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QUANTUM CREATION OF INFLATIONARY UNIVERSE

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It is shown that the process of quantum creation of the universe in a wide class of elementary particle theories with a large probability leads to the creation of the exponentially expanding (inflationary) universe, which after expansion acquires the size $\ell \gtrsim 10^{28}$ cm.

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There is a large interest now in the inflationary universe scenario [1-3], which may help us to solve simultaneously many different cosmological problems. For a review of the present status of this scenario see [4,5]. The main feature of this scenario is the existence of the stage of exponential (or quasiexponential) expansion (inflation) $a(t) \sim a_0 e^{Ht}$ during some time $\Delta t \gtrsim 70\ H^{-1}$ at the very early stages of the universe evolution. Here $a(t)$ is the scale factor of the universe, H is the Hubble "constant", which may vary slowly during the expansion, so that $\dot{H} \ll H^2$.

The standard assumption which is usually made in this scenario is that the exponential expansion starts after some earlier stage of expansion and cooling of the hot singular Friedmann universe [1-5]. However there exists one more possibility to realize this scenario. It is known, that any classical description of the universe evolution becomes impossible due to large quantum fluctuations of metric in the very early universe, when the energy density of the universe ρ exceeds the Planck density $\rho_P = M_P^4 \sim 10^{94}$ g.cm^{-3}. Therefore it is possible that the universe never was in a singular state, but it was created as a whole either from "nothing" or from some other universe [6-10]. The theory of such processes is far from being completely developed, and even the very concept of creation from "nothing" or from "some other universe" deserves a more detailed examination, see a discussion of this question in [4]. Nevertheless some qualitative features of such processes can be easily understood. It is clear, for example, that the process of quantum creation of the universe may effectively occur only if the energy density of the created universe ρ is sufficiently large, $\rho \gtrsim \rho_p = M_p^4$ since only in that case quantum fluctuations of metric at a scale of the size of the universe may become considerable [12]. Presumably the process of quantum creation of the universe is somewhat similar to the barrier tunneling process, and therefore one may expect that the probability of the universe creation with $\rho \ll \rho_p$ is exponentially suppressed, $P \sim e^{-c\frac{\rho_p}{\rho}}$, where $C = O(1)$ [13]. For the same reason the probability of quantum creation of the universe of the size $\ell \gg M_p^{-1}$ also should be exponentially suppressed. Therefore only closed universe of the size $\ell \lesssim M_p \sim 10^{-33}$ cm can be created "from nothing". A question arises, how such a small universe could give rise to our universe of the size $\ell \gtrsim 10^{28}$ cm., with the baryon asymmetry $\frac{n_B}{n_\gamma} \sim 10^{-8}$, with the density perturbations $\frac{\delta\rho}{\rho} \sim 10^{-4}$ etc. A new stage in the development of the idea of quantum creation of the universe is connected with the works of Zeldovich and Grishchuk [7], in which it was pointed out that all these problems can be easily solved if the created universe is described either by the inflationary universe scenario [1-3] or by a similar scenario suggested by Starobinsky and based on the investigation of quantum gravity effects in the very early universe [11].

This observation of ref.[7] was especially important for the development of the Starobinsky scenario, since the only possibility to realize this scenario, which was known at that time, was connected just with the idea of quantum creation of the universe. On the other hand, the possibility of quantum creation of the inflationary universe at that time seemed rather improbable, since the typical vacuum energy $V(\varphi)$ during inflation in the first versions of the inflationary universe scenario [1,2] is many orders of magnitude smaller than ρ_p, and, consequently, quantum gravity effects, which could lead to the creation of the inflationary universe, in these versions of the inflationary universe scenario are negligibly small.

A first attempt of a quantitative investigation of quantum creation of the inflationary universe has been made by Vilenkin [8]. However, as we have written in [4], his approach to this problem, being intuitively appealing, was not well motivated. As a result, he has obtained the expression for the probability of the universe creation $P \sim e^{\frac{3\rho_p}{8\rho}}$, from which it would follow that the quantum gravity effects become stronger at small ρ, and that quantum fluctuations of metric become greater at greater length scales. Such a conclusion would be in a contradiction with the present understanding of quantum gravity effects and with the well-known fact that spacetime is foamlike just at small, and not at large scales [12]. One more attempt to study this question has been made recently by Moss and Wright [10]. However they have investigated the theory of the conformal scalar field φ only, in which, as it was expected before [4], the inflationary universe scenario cannot be realised.

From our point of view, a reasonable estimate for the probability of creation of the inflationary universe is $P \sim e^{-\frac{3\rho_p}{8\rho}}$ [13] (compare with [8]). This result can be obtained by the method suggested recently by Hartle and Hawking [9] after some improvement of quantization of the scale factor a in the minisuperspace approach. For a detailed discussion of this question see ref. [13]. We will not discuss this question here anymore, since the only fact which will be necessary for us in the present paper is that, as we have argued above, the probability of creation of the universe with $\rho \ll \rho_p$ should be strongly suppressed. The main aim of our paper is to show that in the novel version of the inflationary universe scenario [3] there is no exponential suppression of quantum creation of the inflationary universe.

It is clear that the exponential suppression of the probability of quantum creation of the universe should be absent if the energy density of the created universe $\rho \gtrsim \rho_p = M_p^4$. If, however, this energy density is due to the large inhomogeneity of the field φ, or due to its kinetic energy $\sim \frac{\dot{\varphi}^2}{2}$, then the newly born universe does not expand exponentially. The typical lifetime of such a universe is $\Delta t \sim M_p^{-1}$, and therefore in some sense such a universe does not appear as a truly classical object, but looks just like a quantum fluctuation of metric and of the field φ. Let us consider now an opposite case, in which the energy density $\rho \gtrsim \rho_p$ appears due to the large value of the effective potential $V(\varphi)$ of a sufficiently homogeneous (at a scale $\ell \gtrsim H^{-1}$ [3]) and slowly changing ($(\dot{\varphi})^2 \ll V(\varphi)$) field φ. In this case it can be shown, that, under some rather natural assumptions about $V(\varphi)$, the universe filled with such a field φ after its creation exponentially expands, and its size after the expansion exceeds the size of the observable part of our universe $\ell \gtrsim 10^{28}$ cm.

Let us consider as an example the theory $V(\varphi) = \frac{\lambda}{4}\varphi^4$ at $\lambda \ll 10^{-2}$ without any extra terms $\sim \xi R \varphi^2$ in the Lagrangian. As it was argued above (see also [13]), the exponential suppression of the probability of the universe creation disappears if the universe is created with such a field $\varphi = \varphi_0$, that $\frac{\lambda}{4}\varphi_0^4 \gtrsim M_p^4$, whence $\varphi_0 \gtrsim M_p\left(\frac{4}{\lambda}\right)^{1/4} \gg 5M_p$. A subsequent evolution of the homogeneous field φ in the closed universe is governed by equation

$$\ddot{\varphi} + 3\,\frac{\dot{a}}{a}\,\dot{\varphi} = -\,\frac{dV}{d\varphi}\ , \qquad (1)$$

and the scale factor $a(t)$ satisfies the Einstein equation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{a^2} = \frac{8\pi}{3M_p^2}\left(V(\varphi) + \frac{1}{2}\,\dot{\varphi}^2\right), \qquad (2)$$

where $V(\varphi) = \frac{\lambda}{4}\varphi^4$. If initially $\frac{1}{2}\dot{\varphi}^2 \ll V(\varphi)$, as we have assumed, then from eq. (1) it can be shown that at $\varphi \gg \frac{M_p}{3}$ the field φ changes in time very slowly, $\frac{\dot{\varphi}}{\varphi} \ll H$, where $H = \sqrt{\frac{8\pi}{3M_p^2}V(\varphi)}$ [3]. In such a case from (2) it follows that the universe behaves as a closed de Sitter space,

$$a(t) \approx H^{-1}\,\mathrm{ch}\,Ht\ . \qquad (3)$$

It is most likely that the universe creation occurs at the moment $t = 0$, at which the spatial volume of the universe is minimal, $a_0 = H^{-1}$. From (1), (3) it follows that at $\varphi \gtrsim 5M_p$ the field φ does not change considerably during the time $\Delta t \sim H^{-1}$. According to (3), after that time the scale factor $a(t)$ becomes much greater than H^{-1} , the value of $\frac{\dot{a}}{a}$ in (1), (2) becomes equal to H , and the terms $\ddot{\varphi}$ in (1) and $\frac{4\pi}{3M_p^2}\dot{\varphi}^2$ in (2) can be neglected until the field φ becomes smaller than $\frac{M_p}{3}$. Therefore at $\varphi \gtrsim \frac{M_p}{3}$ the field φ behaves as follows [3] :

$$\varphi \approx \varphi_0\,\exp\left(-\frac{\sqrt{\lambda}}{\sqrt{6\pi}}\,M_p t\right), \qquad (4)$$

which, together with eq. (3), implies that the universe, in which initially $\varphi = \varphi_0 \gtrsim 5 M_p$, during the period of exponential expansion grows more than e^{70} times. This observation is a basic point of the chaotic inflation scenario [3]. Now it is clear that the same observation may simplify considerably the realization of the idea of quantum creation of the universe. Indeed, as we have argued above, there is no exponential suppression of the universe creation with $\varphi \gtrsim \left(\frac{1}{\lambda}\right)^{\frac{1}{4}} M_p$, which (for $\lambda \lesssim 10^{-2}$) is sufficient for the realization of the inflationary universe scenario [3].

From the results obtained in ref. [3] it follows that the same result is valid not only for the theory $\frac{\lambda}{4}\varphi^4$ with $\lambda \lesssim 10^{-}$ but for the wide class of theories in which $V(\varphi)$ is not too curved at $\varphi \gtrsim M_p$. Therefore if the process of quantum creation of the universe is possible, in the abovementioned theories this process with a considerable probability leads to the creation of the inflationary universe [3], which during inflation and during the subsequent expansion after reheating acquires the size $\ell \gtrsim 10^{28}$ cm.

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