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Kindest regards.

Carmen Vasini

On the Meaning of Fermi Coupling.

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Various authors have attempted to account for elementary particles, starting from a theory of Fermi-type coupling between some Fermion fields. However, no quantum attempt has been clearly successful so far. We shall here investigate the simplest example, that of a scalar self-coupling of a spinor ψ, and show that this theory is identical with a Yukawa-type theory of a scalar meson whose experimental mass μ and coupling constant G are given functions of the Fermion constants [+]. This identity will be used to express the unrenormalized quantities of the Fermi-theory in terms of the renormalized ones. We assume that both theories have a solution and that the Yukawa-theory still has a meaning when the renormalization constants tend to the critical values given below.

Let $L_F = L_1(m_0, \psi_0) + (g_0/2)(\bar{\psi}_0\psi_0)(\bar{\psi}_0\psi_0)$ be the Fermi-Lagrangian, as a function of the unrenormalized quantities, where L_1 is the free particle of the Dirac Lagrangian; S_F will be the corresponding S-matrix. The Yukawa-Lagrangian $L_Y = L_1(m_0, \psi_0) + L_2(\mu_0, A_0) + G_0 A_0 \bar{\psi}_0 \psi_0$, again in unrenormalized form (L_2 being the free meson part), will lead to an S-matrix S_Y, of which these matrix elements in which no real mesons appear are denoted by $(\langle S_Y \rangle_{0\,\text{meson}})$. Comparing the matrix elements of $(\langle S_Y \rangle_{0\,\text{meson}})$ proportional to G^{2N} with those of S_F proportional to g^N, one sees that one would obtain

(1)
$$S_F = (\langle S_Y \rangle_{0\,\text{meson}})$$

provided that

(2)
$$g_0 = \frac{G_0^2}{p^2 + \mu_0^2}.$$

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(1) B. JOUVET: Journ. de Math., 33, 201 (1954) or Thèse de Doctorat, Paris (1954); Suppl. Nuovo Cimento, 2, 941 (1955); Proceed. Pisa Conf. (1955).

Both G_0^2 and μ_0^2 must be infinite if this equality is to hold; thus the (contact) Fermi-interaction is equivalent to that resulting from the exchange of mesons of infinite mass μ_0, with coupling constant G_0, also infinite. The point of interest is, however, that both these parameters are the unobservable unrenormalized ones. Because of the occurrence of g_0 in the divergent quantities, care must be taken to show that (2) actually leads to (1). By using a limiting process with a cut-off on the Fermion momentum, $p^2 < \Lambda$, one can give this proof providing that $G_0^2(\Lambda)$ and $\mu_0^2(\Lambda)$ satisfy

$$(3) \qquad \left(\frac{\partial}{\partial \Lambda}\left(\frac{1}{G_0^2(\Lambda)}\right) \ll \frac{a}{\Lambda}\right)_{\Lambda \to \infty} ; \qquad a = \text{const} ; \qquad \mu_0^2(\Lambda) = \frac{G_0^2(\Lambda)}{g_0} .$$

Let us now deal with the renormalized quantities. If we assume that the usual renormalization constants of the Yukawa-theory are determined as functions of the observable parameters (m, μ, G), only the meson quantities may be renormalized at first

$$(4) \qquad A_0 = Z_3^{\frac{1}{2}} A , \qquad \mu_0^2 = \mu^2 + (\delta\mu^2/Z_3) , \qquad G_0 = G_1 Z_3^{-\frac{1}{2}} = \frac{Z_1}{Z_2} Z_3^{-\frac{1}{2}} G .$$

Expressing $(\langle S_Y \rangle_{0\,\text{meson}})$ in terms of these quantities, the equality (1) holds when

$$(5) \qquad\qquad\qquad Z_3 = 0 , \qquad \delta\mu^2 = G_1^2/g_0$$

and condition (3) leads to

$$(6) \qquad\qquad\qquad \frac{\partial Z_3(\Lambda)}{\partial \Lambda} \ll aG_1^2/\Lambda .$$

The unitarity of S_F and S_Y can be shown to imply that

$$(7) \qquad\qquad\qquad S_Y |\text{meson}\rangle = 0 ,$$

which is the case if $\mu > 2m$ so that the meson has a finite life-time; Δ_F' then has a complex pole, and $\delta\mu^2$ is defined as the real part of the usual quantity (+). One may then renormalize the ψ field in the usual way. Finally, one may renormalize the constant g_F, which is the value of the total Fermi-interaction between free particles when the exchanged momentum tends to zero, with

$$(8) \qquad\qquad g_0 = g_F \cdot Z_F^2 Z_2^{-2} ; \qquad g_F = G^2\big(\mu^2 + iG^2 K_{\text{ren}}^Y (p^2 = 0)\big)^{-1} .$$

$K_{\text{ren}}^Y(p^2)$ is the meson renormalized polarization operator. From this treatment one obtains two relations that we expect would determine the observable parameters

$$(9) \qquad Z_3(m, \mu, G) = 0 ; \qquad \delta\mu^2(m, \mu, G)Z_F^2(m, \mu, G)g_F = G^2 Z_1^2(m, \mu, G) .$$

(+) A similar case has been studied in detail in conneciton with the Lee model by V. GLASER (*Colloquium*, October 31st, 1955, Copenhagen).

Analogous equations should enable one to calculate, in principle, physical constants of elementary particles, such as electric charge, through the appropriate modifications of the old schematic model.

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