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Institut für Hochfrequenztechnik und Radarsysteme

# Polarimetric SAR Interferometry

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## Polarimetric Interferometry

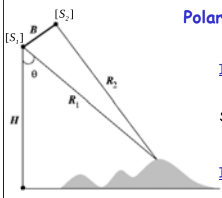


Image 1: Scattering Matrix: $[S_1]=\begin{bmatrix} S^1_{HH} & S^1_{HV} \\ S^1_{VH} & S^1_{VV} \end{bmatrix}$

Scattering Vector: $\vec{k}_1=\frac{1}{\sqrt{2}}[S^1_{HH}+S^1_{VV} \quad S^1_{HH}-S^1_{VV} \quad 2S^1_{HV}]^T$

Image 2: Scattering Matrix: $[S_2]=\begin{bmatrix} S^2_{HH} & S^2_{HV} \\ S^2_{VH} & S^2_{VV} \end{bmatrix}$

Scattering Vector: $\vec{k}_2=\frac{1}{\sqrt{2}}[S^2_{HH}+S^2_{VV} \quad S^2_{HH}-S^2_{VV} \quad 2S^2_{HV}]^T$

Image formation: $i_1=\vec{w}_1^+\cdot\vec{k}_1$ and $i_2=\vec{w}_2^+\cdot\vec{k}_2$ where $\vec{w}_i$ are complex unitary vectors

Interferogram formation: $i_1\ i_2^*=(\vec{w}_1^+\cdot\vec{k}_1)(\vec{w}_2^+\cdot\vec{k}_2)^+=\vec{w}_1(\vec{k}_1\cdot\vec{k}_2^+)\vec{w}_2^+$

Example:
$\vec{w}_1=[1/\sqrt{2} \quad 1/\sqrt{2} \quad 0]^T \rightarrow i_1=\vec{w}_1\cdot\vec{k}_1=S_{1HH}$
$\vec{w}_2=[1/\sqrt{2} \quad 1/\sqrt{2} \quad 0]^T \rightarrow i_2=\vec{w}_2\cdot\vec{k}_2=S_{2HH}$



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## Polarimetric Interferometry

Image 1: $[S_1]=\begin{bmatrix} S^1_{HH} & S^1_{HV} \\ S^1_{VH} & S^1_{VV} \end{bmatrix}\rightarrow\vec{k}_1=\frac{1}{\sqrt{2}}[S^1_{HH}+S^1_{VV} \quad S^1_{HH}-S^1_{VV} \quad 2S^1_{HV}]^T\rightarrow[T_1]:=<\vec{k}_1\cdot\vec{k}_1^+>$

Image 2: $[S_2]=\begin{bmatrix} S^2_{HH} & S^2_{HV} \\ S^2_{VH} & S^2_{VV} \end{bmatrix}\rightarrow\vec{k}_2=\frac{1}{\sqrt{2}}[S^2_{HH}+S^2_{VV} \quad S^2_{HH}-S^2_{VV} \quad 2S^2_{HV}]^T\rightarrow[T_2]:=<\vec{k}_2\cdot\vec{k}_2^+>$

Interferogram: $[T_6]:=\left\langle\begin{bmatrix}\vec{k}_1\\ \vec{k}_2\end{bmatrix}[\vec{k}_1^+ \quad \vec{k}_2^+]\right\rangle=\begin{bmatrix}<\vec{k}_1\cdot\vec{k}_1^+> & <\vec{k}_1\cdot\vec{k}_2^+>\\ <\vec{k}_2\cdot\vec{k}_1^+> & <\vec{k}_2\cdot\vec{k}_2^+>\end{bmatrix}=\begin{bmatrix}[T_1] & [\Omega]\\ [\Omega]^+ & [T_2]\end{bmatrix}$

$[T_1]$ Coherency Matrix for Image 1: 3x3 complex hermitian

$[T_1]$ Coherency Matrix for Image 2: 3x3 complex hermitian

$[\Omega]$ 3x3 complex non-hermitian matrix $\vec{k}_1\neq\vec{k}_2$



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## Polarimetric Interferometry

Image formation: $i_1=\vec{w}_1^+\cdot\vec{k}_1$ and $i_2=\vec{w}_2^+\cdot\vec{k}_2$

Interferogram: $i_1\ i_2^*=(\vec{w}_1^+\cdot\vec{k}_1)(\vec{w}_2^+\cdot\vec{k}_2)^+=\vec{w}_1(\vec{k}_1\cdot\vec{k}_2^+)\vec{w}_2^+=\vec{w}_1[\Omega]\vec{w}_2^+$

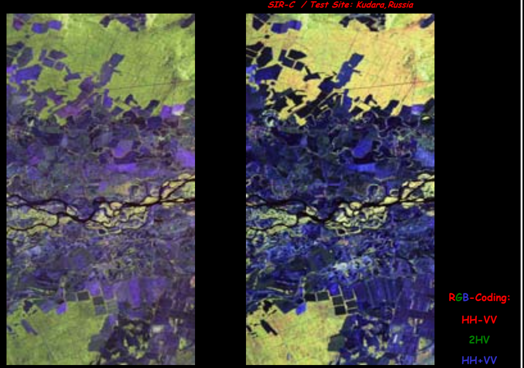
Complex Interferometric Coherence:

$$\tilde{\gamma}(\vec{w}_1,\vec{w}_2)=\frac{<i_1\ i_2^*>}{\sqrt{<i_1\ i_1^*><i_2\ i_2^*>}}=\frac{<\vec{w}_1[\Omega]\vec{w}_2^+>}{\sqrt{<(\vec{w}_1[\Omega]\vec{w}_1^+)><(\vec{w}_2[\Omega]\vec{w}_2^+)>}}$$

- $\arg\{\tilde{\gamma}\}$ ... interferometric phase $\phi(\vec{w}_1,\vec{w}_2)=\arg\{<\vec{w}_1[\Omega]\vec{w}_2^+>\}$
- $|\tilde{\gamma}|=\gamma$ ... normalised complex cross-correlation coefficient
  - If $\vec{w}_1=\vec{w}_2$ then $\gamma=\gamma_{Int}$
  - If $\vec{w}_1\neq\vec{w}_2$ then $\gamma=\gamma_{Int}\gamma_{Pol}$

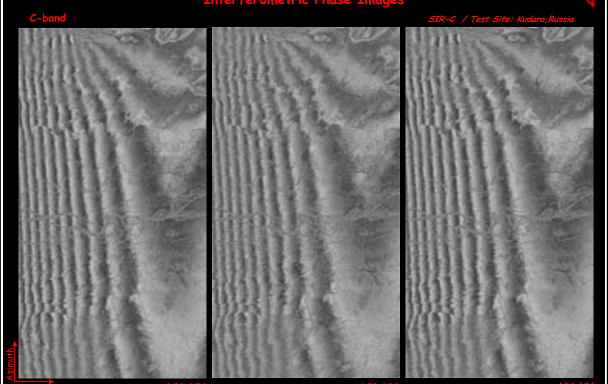


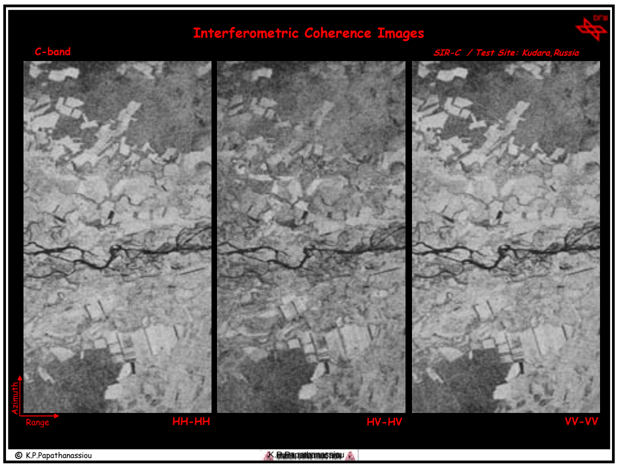
## Pauli Images

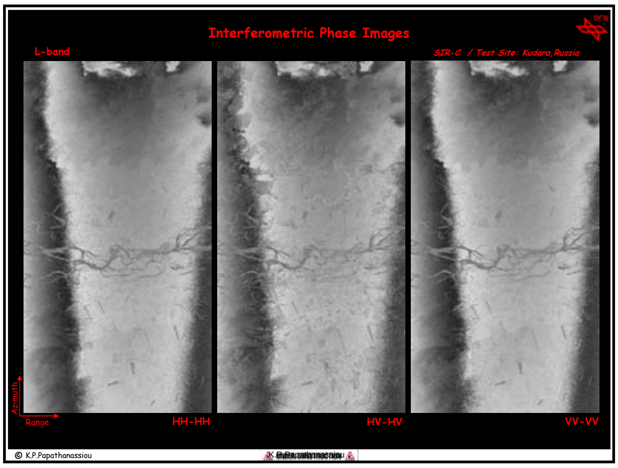


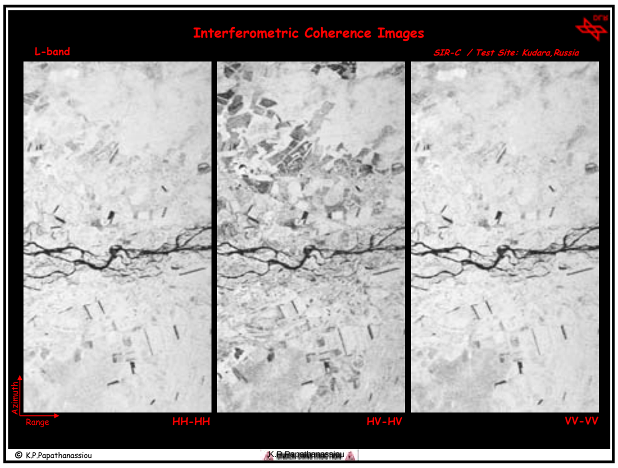



## Interferometric Phase Images



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## Coherence Optimisation

Interferometric Coherence: $\gamma(\vec{w}_1,\vec{w}_2)=\dfrac{<\vec{w}_1[\Omega_{12}]\vec{w}_2^{\,+}>}{\sqrt{<\vec{w}_1[T_{11}]\vec{w}_1^{\,+}><\vec{w}_2[T_{22}]\vec{w}_2^{\,+}>}}$

Question: *Wich polarisation combination leads to the maximum possible interferometric coherence ?*

Optimisation Procedure: $\max_{\vec{w}_1,\vec{w}_2}\{\ L\ L^*\ \}$ $\quad L$ ... Lagrangian function

$L:=\vec{w}_1[\Omega_{12}]\vec{w}_2^{\,+}+\lambda_1(\vec{w}_1[T_{11}]\vec{w}_1^{\,+}-C_1)+\lambda_2(\vec{w}_2[T_{22}]\vec{w}_2^{\,+}-C_2)$ $\quad \lambda_{1,2}$ ... Lagrangian multipliers

$$\frac{\partial L}{\partial \vec{w}_1^+}=[\Omega_{12}]\ \vec{w}_2+\lambda_1[T_{11}]\vec{w}_1=0 \ \rightarrow\ \vec{w}_1^+[\Omega_{12}]\ \vec{w}_2=-\lambda_1\vec{w}_1^+[T_{11}]\vec{w}_1$$
$$\frac{\partial L}{\partial \vec{w}_2^+}=[\Omega_{12}]^+\vec{w}_1+\lambda_2^*[T_{22}]\vec{w}_2=0 \ \rightarrow\ \vec{w}_2^+[\Omega_{12}]^+\vec{w}_1=-\lambda_2^*\vec{w}_2^+[T_{22}]\vec{w}_2$$

$$[T_{22}]^{-1}[\Omega_{12}]^+[T_{11}]^{-1}[\Omega_{12}]\ \vec{w}_2=[A][B]\vec{w}_2=\lambda_1\lambda_2^*\vec{w}_2=\nu\ \vec{w}_2$$
$$[T_{11}]^{-1}[\Omega_{12}]\ [T_{22}]^{-1}[\Omega_{12}]^+\vec{w}_1=[B][A]\vec{w}_1=\lambda_1\lambda_2^*\vec{w}_1=\nu\ \vec{w}_1$$

$[T_{22}]^{-1}[\Omega_{12}]^+[T_{11}]^{-1}[\Omega_{12}]$ is not hermitian but $\nu=\lambda_1\lambda_2^*$ are real

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## Coherence Optimisation

Coherence Optimisation:

$$[T_{22}]^{-1}[\Omega_{12}]^+[T_{11}]^{-1}[\Omega_{12}]\ \vec{w}_2=[A][B]\vec{w}_2=\nu\ \vec{w}_2$$
$$[T_{11}]^{-1}[\Omega_{12}]\ [T_{22}]^{-1}[\Omega_{12}]^+\vec{w}_1=[B][A]\vec{w}_1=\nu\ \vec{w}_1$$

3 real eigenvalues: $\nu_1\geq\nu_2\geq\nu_3\geq 0$

and 3 pairs of eigenvectors: $\{\vec{w}_{11},\vec{w}_{12}\},\{\vec{w}_{21},\vec{w}_{22}\},\{\vec{w}_{31},\vec{w}_{32}\}$

Optimum Coherence Values: $\gamma_i=\sqrt{\nu_i}$ $\qquad$ Optimum Scattering Mechanisms: $\{\vec{w}_{i1},\vec{w}_{i2}\}$

Image Formation: $i_1=\vec{w}_1\cdot\vec{k}_1$ and $i_2=\vec{w}_2\cdot\vec{k}_2$

Eigen Phase Normalisation: $\phi_e=\arg\{\ \vec{w}_{1i}\cdot\vec{w}_{2i}\ \}=0$

$i_1=i_1\exp(-i\phi_e/2)$ and $i_2=i_2\exp(+i\phi_e/2)$

Interferogram Formation: $i_{1i}\cdot i_{2i}^*=(\vec{w}_{1i}\cdot\vec{k}_1)(\vec{w}_{2i}\cdot\vec{k}_2)^+=\vec{w}_{1i}[\Omega]\vec{w}_{2i}^+$

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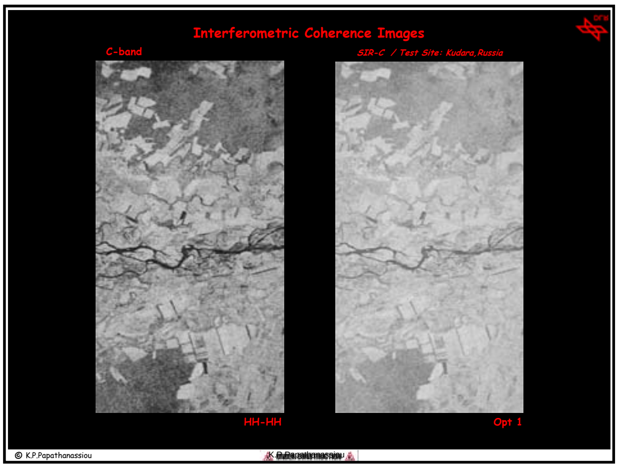

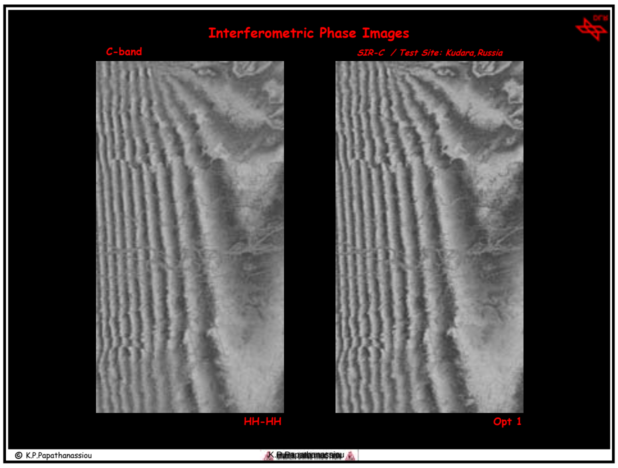


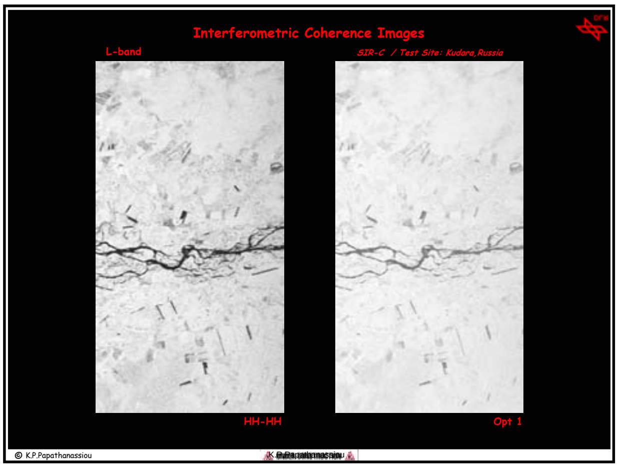


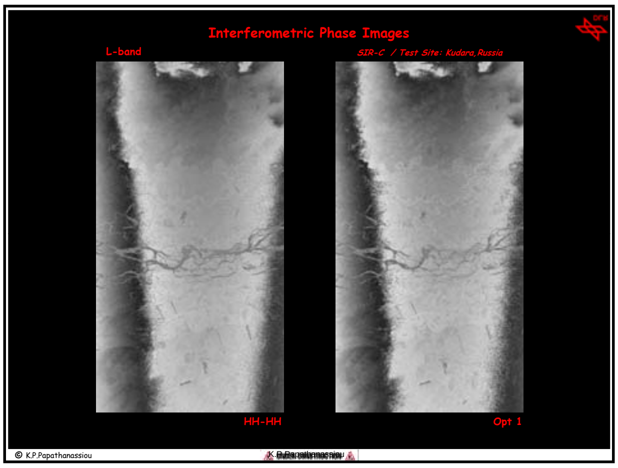


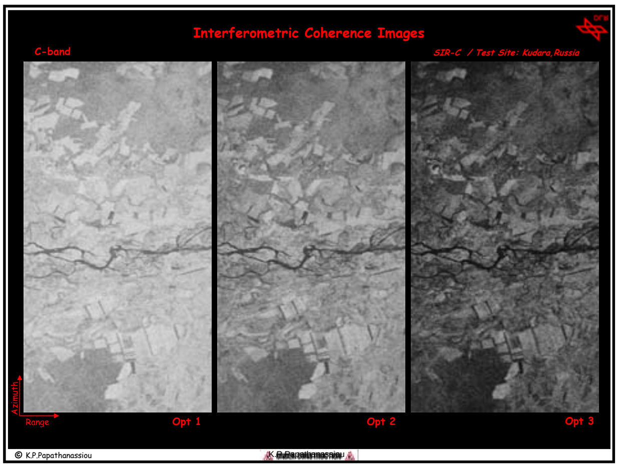


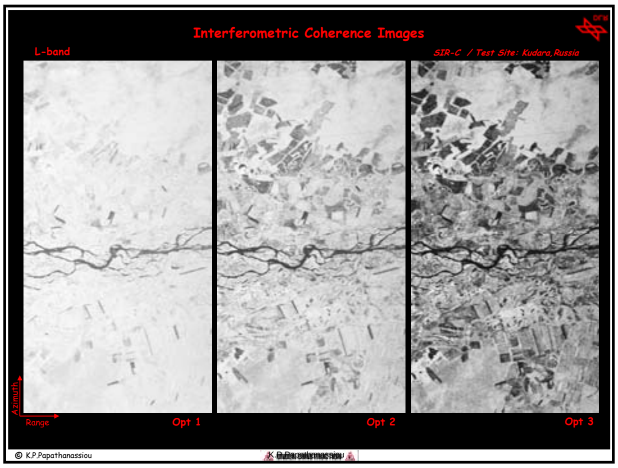


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## Interferometric Decomposition

Coherent Interferometric Decomposition:

$$[T_{22}]^{-1}[\Omega_{12}]^{+}[T_{11}]^{-1}[\Omega_{12}] = \sum \nu_i (\vec{w}'_{i1}\cdot\vec{w}'^{+}_{i1}) = \nu_1(\vec{w}'_{11}\cdot\vec{w}'_{12}) + \nu_2(\vec{w}'_{21}\cdot\vec{w}'_{22}) + \nu_2(\vec{w}'_{31}\cdot\vec{w}'_{32})$$

$$[T_{11}]^{-1}[\Omega_{12}]\,[T_{22}]^{-1}[\Omega_{12}]^{+} = \sum \nu_i (\vec{w}'_{i1}\cdot\vec{w}'^{+}_{i1}) = \nu_1(\vec{w}'_{11}\cdot\vec{w}'_{12}) + \nu_2(\vec{w}'_{21}\cdot\vec{w}'_{22}) + \nu_2(\vec{w}'_{31}\cdot\vec{w}'_{32})$$

Note that $\vec{w}'_{i1} = \sqrt{[T_{11}]}\ \vec{w}_{i1}$ and $\vec{w}'_{i2} = \sqrt{[T_{22}]}\ \vec{w}_{i2}$ are orthonormal

Three Interferograms:

$$i_{11}\cdot i^{*}_{12} = (\vec{w}_{11}\cdot\vec{k}_1)(\vec{w}_{12}\cdot\vec{k}_2)^{+} = \vec{w}_{11}[\Omega]\vec{w}^{+}_{12}$$ 1st Scattering Mechanism

$$i_{21}\cdot i^{*}_{22} = (\vec{w}_{21}\cdot\vec{k}_1)(\vec{w}_{22}\cdot\vec{k}_2)^{+} = \vec{w}_{21}[\Omega]\vec{w}^{+}_{22}$$ 2nd Scattering Mechanism

$$i_{31}\cdot i^{*}_{32} = (\vec{w}_{31}\cdot\vec{k}_1)(\vec{w}_{32}\cdot\vec{k}_2)^{+} = \vec{w}_{31}[\Omega]\vec{w}^{+}_{32}$$ 3rd Scattering Mechanism

The three interferograms are optimised with respect to the interferometric coherence !!!

Phase Difference between scattering mechanisms:

$$\Delta\phi_{ij} = \arg\{\ (\vec{w}_{i1}[\Omega]\vec{w}^{+}_{i2})(\vec{w}_{j1}[\Omega]\vec{w}^{+}_{j2})^{*}\ \}$$

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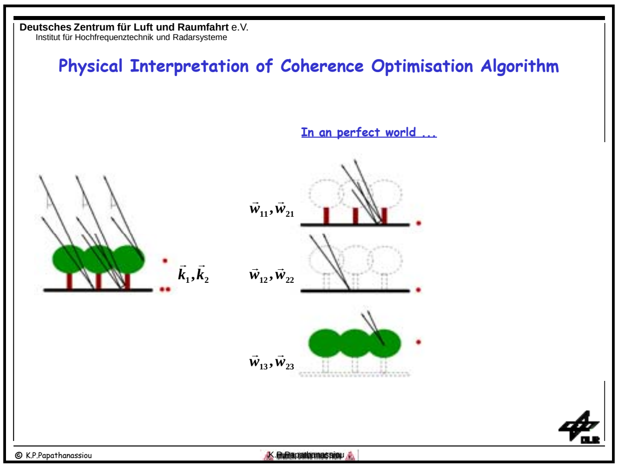
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Physical Interpretation of Coherence Optimisation Algorithm
In an perfect world ...
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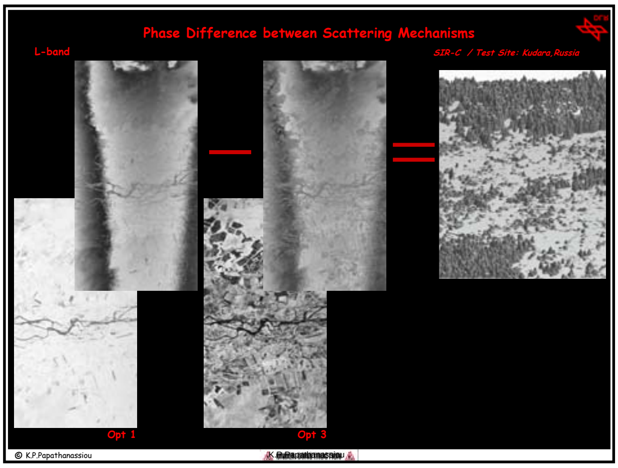
Phase Difference between Scattering Mechanisms
L-band
SIR-C / Test Site: Kudara,Russia
Opt 1
Opt 3
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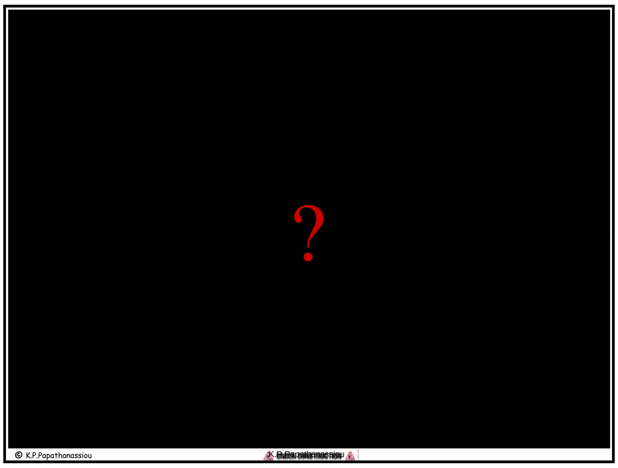
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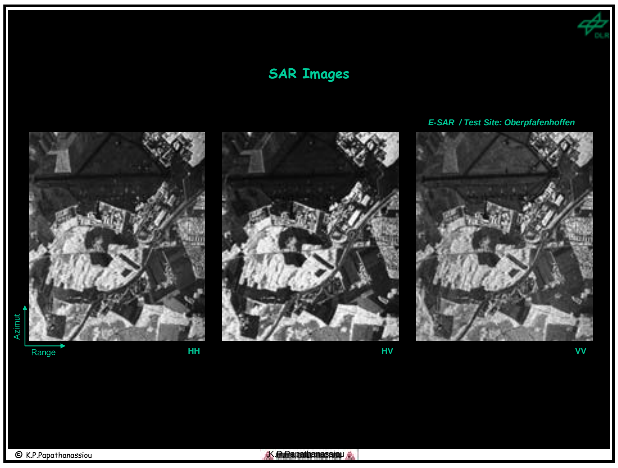
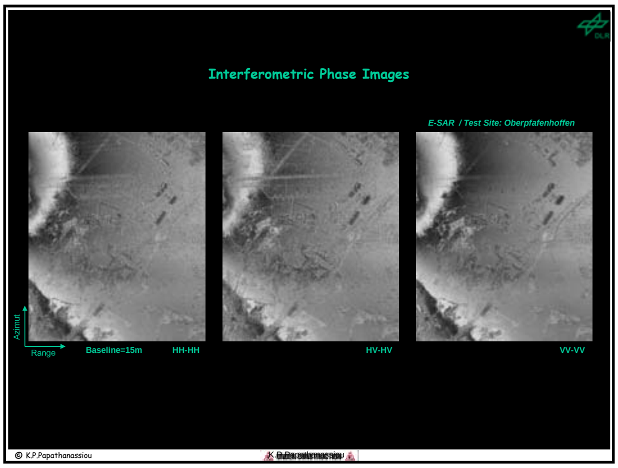
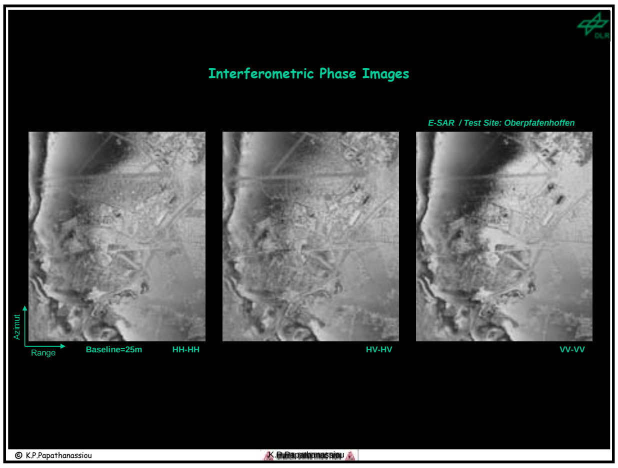
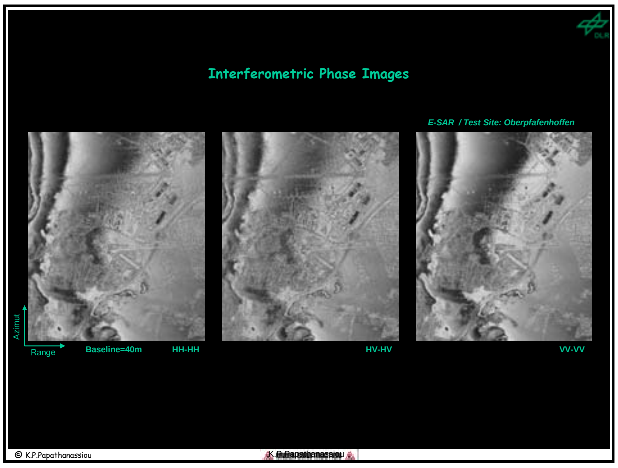

SAR Images
E-SAR / Test Site: Oberpfafenhoffen
Azimut
Range
HH
HV
VV
© K.P.Papathanassiou


Interferometric Phase Images
E-SAR / Test Site: Oberpfafenhoffen
Azimut
Range
Baseline=15m
HH-HH
HV-HV
VV-VV
© K.P.Papathanassiou

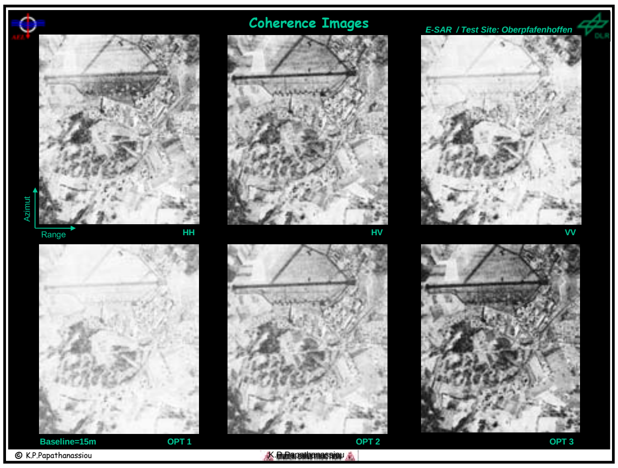

Interferometric Coherence Images
E-SAR / Test Site: Oberpfafenhoffen
Azimut
Range
Baseline=15m
HH-HH
HV-HV
VV-VV
© K.P.Papathanassiou


Interferometric Phase Images
E-SAR / Test Site: Oberpfafenhoffen
Azimut
Range
Baseline=25m
HH-HH
HV-HV
VV-VV
© K.P.Papathanassiou


Interferometric Coherence Images
E-SAR / Test Site: Oberpfafenhoffen
Azimut
Range
Baseline=25m
HH-HH
HV-HV
VV-VV
© K.P.Papathanassiou


Interferometric Phase Images
E-SAR / Test Site: Oberpfafenhoffen
Azimut
Range
Baseline=40m
HH-HH
HV-HV
VV-VV
© K.P.Papathanassiou


Interferometric Coherence Images
E-SAR / Test Site: Oberpfafenhoffen
Azimut
Range
Baseline=40m
HH-HH
HV-HV
VV-VV
© K.P.Papathanassiou


Coherence Images
E-SAR / Test Site: Oberpfafenhoffen
Azimut
Range
HH
HV
VV
Baseline=15m
OPT 1
OPT 2
OPT 3
© K.P.Papathanassiou


Coherence Images
E-SAR / Test Site: Oberpfafenhoffen
Azimut
Range
HH
HV
VV
Baseline=25m
OPT 1
OPT 2
OPT 3
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Coherence Images
E-SAR / Test Site: Oberpfafenhoffen
Azimut
Range
HH
HV
VV
Baseline=40m
OPT 1
OPT 2
OPT 3
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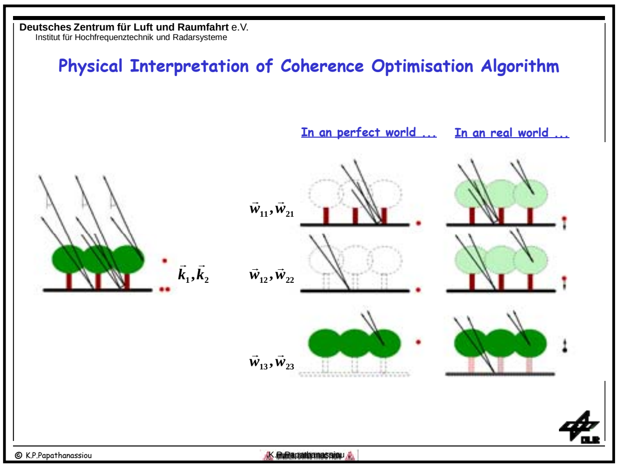
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Institut für Hochfrequenztechnik und Radarsysteme
Physical Interpretation of Coherence Optimisation Algorithm
In an perfect world ...
In an real world ...
$\vec{k}_1, \vec{k}_2$
$\vec{w}_{11}, \vec{w}_{21}$
$\vec{w}_{12}, \vec{w}_{22}$
$\vec{w}_{13}, \vec{w}_{23}$
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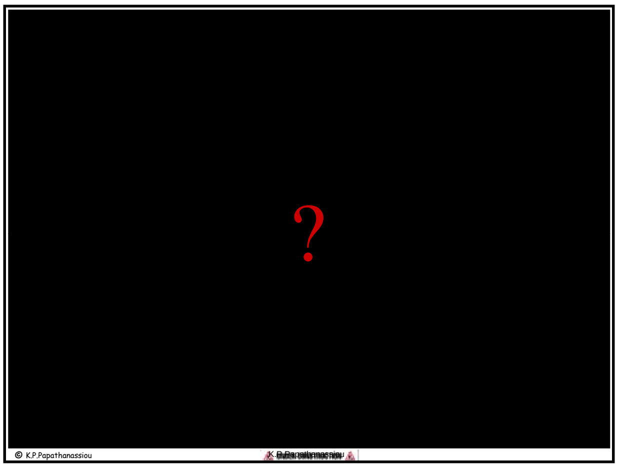
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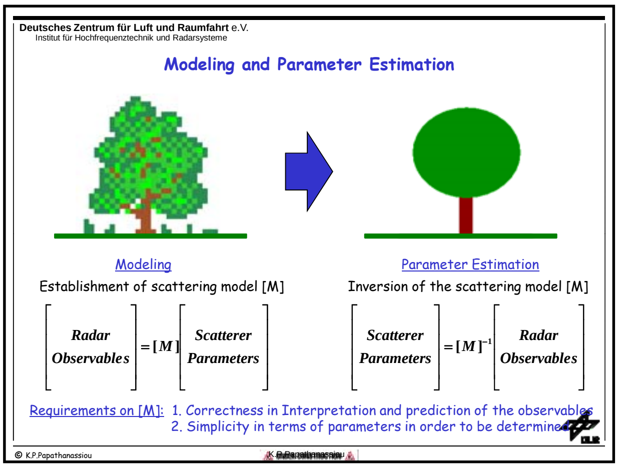
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Institut für Hochfrequenztechnik und Radarsysteme
Modeling and Parameter Estimation
Modeling
Establishment of scattering model [M]
Parameter Estimation
Inversion of the scattering model [M]
$\begin{bmatrix} Radar \\ Observables \end{bmatrix} = [M] \begin{bmatrix} Scatterer \\ Parameters \end{bmatrix}$
$\begin{bmatrix} Scatterer \\ Parameters \end{bmatrix} = [M]^{-1} \begin{bmatrix} Radar \\ Observables \end{bmatrix}$
Requirements on [M]: 1. Correctness in Interpretation and prediction of the observables
2. Simplicity in terms of parameters in order to be determined
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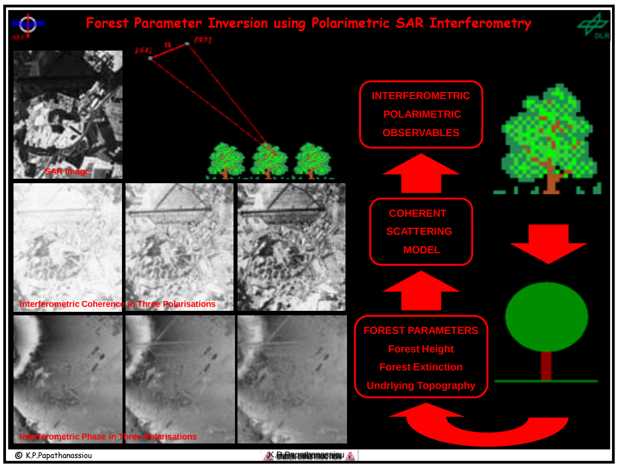
Forest Parameter Inversion using Polarimetric SAR Interferometry
SAR Image
INTERFEROMETRIC
POLARIMETRIC
OBSERVABLES
COHERENT
SCATTERING
MODEL
FOREST PARAMETERS
Forest Height
Forest Extinction
Undrlying Topography
Interferometric Coherence in Three Polarisations
Interferometric Phase in Three Polarisations
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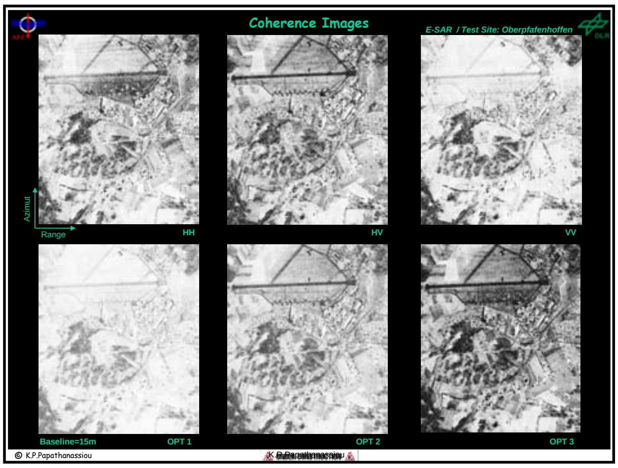
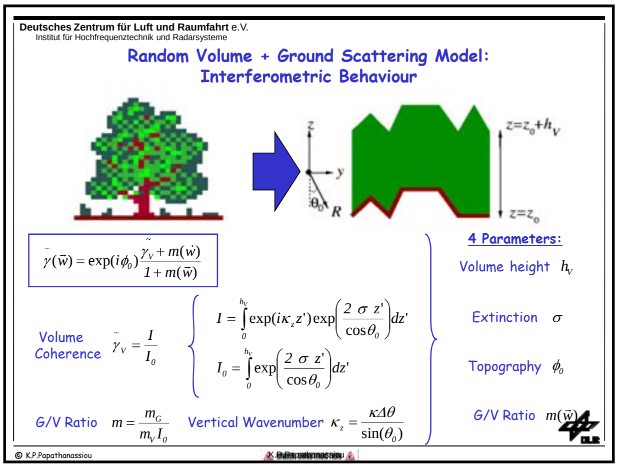

Coherence Images
E-SAR / Test Site: Oberpfafenhoffen
Azimut
Range
HH
HV
VV
Baseline=15m
OPT 1
OPT 2
OPT 3
© K.P.Papathanassiou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Institut für Hochfrequenztechnik und Radarsysteme
Random Volume + Ground Scattering Model: Interferometric Behaviour
$z=z_0+h_V$
$z=z_0$
$\tilde{\gamma}(\vec{w}) = \exp(i\phi_0)\frac{\tilde{\gamma}_V + m(\vec{w})}{1+m(\vec{w})}$
4 Parameters:
Volume height $h_V$
Extinction $\sigma$
Topography $\phi_0$
G/V Ratio $m(\vec{w})$
Volume Coherence $\tilde{\gamma}_V = \frac{I}{I_0}$
$I = \int_0^{h_V} \exp(i\kappa_z z')\exp\left(\frac{2\sigma z'}{\cos\theta_0}\right)dz'$
$I_0 = \int_0^{h_V} \exp\left(\frac{2\sigma z'}{\cos\theta_0}\right)dz'$
G/V Ratio $m = \frac{m_G}{m_V I_0}$
Vertical Wavenumber $\kappa_z = \frac{\kappa\Delta\theta}{\sin(\theta_0)}$
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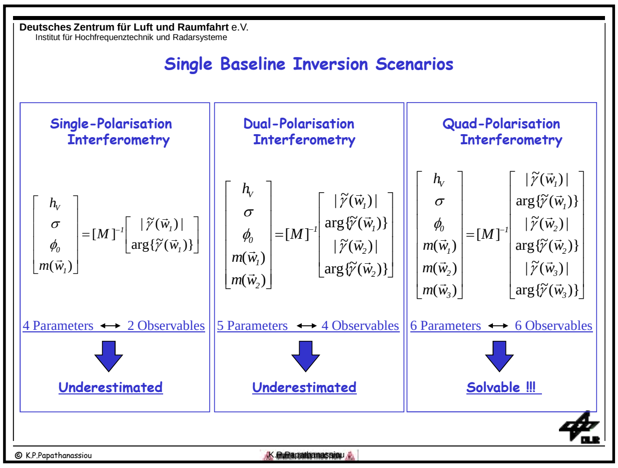
Deutsches Zentrum für Luft und Raumfahrt e.V.
Institut für Hochfrequenztechnik und Radarsysteme
Single Baseline Inversion Scenarios
Single-Polarisation Interferometry
4 Parameters ↔ 2 Observables
Underestimated
Dual-Polarisation Interferometry
5 Parameters ↔ 4 Observables
Underestimated
Quad-Polarisation Interferometry
6 Parameters ↔ 6 Observables
Solvable !!!
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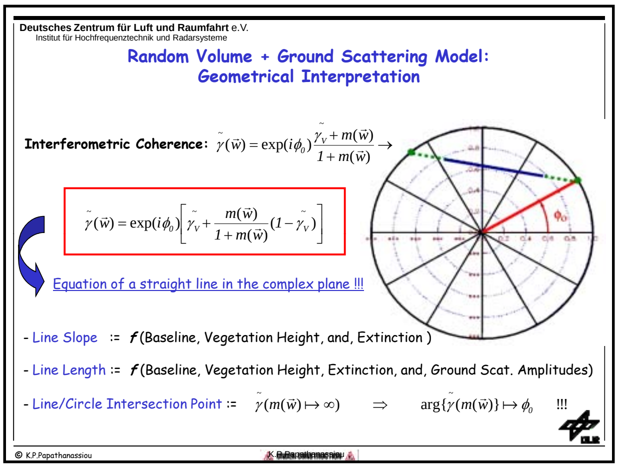
Deutsches Zentrum für Luft und Raumfahrt e.V.
Institut für Hochfrequenztechnik und Radarsysteme
Random Volume + Ground Scattering Model: Geometrical Interpretation
Interferometric Coherence: $\tilde{\gamma}(\vec{w}) = \exp(i\phi_0)\frac{\tilde{\gamma}_V + m(\vec{w})}{1+m(\vec{w})}$ →
$\tilde{\gamma}(\vec{w}) = \exp(i\phi_0)\left[\tilde{\gamma}_V + \frac{m(\vec{w})}{1+m(\vec{w})}(1-\tilde{\gamma}_V)\right]$
Equation of a straight line in the complex plane !!!
- Line Slope := f (Baseline, Vegetation Height, and, Extinction )
- Line Length := f (Baseline, Vegetation Height, Extinction, and, Ground Scat. Amplitudes)
- Line/Circle Intersection Point := $\tilde{\gamma}(m(\vec{w}) \mapsto \infty)$ ⇒ $\arg\{\tilde{\gamma}(m(\vec{w}))\} \mapsto \phi_0$ !!!
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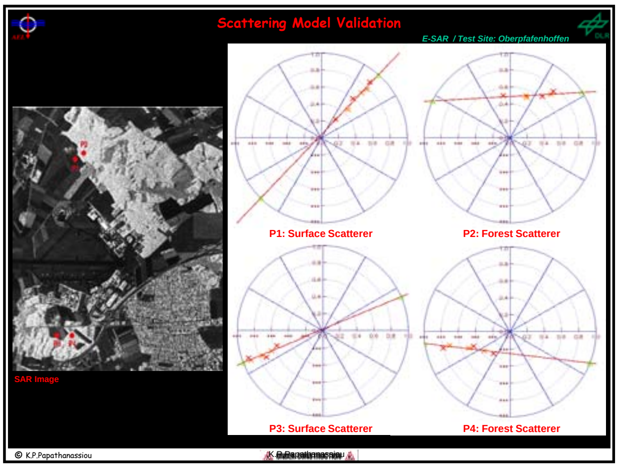
Scattering Model Validation
E-SAR / Test Site: Oberpfaffenhoffen
SAR Image
P1: Surface Scatterer
P2: Forest Scatterer
P3: Surface Scatterer
P4: Forest Scatterer
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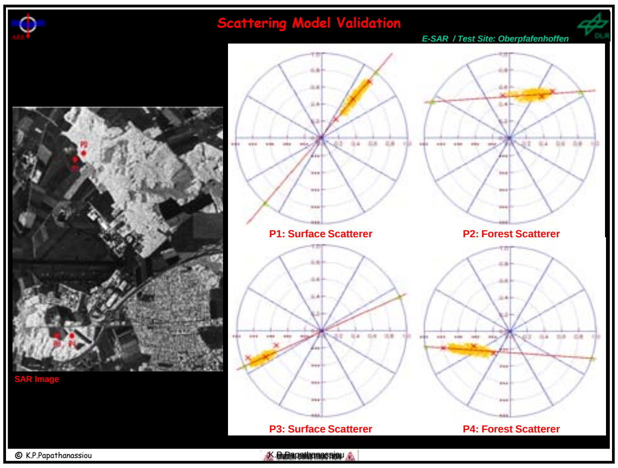
Scattering Model Validation
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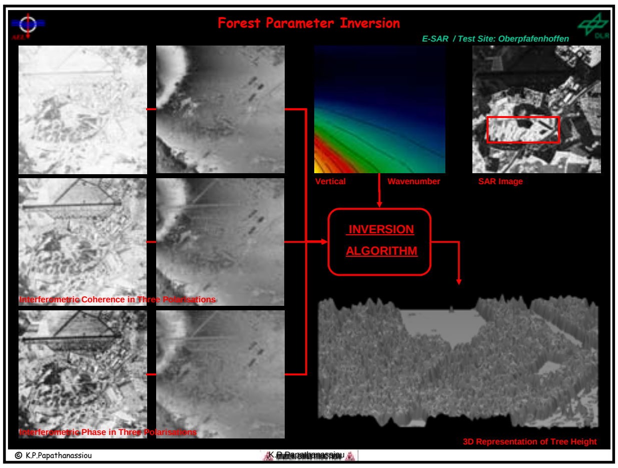
Forest Parameter Inversion
E-SAR / Test Site: Oberpfaffenhoffen
Vertical Wavenumber
SAR Image
INVERSION ALGORITHM
3D Representation of Tree Height
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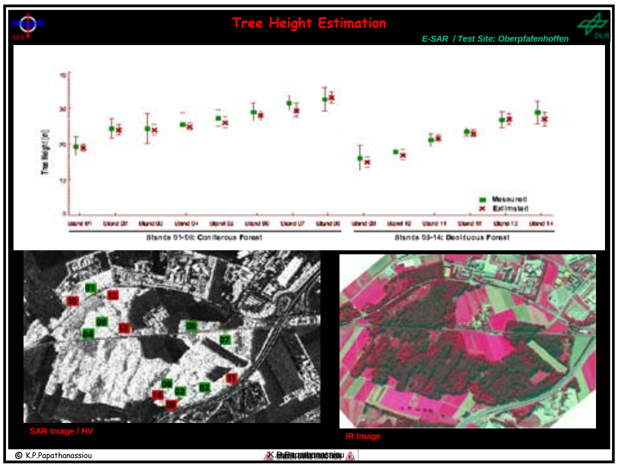


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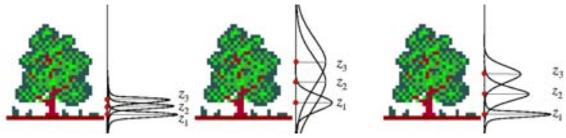
## Conditioning of the Inversion Problem

… or which polarisations lead to the best parameter estimates

| Senario 1 | Senario 2 | Senario 3 |
|---|---|---|
| High Coherences<br>Close phase centers | Low Coherences<br>Separated phase centers | Optimised Coherences<br>Separated phase centers |




| Insufficient Separability | Insufficient Separability | Sufficient Separability |
|---|---|---|
| Ill-Conditioned | Ill-conditioned | Well-conditioned |

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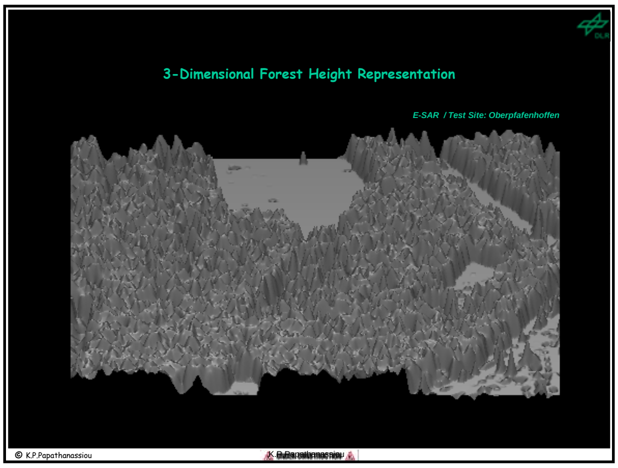


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## Forest Height and Biomass Estimation

**Conventional Biomass Estimators from SAR Data:**

| | Biomass Saturation Limit [T/ha] | [Kg/(m^2)] | % of Earths Vegetated Area | % of Total Biomass Stock |
|---|---|---|---|---|
| C-band | 20 | 2 | 25% | 4% |
| L-band | 40 | 4 | 35% | 8% |
| P-band | 100 | 10 | 60% | 20% |
| Tree Height | 250 | 25 | 75% | 80% |

M. L. Imhoff, "Radar Backscatter and Biomass Saturation: Ramifications for Global Biomass Inventoty ",*IEEE TGARS*, Vol. 33, No. 2, March 1995

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## Alternative Inversion Scenarios

**Multi-Baseline Single-Polarisation Interferometry:**

Baseline 1: $\tilde{\gamma}(\kappa_{z1}) = \exp(i\phi_0)\dfrac{\tilde{\gamma}_V(\kappa_{z1})+m}{1+m}$

Baseline 2: $\tilde{\gamma}(\kappa_{z2}) = \exp\left(i\dfrac{\kappa_{z2}}{\kappa_{z1}}\phi_0\right)\dfrac{\tilde{\gamma}_V(\kappa_{z2})+m}{1+m}$

$$\begin{bmatrix} h_V \\ \sigma \\ \phi_0 \\ m \end{bmatrix} = [M]^{-1}\begin{bmatrix} |\tilde{\gamma}(\kappa_{z1})| \\ \arg\{\tilde{\gamma}(\kappa_{z1})\} \\ |\tilde{\gamma}(\kappa_{z2})| \\ \arg\{\tilde{\gamma}(\kappa_{z2})\} \end{bmatrix}$$

4 Parameters ⟷ 4 Observables — Solvable !!!

**Single-Baseline Dual-Frequency Interferometry:**

Frequency 1: $\tilde{\gamma}(f_1) = \exp(i\phi_0)\dfrac{\tilde{\gamma}_V(f_1)+m(f_1)}{1+m(f_1)}$

Frequency 2: $\tilde{\gamma}(f_2) = \exp\left(i\dfrac{f_2}{f_1}\phi_0\right)\dfrac{\tilde{\gamma}_V(f_2)+m(f_2)}{1+m(f_2)}$

$$\begin{bmatrix} h_V \\ \sigma(f_1) \\ \sigma(f_1) \\ \phi_0 \\ m(f_1) \\ m(f_2) \end{bmatrix} = [M]^{-1}\begin{bmatrix} |\tilde{\gamma}(f_1)| \\ \arg\{\tilde{\gamma}(f_1)\} \\ |\tilde{\gamma}(f_2)| \\ \arg\{\tilde{\gamma}(f_2)\} \end{bmatrix}$$

6 Parameters ⟷ 4 Observables — Underestimated

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?

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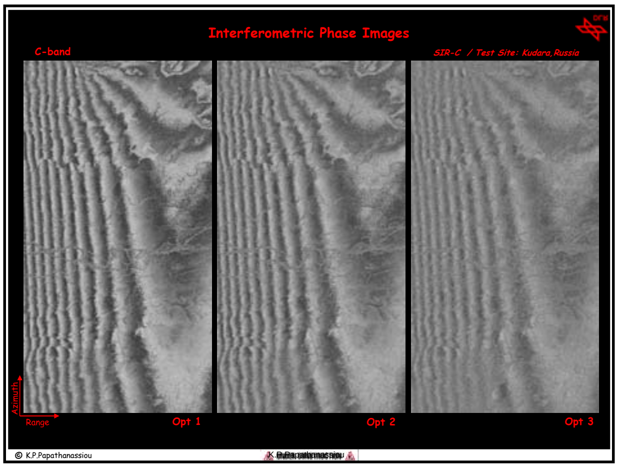


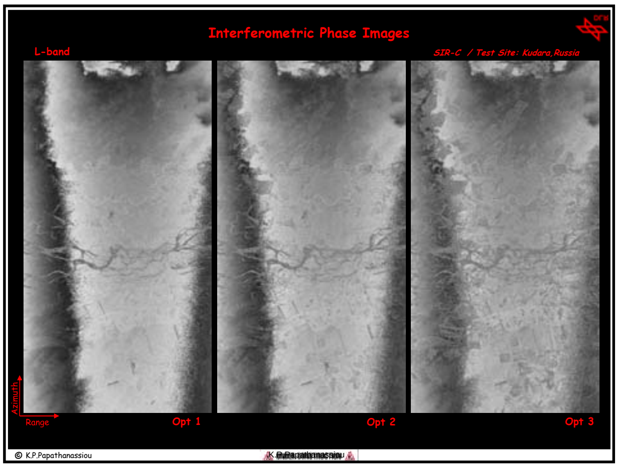


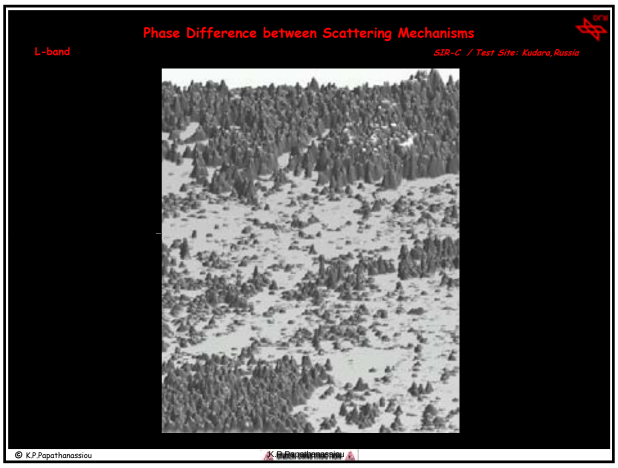


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## From Remote Imaging to Remote Measurement

**Wat has been done:** Estimation of
- Tree Height
- Forest Extinction
- Underlying Topography
- Ground Signature under the Forest

… using a single frequency single baseline sensor

Validation in L-band

Validation in P-band

**Where are we now:** Model extention to account for temporal decorrelation

Model Development and Validation of Biomass Estimation

Multi-Baseline Pol-InSAR: Processing Techniques, Model Development, Inversion Algorithms

**Where we like to go:** Estimation of Surfaces Parameters under Vegetation

Estimation of Forest Structural parameters

Differential Polarimetric Interferometry



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## Coherence Optimisation

Interferometric Coherence:

$$\gamma(\vec{w}_1,\vec{w}_2)=\frac{<\vec{w}_1[\Omega_{12}]\vec{w}_2^{\,+}>}{\sqrt{<\vec{w}_1[T_{11}]\vec{w}_1^{\,+}><\vec{w}_2[T_{22}]\vec{w}_2^{\,+}>}}$$

**Question:** *Which polarisation combination leads to the maximum possible interferometric coherence ?*

Coherence Optimisation:

$$[T_{22}]^{-1}[\Omega_{12}]^{+}[T_{11}]^{-1}[\Omega_{12}]\ \vec{w}_2=[A][B]\vec{w}_2=\nu\ \vec{w}_2$$
$$[T_{11}]^{-1}[\Omega_{12}]\ [T_{22}]^{-1}[\Omega_{12}]^{+}\vec{w}_1=[B][A]\vec{w}_1=\nu\ \vec{w}_1$$

3 real eigenvalues: $\nu_1\geq\nu_2\geq\nu_3\geq 0$

and 3 pairs of eigenvectors: $\{\vec{w}_{11},\vec{w}_{12}\},\{\vec{w}_{21},\vec{w}_{22}\},\{\vec{w}_{31},\vec{w}_{32}\}$

Optimum Coherence Values: $\gamma_i=\sqrt{\nu_i}$ Optimum Scattering Mechanisms: $\{\vec{w}_{i1},\vec{w}_{i2}\}$

Image Formation: $s_1=\vec{w}_1\cdot\vec{k}_1$ and $s_2=\vec{w}_2\cdot\vec{k}_2$

Interferogram Formation: $s_{1i}\cdot s_{2i}^{*}=(\vec{w}_{1i}\cdot\vec{k}_1)(\vec{w}_{2i}\cdot\vec{k}_2)^{+}=\vec{w}_{1i}[\Omega]\vec{w}_{2i}^{+}$

Eigen Phase Normalisation: $\phi_e=\arg\{\ \vec{w}_{1i}\cdot\vec{w}_{2i}\ \}=0$



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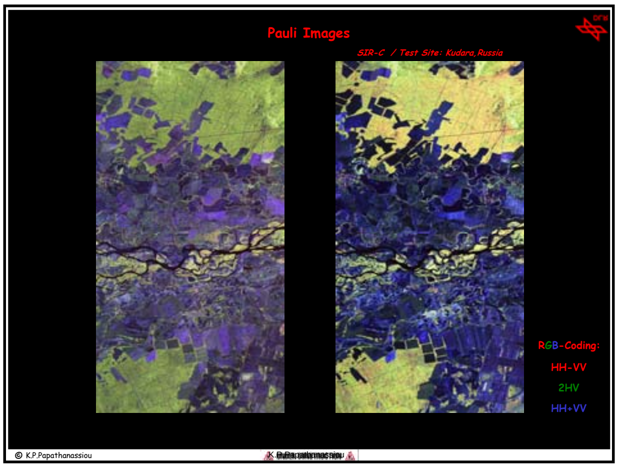
## Random Volume + Ground Scattering Model: Polarimetric Behaviour

$$[T_V]=\begin{bmatrix}t_{V11}&0&0\\0&t_{V22}&0\\0&0&t_{V33}\end{bmatrix}$$

$$[T_G]=\begin{bmatrix}t_{G11}&t_{G12}&0\\t_{G21}&t_{G22}&0\\0&0&t_{G33}\end{bmatrix}$$


Pauli Images
SIR-C / Test Site: Kudara, Russia
RGB-Coding:
HH-VV
2HV
HH+VV
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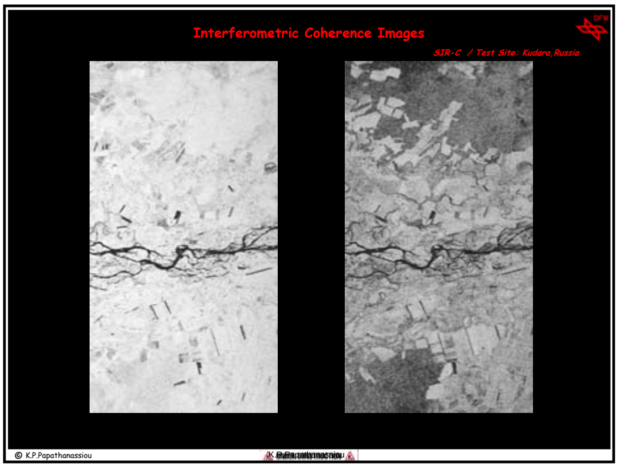
Interferometric Coherence Images
SIR-C / Test Site: Kudara, Russia
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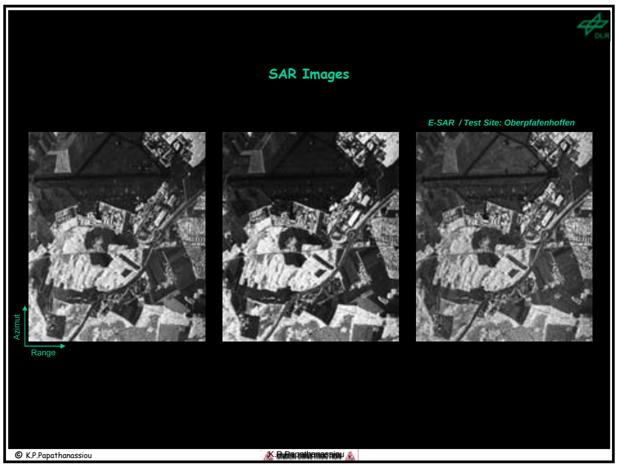
SAR Images
E-SAR / Test Site: Oberpfafenhoffen
Azimut
Range
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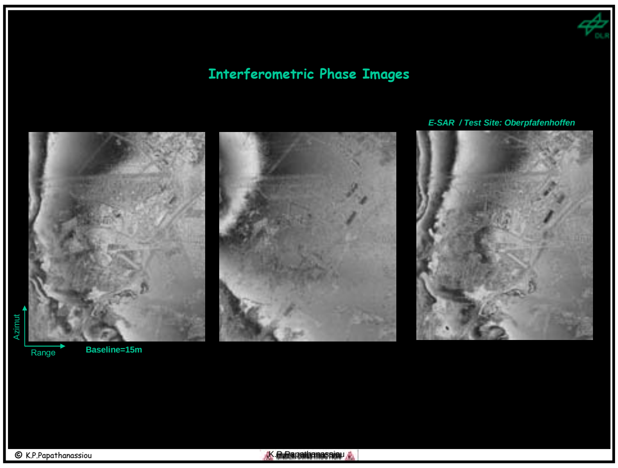
Interferometric Phase Images
E-SAR / Test Site: Oberpfafenhoffen
Azimut
Range
Baseline=15m
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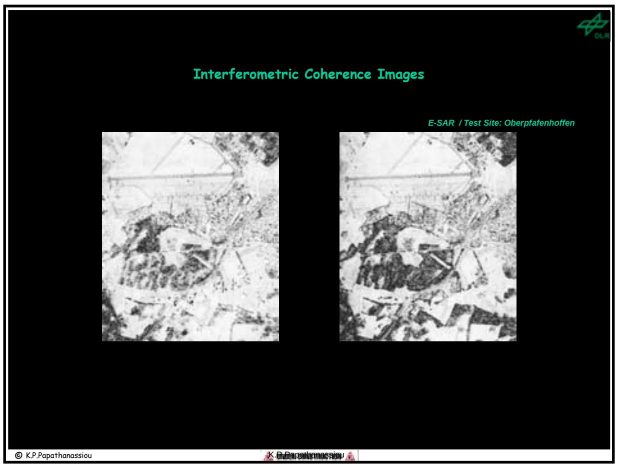
Interferometric Coherence Images
E-SAR / Test Site: Oberpfafenhoffen
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Interferometric Coherence Images
E-SAR / Test Site: Oberpfafenhoffen
Azimut
Range
Baseline=25m
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