BREAKDOWN AND REGENERATION OF TIME REVERSAL SYMMETRY IN NONEQUILIBRIUM STATISTICAL MECHANICS

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Breakdown and regeneration of time reversal symmetry in nonequilibrium Statistical Mechanics

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Abstract: A review and non technical exposition of recent studies on the relevance of time reversal, on its spontaneous breakdowns, and on its possible resurgence as a weaker symmetry that still anticommutes with time evolution.

1. Time reversal and irreversibility.

This review illustrates joint work with F. Bonetto ([BG]) and P. Garrido ([BGG]).

Time reversal symmetry is manifest in most fundamental equations. It is very often associated with the property of motion reversibility and considered the source of various difficulties into which run attempts at fundamental explanations of easily observed irreversibility phenomena.

It is important to realize immediately that a microscopically reversible system can exhibit irreversible behaviour even without invoking thermodynamic limits or suitable *ansatzes.* Consider an example, which is paradigmatic and not really as special as it might look initially: a system of N particles (one or more) moving on a closed surface Σ, interacting via conservative pair forces with potential V and subject to an external *field* of potential $E\,\Phi$ which "drives" the system (in the sense that Φ is not single valued on the surface Σ, *i.e.* the corresponding force is an "electromotive force") and to the forces that keep the particles on Σ and *also* enforce as a *constraint* that the total energy $\mathcal{E} = K + V$ is an exact constant of motion ($K = \frac{1}{2m}\sum_i \underline{p}_i^2$, if m is the particles mass).

Thus the equations of motion are:

$$m\ddot{\underline{q}}_i = -\underline{\partial}_{\underline{q}_i} V(\underline{q}) - E\,\underline{\partial}_{\underline{q}_i}\Phi + \underline{R}_i \tag{1.1}$$

where the *constraints reactions* $\underline{R}_i$ are determined by imposing that the particles stay on the surface and keep constant total energy $\mathcal{E}$. The constraint is anholonomous and it will be supposed *ideal* in the sense that it verifies Gauss' least constraint principle, see appendices of [G1], [BGM].

It realizes a *thermostatted model* in the cases of interest to us, in which the geometry of Σ is nontrivial and the external field is locally conservative but not globally conservative.

For instance Σ could be a torus deprived of a few convex regions, *obstacles*, and the picture will be like:

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* Permanent address: Fisica, Università di Roma, P.le Moro 2, 00185 Roma, Italia. This paper is dedicated to Professor Raoul Gatto for his 65-th birthday, retirement age in Switzerland: his support and example during my scientific formation have been very influential and I have always nostalgy of those days in Firenze, and regret for not having been really able to reach anywhere close to imitate his masterful conception of symmetries and their role.

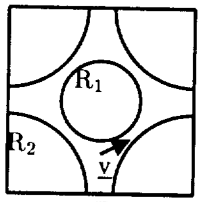

Fig.1

Fig.1: Circular scatterers of radius R_1 and R_2 in a periodic box "horizontal electric field" $\underline{E} = E\underline{j}$ along the axis $\underline{j}$ and 1 particle.

This can be considered a model for electric conduction in a crystal with N ($N = 1$ in the figure) free electrons per cell. After an elementary analysis of the gaussian constraint, if V is (for simplicity) supposed a hard core potential and $\underline{p}_i = m\underline{\dot{q}}_i$, the equations (1.1) for the latter simple model become:

$$m\underline{\ddot{q}}_i = E\underline{j} - \alpha\underline{p}_i, \qquad \alpha = \frac{\underline{E}\cdot\sum_i \underline{p}_i}{\sum_i \underline{p}_i^2} \tag{1.2}$$

plus elastic collision rules (at particle–particle or particle–obstacle collisions). The energy $\mathcal{E}$ equals the kinetic energy K, and if K is written $K = dN\frac{1}{2}k_B\vartheta$ (d being the space dimension and k_B the Boltzmann's constant) this is a model for electric conduction at temperature ϑ. The equation (1.2) should be compared with the one sometimes used in the simplest electric conduction models (simplifications of *Lorentz' model* of conductivity) at temperature ϑ:

$$m\underline{\ddot{q}}_i = E\underline{j} - \nu\underline{p}_i, \qquad \nu = const \tag{1.3}$$

where ν is a phenomenological constant adjusted so that the average kinetic energy is $dN\frac{k_B\vartheta}{2}$.

The two models (1.2) and (1.3) should be essentially equivalent (for a discussion see [G2],[G3],[G4]). But it is clear that they are deeply different in various respects. Namely the first is *time reversal invariant* in the sense that there is an isometry I defined on phase space, *i.e.* the familiar map $I(\underline{q},\underline{p}) = (\underline{q},-\underline{p})$, which *anticommutes* with the time evolution operator $(\underline{q},\underline{p}) \to S_t(\underline{q},\underline{p})$:

$$IS_t = S_{-t}I \qquad \text{for all } t \tag{1.4}$$

while the second is manifestly not I–invariant.

Nevertheless (1.2) shows an irreversible behavior in the sense that if $E \neq 0$:

$$\frac{1}{T}\int_0^T \alpha(S_t x)dt \xrightarrow[T\to+\infty]{} \langle\alpha\rangle_+ > 0 \tag{1.5}$$

for almost all choices of the initial datum x (with respect to the area distibution) and time reversal symmetry implies "only" that the contraction of phase space in the backward direction is also $\langle\alpha\rangle_+ > 0$.

Since the phase space contraction rate is:

$$\sigma_E(x) \equiv (dN-1)\,\alpha(x) = \frac{dN-1}{\sum_i \underline{p}_i^2}\,\underline{E}\cdot\sum_i \underline{p}_i \tag{1.6}$$

we see that $\langle\alpha\rangle_+ > 0$ implies singularity of the probability distribution μ_+ giving us the time averages of observables F followed on the trajectory starting at a point $x = (\underline{q}\,,\,\underline{p})$, randomly chosen with respect to the area on the surface of energy $\mathcal{E}$:

$$\frac{1}{T}\int_0^T F(S_t x)dt \xrightarrow[T\to\infty]{} \int \mu_+(dx)F(x) \tag{1.7}$$

In other words the distribution μ_+ which, for the driven system, is the analogue of the microcanonical distribution *cannot* be expressed in the form (if $x = (\underline{q}\,,\,\underline{p}), dx = d\underline{p}\,d\underline{q}$):

$$\rho_E(x)\,\delta(K+V-\mathcal{E})dx \tag{1.8}$$

for *any* ρ_E, except when $E = 0$ (in which case ρ_E is just a constant and μ_+ is the microcanocial ensemble).

The property $\langle\alpha\rangle_+ > 0$ is of course essential and one has to make sure that it holds. Fortunately this is a theorem in simple cases (*e.g.* $N = 1$), see [CELS],[BGM]. It is also in general true that $\langle\alpha\rangle_+ \ge 0$, as proved by *Ruelle's H-theorem*, [R]. Finally the property $\langle\alpha\rangle_+ > 0$ can be verified numerically quite easily in systems with few degrees of freedom, see for instance [BGG].

Calling μ_- the analogue of μ_+ for the time averages towards the past (clearly μ_- is the time reversal image of μ_+: $\mu_- = I\mu_+$) we can try to visualize (improperly, as it will be seen shortly) the situation by thinking that there is a set A_+ of 0 area such that $\mu_+(A_+) = 1, area(A_+) = 0$ and another set A_- (time reversal image of A_+: $A_- = IA_+$) with $\mu_-(A_-) = 1, area(A_-) = 0$ such that a randomly chosen initial datum x evolves towards the closure of $A_\pm$ as the time tends to $\pm\infty$:

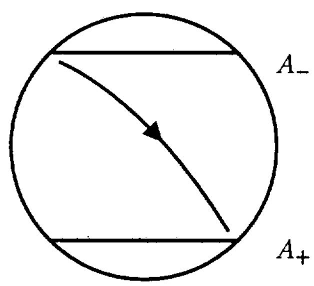

Fig.2

and an initial x randomly chosen (with respect to the uniform area distribution) will evolve towards $\overline{A}_+$ as $t \to +\infty$ and towards $\overline{A}_-$ as $t \to -\infty$, if as usual $\overline{A}$ denotes the closure of the set A.

For very large t the average of the values taken by an observable over the trajectory of x, *i.e.* the average of of $F(S_t x)$, will be the same as that of $F(S_t x_+)$ where x_+ is a (suitably chosen) point on the attracting set $\overline{A}_+$, while as $t \to -\infty$

the history of F will look the same as that of F itself on the trajectory of a suitable point $x_- \in \overline{A}_-$.

For very large positive t the points $S_t x$ and $S_t x_+$ will not be really different, while for very large negative t the points $S_t x$ and $S_t x_-$ will be essentially the same.

But, *no matter how large a prefixed time* t_0 *is taken*, the t-average of he function $F(S_{-t}(S_{t_0} x_+))$ *will still be* $\int F d\mu_+$ (both forward and backward) and that of $F(S_t(S_{t_0} x_-))$ (also both forward and backward) *will still be* $\int F d\mu_-$: this is a mathematically clean way of describing the phenomenon of irreversibility.

We see that time reversal symmetry plays no role except in the fact that it says that the forward attracting set $\overline{A}_+$ and the backward attracting set (*i.e.* the repelling set) $\overline{A}_-$ are linked by the symmetry $I\overline{A}_+ = \overline{A}_-$.

However the picture cannot be so simple: in fact if $E = 0$ the two sets $\overline{A}_+$ and $\overline{A}_-$ *coincide* if one assumes the ergodic hypothesis. Thus we can at most expect that the two sets A_+ and A_- are "very close" when $E \neq 0$ is small.

In fact for driven systems the role of the ergodic hypothesis can be taken by the chaotic hypothesis, [GC], which will imply more: namely that the system is so chaotic that $\overline{A}_+ \equiv \overline{A}_- =$ *whole energy surface* not only for $E = 0$ but *also* for all E's small enough.

For reference the chaotic hypothesis states:

Chaotic hypothesis: A many particle system can be supposed, for the purposes of computing time averages of macroscopic observables, to be a mixing Anosov system.

In spite of its apparent "uselessness", only partly due to the fact that Anosov systems are not really familiar to most of us (se [G4] for instance), it turns out to be, like the similarly useless ergodic hypothesis for equilibrium statistical mechanics, an hypothesis rich of implications.

Anosov systems, also called *hyperbolic*, are systems with highly unstable motions; *the motion in the vicinity of a point* x *seen from an observer that moves with* x *is like the motion near an hyperbolic fixed point.* So unstable that any other system close enough to them shares the same property of showing highly unstable motions. In particular if the system is ergodic and Anosov at $E = 0$ it will remain Anosov at small E and its attracting set will be the whole phase space.

This shows that the above picture was naive and *basically incorrect* close to equilibrium where one cannot really distinguish A_+ from A_- (in a practical sense) because they are dense into each other (being $\overline{A}_+ = \overline{A}_-$)..

Thus reversibility and irreversibility can coexist in a far stronger sense: namely $\overline{A}_+ = \overline{A}_-$ coincide with the whole phase space but $A_+ \neq A_-, \mu_\pm(A_\pm) = 1, \mu_\mp(A_\pm) = 0$. And close to equilibrium the reversible system described by (1.2) behaves irreversibly even though the attracting set is the full phase space.

In [GC] it was pointed out that such property, rather than hindering the theoretical interpretation, does lead to *observable* consequences in the form of relations between observable quantities, and such relations are *exact and* *parameter free*, hence they are general laws. Except for their importance they can be compared to the relations that, in equilibrium statistical mechanics, follow from the ergodic hypothesis and express the exactness of the differential form $(dU + pdV)/T$ (called the *heat theorem* by Boltzmann, [G5]).

In the above example, (1.2), the consequence is a strong restriction on the distribution of the current fluctuations. Let $J(x) = \sum_i \frac{1}{m}\underline{p}_i \cdot \underline{j}$ be the total current in the $\underline{j}$ direction (see Fig.1) and $J_\tau(x)$ be its average over a time interval τ: $J_\tau(x) = \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} J(S_t x)dt$. We observe this quantity as a random variable with the distribution that is assigned to it by the stationary state μ_+ and write it as $J_\tau(x) = p(x)\,J_+$, where J_+ is the stationary average current ($J_+ \equiv \int J\,d\mu_+$) and p is a "dimensionless average current".

Then the distribution of p will be written as $\pi_\tau(p)dp = e^{\zeta(p)\tau}dp$, with $\zeta(p)$ being an entropy function for the fluctuating variable p and the *fluctuation theorem* of [GC] says:

$$\frac{\zeta(p) - \zeta(-p)}{\tau J_+} = 1, \qquad -p^* < p < p^* \tag{1.9}$$

where $p^* \geq 1$ is the maximum observable value of the current (note that by our definitions $\langle p\rangle_+ = \int p\,d\mu_+ \equiv 1$).

In general reversible systems the role of the current is plaid by the phase space contraction rate $\sigma_E(x)$: as noted above the current in the model (1.2) is proportional to $\sigma_E(x)$ (because is is essentially $\sigma_E(x) = \frac{EJ(x)}{k_B\vartheta}$, by (1.6),(1.2)). The quantity $\sigma_E(x)$, however, can be defined *in general* as the divergence of the equations of motion and in driven systems it is expected to have a positive average σ_+: if p and the average contraction rate $\sigma_\tau(x) \equiv \sigma_+ p$ are defined by $\sigma_\tau(x) = \tau^{-1}\int_{-\tau/2}^{\tau/2}\sigma_E(S_t x)dt$ then its stationary distribution verifies (1.9) whatever the meaning of the driving force parameter actually is. This is important for later developments even if one is just interested in the system (1.2).

The above relation holds as long as $\overline{A}_+ = \overline{A}_-$, [GC]. However it has long been known since the observation of the so called *string phase* in similar systems, [EM], (and it is easy to check in various cases, see §6 of [BGG]) that for large E such property cannot be generally true and $\overline{A}_+ \cap \overline{A}_- = \emptyset$ so that the attracting set $\overline{A}_+$ becomes strictly smaller than the phase space (*i.e.* it has an open complement in it).

Time reversibility of the stationary state is *spontaneously broken* in a quite spectacular way; irreversibility becomes obvious (as the above described naive analysis now applies); *but the fluctuation theorem seems to be gone.*

Nevertheless there is some experimental evidence, [BGG], that a relation like (1.9) continues to hold even at large E, with possibly 1 replaced by another constant (still parameterless).[1]

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[1] Time reversal symmetry and density of attractor and repeller imply equal number of positive and negative Lyapunov exponents: therefore a very sensitive test of the density of $A_\pm$ is that the

Therefore a natural idea is that there might still be a kind of time reversal symmetry for the system, a symmetry in which the phase space is exactly the attracting set $\overline{A}_+$: if there was *another* map I^* transforming the invariant set $\overline{A}_+$ into itself and *anticommuting* with time reversal, then a relation "like" the (1.9) could hold. Indeed the derivatiion of such relation is, for the model (1.2), essentially based on:

(1) the assumption of time reversal symmetry and that the system is a mixing Anosov flow on the *whole* phase space.
(2) the proportionality between the phase space contraction σ_E and the electric current, see (1.6) recalling that $K = \frac{1}{2m}\sum_i \underline{p}_i^2$ is constant.

and at least (1) would be verified (see below for what concerns (2)).
Thus we look for a *local time reversal*, *i.e.* for a map I^* such that:

$$I^* S_t = S_{-t} I^*, \qquad I^* \overline{A}_+ = \overline{A}_- \tag{1.10}$$

and the idea is that *under very general settings* one can find a third map $\tilde{I}$ commuting with S_t and transforming $\overline{A}_-$ to $\overline{A}_+$ so that $I^* = \tilde{I} \cdot I$ anticommutes with S_t and changes $\overline{A}_+$ into itself (recall that I exchanges $\overline{A}_+$ and $\overline{A}_-$).

The above reminds us (or at least me) very much of the well known fact that in our Universe time reversal T is not a symmetry but TCP is such: hence I plays the role of T, $\tilde{I}$ that of CP and I^* that of TCP.

In [BG] we discuss a condition (see below) ensuring that every time the time reversal symmetry I is spontaneously broken because the attracting set becomes smaller than phase space, and is therefore mirrored by a repeller set, a new symmetry $I^* = \tilde{I} \cdot I$ anticommuting with the evolution is spawned.
This does not yet make the (1.9) apply immediately because, since only one of the two conditions mentioned before (1.10) is fulfilled, it can be shown to "only" give a relation like (1.9) valid for the fluctuations of the average contraction rate of the area element of $\overline{A}_+$ (see also the comments following (1.9)): we saw that such rate was, *in the small E case*, proportional to the current but there seems to be no *a priori* reason to be so once one compares the current with the area contraction on the attracting set $\overline{A}_+$, when the latter is smaller than the whole phase space.
However in [BG] use is made of *another* recently discovered remarkable property of thermostatted systems like (1.2), the *pairing rule* of the Lyapunov exponents, [ECM1],[DM],[WL] whereby the (non trivial) Lyapunov exponents arranged in decreasing order of size are paired so that the sum of the exponents equidistant[2] from the two central ones is a constant equal to the *average*

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number of positive Lyapunov exponents is less than that of the negative ones, see [BGG]. The parameter values where the density of the attractor no longer holds are values corresponding to which the system shows a "non trivial" *vanishing* Lyapunov exponent: thus they are values in correspondence of which the chaotic hypothesis does not hold. They are however special isolated parameter values, see [BGG].

[2] In systems of particles subject to gaussian constraints there are $2k$ degrees of freedom and at least

friction $\langle\alpha\rangle_+$. And arguments are given leading to a proportionality of the total current to the area contraction rate of $\overline{A}_+$. If $\mathcal{N}$ is the number of pairs of Lyapunov exponents and $\mathcal{M}$ is the number of pairs with *two* negative exponents (which never exist, by time reversal symmetry, when $\overline{A}_+$ is the whole phase space) then the fluctuation theorem (1.10) is modified into:

$$\frac{\zeta(p)-\zeta(-p)}{\tau J_+} = 1 - \frac{\mathcal{M}}{\mathcal{N}}, \qquad -p^* < p < p^* \tag{1.11}$$

a surprising result because one might naively expect a r.h.s. *greater* than 1, se [BGG], [BG].

I conclude by stating in qualitative form the assumptions that insure that time reversal is *an unbreakable symmetry* in the sense that at every spontaneous breakdown it respawns a new ("smaller", *i.e.* defined on an invariant set much smaller than the whole phase space) symmetry which still anticommutes with time evolution.

The assumptions amount at the requirement that the system should verify a geometric property called in [BG] *axiom C*. To introduce the property we note that the chaotic hypothesis has to be interpreted, in the cases in which the attracting set is not the whole phase space, as saying that the attracting set is a smooth surface and the flow restricted to it is a Anosov flow. Such attractors are special cases of the *axiom A attractors* (which essentially are more general only because the attracting set is not supposed to be a smooth surface).

A difficulty with axiom A attractors is that the axiom A property is a *local property* formulated in terms of motions near the attractor. And this is not satisfactory in systems in which what happens near an attractor is related to what happens far from it (because, for instance, there is a time reversal symmetry mapping the attracting set into a repelling one).

This was probably perceived by Smale, [Sm], when he introduced the notion of *axiom B* systems which basically are systems with attracting sets verifying axiom A *and* with some relation between the stable manifolds of the attracting sets and the unstable manifolds of the repeller and of other possibly existing invariant sets.

Smale's axiom B is too weak for our purposes (and it has the disadvantage of *not* being structurally stable, *i.e.* such that a small modification of the equations of motion does not substantially change the nature of the system and keeps it still veryfying axiom B).

The axiom C of [BG] is a strengthening of the axiom B property, which is stable under perturbations. Informally the axiom C property in simple systems with only one attracting set and one repelling set, different from each other, is that

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two trivial Lyapunov exponents: the one associated with the flow direction and the one associated with the constrained variable (the energy in the case (1.1)). Therefore, in absence of other trivial exponents and if the exponents are ordered as $\lambda_1 \ge \lambda_2 \ge \ldots \lambda_{k-1} \ge \lambda_k \ge \ldots \lambda_{2k-2}$, the pairing rule concerns the pairs $(\lambda_{k-1}, \lambda_k) \ldots (\lambda_1, \lambda_{2k-2})$. Under the chaotic hypothesis in equilibrium the sum of each pair is 0 and one element is positive and one negative; in presence of driving force this remains true for small enough force altough the sum of the pairs is now negative, but eventually there may appear pairs consisting of two negative exponents.

the stable manifold of the points on $\overline{A}_+$ reaches the repeller $\overline{A}_-$ approaching it transversally (of course without ever reaching it) and viceversa the unstable manifolds of the repeller reach the attractor transversally, see the schematic picture in Fig.3 below.

In the figure the first picture illustrates a point $x \in \overline{A}_+$ and a local piece of its stable manifold, the second picture a point $x' \in \overline{A}_-$ with a local piece of its unstable manifold; the third picture shows an intersection between the unstable manifold of a point $x \in \overline{A}_+$ and the stable manifold of $\tilde{I}x$, which is a manifold (one–dimensional in the picture but in general of dimension equal to the codimension of $\overline{A}_\pm$): the intersection generates a correspondence between $\overline{A}_+$ and $\overline{A}_-$ which in fact defines $\tilde{I}$. The points "between the two sets" represent most of the phase space points and are *wandering* (or nonrecurrent) points.

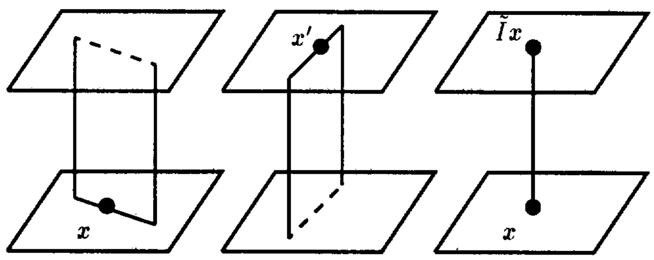

Fig.3

Let δ be the dimension of the surfaces $\overline{A}_\pm$ and u and s be the dimensions of the relative (*i.e.* relative to the dynamics restricted to $\overline{A}_\pm$) stable and unstable manifolds of the points in $\overline{A}_\pm$ so that $u + s = \delta$. In the case of Fig.3 it is $u = s, \delta = 3$.

Then the complete (*i.e.* relative to the dynamics in the full phase space) stable manifolds of the points in $\overline{A}_+$ have dimension $s + m$ for some $m > 0$ (because the stable manifolds of the points of $\overline{A}_+$ "stick out" of $\overline{A}_+$ by the attractivity of $\overline{A}_+$; in Fig.3 it is $m = 1$).

Time reversal symmetry implies that $u = s$ and that the dimension of the unstable manifolds of $\overline{A}_-$ is also $u+m$: hence $u+s+m$ is the dimension of the phase space; and the stable manifolds of points of $\overline{A}_+$ intersect the unstable manifolds of the points in $\overline{A}_-$ locally on manifolds of dimension m. The latter intersect $\overline{A}_+$ and $\overline{A}_-$ in single points and can be regarded as *threads* linking $\overline{A}_+$ and $\overline{A}_-$ and establishing the correspondence $\tilde{I}$ (which is defined only on $\overline{A}_\pm$ and *not* outside them, so that I^* also is defined only on $\overline{A}_\pm$). This shows that the Fig.3 represents correctly, if symbolically, the general case.

The axiom C excludes the possibility that the stable manifold out of $x \in \overline{A}_+$ in the first figure *wraps around* $\overline{A}_-$ instead of "cutting through it", and also it excludes the corresponding possibility for the unstable manifold of $x' \in \overline{A}_-$.

Hence one can view the above axiom as a maximal simplicity assumption. It is not clear whether the axiom C property is really relevant for nonequilibrium statistical mechanics since the spontaneaous breakdown of time reversal symmetry may occurr at extremely large driving fields: at least this seems to be so for (1.2) regarded as a conduction model, [BGG], as well as in the pioneering experiment of [ECM2]), source of all the above theoretical ideas.

But the above philosophy applies also to the theory of fluid motions, [G2], [G3], and leads to properties that might be checkable experimentally (in real or numerical experiments): and in such cases it is certain that the attractors, being finite dimensional, will be much smaller than the total phase space (which is infinite dimensional).

Therefore in [BG] we proposed to modify the Anosov property in the formulation of the chaotic hypothesis into the axiom C property, at least in the cases of reversible systems.

Finally one may object that *in reality* systems are dissipative in the naive sense of the word, like in the example (1.3), and irreversibilty (due to some constant friction coefficients) is always accompanied by an explicit time reversal violation (*i.e.* the equations of motion are not time reversal invariant).

The latter question is examined, giving among other considerations arguments for the effective equivalence between (1.2) and (1.3), in [G2],[G3],[G4]. For irreversible equations that are phenomenological macroscopic descriptions of evolutions microscopically based on reversible equations it is argued that the usual *irreversible* macroscopic equations (like the Navier–Stokes equations) should be equivalent, for practical purposes, to other *time reversible* macroscopic equations: practical purposes should include taking the average of "local observables" in some limiting situation (like the thermodynamic limit in the case of particles systems or the limit of infinite Reynolds number for fluids). Here local means depending only on the coordinates of particles that are in a fixed volume (while the container volume tends to ∞) in the case of particle systems or depending only on finitely many Fourier modes in the case of a fluid (while the Kolmogorov momentum, or equivalently the Reynolds number tend to ∞). And in fact this is used to suggest a natural extension of the notion of *"statistical ensemble"* to nonequilibrium statistical mechanics.

Basically one can think of the phase space of a system described by irreversible equations and with a small attractor as being "half" the phase space of *another* system which is reversible, verifies axiom C and has *the same attractor*. Being reversible it must, however, have (somewhere in its phase space) also a repeller: hence the axiom C will guarantee that there is a time reversal symmetry valid on the attractors of the second system and, therefore, properties that hold for the second system as a consequence of the existence of a time reversal symmetry will also hold for the first. In other words the motion on the attractor of such irreversible systems will still look like a reversible motion.

It is not clear that this is enough for establishing a fluctuation theorem for interesting irreversible systems: doubts have been expressed in [G2],[G3],[G4]. But the arguments against do not seem really compelling and surprises are possible. Independently on the fluctuation theorems the assumption that for instance (1.2) and (1.3) are in some sense equivalent leads, when combined with the chaotic hypothesis, to experimentally testable predictions. And the same is true in the case of the Navier–Stokes equations, see [G2], [G3]. Work on these questions is in slow progress.

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