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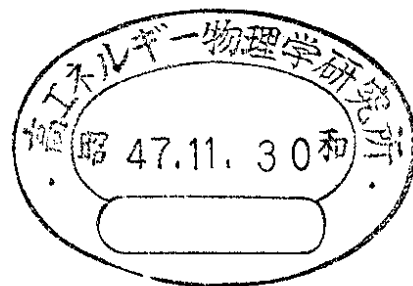
STOCHASTIC DAMPING OF BETATRON OSCILLATIONS

IN THE ISR

by

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SUMMARY

In principle, betatron oscillations could be damped by detecting and compensating statistical variations of the average beam position, caused by the finite number of particles present. It is shown that achieving useful damping in the ISR would be difficult with presently available techniques.

1. STOCHASTIC DAMPING

As is well known, Liouville's theorem predicts that betatron oscillations cannot be damped by the use of electromagnetic fields deflecting the particles. However, this theorem is based on statistics and is only strictly valid either for an infinite number of particles, or for a finite number if no information is available about the position in phase plane of the individual particles. Clearly, if each particle could be separately observed and a correction applied to its orbit, the oscillations could be suppressed. It is also well known to be possible to damp coherent betatron oscillations (where the beam behaves like a single particle) by means of pickup-deflector feedback systems. In the same way, the statistical fluctuations of the average beam position, caused by the finite number of particles, can be detected with pickup electrodes and a corresponding correction applied. In other words, the small fraction of the oscillations that happens to be coherent at any time due to the statistical fluctuations, can be damped.

After the beam would have passed through such a damping system (for which the name "stochastic damping" could perhaps be used), it would no longer present any coherent oscillations, and further damping would seem to be impossible. However, there are two effects that reintroduce randomness, and therefore some coherency:

- a) Not all particles have the same revolution time, due to momentum spread.
- b) Different particles have different Q-values.

As will be shown in the following sections, the first effect would be the most important one in the ISR.

For ISR application, damping of the vertical betatron oscillations would be most interesting, since it would increase the luminosity.

It is clear that for efficient damping the feedback system should have a large bandwidth (i.e. a short response time). In this way smaller samples of the total number of circulating particles will be seen as separate entities by the system, and the ideal of separate treatment of each particle will be more closely approximated. In the following section this will be shown in a more quantitative way.

2. CALCULATION OF THE DAMPING TIME

We shall assume that a damping system can be made that suppresses coherent betatron oscillation of the beam that passes through it once. Of course, only the components of coherent oscillation within the bandwidth of the system will be removed.

We shall first consider the smallest sample of particles that the system will be able to distinguish. This is about equal to the number of particles that passes through the system in one rise-time. We assume that there are n particles in this sample, with oscillation amplitudes $A_1, A_2 \dots A_n$, randomly chosen according to a distribution function $F(A)$, with

$$\int_0^{\infty} F(A) dA = 1$$

The mean square amplitude is

$$\sigma_0^2 = \frac{\sum A^2}{n} \quad (1)$$

We shall describe the position of each particle in phase space, as it enters the damping system, by A and ψ , as shown in Fig. 1. The centre of gravity of the sample in phase space is then at

$$\left[\frac{1}{n} \sum (A \cos \psi) \right], \left[\frac{1}{n} \sum (A \sin \psi) \right]$$

The damping system will now bring this centre of gravity back to the origin by giving the same kick to all particles. The new position of particle i will then be

$$\left[A_i \cos \psi_i - \frac{1}{n} \sum (A \cos \psi) \right], \left[A_i \sin \psi_i - \frac{1}{n} \sum (A \sin \psi) \right]$$

The square of the amplitude of particle i will now become

$$A_i^2 - \frac{2}{n} A_i \left[\cos \psi_i \sum (A \cos \psi) + \sin \psi_i \sum (A \sin \psi) \right] + \frac{1}{n^2} \left[\left\{ \sum (A \cos \psi) \right\}^2 + \left\{ \sum (A \sin \psi) \right\}^2 \right]$$

The new mean square amplitude is

$$\sigma_1^2 = \frac{1}{n} \left[\sum A^2 - \frac{1}{n} \left\{ \left(\sum (A \cos \psi) \right)^2 + \left(\sum (A \sin \psi) \right)^2 \right\} \right] \quad (2)$$

Comparing (1) and (2), we see that the r.m.s. amplitude is reduced by a fraction

$$\epsilon = \frac{\sigma_0 - \sigma_1}{\sigma_0} \approx \frac{\left\{ \sum (A \cos \psi) \right\}^2 + \left\{ \sum (A \sin \psi) \right\}^2}{2n \sum A^2} \quad (\epsilon \ll 1)$$

The average value of this reduction is found by integrating over all possible combinations of the parameters A and ψ :

$$\bar{\varepsilon} = \int_0^{\infty} \dots \int_0^{2\pi} \dots \frac{\{\sum(A \cos \psi)\}^2 + \{\sum(A \sin \psi)\}^2}{2n \sum A^2} \cdot F(A_1) F(A_2) \dots F(A_n) \cdot \frac{1}{(2\pi)^n} \cdot d\psi_1 d\psi_2 \dots d\psi_n dA_1 dA_2 \dots dA_n$$

Now it can be shown that

$$\int_0^{2\pi} \dots \int_0^{2\pi} [\{\sum(A \cos \psi)\}^2 + \{\sum(A \sin \psi)\}^2] d\psi_1 d\psi_2 \dots d\psi_n = (2\pi)^n \sum A^2$$

Therefore

$$\bar{\varepsilon} = \frac{1}{2n} \int_0^{\infty} \dots \int_0^{\infty} F(A_1) F(A_2) \dots F(A_n) dA_1 dA_2 \dots dA_n = \frac{1}{2n}$$

It appears that the average damping achieved by one passage through the system is independent of the amplitude distribution. (This probably means that a simpler derivation of the result above could be given.)

After 2n passages through the damping system, assuming complete randomizing of the sample populations between passages, the r.m.s. amplitude would be reduced by a factor e.

We shall now suppose that all particles pass through the system once per revolution, and that the randomizing during one turn is indeed complete (see par. 3). If the rise-time of the system is τ , we have approximately

$$n = N \frac{\tau}{T_r}$$

where N is the total number of particles in the ring, and T_r the revolution time. The damping time constant is then

$$T_d = 2n \cdot T_r = 2N\tau$$

If the system bandwidth is $F = \frac{1}{2\pi\tau}$, we have $T_d = \frac{N}{\pi F}$.

For instance, if we could obtain a bandwidth of 1 GHz, with 4×10^{14} particles in the ISR, we would obtain a damping time of 35 hours. This does not look attractive. With a smaller number of particles stored, the result might be of marginal interest, although probably much development would be required in order to achieve the required bandwidth. It can easily be seen that in accelerators (such as, for instance, the PS) the available time is insufficient to achieve significant damping.

3. THE RANDOMIZING PROCESS

With a bandwidth of 1 GHz, the rise-time is about 0.16 ns, and the smallest particle samples seen by the system are 5 cm long. If the ISR would contain its maximum momentum bite of 2% (corresponding to the design figure $N = 4 \times 10^{14}$), the circumference of the closed orbits for maximum and minimum momentum would be different by about 25 cm. As a consequence, at energies well above the transition energy, after each turn the 5 cm long samples would contain a quite different population.

Since it seems difficult to analyze what will be the damping time if the randomizing process is less efficient than in the example above, a simple Monte Carlo programme was written, simulating damping in the ISR for different amounts of "smearing out" due to momentum spread. This programme, although necessarily dealing with a very much smaller number of particles (i.e. 2500) than expected in the ISR, is thought to be valid for study of the above mentioned randomizing behaviour. The

results for a constant density stack are shown in Fig. 2. The "mixing factor" is equal to the distance, gained per turn by the particles with lowest momentum on those with the highest momentum, divided by the sample length. For a mixing factor of 50 or larger the programme showed the same damping rate as derived in section 2.

The randomizing effect of the Q-spread was also simulated, and it was found that this effect was quite small for practical Q-spread values, compared with the momentum spread effect.

The examples given at the end of section 2 can now be corrected for non-ideal randomizing. As a result, the damping time required for 4×10^{14} particles would be increased to 45 hours. On the other hand, it would be possible to install five damping systems equally spaced around the ISR circumference. Each of these systems would work with a mixing factor of 1, and the resulting damping time would be $\frac{35 \times 2}{5}$ hours = 14 hours.

The computer results show, however, that the damping only continues at the predicted rate until the amplitudes are reduced by a factor 2 to 3. Then the damping rate decreases. This effect seems to be independent of n or τ . However, it seems nearly sure that this is caused by some approximation made in the computer program. In reality such a behaviour would seem to be quite impossible; it could only be due to the disappearance of randomness and this would be completely re-established in a very short time compared to the damping time.

4. FINAL NOTE

This work was done in 1968. The idea seemed too far-fetched at the time to justify publication. However, the fluctuations upon which the system is based were experimentally observed recently. Although it may still be unlikely that useful damping could be achieved in practice, it seems useful now to present at least some quantitative estimation of the effect.

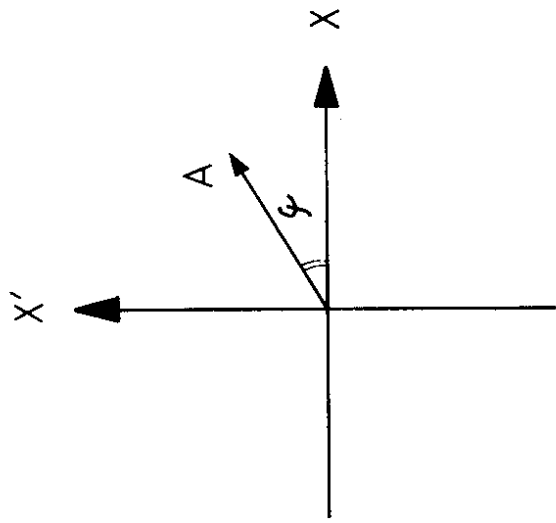


Fig. 1

