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ON THE STRUCTURE OF COMPOSITE GOLDSTONE SUPERMULTIPLETS

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A B S T R A C T

We study the effect of a supersymmetry breaking scalar mass Δ in the supersymmetric Nambu-Jona Lasinio model. For a supersymmetry breaking scale Δ larger than a critical value, $\Delta > \Delta_c$, a spontaneous breaking of chiral invariance is found, and a supersymmetric Dirac mass is generated for the fundamental chiral multiplets. The Goldstone sector of the theory contains two composite chiral multiplets with non-supersymmetric masses of order Δ. The supermultiplet containing the Goldstone boson exhibits a residual self-interaction of current-current type. Possible applications for composite models of quarks and leptons are considered.

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1. - INTRODUCTION

In recent years, the problem of dynamical symmetry breaking in supersymmetric theories[1] has received considerable attention. The connection between chiral symmetry and supersymmetry has been investigated in the context of non-renormalizable models[2)-3)] as well as supersymmetric gauge theories which have been studied on the basis of effective Lagrangians[4)-8)], by exploiting general principles[9)-13)] such as anomaly matching and complementarity, or by explicit dynamical calculations[14)-17)]. It has become clear that, due to their special vacuum structure, supersymmetric theories behave very differently from ordinary ones. Despite considerable efforts, however, our understanding of the complicated non-perturbative structure of confining supersymmetric gauge theories is still in its infancy.

To a large extent this theoretical work is motivated by attempts to construct a dynamical theory of composite quarks and leptons[18]. Such preon theories have to provide a dynamical reason for the occurrence of fermionic bound states whose size is very small compared to their Compton wave length, $\langle r \rangle_f \ll 1/m_f$. Supersymmetry may play a crucial role in generating light composite fermions: chiral symmetries, which have been suggested as a means to keep fermionic bound states light[19),20)], may be protected[2),4),9)], Goldstone fermions[21] may appear, or quasi-Goldstone fermions[22] may be generated as supersymmetric partners of Goldstone bosons. In applying the Goldstone mechanism to supersymmetric preon models, one usually assumes that either one or two Goldstone bosons are part of one chiral supermultiplet[22)-25)], and that these Goldstone multiplets contain essentially all massless bound states. Yet very little is known about the Goldstone sector of supersymmetric theories with dynamical symmetry breaking, and it is conceivable that only a deeper understanding of these dynamical questions will provide the clue to the main issues of composite models: the family problem and the mechanism of fermion mass generation.

In order to learn more about composite Goldstone supermultiplets, we study again the supersymmetric Nambu-Jona Lasinio model[2]. It turns out that a sufficiently large supersymmetry breaking scalar mass Δ, $\Delta \gg \Delta_c$, which can be small compared to the cut-off Λ, $\Delta \ll \Lambda$, leads to spontaneous breaking of chiral invariance. A supersymmetric Dirac mass m is generated for the fundamental chiral multiplets. The Goldstone sector contains two composite chiral multiplets. One of them contains the Goldstone boson and exhibits residual self-interactions familiar from the study of non-linear realizations; it is connected with a second composite chiral multiplet through a Dirac-like mass term. Masses in the Goldstone sector are of order Δ.

The paper is organized as follows. In Section 2 we discuss the ordinary Nambu-Jona Lasinio (NJL) model[26] in terms of the effective potential for a properly chosen auxiliary field. Section 3 deals with the supersymmetric NJL (SNJL) model and its Lagrange multiplier superfields. In Section 4 we compute the effective potential for the SNJL model and discuss the conditions for spontaneous symmetry breaking. Sections 5 and 6 treat the Goldstone multiplets and their physical properties. In Section 7 we summarize our results and present some speculations concerning possible applications for composite models of quarks and leptons. In the Appendix, the propagators for chiral multiplets are listed in the case of a supersymmetry breaking scalar mass term.

2. - THE NAMBU-JONA LASINIO MODEL

For later comparison with the supersymmetric case and in order to establish some notation, we discuss in this section the ordinary NJL model using the effective potential approach[27),28)]. Our treatment is closely related to the Gross-Neveu model[29].

The Lagrangian of the NJL model reads*):

*) We use the conventions of Wess and Bagger[30].

$$\mathcal{L} = i\partial_m \psi_+ \sigma^m \bar{\psi}_+ + i\partial_m \psi_- \sigma^m \bar{\psi}_- + g^2 \psi_+ \psi_- \bar{\psi}_+ \bar{\psi}_- \quad , \tag{2.1}$$

where ψ_+ and ψ_- are two two-component Weyl spinors and the coupling g has mass dimension -1. The model is non-renormalizable and has to be provided with a cut-off Λ *). The Lagrangian (2.1) is invariant under the chiral U(1) transformations:

*) As in the Gross-Neveu model[29], we could have started with N-component fermion fields in order to make our subsequent one-loop approximation to the effective action of the bound states exact to leading order in a 1/N expansion.

$$U_V(1) \quad : \quad \psi_\pm \longrightarrow e^{\pm i\alpha}\, \psi_\pm \quad , \tag{2.2a}$$

$$U_A(1) \quad : \quad \psi_\pm \longrightarrow e^{i\beta}\, \psi_\pm \quad . \tag{2.2b}$$

As Nambu and Jona Lasinio have shown, for appropriate choices of Λ and g, the second U(1) symmetry is spontaneously broken by the formation of a vacuum expectation value of the operator $\psi_+\psi_-$. This phenomenon of dynamical symmetry breaking is conveniently studied in terms of the effective potential of an auxiliary field ϕ [29]:

$$\begin{aligned} \mathcal{L} \rightarrow \mathcal{L} &- (\varphi - g\bar\psi_+\bar\psi_-)(\varphi^* - g\psi_+\psi_-) \\ &= i\partial_m\psi_+\sigma^m\bar\psi_+ + i\partial_m\psi_-\sigma^m\bar\psi_- - \varphi^*\varphi \\ &\qquad + g\varphi\psi_+\psi_- + g\varphi^*\bar\psi_+\bar\psi_- \quad ; \end{aligned} \tag{2.3}$$

by its classical equation of motion ϕ is identical with $g\bar\psi_+\bar\psi_-$ *).

*) For a discussion of the subtle differences between the effective potential of the field ϕ and the effective potential of the composite operator $\psi_+\psi_-$ we refer the reader to the paper of Gross and Neveu[29].

As a consequence of the chiral U(1) invariance of the Lagrangian (2.3), the effective potential of ϕ depends only on $z = \phi^*\phi$. The one-loop contribution to the derivative of the effective potential is given by the tadpole graph[31] of Fig. 1 calculated with the fermion propagator for a constant background field ϕ. Using the propagators of the Appendix, one obtains for the sum of the tree and one-loop contributions with a covariant Euclidean cut-off[26],

$$\frac{\partial}{\partial\varphi^*}V(z) = \varphi - 2ig\,(g\varphi^*)^*\,\Delta_F(0;g^2\varphi^*\varphi)$$

$$= \varphi\left[1 - \frac{g^2\Lambda^2}{8\pi^2}\left(1 - \frac{g^2 z}{\Lambda^2}\ln\left(\frac{\Lambda^2}{g^2 z}+1\right)\right)\right] \quad . \tag{2.4}$$

Integration of (2.4) with the boundary condition V(0) = 0 yields

$$v(\eta) \equiv 16\pi^2\frac{V(z)}{\Lambda^4} = \frac{2}{\alpha}\eta - \eta + \eta^2\ln\left(\frac{1}{\eta}+1\right) - \ln(1+\eta) \quad , \tag{2.5}$$

with

$$\eta = \frac{g^2}{\Lambda^2}z = \frac{g^2}{\Lambda^2}\varphi^*\varphi \quad , \quad \alpha = \frac{g^2\Lambda^2}{8\pi^2} \quad .$$

From (2.4) it is easy to see that the extremum condition

$$\frac{\partial v}{\partial\varphi^*} = \varphi\frac{2g^2}{\Lambda^2}\left[\frac{1}{\alpha} - 1 + \eta\ln\left(\frac{1}{\eta}+1\right)\right] = 0 \tag{2.6}$$

is the familiar gap equation[26)] which admits a symmetry breaking solution $\phi \neq 0$ only for

$$\frac{\partial}{\partial\eta}v(\eta)\Big|_{\eta=0} = 2\left(\frac{1}{\alpha}-1\right) < 0 \quad , \tag{2.7}$$

i.e., for strong enough coupling g satisfying the inequality

$$\alpha = \frac{g^2\Lambda^2}{8\pi^2} > 1 \quad . \tag{2.8}$$

If (2.8) is satisfied, the ground state of the theory is given by a non-vanishing value ϕ^0 of the field $\phi(x)$ and a Dirac mass m, which we may choose to be real, is generated for the spinors ψ_+ and ψ_-,

$$m = g\varphi^0 = g\varphi^{0*} \quad . \tag{2.9}$$

The Goldstone boson associated with this spontaneous breaking of the axial $U_A(1)$ symmetry (cf. (2.2b)) may be studied by computing the effective action of the field $\phi(x)$. After a shift around the minimum value ϕ^0,

$$\varphi = \varphi^0 + \varphi^1 \quad , \tag{2.10}$$

the Lagrangian (2.3) reads

$$\begin{aligned}\mathcal{L} = &- \varphi^{0\,2} - \varphi^0(\varphi' + \varphi'^*) - \varphi'^*\varphi' \\ &+ i\partial_m \psi_+ \sigma^m \bar{\psi}_+ + i\partial_m \psi_- \sigma^m \bar{\psi}_- + m(\psi_+\psi_- + \bar{\psi}_+\bar{\psi}_-) \\ &+ g\varphi'\psi_+\psi_- + g\varphi'^*\bar{\psi}_+\bar{\psi}_- \quad .\end{aligned} \tag{2.11}$$

The one-loop contribution to the effective action of ϕ' contains a tadpole term (cf. Fig. 1) which cancels the linear part in ϕ' and ϕ'^* in (2.11) and two quadratic contributions (cf. Fig. 2). One obtains, after partial integrations, for the total quadratic part

$$\begin{aligned}\Gamma^{(2)}(\varphi'^*, \varphi') = &- \int d^4x\, \varphi'^*(x)\, \varphi'(x) \\ &- ig^2 \int d^4x\, d^4x' \Big\{ \Delta_F^2(x-x'; m^2) \big[-m^2(\varphi'^*(x)\,\varphi'^*(x') + \varphi'(x)\,\varphi'(x')) \\ &\qquad + \varphi'^*(x)\,(\Box' - 2m^2)\,\varphi'(x') \,\big] \\ &\qquad - 2\Delta_F(0; m^2)\, \delta^4(x-x')\, \varphi'^*(x)\, \varphi'(x') \Big\} \quad .\end{aligned} \tag{2.12}$$

As a consequence of the gap equation (2.6) the two local terms in (2.12) cancel and with

$$\varphi(x) = \frac{1}{\sqrt{2}} \left(\sigma(x) + i\pi(x) \right) \tag{2.13}$$

we arrive at

$$\begin{aligned}\Gamma^{(2)}(\sigma, \pi) = -ig^2 \int d^4x\, d^4x'\, \Delta_F^2(x-x'; m^2) \big[&\tfrac{1}{2}\sigma(x)(\Box' - 4m^2)\sigma(x') \\ &+ \tfrac{1}{2}\pi(x)\, \Box'\, \pi(x') \;\big] \quad .\end{aligned} \tag{2.14}$$

Equation (2.14) represents the expected result: quantum corrections generate a kinetic term for the auxiliary field $\phi(x)$ - its imaginary part corresponds to a massless excitation, the Goldstone boson of the spontaneously broken $U_A(1)$ symmetry, and its real part is related to the σ particle of mass 2m. This is the familiar result first obtained by Nambu and Jona Lasinio[26]; as in the Bethe-Salpeter formalism, the occurrence of a massless pion is a direct consequence of the gap equation.

It is easy to see that the kinetic terms (2.14) have the correct sign. In the case $m \ll \Lambda$, to which we restrict ourselves, the expression (2.14) is essentially local and one obtains up to terms of relative order $1/(\ln(\Lambda^2/m^2))$

$$\Gamma^{(2)}(\sigma,\pi) = Z \int d^4x \left[\tfrac{1}{2}\sigma(x)(\Box - 4m^2)\sigma(x) + \tfrac{1}{2}\pi(x)\Box\pi(x) \right] , \tag{2.15}$$

with a wave function renormalization constant Z given by

$$Z = \frac{g^2}{16\pi^2}\left(\ln\frac{\Lambda^2}{m^2} + O(1) \right) . \tag{2.16}$$

3. - THE SUPERSYMMETRIC EXTENSION

The supersymmetric Nambu-Jona Lasinio (SNJL) model has previously been studied in Ref. 2). The Lagrangian reads

$$\mathcal{L} = \int d^4\theta \left[\bar{\Phi}_+\Phi_+ + \bar{\Phi}_-\Phi_- + g^2 \bar{\Phi}_+\bar{\Phi}_-\Phi_+\Phi_- \right] , \tag{3.1}$$

where the chiral multiplets $\Phi_\pm$ contain the Weyl spinors $\psi_\pm$ considered in the previous section,

$$\Phi_\pm(x,\theta,\bar\theta) = A_\pm(y) + \sqrt{2}\,\theta\psi_\pm(y) + \theta\theta F_\pm(y) , \tag{3.2}$$

with

$$y^m = x^m + i\theta\sigma^m\bar\theta .$$

The Lagrangian (3.1) is invariant under the two U(1) symmetries of Section 2 which now act on the entire supermultiplets,

$$U_V(1) : \quad \Phi_\pm \to e^{\pm i\alpha}\,\Phi_\pm \tag{3.3a}$$

$$U_A(1) \quad : \quad \Phi_\pm \rightarrow e^{i\beta}\,\Phi_\pm \tag{3.3b}$$

and an additional R symmetry which may be defined as

$$U_R(1) \; : \quad \begin{aligned} A_\pm &\rightarrow e^{i\gamma} A_\pm \\ \psi_\pm &\rightarrow \psi_\pm \\ F_\pm &\rightarrow e^{-i\gamma} F_\pm \end{aligned} \quad . \tag{3.3c}$$

As shown in Ref. 2), the SNJL model does not exhibit a spontaneous symmetry breaking like the ordinary NJL model, i.e., supersymmetry protects the chiral invariance. This feature of the SNJL model, however, is not expected to persist if an explicit supersymmetry breaking scalar mass term

$$\delta \mathcal{L}_{SB} = -\Delta^2 (A_+^* A_+ + A_-^* A_-) \quad , \tag{3.4}$$

which preserves the invariances (3.3), is added to the Lagrangian (3.1). Indeed, for $\Delta \gg \Lambda$, where the scalar degrees of freedom cease to play any role for the dynamics of the theory, one should recover the ordinary NJL model, i.e., for strong enough coupling g [cf. Eq. (2.8)] the $U_A(1)$ symmetry should be broken. A priori it is not clear what this transition looks like, i.e., whether dynamical chiral symmetry breaking appears for arbitrarily small Δ or only for Δ larger than some critical supersymmetry breaking scale Δ_c.

In order to study the model defined by (3.1) and (3.4), we proceed in analogy to Section 2: we first introduce auxiliary superfields and study their effective potential which, for strong enough coupling g, will again exhibit spontaneous symmetry breaking; in a second step, we then study the composite Goldstone multiplets in terms of the effective action for the fluctuations of the auxiliary superfields.

Equation (2.3) suggests the introduction of a single auxiliary chiral superfield Φ associated with the change of the Lagrangian (3.1)

$$\mathcal{L} \rightarrow \mathcal{L} - \int d^4\theta\, (\bar\Phi - g\bar\Phi_+\bar\Phi_-)(\Phi - g\Phi_+\Phi_-) \quad . \tag{3.5}$$

In Eq. (3.5), however, the scalar and the fermionic components of the superfield Φ appear with kinetic terms, hence they do not play the role of Lagrange multipliers: their equations of motion have non-trivial solutions, i.e., they do not have the form of constraints which would make (3.5) identical with the original Lagrangian at the classical level. Furthermore, the kinetic term has the wrong (negative) sign which does not allow for a functional integration over Φ.

It turns out that one has to introduce two chiral superfields Φ_1 and Φ_2. The corresponding Lagrangian reads

$$\mathcal{L} = \int d^4\theta \left[(\bar{\Phi}_+\Phi_+ + \bar{\Phi}_-\Phi_-)(1-\delta) + \bar{\Phi}_2\Phi_2 \right] + \int d^2\theta \left[\Phi_1\Phi_2 - g\,\Phi_1\Phi_+\Phi_- \right] + \text{c.c.} \quad , \tag{3.6}$$

where $\delta = \Delta^2\theta\theta\bar{\theta}\bar{\theta}$ corresponds to the supersymmetry breaking term introduced in (3.4). The fields Φ_1, Φ_2 carry mass dimensions 2 and 1, respectively. The superfield Φ_1 plays the role of a Lagrange multiplier whose equation of motion yields the constraint

$$\Phi_2 = g\,\Phi_+\Phi_- \quad , \tag{3.7a}$$

which, in components, reads

$$\begin{aligned} A_2 &= g\,A_+A_- \quad , \\ \psi_2 &= g(A_+\psi_- + A_-\psi_+) \quad , \\ F_2 &= g(A_+F_- + A_-F_+ - \psi_+\psi_-) \quad . \end{aligned} \tag{3.7b}$$

After inserting (3.7) into (3.6) one immediately recovers the original Lagrangian (3.1). Despite the fact that (3.6) contains a canonical kinetic energy for Φ_2 the above constraint shows that Φ_2 has to be interpreted as a composite object made out of Φ_+ and Φ_-. The interpretation of Φ_1 can be obtained from the equation of motion of the field Φ_2,

$$(-\tfrac{1}{4})\bar{D}^2\bar{\Phi}_2 + \Phi_1 = 0 \quad . \tag{3.8}$$

Hence, after inserting (3.7) into (3.8), one finds

$$\Phi_1 = -g(-\tfrac{1}{4})\bar{D}^2(\bar{\Phi}_+\bar{\Phi}_-) \quad , \tag{3.9a}$$

or, in components,

$$A_1 = -g\,(A_+^* F_-^* + A_-^* F_+^* - \bar{\psi}_+ \bar{\psi}_-) \quad ,$$
$$\psi_1 = -g i \sigma^m \partial_m (A_+^* \bar{\psi}_- + A_-^* \bar{\psi}_+) \quad ,$$
$$F_1 = -g \,\Box (A_+^* A_-^*) \quad . \tag{3.9b}$$

It is important to observe that it is not possible to use (3.8) in order to eliminate Φ_1 or Φ_2 in (3.6): in both cases one would end up with pathological kinetic energies (with wrong sign or with derivatives in the denominator). Both fields Φ_1 and Φ_2 have to be kept as independent degrees of freedom in order to obtain a consistent theory.

As for the ordinary NJL model, the Lagrangian (3.6) for the auxiliary fields can be found as a limiting case of a renormalizable theory with canonical superfields. Starting with

$$\mathcal{L} = \int d^4\theta \left[(\bar{\Phi}_+ \Phi_+ + \bar{\Phi}_- \Phi_-)(1-\delta) + \bar{\tilde{\Phi}}_1 \tilde{\Phi}_1 + \bar{\Phi}_2 \Phi_2 \right]$$
$$+ \int d^2\theta \left[\mu \tilde{\Phi}_1 \Phi_2 - h \tilde{\Phi}_1 \Phi_+ \Phi_- \right] + \text{c.c.} \quad , \tag{3.10}$$

where the mass dimensions of $\tilde{\Phi}_1$, μ, and h are 1, 1 and 0, one obtains (3.6) after defining

$$\tilde{\Phi}_1 = \frac{1}{\mu} \Phi_1 \quad ,$$
$$h = \mu g \quad , \tag{3.11}$$

in the limit $\mu \to \infty$ with Φ_1 and g fixed.

4. - THE EFFECTIVE POTENTIAL FOR THE SNJL MODEL

In this section we will use the Lagrangian (3.6) in order to investigate the vacuum structure of the SNJL model with explicit supersymmetry breaking. We are interested in possible vacuum expectation values of the scalar fields A_1, A_2, F_1, F_2. The tree level potential for these fields reads:

$$V_0 = -F_2^* F_2 - A_1 F_2 - A_2 F_1 - A_1^* F_2^* - A_2^* F_1^* \quad . \tag{4.1}$$

From (3.6) it is clear that the one-loop contribution to the effective potential will be some function V_1 of A_1 and F_1 only. Hence the equations for the extrema of the complete potential in the directions A_2 and F_2 can already be found from (4.1),

$$0 = \frac{\partial V_0}{\partial A_2} = - F_1 \quad , \tag{4.2a}$$

$$0 = \frac{\partial V_0}{\partial F_2} = - F_2^* - A_1 \quad . \tag{4.2b}$$

As a consequence of (4.2a), we have to evaluate V_1 only for $F_1 = 0$ in order to find the extrema of the potential. After using (4.2), one obtains for the sum of the tree and one-loop contributions of the effective potential

$$V = A_1^* A_1 + V_1 (A_1^* A_1) \quad , \tag{4.3}$$

where V_1 depends only on $z = |A_1|^2$ due to the chiral invariance of the theory. In the next section it will be shown that the solution of (4.2a) actually corresponds to a minimum of V.

The interaction Lagrangian between A_1 and the components of Φ_+ and Φ_- reads (cf. (3.6)),

$$\mathcal{L}_{int} = - g A_1 (A_+ F_- + A_- F_+ - \psi_+ \psi_-) + \text{c.c.} \quad , \tag{4.4}$$

which is analogous to the coupling of the auxiliary field ϕ in Eq. (2.3) except for the additional scalar interactions caused by supersymmetry.

The one-loop part V_1 of the effective potential can now be computed in the same way as in Section 2. In addition to the fermionic tadpole, Fig. 1, one has the scalar contribution of Fig. 3, and using the propagators of the Appendix, one obtains

$$\frac{\partial}{\partial A_1^*} V_1 = -2ig\,(gA_1^*)^*\left[\Delta_F(0;g^2z) - \Delta_F(0;g^2z+\Delta^2)\right]$$

$$= -A_1 \frac{g^2\Lambda^2}{8\pi^2}\left[\frac{g^2z+\Delta^2}{\Lambda^2}\ln\left(\frac{\Lambda^2}{g^2z+\Delta^2}+1\right) - \frac{g^2z}{\Lambda^2}\ln\left(\frac{\Lambda^2}{g^2z}+1\right)\right] , \qquad (4.5)$$

where the same regularization as in Section 2 has been used. Integration of (4.5) yields for the complete potential $V = V_0 + V_1$,

$$v(\eta) \equiv 16\pi^2 \frac{V(z)}{\Lambda^4} = \frac{2}{\alpha}\eta - (\eta+\xi)^2\ln\left(\frac{1}{\eta+\xi}+1\right) + \eta^2\ln\left(\frac{1}{\eta}+1\right)$$

$$- \ln\frac{(1+\eta)(1+\xi)}{1+\eta+\xi} + \xi^2\ln\left(\frac{1}{\xi}+1\right) , \qquad (4.6)$$

with

$$\eta = \frac{g^2}{\Lambda^2}z = \frac{g^2}{\Lambda^2}A_1^*A_1 \quad , \quad \xi = \frac{\Delta^2}{\Lambda^2} \quad , \quad \alpha = \frac{g^2\Lambda^2}{8\pi^2} \quad ;$$

the integration constant in (4.6) has been chosen such that $v(0) = 0$. $v(\eta)$ coincides with its non-supersymmetric analogue (2.5) in the limit $\xi \to \infty$ whereas the quantum contribution to v vanishes for $\xi = 0$.

The supersymmetric gap equation

$$\frac{\partial v}{\partial A_1^*} = A_1\frac{2g^2}{\Lambda^2}\left[\frac{1}{\alpha} - (\eta+\xi)\ln\left(\frac{1}{\eta+\xi}+1\right) + \eta\ln\left(\frac{1}{\eta}+1\right)\right] = 0 , \qquad (4.7)$$

has a symmetry breaking solution $A_1^0 \neq 0$ only for

$$\frac{\partial}{\partial\eta}v(\eta)\Big|_{\eta=0} = 2\left[\frac{1}{\alpha} - \xi\ln\left(\frac{1}{\xi}+1\right)\right] < 0 , \qquad (4.8)$$

i.e., for a coupling α satisfying

$$\alpha \xi \ln\left(\frac{1}{\xi}+1\right) > 1 \qquad . \tag{4.9}$$

The inequality (4.9) replaces the non-supersymmetric bound (2.8), $\alpha > 1$. It is straightforward to see that, if (4.9) is fulfilled, the absolute minimum of $v(\eta)$ is assumed for $\eta_0 \neq 0$ and the axial $U_A(1)$ symmetry (3.3b) is spontaneously broken.

The relation (4.9) determines for any fixed coupling $\alpha > 1$ the critical supersymmetry breaking scale Δ_c, below which chiral symmetry remains unbroken,

$$\alpha \xi_c \ln\left(\frac{1}{\xi_c}+1\right) = 1 \qquad . \tag{4.10}$$

Equation (4.10) reflects the effect of supersymmetry which turns a quadratically divergent expression for the one-loop effective potential into a logarithmically divergent one: for a supersymmetry breaking scale Δ much smaller than the cut-off Λ, $\xi \ll 1$, a very strong coupling is needed,

$$\alpha \sim \frac{1}{\xi \ln \frac{1}{\xi}} \qquad , \tag{4.11}$$

in order to form the condensate which breaks chiral invariance, i.e., supersymmetry indeed protects chiral symmetry.

The connection between α and η, which follows from the gap equation (4.7), is shown in Fig. 4. One should bear in mind that our model is physically sensible only for mass parameters Δ^2, $g^2 z_0$ smaller than the cut-off Λ^2, i.e., ξ, $\eta < 1$. In Fig. 5, we have schematically plotted η, which parametrizes the condensate, as a function of ξ for some fixed coupling α: for $\xi < \xi_c$ one has $\eta = 0$; furthermore we find $(d\eta/d\xi)\big|_{\xi=\xi_c} = 0$. For $\xi \to \infty$, η approaches the value η_∞ which is given by the solution of the non-supersymmetric gap equation (2.6).

If the axial $U_A(1)$ symmetry is broken and the effective potential acquires its minimum at $z_0 = |A_1^0|^2 \neq 0$, we infer from Eq. (3.6) that a <u>supersymmetric</u> Dirac mass m is generated for the two chiral multiplets Φ_+ and Φ_-, which one may choose to be real,

$$m = g A_1^0 = g A_1^{0*} \qquad . \tag{4.12}$$

The explicit supersymmetry breaking scalar mass Δ leads to a non-supersymmetric mass spectrum in the Goldstone sector of the theory which we shall discuss in the following section.

5. - THE GOLDSTONE MULTIPLETS

In order to study the Goldstone sector of the SNJL model, we proceed as in Section 2, i.e., we shift the superfield Φ_1 around the minimum value A_1^0,

$$\Phi_1 = A_1^0 + \Phi_1' \qquad , \tag{5.1}$$

and compute the quadratic part of the effective action for Φ_1' , $\Gamma^{(2)}(\bar\Phi_1', \Phi_1')$, starting from the Lagrangian (cf. (3.6) and (4.12))

$$\mathcal{L} = \int d^4\theta \left[(\bar\Phi_+\Phi_+ + \bar\Phi_-\Phi_-)(1-\delta) + \bar\Phi_2\Phi_2 \right] + \int d^2\theta \left(-m\Phi_+\Phi_- + A_1^0\Phi_2 + \Phi_1'\Phi_2 - g\Phi_1'\Phi_+\Phi_- \right) + \text{c.c.} \quad . \tag{5.2}$$

The various contributing graphs are listed in Fig. 6; the corresponding propagators are given in the Appendix. After some algebra one obtains the following result for the quantum contributions to $\Gamma^{(2)}(\bar\Phi_1', \Phi_1')$:

$$\begin{aligned} \Gamma^{(2)}_{qu}(\bar\Phi_1', \Phi_1') = \\ -ig^2 \int d^4x\, d^4x' \Big\{ & -m^2\left[\Delta_F^2(x-x';m^2) - \Delta_F^2(x-x';m^2+\Delta^2)\right]\left(A_1'^*(x)A_1'^*(x') + A_1'(x)A_1'(x')\right) \\ & + A_1'^*(x)\Big(\Delta_F^2(x-x';m^2)\Box' - 2m^2\left[\Delta_F^2(x-x';m^2) - \Delta_F^2(x-x';m^2+\Delta^2)\right]\Big)A_1'(x') \\ & - 2i\,\Delta_F(x-x';m^2+\Delta^2)\,\partial_\mu\Delta_F(x-x';m^2)\,\psi_1(x)\sigma^\mu\bar\psi_1(x') \\ & + \Delta_F^2(x-x';m^2+\Delta^2)\,F_1^*(x)F_1(x') \\ & - 2\left[\Delta_F(0;m^2) - \Delta_F(0;m^2+\Delta^2)\right]\delta^4(x-x')\,A_1'^*(x)A_1'(x') \Big\} \; ; \end{aligned} \tag{5.3}$$

as in Section 2, the tadpole terms which cancel the part linear in Φ_2 in the Lagrangian (5.2) have been omitted. We note that the expression (5.3) agrees with (2.12) in the limit $\xi = (\Delta^2/\Lambda^2) \to \infty$.

Subsequently, we will limit ourselves to the case of a supersymmetry breaking scale Δ much smaller than the cut-off Λ, i.e., $\xi \ll 1$. According to (4.11) this requires a large coupling α in order to obtain chiral symmetry breaking,

$$\alpha > \frac{1}{\xi \ln\left(\frac{1}{\xi}+1\right)} .$$

Depending on α, the spontaneously generated mass $m = \eta^{\frac{1}{2}}\Lambda$ can take any arbitrary value. Yet in the physically relevant range of parameters,

$$0 < m, \Delta \ll \Lambda \qquad , \tag{5.4}$$

the gap equation (4.7) has the simple solution

$$\frac{m^2}{\Lambda^2} = \eta = e^{-\frac{1}{\alpha\xi}} = e^{-\frac{8\pi^2}{g^2\Delta^2}} . \tag{5.5}$$

In (5.5) corrections $O(1)$ and $O(\ln(1+(\Delta^2/m^2)))$ to $\ln(\Lambda^2/m^2)$ have been neglected, i.e., the above solution is not adequate in the limit $m \to 0$.

For parameters satisfying (5.4) the expression (5.3) is essentially local and one obtains for the quadratic part of the effective action of Φ_1' and Φ_2, up to terms of relative order $1/(\ln(\Lambda^2/m^2))$,

$$\begin{aligned}
&\Gamma^{(2)}(\bar\Phi_1', \bar\Phi_2, \Phi_1, \Phi_2) \\
&= \int d^4x \Big\{ A_2^* \Box A_2 + i\partial_m \psi_2 \sigma^m \bar\psi_2 + F_2^* F_2 \\
&\qquad + A_1' F_2 + A_2 F_1 + A_1'^* F_2^* + A_2^* F_1^* - \psi_1\psi_2 - \bar\psi_1\bar\psi_2 + A_1'^* A_1' \\
&\qquad + Z_A A_1'^* \Box A_1' - 4m^2 Z_m \,\mathrm{Re}^2(A_1') \\
&\qquad + Z_\psi\, i\partial_m \psi_1 \sigma^m \bar\psi_1 + Z_F F_1^* F_1 \Big\} ,
\end{aligned} \tag{5.6}$$

with

$$Z_A = Z_\psi = Z_F = \frac{g^2}{16\pi^2}\left(\ln\frac{\Lambda^2}{m^2} + O(1)\right) , \tag{5.7a}$$

$$Z_M = \frac{g^2}{16\pi^2}\left(\ln\frac{m^2+\Delta^2}{m^2} + O\left(\frac{1}{\Lambda^2}\right)\right) . \tag{5.7b}$$

The solution (5.5) of the gap equation yields for the wave function renormalization constants, up to terms of relative order $1/(\ln(\Lambda^2/m^2))$,

$$Z_A = Z_\psi = Z_F = \frac{1}{2\Delta^2} . \tag{5.8}$$

The effective action (5.6) shows that the components of the superfield Φ_1 have received kinetic terms from quantum corrections. We note that no term involving derivatives of F_1 has been generated. Consequently, F_1 can be eliminated as usual resulting in a positive mass squared for the field A_2. This justifies the use of Eq. (4.2a); the point $\langle F_1\rangle_0 = \langle A_2^*\rangle_0 = 0$, $\langle F_2\rangle_0 = -\langle A_1^*\rangle_0 = gA_1^0$ is indeed a minimum of the effective potential. Hence, no scalar condensate $\langle A_2\rangle_0 = g\langle A_+A_-\rangle_0$ (which carries non-zero R charge) is formed and R invariance remains unbroken.

After eliminating the auxiliary fields F_1 and F_2, introducing

$$A_1'(x) = \frac{1}{\sqrt{2}}\left(\sigma(x) + i\,\pi(x)\right) , \tag{5.9}$$

and rescaling the components of Φ_1 such that they have canonical kinetic energies, one obtains

$$\begin{aligned} &\Gamma^{(1)}(\bar\Phi_1', \bar\Phi_2, \Phi_1', \Phi_2) \\ &= \int d^4x \Big\{ A_2^*(\Box - 2\Delta^2)A_2 + i\partial_m\psi_2\sigma^m\bar\psi_2 \\ &\qquad + \tfrac{1}{2}\sigma(\Box - 4m^2\varepsilon)\sigma + \tfrac{1}{2}\pi\Box\pi + i\partial_m\psi_1\sigma^m\bar\psi_1 \\ &\qquad - \sqrt{2}\,\Delta(\psi_1\psi_2 + \bar\psi_1\bar\psi_2) \Big\} , \end{aligned} \tag{5.10}$$

with

$$\varepsilon = O\left(\frac{1}{\ln\frac{\Lambda^2}{m^2}}\right) \quad .$$

The physical interpretation of this result is obvious: the effective action (5.10) describes a massless Goldstone boson π, a real scalar σ of mass $2m\varepsilon^{1/2}$ (the σ mass is finite although very small for our choice of parameters), a complex scalar A_2 of mass $\sqrt{2}\Delta$ and a "Dirac fermion" of mass $\sqrt{2}\Delta$.

The action (5.10) has been obtained from (5.3) for parameters satisfying the condition (5.4). In the case $m \ll \Lambda \ll \Delta$ one also arrives at (5.10) except for the σ mass term for which one now has $\varepsilon = 1$. We thus see that in the limit $\xi = (\Delta^2/\Lambda^2) \to \infty$ all fields, except π and σ, become infinitely heavy, i.e., the physical degrees of freedom due to supersymmetry are "frozen", and we recover the result of Section 2 for the non-supersymmetric case.

In addition to the fields of the Goldstone sector we have, of course, two complex scalars $A_\pm$ of mass $\sqrt{m^2+\Delta^2}$ and one Dirac fermion $\psi_\pm$ of mass m. We remark that the boson and fermion masses, summed over all fields, obey the relation (up to terms of relative order $1/(\ln(\Lambda^2/m^2))$ and possible contributions from additional bound states)

$$\sum_{i=+,-,1,2} \left(M_{b_i}^2 - M_{f_i}^2 \right) = 0 \quad . \tag{5.11}$$

The structure of the Goldstone boson is particularly interesting. It is part of the multiplet Φ_1 whose "wave function" involves derivatives, a possibility already considered in the literature[9),4),32)]. From (3.9) one obtains

$$\begin{aligned}\pi &\sim \mathrm{Im}\left(\left(-\tfrac{1}{4}\right) \bar{D}^2 (\bar{\Phi}_+ \bar{\Phi}_-) \Big|_{\theta=0} \right) \\ &\sim \bar{\psi}\gamma_5 \psi \left[1 + g^2 (A_+^* A_+ + A_-^* A_-) \right]^{-1} \end{aligned} \quad , \tag{5.12}$$

where ψ is the Dirac spinor formed from the Weyl spinors ψ_+ and ψ_-,

$$\psi = \begin{pmatrix} \psi_{+\alpha} \\ \bar{\psi}_-{}^{\dot{\alpha}} \end{pmatrix} \quad . \tag{5.13}$$

In order to obtain (5.12), we have eliminated $F_\pm$ by using the classical equations of motion (cf. (3.1)). It is interesting that the Goldstone boson appears to be predominantly a fermion-antifermion bound state.

We are thus led to the following picture: the Goldstone boson of the spontaneously broken $U_A(1)$ symmetry is connected with <u>two</u> chiral superfields, Φ_1 and Φ_2. Thus the number of quasi-Goldstone bosons and fermions[25] is larger than usually assumed. The superfields Φ_1 and Φ_2 describe physical states which are created from the vacuum by the operators $(-1/4)\bar{D}^2(\bar{\Phi}_+\bar{\Phi}_-)$ and $\Phi_+\Phi_-$. The case $\Delta \ll m$ is of particular physical interest: a heavy, approximately supersymmetric, sector (Φ_+,Φ_-), whose mass is spontaneously generated, is accompagnied by a light Goldstone sector (Φ_1,Φ_2) with a non-supersymmetric mass spectrum of order Δ.

6. - RESIDUAL INTERACTIONS AND PARITY

Due to the coupling of the composite superfield Φ_1' to $\Phi_+\Phi_-$(cf. (5.2)) higher n-point functions are generated for the components of Φ_1' in addition to kinetic terms. Among them is the four-fermion self-interaction

$$\Gamma^{(4)} = -G\int d^4x\; \bar{\psi}_1(x)\bar{\psi}_1(x)\psi_1(x)\psi_1(x) \quad . \tag{6.1}$$

As a consequence of the explicit supersymmetry breaking, (6.1) is part of a rather complicated expression involving the superfield Φ_1', the spurion field $\delta = \Delta^2\theta\theta\bar{\theta}\bar{\theta}$ and covariant derivatives. After some algebra, one obtains from the one-loop graphs of Fig. 7 in the case $\Delta \ll m$ (cf. (5.8)):

$$G = \frac{1}{192\pi^2}\frac{g^4\Delta^2}{Z_+^2 m^4} = \frac{1}{48\pi^2}\frac{g^4\Delta^6}{m^4} \quad . \tag{6.2}$$

Let us combine the two quasi-Goldstone fermions ψ_1 and ψ_2 into a four-component "Dirac spinor" χ,

$$\chi = \begin{pmatrix} \psi_{1\alpha} \\ \bar{\psi}_2^{\dot{\alpha}} \end{pmatrix} \quad , \tag{6.3a}$$

$$\chi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\,\chi \quad . \tag{6.3b}$$

From Eqs. (5.10) and (6.1), one reads off that the Lagrangian for the fermionic part of the Goldstone sector is given by*)

*) We use the conventions of Wess and Bagger[30].

$$\mathcal{L}_\chi = \bar{\chi}\, i \not\partial \chi + \sqrt{2}\,\Delta\,\bar{\chi}\chi$$
$$+ \frac{1}{2} G\, \bar{\chi}_L \gamma^\mu \chi_L\, \bar{\chi}_L \gamma_\mu \chi_L \quad . \tag{6.4}$$

This is a striking result! We have generated precisely the structure of the low energy effective Lagrangian of the standard model of electroweak interactions: the Goldstone sector contains a massive "Dirac fermion" whose "left-handed" component exhibits a positive current-current self-interaction.

There is, however, a problem with this interpretation which is related to the definition of parity. Conventionally, the parity transformation for the fundamental multiplets Φ_+, Φ_- is given by ($x' = (x_1^0, -\vec{x})$)

$$\mathcal{P}: \quad \begin{aligned} A_\pm(x) &\to A^*_\mp(x') \\ \psi_\pm(x) &\to \sigma^0 \bar{\psi}_\mp(x') \\ F_\pm(x) &\to F^*_\mp(x') \end{aligned} \quad . \tag{6.5}$$

With respect to (6.5) the real (imaginary) parts of $A_\pm$ and $F_\pm$ are scalars (pseudoscalars) and the ordinary scalar and pseudoscalar densities formed out of the Dirac spinor ψ (cf. (5.13)) transform as

$$\bar{\psi}(x)\psi(x) \to \bar{\psi}(x')\psi(x') \quad ,$$
$$\bar{\psi}(x)\gamma_5\psi(x) \to -\bar{\psi}(x')\gamma_5\psi(x') \quad . \tag{6.6}$$

From the constraint equations (3.9) and (3.7),

$$\Phi_1 = -g\left(-\frac{1}{4}\right)\bar{D}^2(\bar{\Phi}_+\bar{\Phi}_-) \quad , \quad \Phi_2 = g\,\Phi_+\Phi_- \quad ,$$

one obtains the transformation properties of the Goldstone fields,

$$\mathcal{P} : \quad \begin{aligned} A_{1,2}(x) &\rightarrow A^*_{1,2}(x') \\ \psi_{1,2}(x) &\rightarrow \sigma^0 \bar{\psi}_{1,2}(x') \\ F_{1,2}(x) &\rightarrow F^*_{1,2}(x') \end{aligned} \quad . \tag{6.7}$$

From (6.7) we infer that under parity the "left-handed" and "right-handed" spinors χ_L and χ_R do not transform into each other. Indeed, the Lagrangian (6.4) is invariant under parity which has to be the case because the fundamental Lagrangian (3.1) as well as the vacuum expectation value (4.12) are invariant under the parity transformation (6.5). Therefore we have to conclude that the two quasi-Goldstone fermions χ_L and χ_R do not form a genuine Dirac fermion whose mass term connects spinors of opposite chirality.

It is conceivable, however, that the peculiar behaviour of the quasi-Goldstone fermions under parity is a special feature of the supersymmetric Nambu-Jona Lasinio model whereas the structure of the Lagrangian (6.4) is more general. The doubling of quasi-Goldstone fermions which occurs in (6.4) can be made plausible by simply considering the effect of a small chiral symmetry breaking mass term ($\mu \ll m$)

$$\delta \mathcal{L}_B = \mu \left(\Phi_+ \Phi_- \big|_{\theta\theta} + \bar{\Phi}_+ \bar{\Phi}_- \big|_{\bar{\theta}\bar{\theta}} \right) \quad . \tag{6.8}$$

From Dashen's theorem[33], we know that the Goldstone boson π will acquire a mass $m_\pi^2 \sim \mu$. Since (6.8) is supersymmetric, one also expects a mass for the Weyl fermion ψ_1. Yet a Majorana mass term $\psi_1 \psi_1$ would violate the unbroken R-invariance because ψ_1 carries R-charge -1. The problem can be circumvented through a Dirac mass term $\psi_1 \psi_2$ with a second quasi-Goldstone fermion ψ_2 whose R-charge is +1. Given the resulting doubling of Goldstone supermultiplets, the asymmetry betwen the "left-handed" and the "right-handed" multiplet is expected because non-linear realizations of broken symmetries require only the supermultiplets containing Goldstone bosons to have residual interactions of the form (6.1). Finally, in a non-parity invariant theory, the two types of Goldstone supermultiplets may very well correspond to states of definite chirality.

7. - CONCLUSIONS

Let us summarize our results. We have studied the supersymmetric Nambu-Jona Lasinio model with a supersymmetry breaking scalar mass Δ. The explicit breaking of supersymmetry strongly influences the vacuum structure of the theory: for large enough breaking, $\Delta > \Delta_c$ (cf. (4.10)), the axial $U_A(1)$ symmetry is spontaneously broken and a supersymmetric Dirac mass m is generated for the two fundamental chiral supermultiplets Φ_+ and Φ_-. For strong enough coupling $\alpha \equiv (g^2\Lambda^2/8\pi^2)$, m can be much larger than Δ.

The Goldstone sector of the theory consists of two composite chiral multiplets, Φ_1 and Φ_2, whose bound state structure is given by

$$\Phi_1 \sim \left(-\tfrac{1}{4}\right) \bar{D}^2 (\bar{\Phi}_+ \bar{\Phi}_-) \quad ,$$

$$\Phi_2 \sim \Phi_+ \Phi_- \quad .$$

The order parameter of the spontaneous chiral symmetry breaking is the F-component of $\Phi_+\Phi_-$. This does not indicate a spontaneous breaking of supersymmetry because the vacuum expectation value disappears for vanishing supersymmetry breaking Δ. The pion field π has the interesting structure (cf. (5.12))

$$\pi \sim \bar{\psi}\gamma_5\psi \left[1 - g^2 (A_+^* A_+ + A_-^* A_-) + \ldots\right] \quad ,$$

where ψ is the Dirac spinor built from the Weyl spinors ψ_+ and ψ_-; thus the Goldstone boson appears to be predominantly a fermion-antifermion bound state. The residual self-interaction of the supermultiplet Φ_1, which contains the Goldstone field π, leads to a Lagrangian for the Goldstone sector whose fermionic part strongly resembles the low energy Lagrangian of the standard model of electroweak interactions: a "left-handed" spinor χ_L with current-current self interaction is coupled to a "right-handed" spinor χ_R through a "Dirac mass term".

Due to the peculiar transformation property of the "Dirac fermion" χ under the conventionally defined parity, however, the above interpretation cannot be maintained. Nevertheless, it seems to us that the following features of our model could be part of a preon theory of quarks and leptons:

- the strong influence of supersymmetry breaking terms on the vacuum structure;
- the doubling of Goldstone supermultiplets, i.e., the appearance of two quasi-Goldstone fermions for one Goldstone boson;
- the occurrence of "residual weak interactions" for left-handed particles only.

Our investigation of the dynamical symmetry breaking in the supersymmetric NJL model revealed a Goldstone sector with interesting structure. We expect an even richer and more surprising structure in supersymmetric confining gauge theories which are the most promising candidates for a description of the substructure of quarks and leptons.

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APPENDIX

In the following, we list the propagators which are needed for the calculations in Sections 2-6. The action

$$S = \int d^4x \left\{ \int d^4\theta \, (\bar{\Phi}_+ \Phi_+ + \bar{\Phi}_- \Phi_-)(1-\delta) + m \int d^2\theta \, \Phi_+ \Phi_- + \text{c.c.} \right\}$$

$$= \int d^4x \Big\{ A_+^*(\Box - \Delta^2) A_+ + A_-^*(\Box - \Delta^2) A_-$$

$$+ i\partial_m \psi_+ \sigma^m \bar{\psi}_+ + i\partial_m \psi_- \sigma^m \bar{\psi}_- + F_+^* F_+ + F_-^* F_-$$

$$+ m(F_+ A_- + A_+ F_- - \psi_+ \psi_-) + \text{c.c.} \Big\} , \qquad \text{(A.1)}$$

with

$$\delta = \theta\theta\bar{\theta}\bar{\theta} \, \Delta^2 ,$$

yields the following propagators

$$\langle T(A_\pm(x) A_\pm^*(x')) \rangle_0 = i \, \Delta_F(x-x'; |m|^2 + \Delta^2) , \qquad \text{(A.2)}$$

$$\langle T(F_\pm(x) F_\pm^*(x')) \rangle_0 = (\Box - \Delta^2) \, i \, \Delta_F(x-x'; |m|^2 + \Delta^2) , \qquad \text{(A.3)}$$

$$\langle T(A_\pm(x) F_\mp(x')) \rangle_0 = -i m^* \Delta_F(x-x'; |m|^2 + \Delta^2) , \qquad \text{(A.4)}$$

$$\langle T(A_\pm^*(x) F_\mp^*(x')) \rangle_0 = -i m \, \Delta_F(x-x'; |m|^2 + \Delta^2) , \qquad \text{(A.5)}$$

$$\langle T(\psi_{\pm\alpha}(x) \psi_\mp^\beta(x')) \rangle_0 = i \, \delta_\alpha{}^\beta \, m^* \, \Delta_F(x-x'; |m|^2) , \qquad \text{(A.6)}$$

$$\langle T(\bar{\psi}_\pm^{\dot{\alpha}}(x) \bar{\psi}_{\mp\dot{\beta}}(x')) \rangle_0 = i \, \delta^{\dot{\alpha}}{}_{\dot{\beta}} \, m \, \Delta_F(x-x'; |m|^2) , \qquad \text{(A.7)}$$

$$\langle T(\psi_{\pm\alpha}(x)\bar{\psi}_{\pm\dot{\beta}}(x'))\rangle_0 = \sigma^m_{\alpha\dot{\beta}}\frac{\partial}{\partial x^m}\Delta_F(x-x';|m|^2) \qquad , \tag{A.8}$$

with

$$\Delta_F(x;M^2) = \frac{1}{\Box - M^2} = -\int\frac{d^4p}{(2\pi)^4}\frac{e^{ipx}}{p^2+M^2-i\varepsilon} \quad .$$

For $\Delta = 0$, (A.2)-(A.8) are the various components of the familiar superfield propagators

$$\begin{aligned}&\langle T(\phi_{\pm}(x,\theta,\bar{\theta})\,\phi_{\mp}(x',\theta',\bar{\theta}'))\rangle_0\\ &= -im^*\delta^2(\theta-\theta')\exp\left[i(\theta\sigma^m\bar{\theta}-\theta'\sigma^m\bar{\theta}')\frac{\partial}{\partial x^m}\right]\Delta_F(x-x';|m|^2) \quad ,\end{aligned} \tag{A.9}$$

$$\begin{aligned}&\langle T(\phi_{\pm}(x,\theta,\bar{\theta})\,\bar{\phi}_{\pm}(x',\theta',\bar{\theta}'))\rangle_0\\ &= i\exp\left[i(\theta\sigma^m\bar{\theta}+\theta'\sigma^m\bar{\theta}'-2\theta\sigma^m\bar{\theta}')\frac{\partial}{\partial x^m}\right]\Delta_F(x-x';|m|^2) \quad .\end{aligned} \tag{A.10}$$

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FIGURE CAPTIONS

Figure 1 Tadpole graph which leads to vev of ϕ in the NJL model.

Figure 2 One-loop contributions to effective action of ϕ.

Figure 3 Additional bosonic tadpole graph which contributes to vev of A_1 in the SNJL model.

Figure 4 Relation between the coupling $\alpha = (g^2\Lambda^2/8\pi^2)$ and the spontaneously generated mass, $\eta = (m^2/\Lambda^2)$, implied by the supersymmetric gap equation for different values of the supersymmetry breaking scale, $\xi = (\Delta^2/\Lambda^2)$.

Figure 5 $\eta = (m^2/\Lambda^2)$ as a function of the supersymmetry breaking scale $\xi = (\Delta^2/\Lambda^2)$ (schematic).

Figure 6 One-loop contributions to effective action of A_1, ψ_1, F_1.

Figure 7 One-loop graphs contributing to the residual interaction of ψ_1.

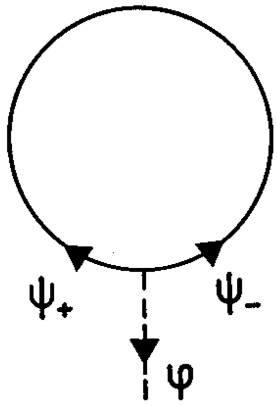


Fig. 1

Fig. 2

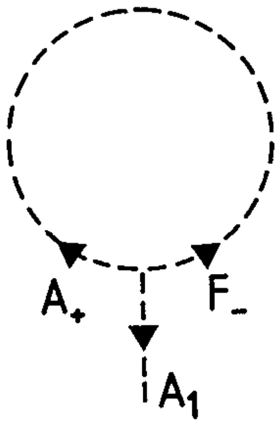


Fig. 3

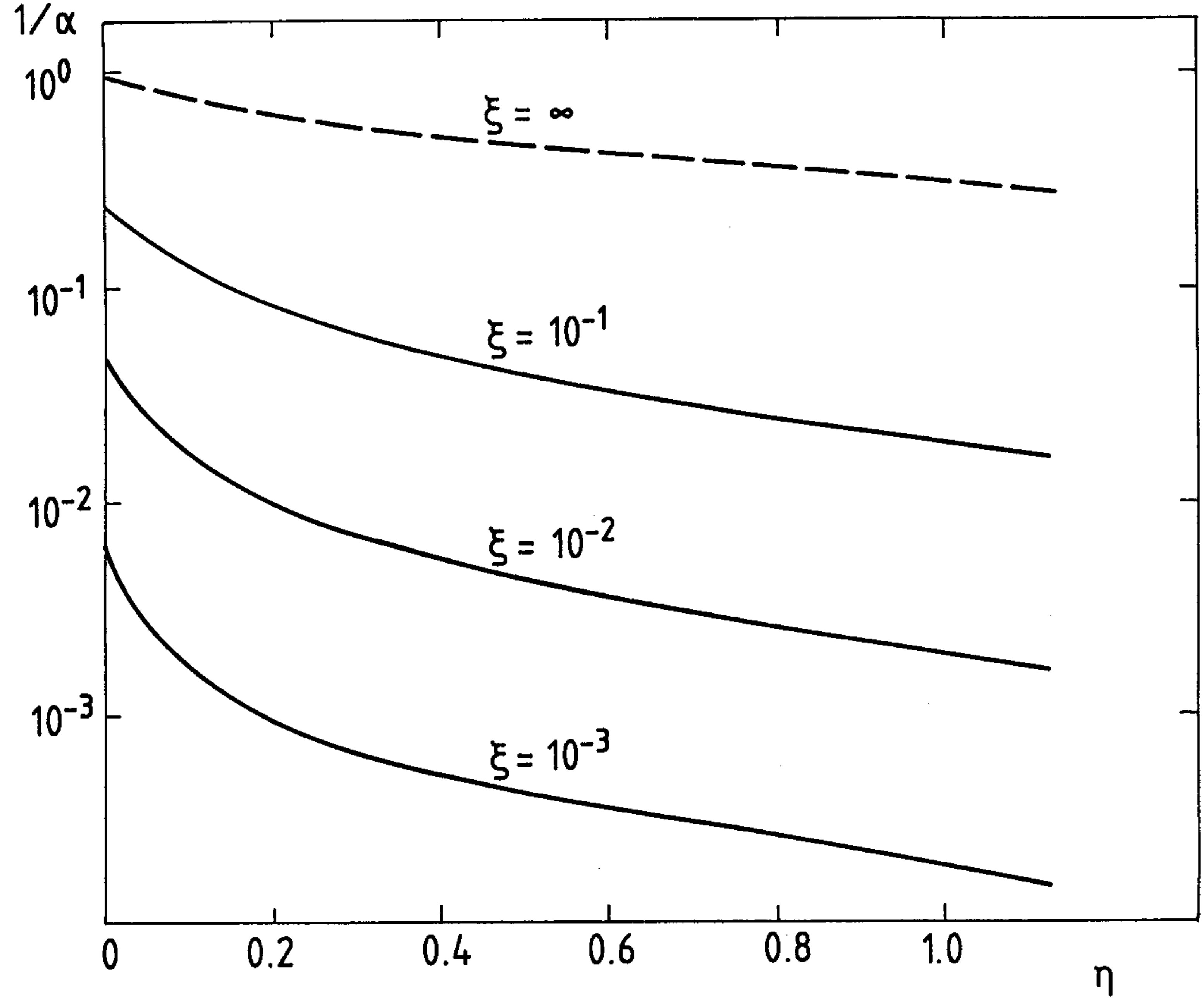


Fig. 4

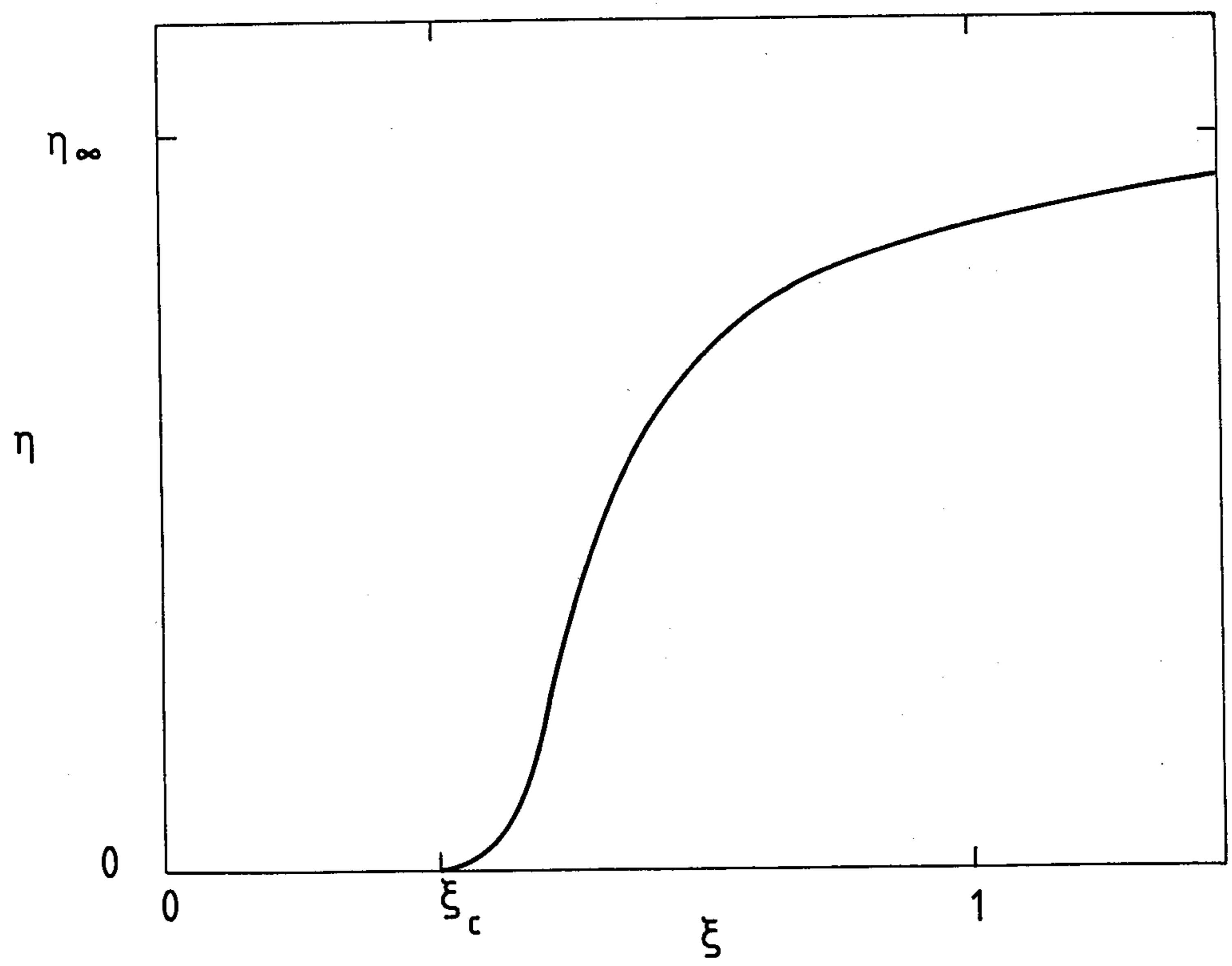
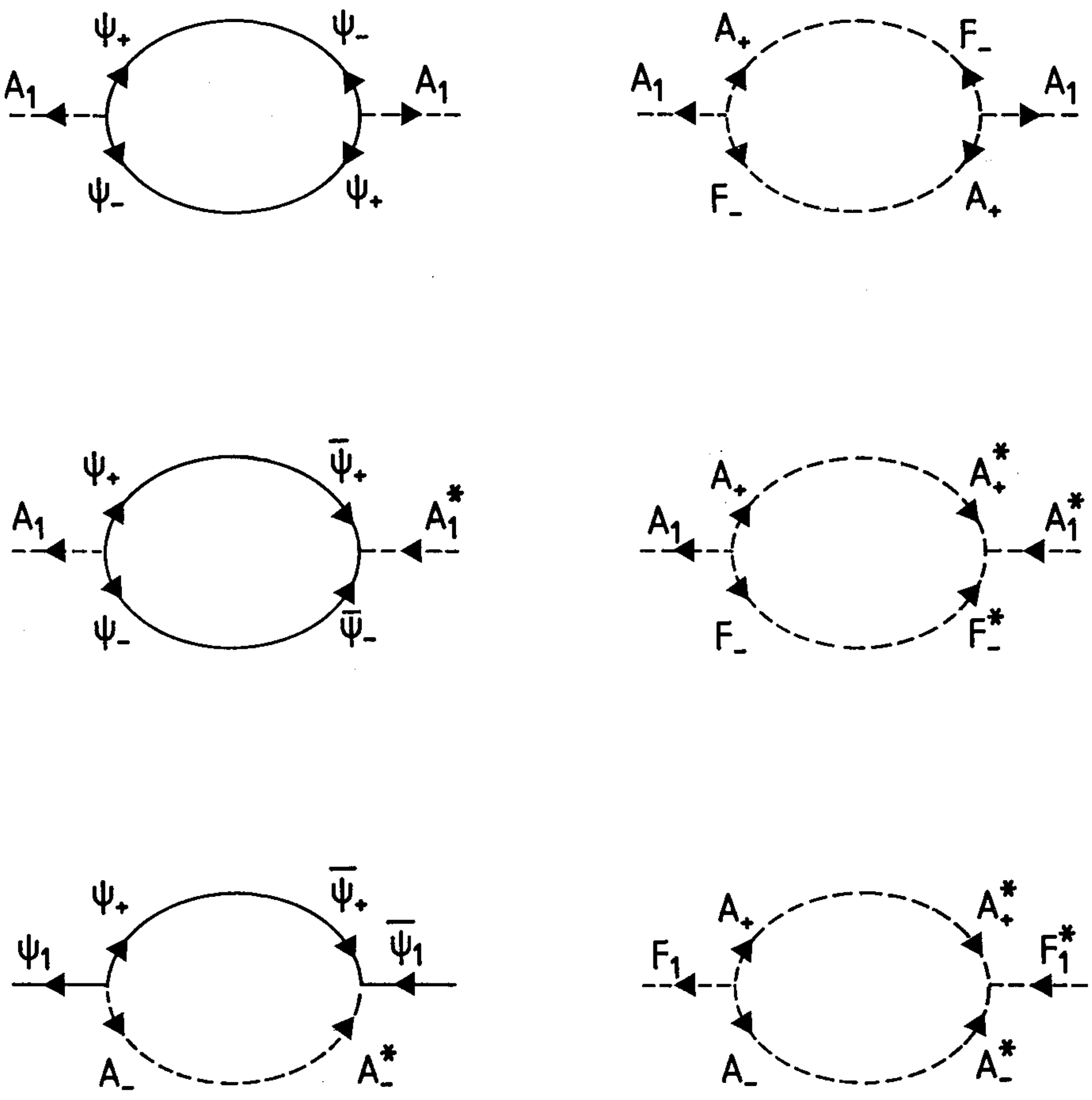


Fig. 5


ψ_+
ψ_-
A_1
A_1
ψ_-
ψ_+
A_+
F_-
A_1
A_1
F_-
A_+
ψ_+
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A_1
A_1^*
ψ_-
$\bar{\psi}_-$
A_+
A_+^*
A_1
A_1^*
F_-
F_-^*
ψ_+
$\bar{\psi}_+$
ψ_1
$\bar{\psi}_1$
A_-
A_-^*
A_+
A_+^*
F_1
F_1^*
A_-
A_-^*

Fig. 6

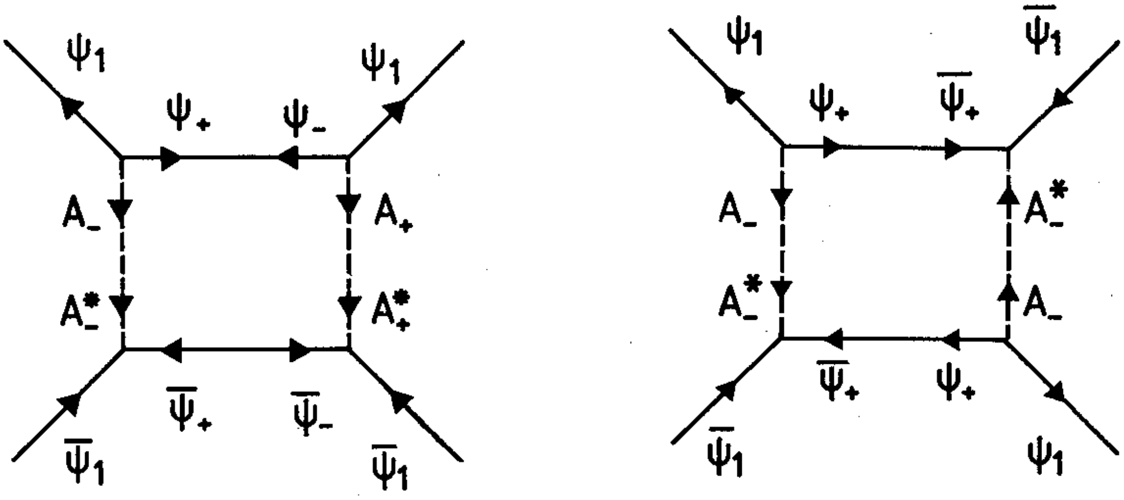

FIG. 7