CERN-TH.4393/86
PITHA 86/03

# THE INTERQUARK POTENTIAL: SU(2) COLOUR GAUGE THEORY WITH FERMIONS*)

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## A B S T R A C T

Including dynamical fermions in the Monte Carlo simulation of the SU(2) gauge theory, we investigate the potential between static quarks on a $16*8^3$ lattice. The Coulomb part of the force is stronger as compared with the pure gauge field theory. At large distances we find indications for a deviation from the linear rise of the potential, expected from a break-up of the flux tube between the heavy quarks through spontaneous creation of light quark pairs.

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*) Supported in part by the W. German Bundesministerium für Forschung und Technologie, Studienstiftung des deutschen Volkes and the (U.S.) National Science Foundation under Grant No. PHY 82-17853

CERN-TH.4393/86
PITHA 86/03
March 1986

1. INTRODUCTION. Compelling evidence has been accumulated in recent years that the potential between static quarks in pure non-abelian gauge theories of the strong interactions rises linearly with distance R at large R [1]. The presence of light dynamical quarks, however, should alter this picture: spontaneous quark-pair creation in the field stretched between the static quarks, screens the charge of the color sources at a scale O(1fm) and thus turns the linearly rising potential at large distances into the short range potential between open flavor bound states of heavy and light quarks. The analysis of the interquark potential at distances O(1fm) thus allows us a first glimpse of the break-up of a flux-tube through spontaneous quark-antiquark creation. Going from medium down to small distances, the decreasing vacuum polarization charge should in turn affect a stronger Coulomb part of the force than in a pure gauge theory.

The lattice formulation provides, at present, the only calculational scheme in which quantum chromodynamics can be solved at large distances. Tackling the problem of color screening needs to go beyond the quenched approximation. Including dynamical fermions, however, requires a formidable computational effort because highly nonlocal terms are introduced into the effective gauge field action through the fermion determinant. We have approached this problem for 4-fold degenerate Kogut-Susskind fermions in color SU(2) [F1], [3,4] on a $16*8^3$ lattice by employing the pseudofermion method [5]. Data were accumulated at $\beta = 4/g^2$ values between 1.85 and 2.5 with quark masses generally chosen between .05 and .2 in units of the inverse lattice spacing $a^{-1}$. We expect to approach approximate asymptotic scaling in this $\beta$ range, allowing us to map the values of the potential measured at various couplings $\beta$ onto one curve. The scale is fixed by assigning the string tension the experimental value $\sqrt{\sigma} = 400$ MeV.

Our attempt should only be understood as a very modest step in regard of the lattice size and the necessary extrapolation to small quark masses. The primary target is of qualitative nature: to see that at small to medium distances the Coulomb part of the force is strengthened and to look for indications of color screening at large distances when light quarks are included. Measurements of the chiral condensate and $\pi,\rho$ masses [for details see ref [6] ] complement our results.

2. TECHNICAL SET-UP. Including four degenerate Kogut-Susskind quarks with mass m, the pure gauge field action is supplemented by the logarithm of the Dirac determinant once the fermionic degrees of freedom are integrated out [F2].

$$S_{eff} = \beta \cdot \sum_{\square}' [1 - \frac{1}{2} \mathrm{tr}\, U_{\square}] - \mathrm{tr}\log[D(U)+m] \tag{1}$$

where

$$[D(U)+m]_{mn} = \frac{1}{2} \sum_{\mu} \Gamma_{\mu}(m)[U_{\mu}(m)\delta_{m+\mu,n} - U^{+}_{\mu}(n)\delta_{m-\mu,n}] + m\delta_{mn} \tag{2}$$

The sum in (1) runs over the ordered plaquettes $U_{\square}$; $U_{\mu}(m)$ denotes the link variable and $\Gamma_{\mu}(m) = (-)^{m_1+m_2+\ldots+m_{\mu-1}}$.

To generate the equilibrium gauge field configurations we employed the Metropolis algorithm. Allowing only for a small variation in the tentative upgrade of a link $U \to U + \delta U$, the increment of the fermionic part of the action can be linearized,

$$\delta S_F = \frac{1}{2} \mathrm{tr}\, J_{\mu}(n)\, \delta U_{\mu}(n) + \mathrm{h.c.} + O([\delta U]^2) \ , \tag{3a}$$

and the flux change $\delta U_{\mu}(n)$ interacts only locally with the current

$$J_{\mu}(n) = \frac{1}{2} \Gamma_{\mu}(n) \ \{[D(U)+m]^{-1} - \mathrm{h.c.}\}_{n+\mu,n} \tag{3b}$$

Keeping $\delta U_{\mu}(n)$ sufficiently small, all the required elements of the propagator $[D+m]^{-1}$ might be computed before performing a complete link upgrade. This leaves us with an error $O([\delta U]^2)$ consistent with the error in the previous linearization of $\delta S$. We have calculated the propagators by adopting the pseudofermion method [5]. Since [D+m] is not a positive definite matrix, complex bosonic pseudofermion fields $\varphi(n)$ must be generated with a distribution according to the weight

$$\exp\ [-S_{PF}] = \exp\,[\,-\varphi^{+}(D^{+}+m)(D+m)\varphi] \tag{4a}$$

The correlation functions of the pseudofermion fields are then related to the fermion propagator by

$$[D(U)+m]^{-1}_{m+\mu,m} = \sum_{j} < \varphi_{m+\mu}\ \varphi^{*}_{j} > [D^{+}+m]_{jm} \tag{4b}$$

Since the action for a single component $\varphi_i$ is of the form $\alpha|\varphi_i|^2 + \mathrm{Re}\,\varphi_i b$, the updating of the bosonic fields can be carried out very efficiently by means of a heat bath algorithm [7],

$$\varphi_i = -\frac{b^*}{2a} + \sqrt{-\frac{1}{a}\log r_1}\; e^{2\pi i r_2} \tag{4c}$$

where $r_1$ and $r_2$ are chosen randomly between 0 and 1.

A typical run then proceeds as follows. At each β-value we start with 5000 pure gauge field sweeps. Thereafter the fermionic part of the action is included into the updating procedure. Having updated all the links once, we allow the pseudofermions in 100 sweeps to adjust to the present gauge field configuration, followed by 100 measurements of the currents $J_\mu(n)$. Taking

$$U \to U+\delta U = U\{\cos\frac{\vartheta}{2} + i\,\vec{\sigma}\,\vec{n}\,\sin\frac{\vartheta}{2}\} \tag{5}$$

as the trial link, we restrict $\delta U$ to small values by choosing $\vartheta/2$ randomly between 0 and $\pi/12$. $\vec{n}$ is a vector randomly distributed over the unit sphere. This yields an acceptance rate of 80 to 90 %, slightly varying over the β range investigated here. Typically it takes $\sim 400$ gauge field sweeps to reach equilibrium in the combined system. The time development of plaquettes and other observables are continuously monitored. Long range correlations over 100 to 300 sweeps have been observed, depending on the non-locality of the measured operator. To control systematic errors, we varied the maximum angle $(\vartheta/2)_{max}$ and the number of pseudofermion upgradings $N_{PF}$ in separate test runs. Choosing a smaller $(\vartheta/2)_{max}$ and/or larger $N_{PF}$ (up to several thousand) we found no noteworthy systematic deviations. After equilibrium was reached data were taken during about 2000 sweeps for each value of the coupling constant and quark mass in the parameter set. Measurements were performed every 50th sweep.

3. MEASUREMENTS. In this note we will concentrate on the determination of the interquark potential. Measurements of the chiral condensate and the meson masses are subject of a separate publication [6].

The average plaquette value

$$E = \langle 1 - \frac{1}{2}\,\mathrm{tr}\,U_\Box \rangle \tag{6}$$

does not change dramatically if dynamical quarks are attached to the gauge boson system, Fig. 1. At $\beta = 2.1$, for instance, it decreases by 10 % for $m = 0.1$ compared with the quenched value [8]. This has also been observed in the microcanonical simulation of the fermionic system [4], [F3]. The presence of fermions thus tends to order the system as expected from the highly nonlocal determinantal interaction. Fermions act similarly to a magnetic field on a spin system. This effect was theoretically anticipated in a weak coupling expansion of the unquenched plaquette [11]. With increasing $\beta$ the data slowly approach the perturbative value but still deviate significantly even for the largest $\beta$'s considered here. For large quark masses, the average plaquette converges to its quenched value as the fluctuations in the fermion determinant $\sim \Pi(1+\lambda^2/m^2)$ get more and more damped. In fact, choosing a quark mass $m = 10$, we observe that the fermions have been frozen out.

By contrast, large Wilson loops are strongly affected by the presence of fermions, raising their values by up to on order of magnitude at a given $\beta$. We extract the potential from the Wilson loops by standard procedures [1,2]

$$V(R) = - \lim_{T\to\infty} \frac{1}{T} \log W(R,T) \tag{7a}$$

$$W(R,T) = \left\langle \frac{1}{2} \operatorname{tr} \prod_{\tau} U \right\rangle \tag{7b}$$

$\tau$ : rectangular R*T loop

The unquenched potential aV depends on the fermion mass am. This dependence has been approximated by a linear function to extrapolate the aV values down to zero quark mass. This is shown in Fig. 2. for $\beta = 2.25$ and 1.95. The mass dependence becomes stronger, as expected, if $\beta$ decreases. In order to map the values of the potential, measured for different couplings $\beta$ [ which infer different lattice spacings $a(\beta)$ ] onto one curve, we assume that asymptotic scaling holds approximately in the $\beta$ range covered in the analysis, for SU(2)

$$a(\beta) = \frac{1}{\Lambda_L(N_F)} \left(\frac{\beta}{4\beta_0}\right)^{\frac{\beta_1}{2\beta_0^2}} e^{-\frac{\beta}{8\beta_0}} \tag{8a}$$

where

$$\beta_0 = \frac{1}{16\pi^2} \left(\frac{22}{3} - \frac{2N_F}{3}\right) \tag{8b}$$

$$\beta_1 = \left(\frac{1}{16\pi^2}\right)^2 \left(\frac{136}{3} - \frac{49N_F}{6}\right)$$

and $\Lambda_{\overline{MS}}/\Lambda_L$ = 19.8 and 91.4 for $N_F$ = 0 and 4, respectively, [12]. The scaling assumption is backed by an analysis of the quark condensate in refs [4,6]. It allows us to construct the potential as a function of the distance in one common, yet to be determined, physical unit. The $\beta$ dependent quark self-energy is subtracted off by adjusting one point of the potential values in the region where they overlap. To confront the unquenched calculation with the quenched approximation we first applied this procedure to the quenched data. [It can be expected that conclusions drawn from such a comparison remain qualitatively true on larger lattices than ours, even though detailed numbers might be subject to corrections.] The result for the quenched potential is shown in Fig. 3a. The curve represents a fit to a superposition of a Coulombic and a confinement term.

$$V_Q(R) = \text{const} - \frac{\alpha_Q}{R} + \sigma R \tag{9}$$

With a string tension of $\sigma = (400\ \text{MeV})^2$ taken as input, we find $\alpha_Q = 0.21 \pm 0.01$ and a lattice spacing of $a_Q(\beta=2.25) = 0.24 \pm 0.01$ fm. This corresponds to $\Lambda_{\overline{MS}}(N_F=0) = 110 \pm 10$ MeV. Within an error margin of $\sim 10\%$ these values coincide with Stack's SU(2) analysis [1] [as well as the measurement of meson masses in ref [13]]. Doing the analysis at $\beta$ = 2.25, 2.375 and 2.5 separately, the data for $\sigma a^2(\beta)$ nicely follow the scaling prediction.

If light quarks are incorporated in the system we get the results presented in Fig. 3b. We have parametrized the potential by an ansatz that corresponds to a cut-off confinement form with screening length $\mu^{-1}$,

$$V(R) = \text{const} + \left[-\frac{\alpha}{R} + \sigma R\right] \frac{1-e^{-\mu R}}{\mu R} \tag{10}$$

While at small to medium range R this parametrization maintains a Coulombic plus linear behavior [slope = $\sigma - \frac{1}{6}\alpha\mu^2 \doteq \sigma$ within less than 5 %], the linear part is damped and approaches a constant $\delta = \sigma/\mu$, the splitting energy of the heavy quark pair, at large distances. Because of strong correlations we have refrained from a simultaneous fit of the parameters $\sigma a^2$, $\mu a$ and $\alpha$. Instead, setting the string tension to $\sigma = (400\ \text{MeV})^2$, $\mu$

and $\alpha$ have been determined by varying the lattice spacing a over a range suggested by the calculation of the $\rho$ mass in ref [6]. The Coulomb coefficient does not change much over the range covered, and is found to be $\alpha = 0.28 \pm 0.02$. However, the screening length does depend on the lattice spacing. The result is shown in the insert of Fig. 3b.

The Coulomb coefficient $\alpha$ is larger than the analogous quenched value of 0.21 This is plausible since $\alpha$ is an effective parameter accounting for string as well as short distance effects in which the leading term of the coupling constant is proportional to $1/(22-2N_F)$. This increase of the Coulombic force [which affects the potential up to medium distances] provides an explanation for phenomenological suggestions[14] based on quarkonium spectroscopy. [Though the quark loops push $\alpha$ into the right direction, it does not match yet the coefficient required e.g. in the Cornell potential, $\sim 0.48$ for color $SU_3$.] For small a it is not possible to measure the screening length since the separation of the static quark pair is not wide enough. However, if the lattice spacing is restricted to the values set by the measurement of the $\rho$ mass [with the preferred linear extrapolation of $am_\rho$ in the quark mass], a $(\beta=1.95) = 0.19 \pm 0.02$ fm, we find $\mu^{-1} = 0.8 \pm 0.2$ fm. We have drawn the curve in Fig. 3b for the mean values of this parameter set. The splitting energy $\delta = \sigma/\mu \sim 600$ MeV falls into the range expected from heavy quarkonium spectroscopy[15].

Attempts to describe the unquenched data only by a renormalization of the lattice spacing while leaving all other quenched parameters unchanged, fail. In addition, a purely perturbative parametrization with a running coupling constant as given in ref [16], could not describe the data beyond 0.15 fm though at shorter distances this parametrization was found to be compatible with the data [F4].

4. SUMMARY. Two results have emerged from the analysis of the potential between heavy quarks in lattice gauge theories including dynamical fermions. For small to medium distances the Coulombic force is stronger than in the quenched approximation. At large distances we have found indications for a break-up of the color flux tube between the quarks as expected from spontaneous creation of light quark pairs. This conclusion is tied to a measurement of the lattice spacing through the $\rho$ mass. Even though this note could illuminate only a few points, the results support the general physical

picture of the force between heavy quarks in non-abelian gauge theories when light fermions are included.

We thank the Computer Centers at RWTH Aachen and Bochum University for their support. P.Z. is grateful to S. Brodsky and J.R. Schrieffer for the warm hospitality extended to him at the QCD Workshop in Santa Barbara, likewise to the CERN Theory Division. We have greatly benefited from discussions with A. Burkitt and T. Neuhaus. E.L. acknowledges clarifying discussions with M. Creutz at the beginning of the project.

FOOTNOTES

[F1] The problem has been investigated in the hopping parameter expansion for Wilson fermions in ref [2].

[F2] All quantities are given in lattice units if not explicitly stated otherwise.

[F3] Note that the measurement of the plaquette in ref [4] [shown as crosses in Fig. 1] and the data presented here nicely agree with each other. The matching of results, obtained by means of different algorithms, is an important cross check of the systematic errors involved. This point is strengthened further by observing a similar agreement of the vacuum condensates [6] and the Polyakov-Wilson lines on $4*8^3$ lattices. A more elaborate comparison of thermodynamic quantities is described in ref [9]. Since present computer capacities do not allow to compare "inexact" and "exact" algorithms directly with each other on large lattices [10] these cross checks are mutually reassuring and appear very encouraging.

[F4] No obvious deconfinement effects have been detected in our "symmetric" lattice. The expectation values of Polyakov-Wilson lines in spatial as well as timelike directions are very small, $\sim 0.02 \pm 0.01$ in the $\beta$ range considered here. This is to be contrasted with a typical finite temperature lattice of size $4*8^3$ where we reproduced, in a low statistics measurement, the results reported in ref. [17].

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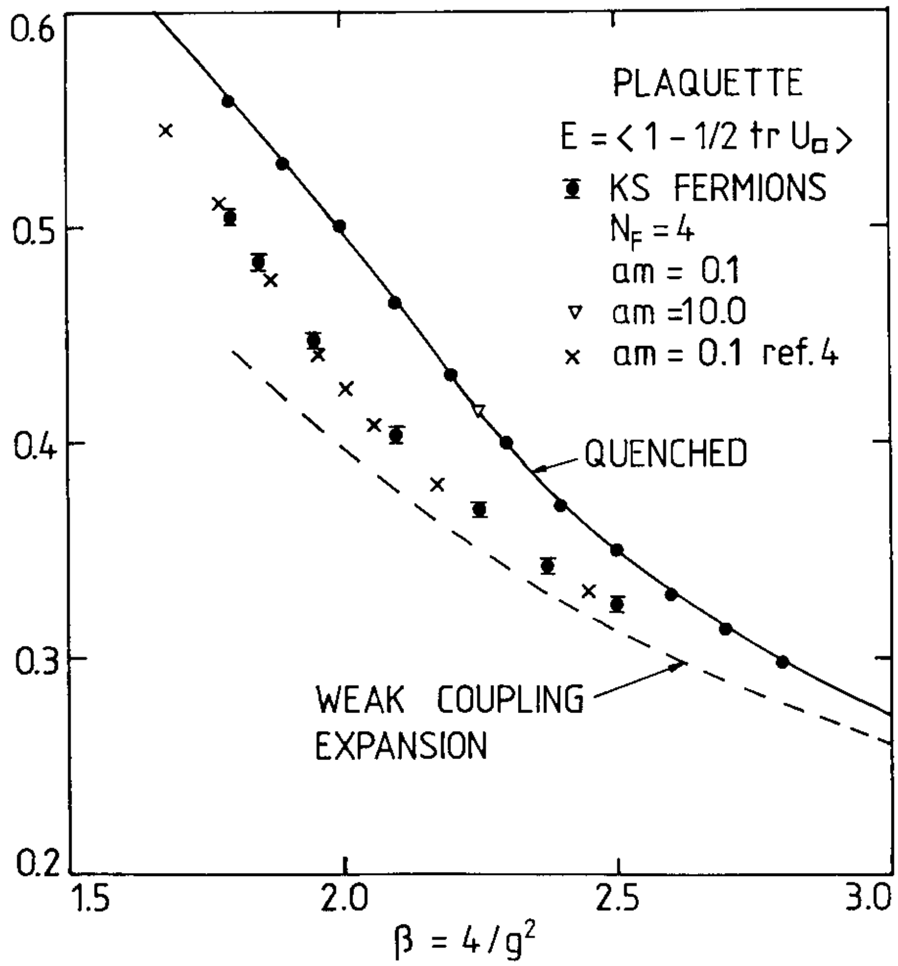
FIGURE CAPTIONS

Fig. 1. Plaquette values as a function of the coupling constant $\beta=4/g^2$, without and with Kogut-Susskind fermions. The crosses show the values obtained in the microcanonical simulation of ref [4]. The weak coupling expansion is adopted from ref [11].

Fig. 2. The potential vs. the quark mass in lattice units at (a) $\beta= 2.25$ and (b) $\beta=1.95$ including the linear extrapolations to zero-mass quarks.

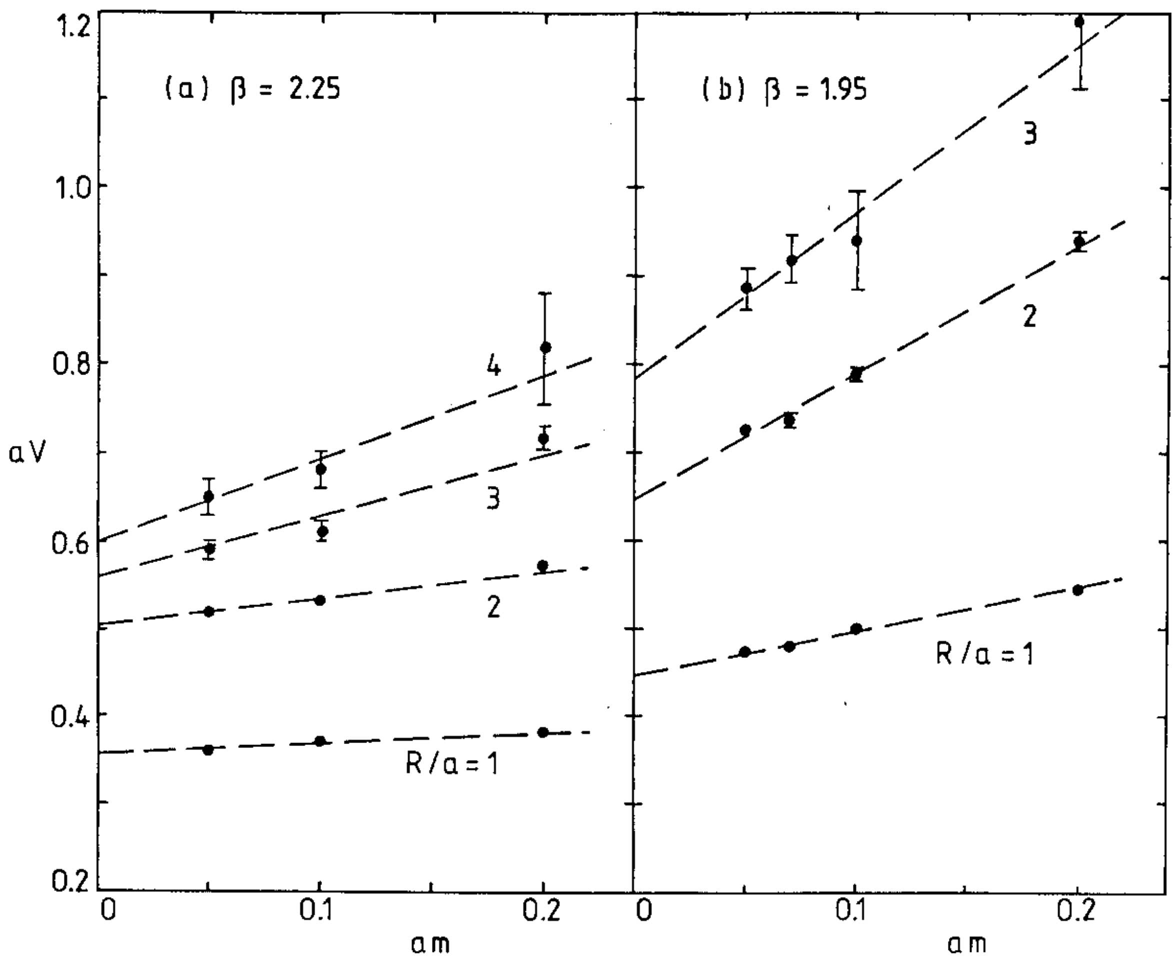
Fig. 3(a) The quenched potential in physical units, $1/\sqrt{\sigma} \approx \frac{1}{2}$ fm; The curve represents the fit described in the text.

(b) The potential of the complete, unquenched theory. The full line corresponds to the screened form of the confinement potential. The insert shows the dependence of the upper and lower limits ($1\sigma$) of the inverse screening lenght $\mu$ on the lattice spacing at $\beta=1.95$. The range preferred by a measurement of the $\rho$ mass in ref. [6] is indicated by arrows. The mean values in this domain are the parameters for which the form of the potential is compared with the data.



PLAQUETTE
E = < 1 - 1/2 tr U□ >
KS FERMIONS
N_F = 4
am = 0.1
am = 10.0
am = 0.1 ref. 4
QUENCHED
WEAK COUPLING
EXPANSION
0.6
0.5
0.4
0.3
0.2
1.5
2.0
2.5
3.0
β = 4/g²


Fig. 1



(a) β = 2.25
(b) β = 1.95
aV
4
3
2
R/a = 1
3
2
R/a = 1
1.2
1.0
0.8
0.6
0.4
0.2
0
0.1
0.2
am
0
0.1
0.2
am


Fig. 2

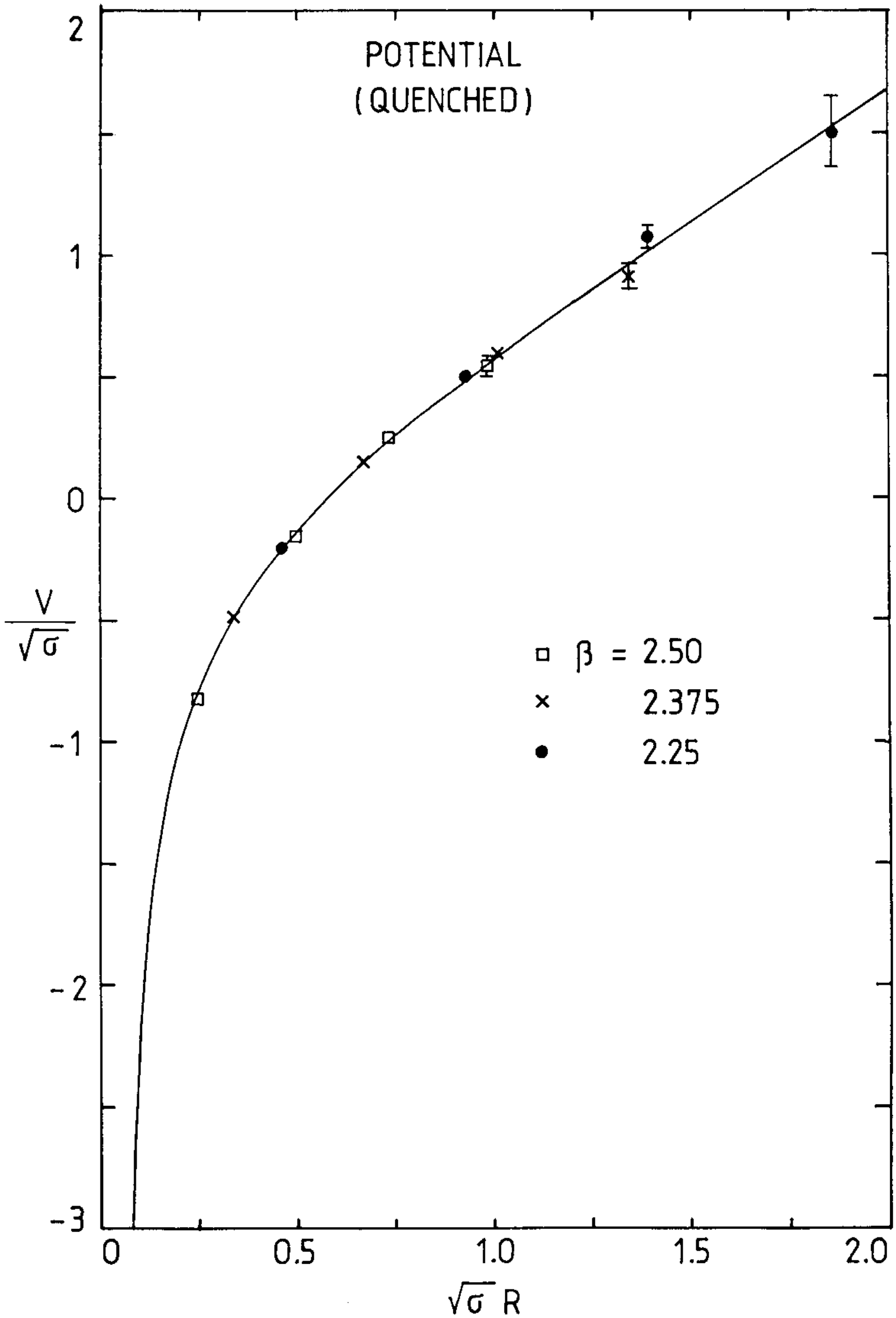
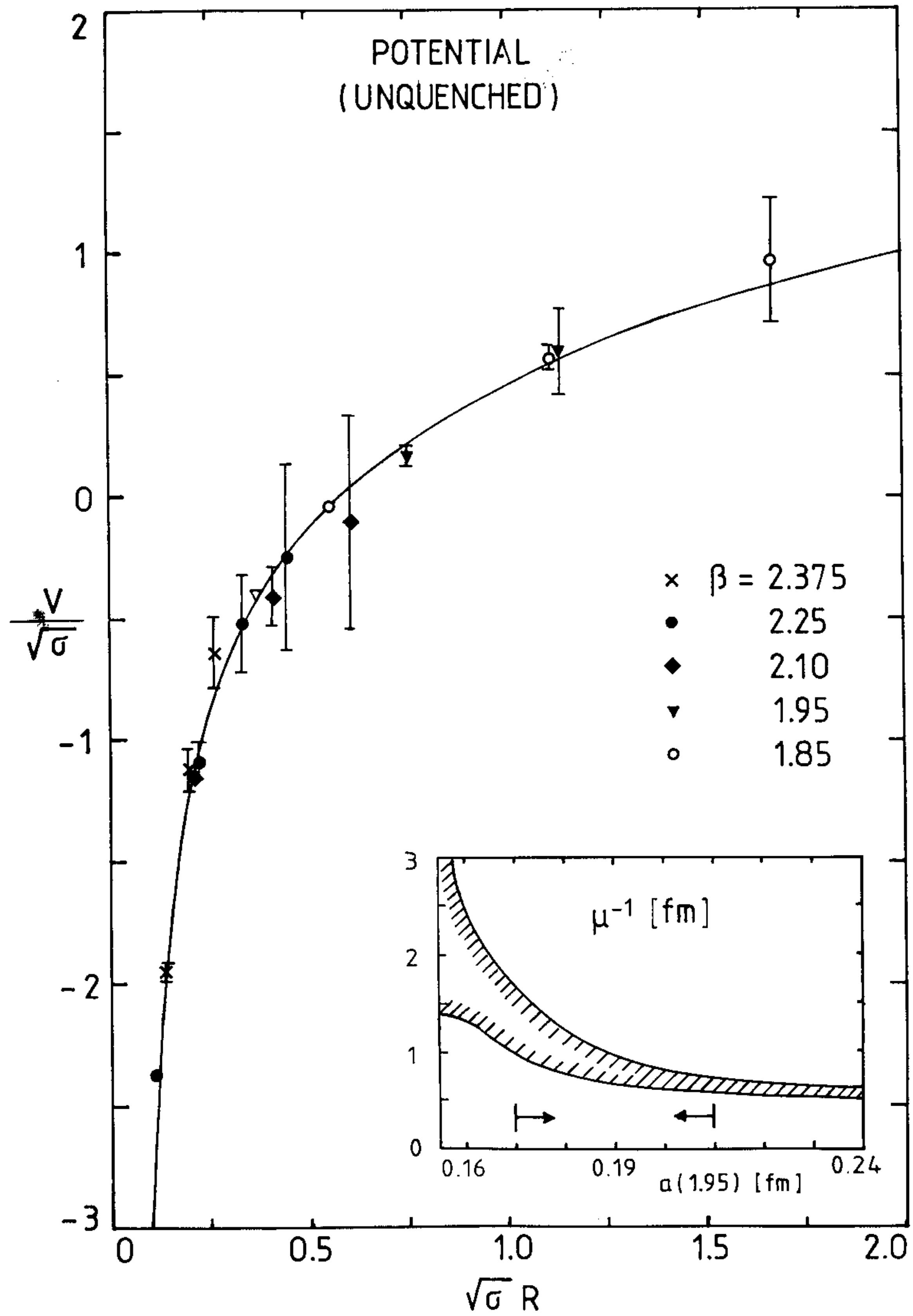



Fig. 3a



POTENTIAL
(UNQUENCHED)
2
1
0
-1
-2
-3
$\frac{V}{\sqrt{\sigma}}$
× β = 2.375
● 2.25
◆ 2.10
▼ 1.95
○ 1.85
0
0.5
1.0
1.5
2.0
$\sqrt{\sigma}$ R
3
2
1
0
$\mu^{-1}$ [fm]
0.16
0.19
0.24
a(1.95) [fm]


Fig. 3b