dot\gamma_1\dot\gamma_2} C_5^{\dot\gamma_1\dot\gamma_2}) \\ &- C_8^{\gamma_1\gamma_2\dot\gamma_1\dot\gamma_2}(4\lambda^2\bar{f}f\varepsilon_{\gamma_1\gamma_2}\varepsilon_{\dot\gamma_1\dot\gamma_2})\} \\ &\varepsilon^{\beta\gamma_1}\varepsilon^{\dot\alpha\dot\beta}\{C_7^{\alpha\dot\gamma_1\dot\gamma_2} 16\lambda\bar{f} m P_{\gamma_1\dot\gamma_1}\bar\theta_{\dot\gamma_2}\} \\ &+ \varepsilon^{\gamma_2\gamma_1}\varepsilon^{\dot\alpha\dot\beta}\{C_8^{\beta\alpha\dot\gamma_1\dot\gamma_2} 8m^2 P_{\gamma_2\dot\gamma_1} P_{\gamma_1\dot\gamma_2}\}\Big] \end{aligned} \tag{A.9}$$

At this stage, a single assumption has been made, that C_5 and C_8 are antisymmetric in their $\dot{\gamma}$ indices and γ indices (individually). (This is finally confirmed when a solution is obtained, relying upon its uniqueness.)

Besides their size, these equations are difficult to solve because of the coupling of spinor indices. However, we now call upon the fact that C_i are only needed at $\theta = 0$, hence may always be given an expansion

$$C_i = A_i^{\cdots\cdot} + B_i^{\cdots\cdot} \bar{\theta}^{\dot{\alpha}} + D_i^{\cdots\cdot} \bar{\theta}^2 \tag{A.10}$$

(recall $\bar{\theta}_{\dot{\alpha}} \bar{\theta}_{\dot{\beta}} = -\frac{1}{2}\epsilon_{\dot{\alpha}\dot{\beta}} \bar{\theta}^2$).

A_i, B_i and D_i can only be constructed from the remaining objects carrying spinor indices, namely,

$$P_{\alpha\dot{\alpha}} \,,\ \varepsilon^{\dot{\alpha}\dot{\beta}} \,,\ \varepsilon^{\alpha\beta} \tag{A.11}$$

and $\square$ is available (in integer powers only) for dimension control. As a result, dimensional analysis shows $B_5 = B_8 = 0$ (as they require fractional dimension) and similarly $A_7 = D_7 = 0$. It is then easy to prove that C_5 <u>must</u> have the form

$$C_5^{\dot{\alpha}\dot{\beta}} = \varepsilon^{\dot{\alpha}\dot{\beta}} Z_5 \tag{A.12}$$

It is a little more difficult to place restrictions on C_8 and C_7, however the following are found to provide a consistent solution,

$$\begin{aligned} C_7^{\alpha\dot{\alpha}\dot{\beta}} &= \varepsilon^{\dot{\alpha}\dot{\beta}} \bar{P}^{\dot{\gamma}\alpha} \bar{\theta}_{\dot{\gamma}} Z_7 \\ C_8^{\alpha\beta\dot{\alpha}\dot{\beta}} &= \varepsilon^{\alpha\beta} \varepsilon^{\dot{\alpha}\dot{\beta}} Z_8 \end{aligned} \tag{A.13}$$

(a $\bar{p}^{\dot{\alpha}\alpha} \bar{\theta}^{\dot{\beta}}$ contribution to C_7 is finally found to have a zero coefficient so I exclude it here). With these in (A.7), (A.8) and (A.9), one obtains the final solutions in (4.21).

This method is straightforward but rather long winded, something equivalent to a p operator approach would be welcome, however the present author was not able to succeed there !

TABLE : The tensor basis employed in the inversion of $\mathcal{O}$, see Eq. (4.19).

i	Ω_i	i	Ω_i
0	1	5	$\bar{D}_{\dot\alpha}\bar{D}_{\dot\beta}$
1	D_α	6	$D_\alpha D_\beta \bar{D}_{\dot\alpha}$
2	$\bar{D}_{\dot\alpha}$	7	$D_\alpha \bar{D}_{\dot\alpha}\bar{D}_{\dot\beta}$
3	$D_\alpha D_\beta$	8	$D_\alpha D_\beta \bar{D}_{\dot\alpha}\bar{D}_{\dot\beta}$
4	$D_\alpha \bar{D}_{\dot\alpha}$		

****

FIGURE CAPTIONS

Figure 1 : The one-loop ϕ tadpole. To utilize superfield propagators we must introduce θ_1 and θ_2 on either side of θ_o and then take the limit as θ_1 and θ_2 approach θ_o.

Figure 2 : In the broken theory, it is still possible to associate $\bar{D}^2$ terms with the vertices rather than with the $\phi\phi$ propagator itself.

Figure 3 : The formal propagator Feynman rule for a $\phi\phi$ propagator in the broken SUSY theory, $\mathcal{O}$ is defined in (4.14).

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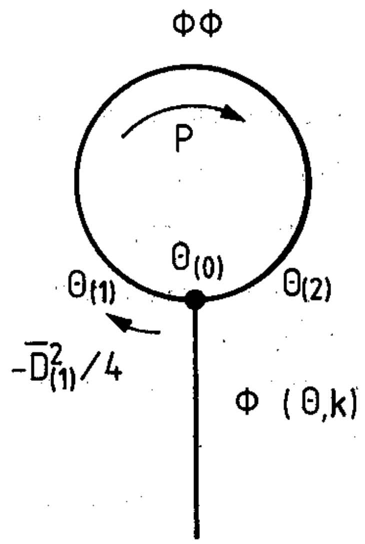


Fig. 1

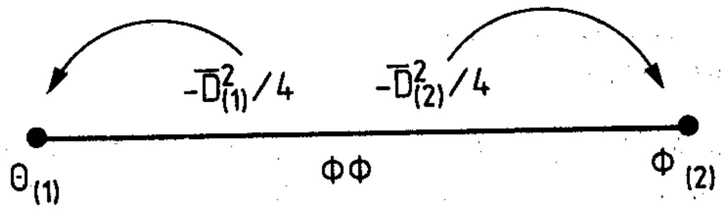
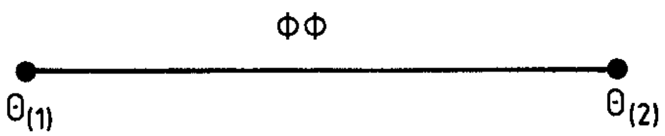


Fig. 2



$$= \frac{1}{\Box} \frac{\bar{m}_{(2)}}{\Box^{1/2}} \; P^{(2)}_{1} \; O^{-1}_{(2)} \; P^{(2)}_{+} \; S(\Theta_{12})$$

Fig. 3