Assessing an approximation algorithm for the minimum fill-in problem in practice

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During a Cholesky factorization, a zero matrix element may become nonzero, a phenomenon called fill-in. This can increase the storage requirement and computing time by orders of magnitude. It is well-known that a permutation of the coefficient matrix influences the size of the fill-in. So, one is interested in the minimum fill-in problem, finding a permutation that leads to the least possible fill-in over all permutations. The formulation of the underlying phenomena is based on representing the coefficient matrix as an undirected graph, where the nodes of the graph correspond to the rows and an edge between node i and j is associated with each nonzero matrix element at position (i, j). Performing one step of the Cholesky factorization corresponds to the elimination of a node and making its neighbors a clique, potentially inserting new "fill-edges." Then, the minimum fill-in problem is equivalent to finding an elimination ordering such that the number of fill-edges is minimized over all possible elimination orderings. Yannakakis [1] showed that the minimum fill-in problem is NP-complete leading to extensive further research. However, most of the work is concerned with the design of new heuristics to cope with the NP-completeness. While many heuristic approaches such as minimum degree and nested dissection are effective, they do not give any hint on the quality of the solution, that is, they do not specify the difference of the size of the computed fill-in and the minimum fill-in. In 2000, theoretical computer scientists [2] have come up with an approximation algorithm for the minimum fill-in problem. The goal of an approximation algorithm is to find a near-optimal solution in polynomial time together with a provable bound for the quality of the solution. The algorithm in [2] finds an elimination ordering such that the size of the fill-in is bounded by $8k^2$ where k is the size of the minimum fill-in. The practicability of this algorithm is still unclear. The algorithm has some degree of freedom since it is composed of several subtasks for which one can choose between different algorithms. The goal of the present work is to study the impact of theses components and carefully examine the practicability of the overall approximation algorithm by a set of numerical examples.

References

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Dagstuhl Seminar Proceedings 09061
Combinatorial Scientific Computing
http://drops.dagstuhl.de/opus/volltexte/2009/2119