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TRANSVERSE MOMENTUM DISTRIBUTION IN DRELL-YAN PAIR AND W AND Z BOSON PRODUCTION

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A B S T R A C T

We exhibit and discuss the QCD prediction for the transverse momentum distribution of W bosons, Z bosons and high mass virtual photons produced in high energy hadron-hadron collisions. Recent work has shown that this prediction is consistent with the structure of leading twist initial state interactions.

The expression we present is expected to give results correct up to order $\alpha_s^N(Q)$ for any Q_T when the boson mass Q is very large ($>10^8$ GeV!), given only input from perturbative calculations at order α_s^{N+2} and deeply inelastic scattering structure functions. We specify the required N = 0 coefficients, employing the order α_s^2 results of Kodaira and Trentadue and of Davies and Stirling. We then show how the expression should be modified to deal with current energy scales. We also discuss the connection between low Q_T and high Q_T formulae.

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1. INTRODUCTION

The transverse momentum distribution in the Drell-Yan process is of great interest because its behaviour is sensitive to the presence of gauge bosons in the strong interactions.

To illustrate, we may compare QCD with ϕ^3 theory in 5+1 dimensions. This non-gauge theory is asymptotically free, like QCD, and thus mimics the parton model in hard hadronic collisions. In ϕ^3 theory, the Q_T distribution of dimuon pairs reflects only the primordial k_T distribution of the incoming partons (as long as we restrict our attention to the small Q_T region, $Q_T << Q$). Thus, for $Q_T << Q$, the shape of the Q_T distribution is independent of the dimuon mass Q[1].

The situation in QCD is entirely different. The annihilating quarks easily radiate colour gauge bosons, just as accelerated electrically charged particles radiate photons. As the time 1/Q available for the annihilation is made smaller, the gluon radiation increases. Since the gluons carry away transverse momentum, the observed dimuon transverse momentum distribution must become broader as Q is made larger[*].

This broadening of the Q_T distribution is in fact seen in the experimental results. Drell-Yan experiments at $Q \sim 10$ GeV find a typical Q_T of about 1 GeV[2]. But when one looks at W production at $Q \sim 80$ GeV, one finds that the typical Q_T has increased to $Q_T \sim 5$ GeV[3]. There have been a number of papers[4] comparing various versions of QCD theory to the measured Drell-Yan distributions, generally with encouraging results. While we were completing this paper, we received a preprint by Altarelli, R.K. Ellis, Greco and Martinelli[5], which contains a comparison of theory to the Q_T distribution of W's observed at CERN, again with encouraging results, along with an appraisal of the theoretical situation.

It will be helpful to review briefly some of the most important theoretical developments relating to the Drell-Yan Q_T distribution. With

*) However, the increase of the mean transverse momentum $\langle Q_T \rangle$ with Q is not by itself a good measure of this effect. One finds $\langle Q_T \rangle \propto Q$, up to logarithms, in perturbation theory for any theory with a dimensionless coupling, whether a gauge theory or not. This is because $\langle Q_T \rangle$ is sensitive to the $Q_T \sim Q$ region, but not the $Q_T << Q$ region.

the advent of the QCD factorization theorem[6], one was able[7] to write the Drell-Yan cross-section at measured Q_T in a form that was useful at large Q_T, $Q_T \sim Q$ (except for the problem of initial state interactions, which was largely unrecognized at the time). However, this factorized form was not useful at $Q_T << Q$ because the perturbative coefficients contain large logarithms: the coefficient of $\alpha_s^N(Q^2)$ contains $1/Q_T^2$ times a series of logarithms $\ln^n(Q^2/Q_T^2)$, with $n = 0,1,\ldots,2N-1$. Dokshitser, D'yakonov and Troyan[8] showed that the leading logarithms, $n = 2N-1$, came from dressed ladder graphs in a physical gauge. Thus the leading terms could be calculated and summed. There was an error in the initial result[8], which was corrected by Parisi and Petronzio[9] and, independently, by Curci, Greco and Srivastava[10]. These authors introduced more powerful techniques: they worked with the Fourier transform, with respect to Q_T, of the cross-section and they showed the usefulness of soft gluon methods.

The leading logarithms, $n = 2N-1$, dominate the sum of perturbation theory in the kinematic region $\ln(Q/\Lambda) >> 1$, $\ln(Q/Q_T) \lesssim [\ln(Q/\Lambda)]^{\frac{1}{2}}$. For example, if we take $\Lambda = 150$ MeV and $Q = M_W = 80$ GeV, then the requirement $\ln(Q/Q_T) < [\ln(Q/\Lambda)]^{\frac{1}{2}}$ implies $Q_T > 7$ GeV. Unfortunately, most of the interesting physics, and most of the data[4], lies outside of this region of validity of the leading logarithm approximation.

An improved approximation, valid for $\ln(Q/\Lambda) >> 1$ with <u>any</u> value of Q_T ($0<Q_T\lesssim Q$), was given in Ref. 11) for the process $e^+e^- \to A+B+X$, which is closely analogous to the Drell-Yan process. The result was stated for the Drell-Yan process in Ref. 12). However, at that time, we did not pursue the Drell-Yan case further because of theoretical problems[11)-14)] arising from the interactions between the initial hadrons before the annihilation takes place. We now believe that these initial state interaction problems are under theoretical control[15)-18)].

It therefore seems appropriate to describe the full QCD result in some detail at this time. We express this result in a new form that seems more convenient than that used in Refs. 11) and 12). We also list the perturbative coefficients that occur in the general form, insofar as they are known. In particular, we employ the second order coefficients that have been calculated by Kodaira and Trentadue[19] and by Davies and Stirling[20].

The result that we will describe has quite a simple form in the extreme high Q^2 limit:

$$
\begin{aligned}
\frac{d\sigma}{dQ^2\,dy\,dQ_T^2} \sim \frac{4\pi^2\alpha^2}{9Q^2 s}\,(2\pi)^{-2}\int d^2b\; e^{i\mathbf{Q}_T\cdot\mathbf{b}}\sum_j e_j^2 \\
\times\sum_a\int_{x_A}^1 \frac{d\xi_A}{\xi_A}\, f_{a/A}(\xi_A; 1/b) \\
\times\sum_b\int_{x_B}^1 \frac{d\xi_B}{\xi_B}\, f_{b/B}(\xi_B; 1/b) \\
\times \exp\left\{-\int_{1/b^2}^{Q^2}\frac{d\bar\mu^2}{\bar\mu^2}\left[\ln\left(\frac{Q^2}{\bar\mu^2}\right)A(g(\bar\mu)) + B(g(\bar\mu))\right]\right\} \\
\times C_{ja}\left(\frac{x_A}{\xi_A}; g(1/b)\right)\; C_{\bar{j}b}\left(\frac{x_B}{\xi_B}; g(1/b)\right) \\
+ \frac{4\pi^2\alpha^2}{9Q^2 s}\, Y(Q_T; Q, x_A, x_B)\ .
\end{aligned}
\tag{1.1}
$$

In this formula, the sums run over parton species, that is, gluons and flavours of quarks and antiquarks. The functions f are parton distribution functions[21),22)] evaluated at a renormalization scale $\mu = 1/b$. The notation for the kinematics is given in Section 2. For the sake of simplicity, two arbitrary constants, denoted by C_1 and C_2 in the following sections, have been set equal to one here.

The first term in (1.1) dominates the cross-section when $Q_T \ll Q$. The Y term provides a needed correction in the region $Q_T \sim Q$.

The construction of Y is described in Section 2, where we also give the results of our calculation of the first order perturbative contribution to Y.

The first term, containing the functions A, B, and C, is described in Section 3. There, we explain how the structure indicated in Eq. (1.1) arises from solving an evolution equation for the cross-section. The functions A, B, and C have perturbative expansions in powers of α_s. The

known low order coefficient (including the second order coefficients calculated in Refs. 19) and 20)) are also listed in Section 3.

When Q is very large ($> 10^8$ GeV, see Section 4), the values of b that are important in the b integral in Eq. (1.1) are small, $b \ll 1/\Lambda$. Then the form given in Eq. (1.1) is adequate. When Q is not so large, it is essential to understand the structure of the integrand for $b \gtrsim 1/\Lambda$. This structure is described in Section 4. It involves two functions, $g_1(b)$ and $g_{j/A}(x,b)$, that cannot be computed in perturbation theory, but must be measured experimentally.

Finally, a method for smoothly joining the two regions $b \ll 1/\Lambda$ and $b \gtrsim 1/\Lambda$ is given in Section 5. The complete result, including the treatment of the $b \gtrsim 1/\Lambda$ region, is given in Eq. (5.8).

We include here a few comments concerning Eq. (1.1).

i) The behaviour of the cross-section for $Q \to \infty$, $Q_T \approx 0$ is controlled by a saddle point[9),11)] in the b integration at $\ln(Q^2b^2) = \underline{\text{const}}.\times \ln(Q^2/\Lambda^2)$. Thus all logarithms may be counted as being equally large in the $Q \to \infty$ limit. There is one such large logarithm that is explicit in the exponent in Eq. (1.1), namely $\ln(Q^2/\bar\mu^2)$. Another large logarithm is implicit in the integration over $\bar\mu$. Suppose that one wants to evaluate the cross-section at $Q_T \approx 0$ in an approximation of "degree N", meaning that any corrections are suppressed by a factor of $[\ln(Q^2/\Lambda^2)]^{-(N+1)}$. Then, since two large logarithms multiply A, one must evaluate A to order α_s^{N+2}. Similarly, one needs B to order α_s^{N+1}, C to order α_s^N and, for the evolution of α_s, the β-function to order α_s^{N+2}.

ii) In particular, if one wants an approximate result that will converge to the exact $Q_T \approx 0$ cross-section as $Q \to \infty$, one needs a degree 0 approximation: A to order α_s^2, B to order α_s^1, C to order α_s^0, and β to order ${\alpha^2}_s$.

iii) If A, B, C, β, and Y are evaluated up to at least order α^N, then both the cross-section at $Q_T \sim Q$ and the normalization will be correct to order α_s^N. In the normalization integral, one integrates over the allowed kinematic range of Q_T. The values of b that are important are of order $b \sim 1/Q$, so there are no large logarithms in the exponent and a straight perturbative expansion of the exponential gives a good approximation.

iv) In many treatments, following Refs. 9) and 10), the integration variable $\bar{\mu}$ in the exponent in (1.1) is a transverse momentum k_T^2 and the integration represents a Fourier transformation of some aproximation to the perturbative cross-section. Then the lower endpoint of the $\bar{\mu}$ integral is 0 instead of 1/b and a factor $[1-J_0(\bar{\mu}b)]$ appears in the integrand. This formulation can be correct at the -1 or 0 degree of approximation discussed under i) above. However, the generalization to higher orders of approximation is not known.

v) In our treatment, there is a Fourier transformation from transverse momentum to b, but the Fourier transform integral is not seen explicitly in the exponent of Eq. (1.1) because it has already been performed in computing the coefficients in the perturbative expansions of A(g) and B(g). The integration over $\bar{\mu}$ arises in our treatment from solving a certain evolution equation and making use of the renormalization group.

vi) The exponent in Eq. (1.1) has the structure of a renormalization group result modified by the appearance of one explicit logarithm of momentum, $\ln(Q^2/\bar{\mu}^2)$.

vii) The cross-section $d\sigma/dy\, dQ_T^2$ for W or Z boson production can be obtained by making two substitutions in Eq. (1.1). First, one changes the normalization:

$$\frac{4\pi^2\alpha^2}{9Q^2 s} \longrightarrow \frac{4\pi^3\alpha}{3s} \quad . \tag{1.2}$$

Second, one changes the quark "charges"

$$\sum_j e_j^2\, C_{ja}\, C_{\bar{j}b} \cdots \rightarrow \sum_{jj'} Q_{jj'}\, C_{ja}\, C_{j'b} \cdots \quad , \tag{1.3}$$

with a corresponding change in the Y term. For Z^0 production, the charges are

$$Q^Z_{jj'} = \delta_{j'\bar{j}}\, \frac{[1-4|e_q|\sin^2\theta_W]^2+1}{16\sin^2\theta_W\cos^2\theta_W} \quad . \tag{1.4}$$

For W^+ production, they are

$$\begin{aligned} Q^{W^+}_{u\bar{d}} &= Q^{W^+}_{\bar{d}u} = Q^{W^+}_{c\bar{s}} = Q^{W^+}_{\bar{s}c} = \frac{\cos^2\theta_c}{4\sin^2\theta_W} \\ Q^{W^+}_{u\bar{s}} &= Q^{W^+}_{\bar{s}u} = Q^{W^+}_{c\bar{d}} = Q^{W^+}_{\bar{d}c} = \frac{\sin^2\theta_c}{4\sin^2\theta_W} \\ Q^{W^+}_{jj'} &= 0 \quad \text{otherwise} \quad . \end{aligned} \tag{1.5}$$

For W^- production, $Q^{W^-}_{jj'} = Q^{W^+}_{\bar{j}\bar{j}'}$. In these formulae, θ_W is the Weinberg angle and θ_C is the Cabibbo angle.

2. SEPARATION OF LARGE AND SMALL Q_T CONTRIBUTIONS

We start with the form given in Ref. 11), transcribed from $e^+ + e^- \to A+B+X$ to $A+B \to e^+ + e^- + X$. We use as independent variables s, Q^2, $y = \frac{1}{2}\ln(Q^+/Q^-)$, and Q_T. In terms of these independent variables, we define

$$x_A = e^{y}\, Q/s^{1/2} \quad , \quad x_B = e^{-y}\, Q/s^{1/2} \quad . \tag{2.1}$$

Then, when $Q \gg \Lambda$, $0 \leqslant Q_T \lesssim Q$ (and x_A and x_B are not near to their kinematic limits), we assert that the cross-section has the form

$$\frac{d\sigma}{dQ^2\,dy\,dQ_T^2} \sim \frac{4\pi\alpha^2}{9Q^2 s} \times \Big\{ (2\pi)^{-2} \int d^2b\; e^{i\underset{\sim}{Q}_T\cdot\underset{\sim}{b}}\; \widetilde{W}(b;Q,x_A,x_B) \tag{2.2}$$
$$+ Y(Q_T;Q,x_A,x_B) \Big\} \; .$$

The W term in this formula gives the dominant contribution when $Q_T \ll Q$. The structure of $\widetilde{W}(b;Q,x_A,x_B)$ will be described in the following sections.

The function Y gives corrections that are negligible for small Q_T but become important when $Q_T \sim Q$. In this section we describe the separation of large and small Q_T contributions, or, equivalently, the construction of Y.

Let us recall the usual formula[6)] for the Drell-Yan cross-section with measured Q_T:

$$\frac{d\sigma}{dQ^2\,dy\,dQ_T^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{a,b} \int_{x_A}^{1} \frac{d\xi_A}{\xi_A} \int_{x_B}^{1} \frac{d\xi_B}{\xi_B}$$
$$\times\, T_{ab}\big(Q_T, Q, x_A/\xi_A, x_B/\xi_B; g(\mu), \mu\big) \tag{2.3}$$
$$\times\, f_{a/A}(\xi_A;\mu)\; f_{b/B}(\xi_B;\mu) \quad .$$

This formula is valid (up to m/Q corrections) when Q_T is of order Q and also when one integrates over Q. The sum runs over all species a and b of

partons (viz., gluon, and flavours of quark and antiquark). Both the hard scattering function T and the parton distribution functions f depend on the renormalization scale μ and the corresponding value, $g(\mu)$, of the QCD coupling. The hard scattering function T, but <u>not</u> f, has a perturbative expansion in powers of $\alpha_s(\mu)$. Normally, in applications of Eq. (2.3) one sets $\mu \sim Q$ to avoid large logarithms, $\ln(Q/\mu)$, that would otherwise spoil the usefulness of a low order perturbative approximation to T.

Let us make explicit the perturbative expansion of T:

$$T_{ab}(Q_T, Q, x_A/\xi_A, x_B/\xi_B; g(\mu), \mu)$$
$$= \sum_{N=0}^{\infty} \left[\frac{\alpha_s(\mu)}{\pi}\right]^N T_{ab}^{(N)}(Q_T, Q, x_A/\xi_A, x_B/\xi_B; \mu) \quad . \tag{2.4}$$

Our normalization is such that in lowest order one has

$$T_{ab}^{(0)} = e_a^2\, \delta_{a\bar{b}}\; \delta\!\left(\frac{x_A}{\xi_A}-1\right) \delta\!\left(\frac{x_B}{\xi_B}-1\right) \delta(\underset{\sim}{Q}_T) \tag{2.5}$$

which corresponds to the Drell-Yan model consisting of simple quark-antiquark annihilation.

The coefficients $T^{(N)}$ are singular as $Q_T \to 0$. They have the form[8)]:

$$T_{ab}^{(N)}(Q_T, Q, x_A/\xi_A, x_B/\xi_B; \mu)$$
$$= \eta_{ab}^{(N)}(Q, x_A/\xi_A, x_B/\xi_B; \mu)\; \delta(\underset{\sim}{Q}_T)$$
$$+ \sum_{m=0}^{2N-1} T_{ab}^{(N,m)}(Q, x_A/\xi_A, x_B/\xi_B; \mu) \left[\frac{\ln^m(Q^2/Q_T^2)}{Q_T^2}\right]_{\rm reg} \tag{2.6}$$
$$+ R_{ab}^{(N)}(Q_T, Q, x_A/\xi_A, x_B/\xi_B; \mu) \; .$$

The terms that are less singular as $Q_T \to 0$ than $Q_T^{-2} \times$ (<u>logs</u> or 1) or $\delta(\underset{\sim}{Q}_T)$ are included in the "regular" part $R_{ab}^{(N)}$ of $T_{ab}^{(N)}$.

The separation of $T_{ab}^{(N)}$ into a singular part and a regular part as

$Q_T \to 0$ is to be made after integration over ξ_A and ξ_B; that is, in the sense of distributions. We have also indicated a distribution theoretic regulation, denoted by reg, of the $\ln^m(Q_T)/Q_T^2$ singularity at $Q_T = 0$. (Such a regulation is possible since the integral of $T_{ab}^{(N)}$ over Q_T is finite, because of cancellations between real and virtual diagrams.) For the sake of definiteness, we adopt the definition

$$\int_0^{k_T^2} dQ_T^2 \left[\frac{\ln^m(Q^2/Q_T^2)}{Q_T^2}\right]_{reg} = -\frac{1}{m+1} \ln^{m+1}(Q^2/k_T^2) \;. \qquad (2.7)$$

We use the "regular" parts $R^{(N)}$ of the perturbation coefficients to define the function Y in Eq. (2.2):

$$Y(Q_T;Q,x_A,x_B) = \sum_{a,b} \int_{x_A}^1 \frac{d\xi_A}{\xi_A} \int_{x_B}^1 \frac{d\xi_B}{\xi_B} \sum_{N=1}^{\infty} \left[\frac{\alpha_s(\mu)}{\pi}\right]^N \times R_{ab}^{(N)}\left([Q_T^2+(Q_T^{min})^2]^{1/2}, Q, x_A/\xi_A, x_B/\xi_B; \mu\right) \times f_{a/A}(\xi_A;\mu)\; f_{b/B}(\xi_B;\mu) \;. \qquad (2.8)$$

Here Q_T has been replaced by $[Q_T^2+(Q_T^{min})^2]^{\frac{1}{2}}$ as a simple method of cutting off certain weak singularities in the coefficients $R_{ab}^{(N)}(Q_T)$ as $Q_T \to 0$. One might choose $Q_T^{min} \approx 300$ MeV, for instance. The behaviour of Y in the region $Q_T \sim Q$ is not affected by this cut-off (see below).

In applications, we would use one or two terms in the perturbative sum in Eq. (2.8) and set $\mu \sim Q$ so that logarithms of Q/μ in the higher order $R^{(N)}$ would not be large. More precisely, we would set $\mu = C_2 Q$ where C_2 is an arbitrary constant of order 1.

We now discuss the motivation for this definition of Y. Consider first the kinematic region $Q_T \sim Q$. Here the factorization formula (2.3), with T expanded perturbatively, is reliable. The definition that we will give for W is such that, when $Q_T \sim Q$, the W term in Eq. (2.2) has a

factorized form like that in Eq. (2.3), but with $T_{ab}^{(N)}$ replaced by just its singular part. The Y term in Eq. (2.2) then restores the contributions from the regular part, $R_{ab}^{(N)}$, of $T_{ab}^{(N)}$. Thus one recovers the standard perturbative result (2.3), (2.4), correct up to the order of perturbation theory used in the calculation of W and Y, in the kinematic region in which the standard result should be reliable.

Consider next the region $0 \leq Q_T \ll Q$. Our discussion of this region has two parts. We first argue that the W term by itself is a good approximation to the cross-section in this region. To see this, we must define W without assuming that $\Lambda \ll Q_T$. (Here Λ stands for the QCD mass scale, hadron or quark masses, or quark transverse momenta in Bethe-Salpeter wave functions. In the definition of Y above, we began with the $\Lambda \ll Q_T$ factorization.) We define W at each order of perturbation theory, with Bethe-Salpeter wave functions for the hadrons. We consider the limit of each perturbative contribution to the cross-section as $Q/Q_T \to \infty$ with Q_T/Λ fixed, dropping all terms that are suppressed by powers of Q_T/Q. This gives the corresponding perturbative contribution to W. Thus each perturbative contribution to W is a good approximation to the corresponding contribution to the cross-section when $Q_T \ll Q$. We assume that the W term remains a good approximation to the cross-section when one has summed the perturbative contributions. This seems a safe enough assumption, since the corrections are suppressed not just by logarithms but by powers of Q_T/Q in perturbation theory. However, we cannot supply a proof that this is so.

We have argued that, in the region $0 \leq Q_T \ll Q$, the W term in Eq. (2.2) provides, by itself, a good approximation to the cross-section. We now argue that it does no harm to include the Y term in Eq. (2.2) when $Q_T \ll Q$ because Y, evaluated at some finite order of perturbation theory, is negligible compared to W when $Q_T \ll Q$. Indeed, as $Q \to \infty$ with Q_T fixed, $Q^2 W(Q_T;Q,x_A,x_B)$ grows like Q^{α}, $\alpha \approx 0.8$ [9),11)], while the first order contribution to $Q^2 Y$ grows only like $\ln(Q/Q_T')$. Here $Q_T' = [Q_T^2+(Q_T^{min})^2]^{\frac{1}{2}}$ is the variable used in the definition of Y, Eq. (2.8). There could have been a small problem at $Q_T = 0$ because of the logarithmic singularity, $\ln(Q/Q_T')$, in Y. It was for this reason that we inserted the rather ad hoc, but calculationally simple, Q_T^{min} cut-off in Eq. (2.8). At higher orders of perturbation theory for Y, the general situation should be similar, although the small Q_T divergence problem might be more severe.

A discussion of this kind of decomposition of the cross-section into a small Q_T W-term and a large Q_T correction Y, applied to ϕ^3 theory in six dimensions, can be found in Ref. 23).

We have evaluated the functions $R^{(1)}_{ab}$ that contribute to Y in Eq. (2.7) at first order in α_s. Using parton distribution functions as defined in Refs. 21) and 22), so that moments of these functions are exactly the hadron matrix elements of $\overline{\mathrm{MS}}$ renormalized local operators, we find:

$$
\begin{aligned}
R^{(1)}_{q\bar q} &= R^{(1)}_{\bar q q} \\
&= \frac{2e_q^2}{3\pi Q_T^2}\Big\{ \frac{(Q^2-\hat t)^2+(Q^2-\hat u)^2}{\hat s}\,\delta(\hat s+\hat t+\hat u-Q^2) \\
&\qquad - 2\,\delta(1-z_A)\,\delta(1-z_B) \\
&\qquad\quad \times\left[\ln(Q^2/Q_T^2) - 3/2\right] \\
&\qquad - \delta(1-z_A)\left[\frac{1+z_B^2}{1-z_B}\right]_+ \\
&\qquad - \delta(1-z_B)\left[\frac{1+z_A^2}{1-z_A}\right]_+ \Big\}
\end{aligned}
\tag{2.9}
$$

$$
\begin{aligned}
R^{(1)}_{qg} &= R^{(1)}_{\bar q g} \\
&= \frac{e_q^2}{4\pi}\Big\{ \frac{(\hat s+\hat t)^2+(\hat t+\hat u)^2}{-\hat s\hat u}\,\delta(\hat s+\hat t+\hat u-Q^2) \\
&\qquad - \frac{1}{Q_T^2}\left[z_B^2+(1-z_B)^2\right]\delta(1-z_A)\Big\}
\end{aligned}
\tag{2.10}
$$

$$
\begin{aligned}
R^{(1)}_{gq} &= R^{(1)}_{g\bar q} \\
&= \frac{e_q^2}{4\pi}\Big\{ \frac{(\hat s+\hat u)^2+(\hat t+\hat u)^2}{-\hat s\hat t}\,\delta(\hat s+\hat t+\hat u-Q^2) \\
&\qquad - \frac{1}{Q_T^2}\left[z_A^2+(1-z_A)^2\right]\delta(1-z_B)\Big\}
\end{aligned}
\tag{2.11}
$$

$$
\begin{aligned}
&R^{(1)}_{gg} = R^{(1)}_{qq'} = R^{(1)}_{\bar q\bar q'} = 0 \\
&R^{(1)}_{q\bar q'} = R^{(1)}_{\bar q q'} = 0 \quad \text{if flavors } q \text{ and } q' \text{ are different.}
\end{aligned}
\tag{2.12}
$$

Here

$$
\begin{aligned}
&z_A = x_A/\xi_A \qquad z_B = x_B/\xi_B \\
&\hat s = z_A^{-1} z_B^{-1} Q^2 \\
&\hat t = Q^2 - z_B^{-1} Q^2\left[1+Q_T^2/Q^2\right]^{1/2} \\
&\hat u = Q^2 - z_A^{-1} Q^2\left[1+Q_T^2/Q^2\right]^{1/2}
\end{aligned}
\tag{2.13}
$$

3. $\tilde W(b)$ FOR $b \ll 1/\Lambda$

We now discuss the structure of the $Q_T \ll Q$ term in the formula, Eq. (2.1), for the cross-section. We work not with the $Q_T \ll Q$ part of the cross-section itself, but with its Fourier transform [9),10)], $\tilde W(b;Q,x_A,x_B)$. There are two cases to consider: $b \ll 1/\Lambda$ and $b \gtrsim 1/\Lambda$. The structure in the case $b \ll 1/\Lambda$ is simpler because in this case we can neglect contributions to $\tilde W(b)$ that are of order mass$\times$b compared to those that we keep. We shall consider this case first, and relate $\tilde W(b)$ for $b \ll 1/\Lambda$ to the singular parts of the coefficients $T^{(N)}$ given in Eq. (2.3).

In the following section we shall turn to the case $b \gtrsim 1/\Lambda$. The two cases will be unified by an equation for the evolution of $\tilde W$ with Q. The same equation holds for any value of b.

The argument below is self-contained except that Eqs. (3.8), (3.3) and (3.6) are given without proof. We assert, based on our work[16)], that these results for the Drell-Yan process may be derived in much the same way as in e^+e^- annihilation[11)]. We leave the required derivations for future publications.

When $b \ll 1/\Lambda$, $\tilde{W}(b)$ is determined by the singular part, as defined in Eq. (2.6), of the hard scattering function T. Thus we can write

$$\begin{aligned}\tilde{W}(b;Q,x_A,x_B) = \sum_{a,b}\int_{x_A}^{1}\frac{d\xi_A}{\xi_A}\int_{x_B}^{1}\frac{d\xi_B}{\xi_B}\, f_{a/A}(\xi_A;\mu)\, f_{b/B}(\xi_B;\mu)\\ \times\sum_{N=0}^{\infty}\left[\frac{\alpha_s(\mu)}{\pi}\right]^N \int d^2Q_T\, e^{-i\underline{Q}_T\cdot\underline{b}}\\ \times\Big\{\eta_{ab}^{(N)}(Q,x_A/\xi_A,x_B/\xi_B;\mu)\,\delta(\underline{Q}_T)\\ +\sum_{m=0}^{2N-1}T_{ab}^{(N,m)}(Q,x_A/\xi_A,x_B/\xi_B;\mu)\\ \times\left[\frac{\ln^m(Q^2/Q_T^2)}{Q_T^2}\right]_{reg}\Big\}\end{aligned}\tag{3.1}$$

An important simplification arises in Eq. (3.1) because of the collinear kinematic configuration that gives T its $Q_T \to 0$ singularity. The x_A/ξ_A and $\underline{a}$ dependence factorizes from the x_B/ξ_B and $\underline{b}$ dependence, so that $\tilde{W}(b)$ has the form:

$$\begin{aligned}\tilde{W}(b;Q,x_A,x_B) = \sum_{a,b}\int_{x_A}^{1}\frac{d\xi_A}{\xi_A}\int_{x_B}^{1}\frac{d\xi_B}{\xi_B}\\ \times f_{a/A}(\xi_A;\mu)\, f_{b/B}(\xi_B;\mu)\\ \times\sum_j e_j^2\, C_{ja}(x_A/\xi_A,b;Qb;g(\mu),\mu)\, C_{\bar{j}b}(x_B/\xi_B,b;Qb;g(\mu),\mu).\end{aligned}\tag{3.2}$$

Here $j = u,\bar{u},d,\bar{d},\ldots$ is the flavour of the annihilating quark or antiquark from hadron A and e_j is its charge in units of e.

Notice that Eq. (3.2) is not yet useful in the region $1/Q \ll b \ll 1/\Lambda$ because the functions C contain large logarithms, $\ln(Q^2b^2)$, that cannot be controlled by simple use of the renmormalization group (i.e., setting $\mu = Q$). These logarithms render invalid the approximation of C by its first few perturbative terms unless $\alpha_s(Q^2)\ln^2(Q^2b^2) \ll 1$. Furthermore, there is no apparent justification for approximating $\tilde{W}$ by the sum of the "leading

log" terms[8)-10)] in (3.1) - the terms with $m = 2N-1$.

In fact, $\tilde{W}$ has a structure that is much simpler than that expected in Eq. (3.2). This simplicity arises because $\tilde{W}(b;Q,x_A,x_B)$ obeys the following evolution equation*):

$$\begin{aligned}\frac{\partial}{\partial\ln Q^2}\tilde{W}(b;Q,x_A,x_B)\\ = \{K(b\mu;g(\mu)) + G(Q/\mu;g(\mu))\}\\ \times\tilde{W}(b;Q,x_A,x_B).\end{aligned}\tag{3.3}$$

Here K and G have perturbative expansions

$$\begin{aligned}K(b\mu;g(\mu)) &= \sum_{N=1}^{\infty}\left[\frac{\alpha_s(\mu)}{\pi}\right]^N K^{(N)}(b\mu)\\ G(Q/\mu;g(\mu)) &= \sum_{N=1}^{\infty}\left[\frac{\alpha_s(\mu)}{\pi}\right]^N G^{(N)}(Q/\mu)\quad,\end{aligned}\tag{3.4}$$

with (using $\overline{\text{MS}}$ renormalization)

$$\begin{aligned}K^{(1)}(b\mu) &= -\tfrac{4}{3}\left[\ln(b^2\mu^2/4) + 2\gamma_E\right]\\ G^{(1)}(Q/\mu) &= -\tfrac{4}{3}\left[\ln(Q^2/\mu^2) - \tfrac{3}{2}\right]\quad,\end{aligned}\tag{3.5}$$

where $\gamma_E = 0.577\ldots$ is Euler's constant.

The crucial point about Eq. (3.2) is that the b and Q dependences are separated on the right-hand side. Thus the only large logarithms in $\tilde{W}^{-1}\partial\tilde{W}/\partial\ln Q$ are logarithms of $b\mu$ in K and Q/μ in G. Furthermore, these large logarithms can be controlled because K and G obey renormalization group equations

*) We have changed the notation slightly from that of Ref. 11). The function G contains contributions from both $\mathcal{B}$ and H of Ref. 11); K and γ_K are unchanged, except for going from e^+e^- annihilation to the Drell-Yan process.

$$\mu \frac{d}{d\mu} K(b\mu ; g(\mu)) = -\gamma_K(g(\mu)) \tag{3.6}$$

$$\mu \frac{d}{d\mu} G(b\mu ; g(\mu)) = +\gamma_K(g(\mu)) ,$$

with

$$\gamma_K(g) = \sum_{N=1}^{\infty} \gamma_K^{(N)} \left[\frac{\alpha_s}{\pi}\right]^N , \tag{3.7}$$

$$\gamma_K^{(1)} = \frac{8}{3} . \tag{3.8}$$

We can use these renormalization group equations to change μ in K to be of order 1/b and μ in G to be of order Q:

$$\begin{aligned} K(b\mu;g(\mu)) &+ G(Q/\mu;g(\mu)) \\ &= -\int_{C_1^2/b^2}^{C_2^2Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \frac{1}{2} \gamma_K(g(\bar{\mu})) \\ &\quad + K(C_1 ; g(C_1/b)) + G(1/C_2 ; g(C_2Q)) . \end{aligned} \tag{3.9}$$

One can now approximate $K(C_1;g(C_1/b))$, $G(1/C_2;g(C_2Q))$ and $\gamma_K(g(\bar{\mu}))$ at some finite order in perturbation theory and rely on higher orders not to dominate lower orders.

In general, μ need not be set to exactly 1/b in K or to exactly Q in G. The exact values can be chosen to "optimize" the perturbation expansion - that is, to keep higher order corrections moderately small. We have left the possibility open by including the constants C_1 and C_2 in Eq. (3.9).

The value of γ_K at second order is important because γ_K multiplies an extra logarithm of Q^2b^2 in Eq. (3.8), arising from the integration over $\bar{\mu}$. Kodaira and Trentadue[19] have given a calculation of e^+e^- annihilation from which the value (C_F=4/3, T_F=1/2, C_A=3 for QCD)

$$\begin{aligned} \gamma_K^{(2)} &= -\frac{10}{9} C_F T_F N_F + C_F C_A \left(\frac{67}{18} - \frac{\pi^2}{6}\right) \\ &= -\frac{20}{27} N_F + \frac{134}{9} - \frac{2\pi^2}{3} \end{aligned} \tag{3.10}$$

is obtained. Note the absence of a C_F^2 term. One may have had some doubts about using this value for the Drell-Yan cross-section because there could be differences between $\gamma_K(g)$ in e^+e^- annihilation and $\gamma_K(g)$ in the Drell-Yan process and also because the calculation of Ref. 19) rests on some plausible but unproved assumptions. Recently, however, Davies and Stirling[20] have calculated $\gamma_K^{(2)}$ direclty by extracting the singular part, as in Eq. (2.6), of the Drell-Yan $T^{(2)}$ given by R.K. Ellis, Martinelli and Petronzio[24]. Davies and Stirling confirm the result (3.10)*). We have performed our own direct calculation of $\gamma_K^{(2)}$ in an Abelian theory[26] by using our Feynman rules for $K(b\mu;g(\mu))$. The result agrees with (3.10) (with C_A=0, but confirming the lack of a C_F^2 term).

We now return to the function $K(b\mu;g(\mu))+G(Q/\mu;g(\mu))$ that appears in the evolution equation (3.3). The representation (3.9) of this function can be further simplified as follows. First write the identity

$$\begin{aligned} F(b,Q) = &-\int_{1/b^2}^{C_2^2Q^2/C_1^2} \frac{d(1/\bar{b}^2)}{(1/\bar{b}^2)} \frac{\partial}{\partial \ln(1/\bar{b}^2)} F(\bar{b},Q) \\ &+ F(C_1/(C_2Q), Q) . \end{aligned} \tag{3.11}$$

Let F(b,Q) be $K(b\mu;g(\mu))+G(Q/\mu;g(\mu))$. Substitute the right-hand side of the representation (3.9) for F into the right-hand side of (3.11). This gives

*) As of this writing, however, we do not understand why the result (3.10) of Ref. 19) is not confirmed (in the e^+e^- case for which it was intended) by the numerical calculation of Ref. 25).

$$K(b\mu;g(\mu)) + G(Q/\mu;g(\mu)) = -\int_{c_1^2/b^2}^{c_2^2Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\, A(g(\bar{\mu});c_1) - B(g(c_2Q);c_1,c_2)\,, \tag{3.12}$$

where, using $\bar{\mu} = c_1/\bar{b}$ and $\partial g(\mu)/\partial \ln\mu = \beta(g(\mu))$, one finds

$$A(g;c_1) = \tfrac{1}{2}\gamma_K(g) + \tfrac{1}{2}\beta(g)\frac{\partial}{\partial g}K(c_1;g) \tag{3.13}$$

$$B(g;c_1,c_2) = -K(c_1;g) - G(1/c_2;g)\,. \tag{3.14}$$

We can now write the differential equation (3.2) as

$$\frac{\partial}{\partial \ln Q^2}\tilde{W}(b;Q,x_A,x_B) = -\left\{\int_{c_1^2/b^2}^{c_2^2Q^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}A(g(\bar{\mu});c_1) + B(g(c_2Q);c_1,c_2)\right\} \times \tilde{W}(b;Q,x_A,x_B)\,. \tag{3.15}$$

The solution of this equation is

$$\tilde{W}(b;Q,x_A,x_B) = \exp\left\{-\int_{c_1^2/b^2}^{c_2^2Q^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{c_2^2Q^2}{\bar{\mu}^2}\right)A(g(\bar{\mu});c_1) + B(g(\bar{\mu});c_1,c_2)\right]\right\} \times \tilde{W}\left(b;\frac{c_1}{c_2 b},x_A,x_B\right)\,. \tag{3.16}$$

Using Eq. (3.2) for $\tilde{W}$ at $Q \propto 1/b$ on the right-hand side of Eq. (3.16), we obtain the form of $\tilde{W}$ presented in the Introduction:

$$\tilde{W}(b;Q,x_A,x_B) = \sum_{a,b}\int_{x_A}^1\frac{d\xi_A}{\xi_A}\int_{x_B}^1\frac{d\xi_B}{\xi_B} f_{a/A}(\xi_A;\mu)\, f_{b/B}(\xi_B;\mu) \times \exp\left\{-\int_{c_1^2/b^2}^{c_2^2Q^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{c_2^2Q^2}{\bar{\mu}^2}\right)A(g(\bar{\mu});c_1) + B(g(\bar{\mu});c_1,c_2)\right]\right\} \times \sum_j e_j^2\, C_{ja}\left(\frac{x_A}{\xi_A},b;\frac{c_1}{c_2};g(\mu),\mu\right) C_{jb}\left(\frac{x_B}{\xi_B},b;\frac{c_1}{c_2};g(\mu),\mu\right). \tag{3.17}$$

In applications, we set the arbitrary renormalization scale μ equal to c_1/b.

The functions A, B, and C have perturbative expansions $A = \Sigma A^{(N)}(\alpha_s/\pi)^N$, etc. The low order terms in A and B are, with $\overline{\text{MS}}$ renormalization (from Eqs. (3.5), (3.6), (3.10), (3.13), (3.14), including the Kodaira-Trentadue result for $A^{(2)}$) are:

$$A^{(1)}(c_1) = \frac{4}{3} \approx 1.3 \tag{3.18}$$

$$A^{(2)}(c_1) = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}N_f + \frac{8}{3}\beta_1 \ln\left(c_1\tfrac{1}{2}e^{\gamma}\right) \approx 2.7 - 0.3\,(N_f - 4) + [5.5 - 0.4(N_f-4)]\ln\left(c_1\tfrac{1}{2}e^{\gamma}\right) \tag{3.19}$$

$$B^{(1)}(c_1,c_2) = \frac{8}{3}\ln\left(\frac{c_1}{2c_2}e^{\gamma-3/4}\right) \approx 2.7\ln\left(\frac{c_1}{2c_2}e^{\gamma-3/4}\right) \tag{3.20}$$

where

$$\beta_1 = \frac{33-2N_f}{12} \quad , \tag{3.21}$$

N_F is the number of quark flavours and $\gamma \approx 0.577$ is Euler's constant.

Davies and Stirling[20] have calculated also the second order contribution to B. Their result is calculated using $C_1 = 2e^{-\gamma}$, $C_2 = 1$. We can recover the C_1 and C_2 dependence from the previous results and the requirement that the cross-section does not depend on C_1 or C_2. This gives

$$\begin{aligned} B^{(2)}(C_1,C_2) &= 2\left[\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}N_f\right] \ln\left(\frac{C_1}{2C_2}e^{\gamma-3/4}\right) \\ &+ \frac{8}{3}\beta_1\left\{\ln^2\left(C_1\tfrac{1}{2}e^{\gamma}\right) - \ln^2\left(C_2 e^{3/4}\right)\right\} \\ &- \frac{9}{8} + \frac{7\pi^2}{6} + \frac{2}{3}\zeta(3) + \left(\frac{5}{36} - \frac{2\pi^2}{27}\right)N_f \\ &\approx \left[5.3 - 0.7(N_f-4)\right]\ln\left(\frac{C_1}{2C_2}e^{\gamma-3/4}\right) \\ &+ \left[5.5 - 0.4(N_f-4)\right] \\ &\quad \times\left\{\ln^2\left(C_1\tfrac{1}{2}e^{\gamma}\right) - \ln^2\left(C_2 e^{3/4}\right)\right\} \\ &+ 8.8 - 0.6\,(N_f-4) \quad . \end{aligned} \tag{3.22}$$

We have calculated the lowest order contributions to C and find, using the definition[21),22)] of parton distribution functions,

$$C^{(0)}_{jk}\left(z,b;\frac{C_1}{C_2};\mu\right) = \delta_{jk}\,\delta(z-1) \tag{3.23}$$

$$C^{(0)}_{jg}\left(z,b;\frac{C_1}{C_2};\mu\right) = 0 \tag{3.24}$$

$$\begin{aligned} C^{(1)}_{jk}\left(z,b;\frac{C_1}{C_2};\mu\right) = \delta_{jk}\Big\{ &\frac{2}{3}(1-z) - \frac{4}{3}\ln\left(\mu b\tfrac{1}{2}e^{\gamma}\right)\left[\frac{1+z^2}{1-z}\right]_+ \\ &+ \delta(1-z)\Big[-\frac{4}{3}\ln^2\left(\frac{C_1}{2C_2}e^{\gamma-3/4}\right) \\ &+ \frac{\pi^2}{3} - \frac{23}{12}\Big]\Big\} \end{aligned} \tag{3.25}$$

$$\begin{aligned} C^{(1)}_{jg}\left(z,b;\frac{C_1}{C_2};\mu\right) &= \frac{1}{2}\left[z^2 - z + 1\right] \\ &- \frac{1}{2}\ln\left(\mu b\tfrac{1}{2}e^{\gamma}\right)\left[z^2 + (1-z)^2\right] . \end{aligned} \tag{3.26}$$

Here j and k represent quark flavours or antiflavours and g represents a gluon.

Our calculation of $C^{(1)}_{jk}$, Eq. (3.25), agrees with that of Davies and Stirling[20]. In addition, the calculation of Davies and Stirling confirms the structure of $\tilde{W}(b)$, Eq. (3.17) at second order of perturbation theory, taking the (n,n) moments of the flavour non-singlet cross-section and extracting the terms proportional to $Q_T^{-2}\ell n^3(Q_T)$, $Q_T^{-2}\ell n^2(Q_T)$, $Q_T^{-2}\ell n(Q_T)$ and Q_T^{-2}. At this level one sees the factorization of the x dependence from the Q_T dependence as indicated in Eq. (3.17), including the appearance of the second order Altarelli-Parisi kernel that generates the evolution of the parton distribution functions $f(x;\mu)$, in agreement with the calculation of this kernel by Curci et al.[22]. We present a more detailed discussion of the relation of the general result (3.17) to second order perturbation theory in Appendix A.

4. $\tilde{W}(b)$ FOR $b \gtrsim 1/\Lambda$

A consequence of Eq. (3.17) is that when Q is larger than 1/b, $\tilde{W}(b;Q,x_A,x_B)$ decreases as Q increases. Indeed, when Q is large enough, the integral

$$\int d^2b\; e^{i\mathbf{Q}_T\cdot\mathbf{b}}\; \tilde{W}(b;Q,x_A,x_B)$$

that determines the cross-section is dominated by small b[9),11)].

Let us follow the argument of Parisi and Petronzio[9] to see this in the simplest case: that $Q_T = 0$ and that $\tilde{W}(b)$ is approximated by just the leading factor in Eq. (3.17),

$$W(b) \propto \exp\left\{-\int_{1/b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \ln(Q^2/\bar{\mu}^2)\, A^{(1)}\, \frac{\alpha_s(\bar{\mu})}{\pi}\right\} ,$$

with $\alpha_s(\bar{\mu})/\pi = [\beta_1 \ln(\bar{\mu}^2/\Lambda^2)]^{-1}$. The integral is

$$\int d\ln(b^2\Lambda^2)\ \exp\left\{\ln(b^2\Lambda^2) - \frac{A^{(1)}}{\beta_1}\int_{\ln(1/b^2\Lambda^2)}^{\ln(Q^2/\Lambda^2)} d\ln\left(\frac{\mu^2}{\Lambda^2}\right) \frac{\ln(Q^2/\mu^2)}{\ln(\mu^2/\Lambda^2)}\right\} .$$

The integral is dominated by a saddle point at

$$0 = 1 + \frac{A^{(1)}}{\beta_1}\, \frac{\ln(Q^2 b^2)}{\ln(b^2\Lambda^2)} .$$

That is

$$b_{SP} = \frac{1}{\Lambda}\left(\frac{Q}{\Lambda}\right)^{-A^{(1)}/[A^{(1)}+\beta_1]} = \frac{1}{\Lambda}\left(\frac{Q}{\Lambda}\right)^{-0.41} , \qquad (4.1)$$

where the numerical value is for five quark flavours. Thus when Q is large enough, the integral over b is dominated by b near b_{SP}, which is much smaller that $1/\Lambda$. For instance when $Q = 10^8$ GeV and $\Lambda = 150$ MeV, one finds $b_{SP} = 2\times10^{-3}\ \mathrm{GeV}^{-1}$. Then it suffices*) to use the form of $\tilde{W}(b)$ given in the previous section, with A, B, and C evaluated at a finite order of perturbation theory, as long as the qualitative behaviour of W(b) for large b is reasonable.

Unfortunately, present accelerators do not produce large enough energies to allow us to neglect what happens to $\tilde{W}(b)$ when $b \gtrsim 1/\Lambda$. For instance, at Q = 15 GeV, Λ = 150 MeV one has $b_{SP} = 1.0\ \mathrm{GeV}^{-1}$, and values of

*) One needs quite a tiny value of b_{SP} because the saddle point is not very sharp and because any errors in $A(g(\bar{\mu}))$ near $\bar{\mu} \sim 1/b_{SP}$ are magnified by the large factor $\ln(Q^2/\bar{\mu}^2)$ multiplying A. The numerical value of 10^8 GeV suffices to reduce to a negligible (20%) level the contribution at $b = 3b_{SP}$ from a possible non-perturbative term in A of the form $\Delta A = (1\ \mathrm{GeV})/\bar{\mu}$.

b out to 2 or 3 GeV^{-1} may be important. Thus one must understand $\tilde{W}(b)$ also when b is large.

When $b \gtrsim 1/\Lambda$, $\tilde{W}(b;Q,x_A,x_B)$ obeys (in perturbation theory) the same evolution equation (3.3) discussed in the previous section:

$$\frac{\partial}{\partial \ln Q^2}\tilde{W}(b;Q,x_A,x_B) = \{K(b\mu, bm_q; g(\mu)) + G(Q/\mu; g(\mu))\} \times \tilde{W}(b;Q,x_A,x_B) . \qquad (4.2)$$

The only change from the $b \ll 1/\Lambda$ case is that now K depends on the masses of quarks that occur in quark loops, denoted collectively by m_q. This dependence can be neglected when $bm_q \ll 1$.

The solution of the evolution equation (4.2) can be written[11] as

$$\tilde{W}(b;Q;x_A,x_B) \sim \exp\left\{-\int_{C_1^2/b^2}^{C_2^2Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{C_2^2Q^2}{\bar{\mu}^2}\right) A(g(\bar{\mu}), m_q/\bar{\mu}; C_1) + B(g(\bar{\mu}), m_q/\bar{\mu}; C_1, C_2)\right]\right\} \qquad (4.3)$$
$$\times \sum_j e_j^2\, \bar{\mathcal{P}}_{j/A}(x_A, b; C_1/C_2)\, \bar{\mathcal{P}}_{\bar{j}/B}(x_B, b; C_1/C_2) .$$

Two differences may be noted between this result, valid for any b, and the previous result (3.17), valid only for $b \ll 1/\Lambda$. First the functions A and B depend in general on quark masses m_q, while $m_q/\bar{\mu}$ can be approximated by zero when 1/b, and hence $\bar{\mu}$ in (4.3), is large. The second difference concerns the functions $\bar{\mathcal{P}}(x,b)$. These functions arise as the integration constant for the differential equation (4.2). They may be interpreted as the Fourier transforms of functions $\mathcal{P}_{j/A}(x,k_T)$ that give parton distributions as a function of x and k_T. When $b \ll 1/\Lambda$, the functions $\bar{\mathcal{P}}(x,b)$ may be factored, neglecting corrections suppressed by powers of $b\Lambda$, into a convolution C*f. Thus Eq. (3.17) is recovered in the $b\Lambda \to 0$ limit.

Equation (4.3) is derived (in Ref. 11), for the e^+e^- case) by considering Feynman graphs at any finite order, coupled to Bethe-Salpeter wave functions for the hadronic bound states. The functions A and B have perturbative definitions. The functions P contain a perturbative component coupled with a Bethe-Salpeter wave function component.

Possible non-perturbative effects, for example instanton effects, are ignored in the derivation of Eq. (4.3), except insofar as such effects are reflected in the Bethe-Salpeter wave functions. We assume that any effects that cannot be seen in perturbation theory do not alter the general form of Eq. (4.3), although at present we cannot verify this assumption.

We can, however, note that the general form of Eq. (4.3) does appear to be sensible in at least one non-perturbative model. Let us divide the integration region in Eq. (4.3) into a perturbative region $\mu_{min} < \bar{\mu} < Q$ and a non-perturbative region, $C_1/b < \bar{\mu} < \mu_{min}$. Then the non-perturbative region contributes factors to Eq. (4.3) that have the form

$$\exp\left[-\ln(Q^2/m^2)\, g_1(b)\right] \quad \exp\left[-g_2(b)\right] \tag{4.4}$$

The second factor may be absorbed into the functions $\bar{P}(x,b)$. These functions may then be interpreted as the Fourier transforms of the "primordial" transverse momentum distributions of the incoming partons.

The first factor in (4.4) may be interpreted as resulting from the emission of low transverse momentum particles from the debris left over from the hard process. The total rapidity range available for particles of mass m is $Y = \ln(Q^2/m^2)$. Suppose that the particles in each small rapidity interval Δy are emitted independently of those in other rapidity intervals. Let the distribution of the total transverse momentum k_T carried by the particles in interval Δy be

$$\rho_{\Delta y}(k_T) = \int \frac{d^2b}{(2\pi)^2}\, e^{i\underline{k}_T\cdot\underline{b}}\quad e^{-g_1(b)\Delta y}$$

Then the total transverse momentum K_T of all the soft particles in all of the rapidity intervals is

$$\rho_{TOT}(K_T) = \int \frac{d^2b}{(2\pi)^2}\, e^{i\underline{K}_T\cdot b}\, \left[e^{-g_1(b)\Delta y}\right]^N ,$$

where $N = Y/\Delta y = \ln(Q^2/m^2)/\Delta y$ is the number of rapidity intervals. In b space, then, the soft particles contribute a factor $\exp\{-\ln(Q^2/m^2)g_1(b)\}$ to $\tilde{W}(b)$. The structure of this factor, deduced in a model of soft hadronic physics, precisely matches the structure of the first factor in Eq. (4.4), deduced from perturbative quark-gluon physics.

5. JOINING LARGE AND SMALL b

We have now stated what we know about the structure of the Drell-Yan cross-section at measured Q_T. Here, we briefly address a question that arises when using this information to compare QCD with Drell-Yan experiments. We wish to join the expression (3.17) for $\tilde{W}(b)$ in the $b \ll 1/\Lambda$ region with the general form (4.3).

Let us define functions $\tilde{W}_j$ that denote the contribution to $\tilde{W}$ from the annihilation of quarks of flavour j, $\bar{j}$ from hadrons A and B respectively. That is (from Eq. (4.3))

$$\begin{aligned}\tilde{W}_j(b;Q;x_A,x_B) &= \exp\left\{-\int_{C_1^2/b^2}^{C_2^2Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{C_2^2Q^2}{\bar{\mu}^2}\right) A(g(\bar{\mu}), m_q/\mu; C_1)\right.\right.\\ &\qquad \left.\left. + B(g(\bar{\mu}), m_q/\mu; C_1, C_2)\right]\right\} \\ &\quad \times \bar{P}_{j/A}(x_A, b; C_1/C_2)\; \bar{P}_{\bar{j}/B}(x_B, b; C_1/C_2)\end{aligned} \tag{5.1}$$

for any b, and (from Eq. (3.17))

$$\tilde{W}_j(b;Q;x_A,x_B)$$
$$\sim \exp\left\{-\int_{C_1^2/b^2}^{C_2^2Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{C_2^2Q^2}{\bar{\mu}^2}\right)A(g(\bar{\mu});C_1) + B(g(\bar{\mu});C_1,C_2)\right]\right\} \tag{5.2}$$
$$\cdot\sum_a \int_{x_A}^1 \frac{d\xi_A}{\xi_A}\, C_{ja}\left(\frac{x_A}{\xi_A},b;\frac{C_1}{C_2};g\left(\frac{C_1}{b}\right),\frac{C_1}{b}\right) f_{a/A}\left(\xi_A;\frac{C_1}{b}\right)$$
$$\cdot\sum_b \int_{x_B}^1 \frac{d\xi_B}{\xi_B}\, C_{\bar{j}b}\left(\frac{x_B}{\xi_B},b;\frac{C_1}{C_2};g\left(\frac{C_1}{b}\right),\frac{C_1}{b}\right) f_{b/B}\left(\xi_B;\frac{C_1}{b}\right)$$

for $b \ll 1/\Lambda$. Then

$$\tilde{W}(b;Q;x_A,x_B) = \sum_j e_j^2\, \tilde{W}_j(b;Q;x_A,x_B)\ . \tag{5.3}$$

One simple prescription for joining the two forms (5.1) and (5.2) is as follows. Let $\tilde{W}_j(b;Q;x_A,x_B)_{pert}$ denote the small b form of $\tilde{W}_j$, with the functions A, B, and C and the Altarelli-Parisi kernel used in computing the evolution of $f(x;\mu)$ evaluated up to some finite order of perturbation theory. Then $\tilde{W}_{pert}$ should be a good approximation to $\tilde{W}$ when b is small enough. Let b_{max} be a distance that is small enough so that $\tilde{W}(b) \approx \tilde{W}(b)_{pert}$ when $b \lesssim b_{max}$. Of course the selection of b_{max} requires some judgement and can be a matter of debate.

Now let b_* be a function of b such that $b_* \approx b$ when $b \ll b_{max}$ and $b_* \lesssim b_{max}$ for all b. For example, let

$$b_* = b/\left[1+b^2/b_{max}^2\right]^{1/2}\ . \tag{5.4}$$

Let us now write $\tilde{W}_j(b)$ in the form

$$\tilde{W}_j(b) = \tilde{W}_j(b_*)\ \frac{\tilde{W}_j(b)}{\tilde{W}_j(b_*)}\ . \tag{5.5}$$

No matter whether b is large or small, b_* is always small, so we can use $\tilde{W}_j(b_*)_{pert}$ for the first factor in Eq. (5.5). For the ratio $\tilde{W}_j(b)/\tilde{W}_j(b_*)$, we can use the structural information present in Eq. (5.1), which yields the structure

$$\frac{\tilde{W}_j(b;Q;x_A,x_B)}{\tilde{W}_j(b_*;Q;x_A,x_B)} = \exp\{-\ln(Q^2/Q_0^2)\ g_1(b;b_{max}) - g_{j/A}(x_A,b;b_{max},Q_0) - g_{\bar{j}/B}(x_B,b;b_{max},Q_0)\}\ . \tag{5.6}$$

Here Q_0 is an arbitrary parameter with the dimension of mass; a change in Q_0 may be exactly compensated by a change in $g_{j/A}$ and $g_{\bar{j}/B}$. The functions $g_1(b)$ and $g_{j/A}(x,b)$ depend also on the conventional parameters b_{max} and Q_0, as indicated. In what follows, we will not indicate this dependence explicitly.

Since $b_* \approx b$ for small b, the left-hand side of Eq. (5.6) approaches one as b approaches zero. Thus

$$g_1(b) \to 0$$
$$g_{j/A}(x,b) \to 0 \tag{5.7}$$

as $b \to 0$.

The function $A(b(\bar{\mu}),m_q/\bar{\mu};C_1)$ in $\tilde{W}$ (or the function B) may contain terms of the form $\Delta A \propto \bar{\mu}^{-N}$ that are power suppressed in the $\bar{\mu} \to \infty$ limit compared to the $\alpha_s^n(\bar{\mu})$ perturbative terms (at $m_q=0$). Such terms could arise from finite quark mass corrections or from non-perturbative contributions of the form $\exp[-\text{const.}/\alpha_{j/A}(x,b)]$. In applications, these g functions can also help absorb the effect of the uncomputed higher order perturbative corrections in the functions A, B and C.

Our final result is then

$$\frac{d\sigma}{dQ^2\,dy\,dQ_T^2} \sim \frac{4\pi^2\alpha^2}{9Q^2 s}\left\{(2\pi)^{-2}\int d^2b\; e^{i\underset{\sim}{Q_T}\cdot\underset{\sim}{b}} \sum_j e_j^2 \right.$$
$$\times \tilde{W}_j(b_*;Q,x_A,x_B)_{pert}$$
$$\times \exp\left[-\ln(Q^2/Q_0^2)\,g_1(b) - g_{j/A}(x_A,b) - g_{\bar{j}/B}(x_B,b)\right] \tag{5.8}$$
$$\left. + Y(Q_T;Q,x_A,x_B)\right\} .$$

Here $\tilde{W}_j(b_*)_{pert}$ is obtained by using finite order perturbation theory for the functions A, B and C in $\tilde{W}_j$ as given in Eq. (5.2). The presently known perturbative coefficients $A^{(N)}$, $B^{(N)}$ and $C^{(N)}$ are given in Eqs. (3.18)-(3.26). Similarly, Y is evaluated by setting $\mu = C_2 Q$ in Eq. (2.8) and retaining a finite number of the perturbative coefficients $R^{(N)}$. The $R^{(1)}$ are given in Eqs. (2.9)-(2.13). One also uses perturbation theory in computing the μ^2 evolution of the parton distribution functions $f_{a/A}(x;\mu^2)$ and the running coupling $\alpha_s(\mu)$.

In Eq. (5.8) one integrates over all values of b, but one does not use low order perturbation theory for $b \gtrsim b_{max}$. Thus $\tilde{W}_{pert}$ is evaluated at $b_* = b/[1+b^2/b_{max}^2]^{\frac{1}{2}}$, which is always less than b_{max}. The large b behaviour is given by the functions $g_1(b)$ and $g_{j/A}(x,b)$. These functions are similar to the quark distribution functions $f_{j/A}(x;\mu)$ in that they cannot be calculated using perturbation theory, but instead must be measured experimentally. In the case of the quark distribution functions, the measurement is normally made in deeply inelastic lepton scattering*). The

*) We emphasize, however, that we define $f_{j/A}(x;\mu^2)$ as in Refs. 21) and 22), so that moments of these functions are exactly hadron matrix elements of $\overline{MS}$ renormalized local operators. Thus the structure function $f_2(x;Q^2)$ is not exactly $\Sigma e_j^2 f_{j/A}(x;Q^2)$, but has order α_s corrections.

large-b functions can be determined from Drell-Yan experiments at two different energies.

For W or Z boson production instead of virtual photon production, one uses Eq. (5.8) with the substitutions described in Eqs. (1.2)-(1.5).

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APPENDIX

RELATION TO STANDARD PERTURBATION THEORY

Let us recall from the discussion in Section 2 that the standard factorized form for the Drell-Yan cross-section at measured Q_T can be rewritten as

$$\frac{d\sigma}{dQ^2\,dy\,dQ_T^2} \sim \frac{4\pi^2\alpha^2}{9Q^2 s}(2\pi)^{-2}\int d^2b\; e^{iQ_T\cdot b}\sum_j e_j^2 \sum_{a,b} \times\int_{x_A}^1\frac{d\xi_A}{\xi_A}f_{a/A}(\xi_A;\mu)\int_{x_B}^1\frac{d\xi_B}{\xi_B}f_{b/B}(\xi_B;\mu) \times\tilde{T}^{\rm sing}_{jab}\left(b;Q,\frac{x_A}{\xi_A},\frac{x_B}{\xi_B};g(\mu),\mu\right) + \frac{4\pi^2\alpha^2}{9Q^2 s}Y(Q_T;Q,x_A,x_B) \quad , \tag{A.1}$$

where the function $\tilde{T}^{\rm sing}$ is the Fourier transform of the singular part (as $Q_T \to 0$) of the usual hard scattering function T. (That is, $\tilde{T}^{\rm sing}$ is the Fourier transform of the first two terms in Eq. (2.6).)

We wish to discuss the structure of the coefficients in the parturbative expansion of $\tilde{T}^{\rm sing}$. In order to display the physics of the structure while keeping the notation reasonably simple, we take moments of $\tilde{T}^{\rm sing}$ and project onto its flavour non-singlet part. That is, we define

$$\tilde{T}^{\rm sing}_{NS}(b;Q,I,J;g(\mu),\mu) = \int_0^1 dz_A\,(z_A)^I\int_0^1 dz_B\,(z_B)^J \times\frac{1}{2}\sum_{ab}(\delta_{aj}-\delta_{a\bar{j}})(\delta_{b\bar{j}}-\delta_{bj}) \times\tilde{T}^{\rm sing}_{jab}(b;Q,z_A,z_B;g(\mu),\mu) \quad , \tag{A.2}$$

where the normalization is such that $\tilde{T}^{\rm sing}_{NS} = 1$ at order zero in α_s.

Using ordinary perturbation theory, one can calculate the coefficients of $\ell n^m(Q^2b^2C_2^2/C_1^2)[\alpha_s(\mu)/\pi]^N$ in the expansion of $\ell n\tilde{T}^{\rm sing}_{NS}$. We call these coefficients ${}_N D_m$, following the notation of Refs. 25), 20) (except that 20) expands in powers of $\alpha_s/2\pi$):

$$\tilde{T}^{\rm sing}_{NS}(b;Q,I,J;g(\mu),\mu) = \exp\left\{\sum_{N=1}^{\infty}\sum_{m=0}^{2N}{}_N D_m(I,J,t) \times\ln^m(Q^2b^2C_2^2/C_1^2)\left[\frac{\alpha_s(\mu)}{\pi}\right]^N\right\} \quad , \tag{A.3}$$

where

$$t = \ln(C_2^2Q^2/\mu^2) \quad . \tag{A.4}$$

In writing Eq. (A.3), we have used the information that one can get two powers of $\ell n(Q^2b^2)$ for each power of α_s in the expansion of T. Thus $\underline{m}$ can run over the range $0 \leq m \leq 2N$. If we add the information that the leading logarithms "exponentiate", we gain the information that ${}_N D_m = 0$ for $m = 2N$, $N \geq 2$. If we add the information that the next-to-leading logarithms also exponentiate, we gain the information that ${}_N D_m = 0$ also for $m = 2N-1$, $N \geq 3$.

We now ask, what does the representation (3.17) for T tell us, in addition, about the ${}_N D_m$? One important conclusion is immediately visible in (3.17): ${}_N D_m = 0$ for $m > N+1$. More information is also present in (3.17).

There are N+1 coefficients ${}_N D_m$ at order α^N. However, Eq. (3.17) tells us that there are only three independent new coefficients at order α_s^N: $A^{(N)}$, $B^{(N)}$, and $C^{(N)}$. The dependence of the coefficients on the moment indices is also simpler than one might have expected on general grounds.

It is instructive to display the relation between the ${}_N D_m$ and the A, B and C coefficients for the cases N = 1 and N = 2. To do this, we first use the renormalization group to extract the μ dependence of C:

$$C_{NS}(I,b;\tfrac{C_1}{C_2};g(\mu),\mu) = \exp\left\{-\int_{C_1^2/b^2}^{\mu^2}\frac{d\bar\mu^2}{\bar\mu^2}\,\frac{1}{2}\gamma_{NS}(I;g(\bar\mu))\right\} \times C_{NS}(I,b;\tfrac{C_1}{C_2};g(\tfrac{C_1}{b}),\tfrac{C_1}{b}) \quad . \tag{A.5}$$

Here the $\gamma_{NS}(I)$ are the moments of the non-singlet Altarelli-Parisi kernels. Next, we expand A, B, γ and $\ln C$ in powers of α_s. For A, B and γ we use our usual notation $A = \Sigma A^{(N)}[\alpha_s/\pi]^N$. For $\ln C$, we denote, for any $\bar\mu$,

$$C_{NS}(I,b;\tfrac{C_1}{C_2};g(\bar\mu),\bar\mu) = \exp\left\{\sum_{N=1}^{\infty}\hat C_{NS}(I,b\bar\mu;\tfrac{C_1}{C_2})\left[\frac{\alpha_s(\bar\mu)}{\pi}\right]^N\right\}. \tag{A.6}$$

After these steps, we have

$$\begin{aligned}\sum_{N=1}^{\infty}\sum_{m=0}^{N+1} {}_N D_m(I,J,t)\ \ln^m(Q^2b^2C_2^2/C_1^2)\left[\frac{\alpha_s(\mu)}{\pi}\right]^N &= -\sum_{N=1}^{\infty}\int_{C_1^2/b^2}^{C_2^2Q^2}\frac{d\bar\mu^2}{\bar\mu^2}\left[\frac{\alpha_s(\bar\mu)}{\pi}\right]^N \\ &\quad\times\left[\ln\left(\frac{C_2^2Q^2}{\bar\mu^2}\right)A^{(N)}(C_1) + B^{(N)}(C_1,C_2)\right] \\ &\quad-\sum_{N=1}^{\infty}\int_{C_1^2/b^2}^{\mu^2}\frac{d\bar\mu^2}{\bar\mu^2}\left[\frac{\alpha_s(\bar\mu)}{\pi}\right]^N \\ &\quad\times\left[\frac{1}{2}\gamma_{NS}^{(N)}(I) + \frac{1}{2}\gamma_{NS}^{(N)}(J)\right] \\ &\quad+\sum_{N=1}^{\infty}\left[\frac{\alpha_s(C_1/b)}{\pi}\right]^N\left[\hat C_{NS}(I,C_1;C_1/C_2)\right. \\ &\quad\left.+\hat C_{NS}(J,C_1;C_1/C_2)\right] \quad .\end{aligned} \tag{A.7}$$

Three more small steps are needed. First, we expand $\alpha_s(\bar\mu)$ and $\alpha_s(C_1/b)$ in powers of $\alpha_s(\mu)$, using

$$\begin{aligned}\frac{\alpha_s(\bar\mu)}{\pi} &= \frac{\alpha_s(\mu)}{\pi} - \beta_1\ln\left(\frac{\bar\mu^2}{\mu^2}\right)\left[\frac{\alpha_s(\mu)}{\pi}\right]^2 + \cdots \\ \beta_1 &= (33-2N_f)/12 \quad .\end{aligned} \tag{A.8}$$

Second, we perform the integrals over $\bar\mu$. Third, we match the coefficients of $\ln^m(Q^2b^2C_2^2/C_1^2)[\alpha_s(\mu)/\pi]^N$. This gives

$$
\begin{aligned}
{}_1D_2 &= -\tfrac{1}{2} A^{(1)}(C_1) \\
{}_1D_1 &= -B^{(1)}(C_1, C_2) - \tfrac{1}{2}\gamma_{NS}^{(1)}(I) - \tfrac{1}{2}\gamma_{NS}^{(1)}(J) \\
{}_1D_0 &= \hat{C}_{NS}^{(1)}(I, C_1; C_1/C_2) + \tfrac{1}{2}\gamma_{NS}^{(1)}(I)\, t \\
&\quad + \hat{C}_{NS}^{(1)}(J, C_1; C_1/C_2) + \tfrac{1}{2}\gamma_{NS}^{(1)}(J)\, t
\end{aligned}
$$

$$
\begin{aligned}
{}_2D_3 &= -\tfrac{1}{3} A^{(1)}(C_1)\beta_1 \\
{}_2D_2 &= -\tfrac{1}{2} A^{(2)}(C_1) + \tfrac{1}{2} A^{(1)}(C_1)\beta_1 t - \tfrac{1}{2} B^{(1)}(C_1, C_2)\beta_1 \\
&\quad - \tfrac{1}{4}\gamma_{NS}^{(1)}(I)\beta_1 - \tfrac{1}{4}\gamma_{NS}^{(1)}(J)\beta_1 \\
{}_2D_1 &= -B^{(2)}(C_1, C_2) + B^{(1)}(C_1, C_2)\beta_1 t \\
&\quad - \tfrac{1}{2}\gamma_{NS}^{(2)}(I) + \tfrac{1}{2}\gamma_{NS}^{(1)}(I)\beta_1 t + \hat{C}_{NS}^{(1)}(I, C_1; C_1/C_2)\beta_1 \\
&\quad - \tfrac{1}{2}\gamma_{NS}^{(2)}(J) + \tfrac{1}{2}\gamma_{NS}^{(1)}(J)\beta_1 t + \hat{C}_{NS}^{(1)}(J, C_1; C_1/C_2)\beta_1
\end{aligned}
\tag{A.9}
$$

$$
\begin{aligned}
{}_2D_0 = {} & \hat{C}_{NS}^{(2)}(I, C_1; C_1/C_2) + \tfrac{1}{2}\gamma_{NS}^{(2)}(I)\, t \\
& - \hat{C}_{NS}^{(1)}(I, C_1; C_1/C_2)\beta_1 t - \tfrac{1}{4}\gamma_{NS}^{(1)}(I)\beta_1 t^2 \\
& + \hat{C}_{NS}^{(2)}(J, C_1; C_1/C_2) + \tfrac{1}{2}\gamma_{NS}^{(2)}(J)\, t \\
& - \hat{C}_{NS}^{(1)}(J, C_1; C_1/C_2)\beta_1 t - \tfrac{1}{4}\gamma_{NS}^{(1)}(J)\beta_1 t^2 .
\end{aligned}
$$

The results of Davies and Stirling[20)] at second order, with $I = J$, can be compactly summarized using Eq. (A.9). The leading coefficient ${}_2D_3$ is found to be as expected. In ${}_2D_2$ one meets the first order anomalous dimensions $\gamma^{(1)}(I)$. Davies and Stirling confirm this structure and calculate thereby the important coefficient $A^{(2)}$, confirming the Kodaira and Trentadue[19)] result for this coefficient. In ${}_2D_1$, one meets the second order anomalous dimensions $\gamma_{NS}^{(2)}(I)$. Davies and Stirling confirm this structure (and thereby confirm the calculation of $\gamma_{NS}^{(2)}(I)$ by Curci et al.[22)]) and extract the previously unknown value of $B^{(2)}$, as quoted in Eq. (3.22). The final second order coefficient, ${}_2D_0$, requires the calculation of purely virtual graphs and has not been computed.

REFERENCES

1) J.C. Collins, Phys.Rev. D21 (1980) 2962.

2) Moriond Workshop on Lepton Pair Production, Les Arcs, France (1981); Fermilab Workshop on the Drell-Yan Process, Fermilab, Batavia, IL, USA (1982).

3) UA1 Collaboration, Proceedings 4th $p\bar{p}$ Workshop, Bern (1984); UA2 Collaboration, ibid.

4) P. Chiappetta and M. Greco, Nuclear Phys. B199 (1982) 77; B221 (1983) 269; Phys.Letters 135 (1984) 187;
F. Halzen, A.D. Martin and D.M. Scott, Phys.Rev. D25 (1982) 754;
P. Aurenche and R. Kinnunen, LAPP preprint TH-78 (1983);
A. Nakamura, G. Pancheri and Y. Srivastava, Z.Physik 21 (1984) 243.

5) G. Altarelli, R.K. Ellis, M. Greco and G. Martinelli, CERN preprint TH.3851 (1984).

6) D. Amati, R. Petronzio and G. Veneziano, Nuclear Phys. B140 (1978) 54; B146 (1978) 29;
S. Libby and G. Sterman, Phys.Rev. D18 (1978) 3252, 4737;
A.H. Mueller, Phys.Rev. D18 (1978) 3705;
R.K. Ellis, H. Georgi, M. Machacek, H.D. Politzer and G.G. Ross, Nuclear Phys. B152 (1979) 285.

7) G. Altarelli, G. Parisi and R. Petronzio, Phys.Letters 76B (1978) 351.

8) Yu.L. Dokshitser, D.I. D'yakonov and S.I. Troyan, Phys.Letters 78B (1978) 290; Proceedings 13th Winter School of the LPNI, Leningrad (1978); Phys.Reports 58 (1980) 271.

9) G. Parisi and R. Petronzio, Nuclear Phys. B154 (1979) 427.

10) G. Curci, M. Greco and R. Srivastava, Phys.Rev.Letters 43 (1979) 834; Nuclear Phys. B159 (1979) 451.

11) J.C. Collins and D.E. Soper, Nuclear Phys. B193 (1981) 381, erratum B213 (1983) 545; B197 (1982) 446.

12) J.C. Collins and D.E. Soper, in Moriond Workshop on Lepton Pair Production, Les Arcs, France (1981).

13) S.D. Drell and T.-M. Yan, Ann.of Phys. 66 (1971) 578.

14) G.T. Bodwin, S.J. Brodsky and G.P. Lepage, Phys.Rev.Letters 47 (1981) 1799.

15) J.L. Cardy and G.A. Winbow, Phys.Letters 52B (1974) 95;
C. De Tar, S.D. Ellis and P.V. Landshoff, Nuclear Phys. B87 (1975) 176.

16) J.C. Collins, D.E. Soper and G. Sterman, Phys.Letters 109B (1982) 388; Nuclear Phys. B223 (1983) 381; Phys.Letters 126B (1983) 275; 134B (1984) 263.

17) W.W. Lindsay, D.A. Ross and C.T. Sachrajda, Nuclear Phys. B214 (1983) 61;
A. Sen and G. Sterman, Nuclear Phys. B229 (1983) 231.

18) J. Frenkel, J.G.M. Gatheral and J.C. Taylor, Sao Paulo preprint (1983).

19) J. Kodaira and L. Trentadue, Phys.Letters 112B (1982) 66.

20) C.T.H. Davies and W.J. Stirling, CERN preprint TH.3853 (1984), Nuclear Phys. (to be published).

21) J.C. Collins and D.E. Soper, Nuclear Phys. B194 (1982) 445.

22) G. Curci, W. Furmanski and R. Petronzio, Nuclear Phys. B175 (1980) 27.

23) P.W. Johnson, M. Medina-Garcia and W.-K. Tung, Ann.Phys.(N.Y.) 153 (1984) 205.

24) R.K. Ellis, G. Martinelli and R. Petronzio, Phys.Letters 104B (1981) 45; Nuclear Phys. B211 (1983) 106.

25) S.D. Ellis, D.G. Richards and W.J. Stirling, Phys.Letters 136B (1984) 99.

26) J.C. Collins, "Sudakov Form Factors", Argonne National Lab. preprint (1984).