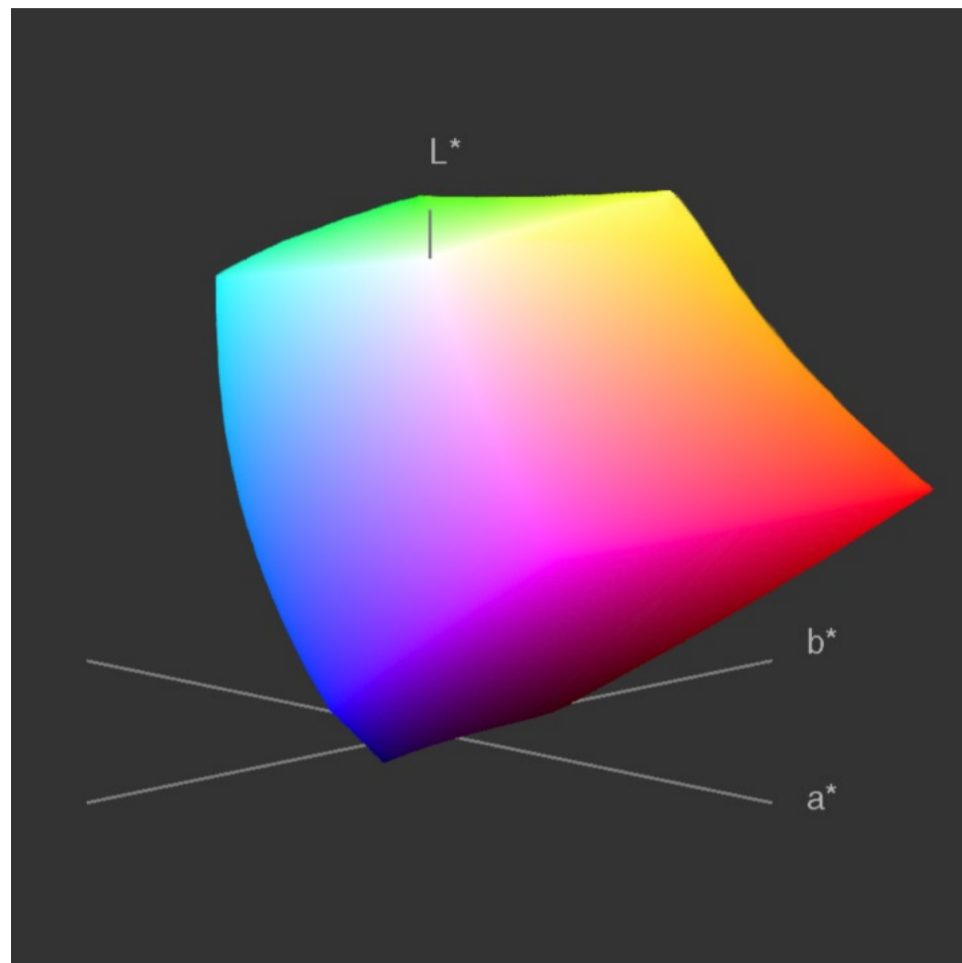


# Gernot Hoffmann

## CIE Lab Color Space



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# 1.1 Introduction

CIE XYZ is an absolute color space (not device dependent). Each visible color has non-negative coordinates X,Y,Z.

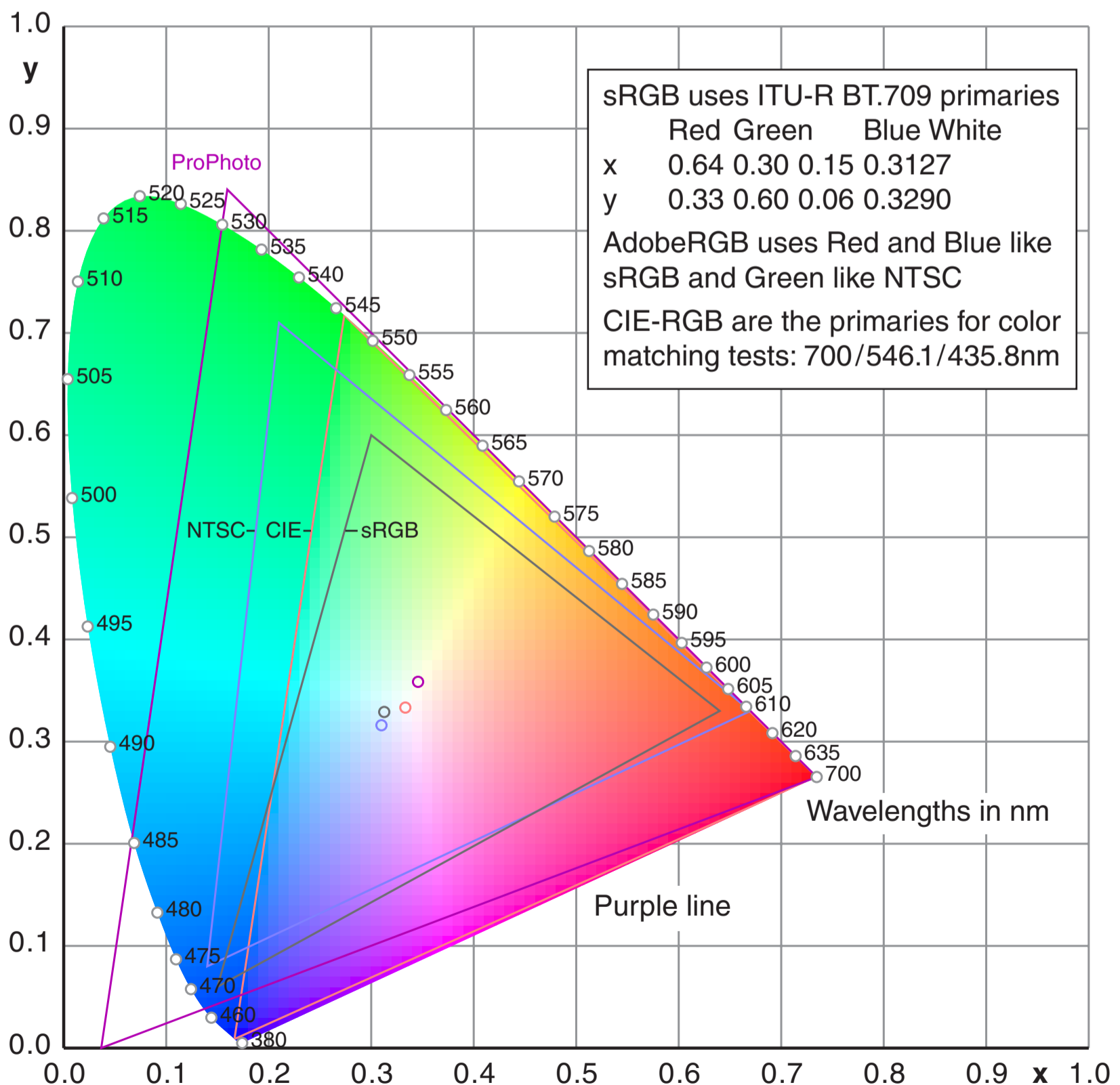
CIE xyY, the horseshoe diagram as shown below, is a perspective projection of XYZ coordinates onto a plane xy. The luminance is missing.

CIELab is a nonlinear transformation of XYZ into coordinates  $L^*, a^*, b^*$ .

The gamut for any RGB color system is a triangle in the CIE xyY chromaticity diagram, here shown for the CIE primaries, the NTSC primaries, the Rec.709 primaries (which are also valid for sRGB and therefore for many PC monitors) and the non-physical working space ProPhotoRGB. The white points are individually defined for the color spaces.

The CIELab color space was intended for equal perceptual differences for equal changes in the coordinates  $L^*, a^*$  and  $b^*$ . Color differences  $\Delta E$  are defined as Euclidian distances in CIELab.

This document shows color charts in CIELab for several RGB color spaces.



## 1.2 Introduction

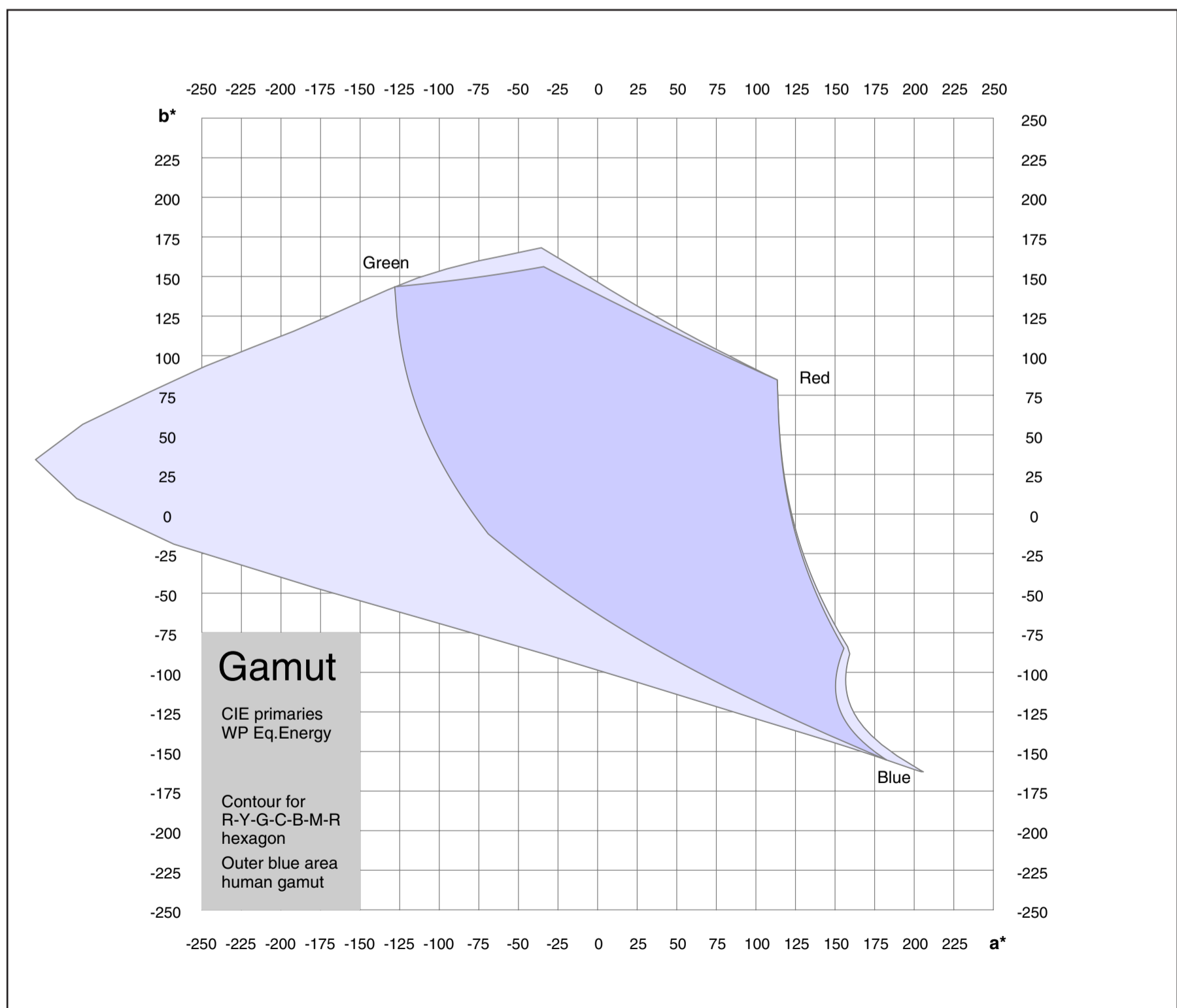
CIELab uses new coordinates  $L^*$ ,  $a^*$  and  $b^*$  by functions as described in the next chapter. The true shape of the human gamut in CIELab is shown in chapter 13. This results in huge areas for higher luminance.

Just to show the orientation together with the gamut triangle for the primaries we proceed as follows:

Convert the contour points  $x, y$  for  $Y=1$  into RGB. Divide R,G,B by the maximum of R,G,B. Convert R,G,B to  $L^*, a^*, b^*$ .

The gamut triangle in  $xyY$  has to be replaced by a representation of a color cube with corners R,G,B and Y(yellow),C,M. The gray axis is at  $a^*=b^*=0$ . Therefore the area is confined by a distorted hexagon. A similar diagram can be found in [1].

Later it will be shown that the actual gamut, which depends on the luminance, is a smaller area inside the hexagon.



## 2.1 Formulas / From RGB' to CIELab

R'G'B' are nonlinearly distorted values for each channel in physical tristimulus systems like CIE, NTSC or working spaces like sRGB or AdobeRGB(98).

RGB are undistorted values which are linearly related to CIE XYZ.

Each RGB system has a white point(w). The transformation to CIELab requires a reference white point (n) which is either (w) or D50.

Issues of adaptation are taken into account by the linearized Bradford transform.

The Bradford matrix maps the XYZ-values for a color in e.g. D65 to another set in D50. The colors should look alike under D65 for the first set and under D50 for the second.

Generic gamma correction,  $G=2.2$ ,  $C=R,G,B$

$$C = C'^G$$

sRGB gamma correction,  $C=R,G,B$

$$C = \begin{cases} C'/12.92 & \text{if } C' \leq 0.03928 \\ ((0.055 + C')/1.055)^{2.4} & \text{else} \end{cases}$$

RGB to XYZ (same white point D65)

$$\mathbf{X} = \mathbf{C}_{xr} \mathbf{R}$$

RGB to XYZ (new white point D50, Bradford correction)

$$\mathbf{X} = \mathbf{B} \mathbf{C}_{xr} \mathbf{R}$$

XYZ to L\*a\*b\* (reference white  $\mathbf{X}_n$ )

$$X_1 = \frac{X}{X_n}$$

$$Y_1 = \frac{Y}{Y_n}$$

$$Z_1 = \frac{Z}{Z_n}$$

$$X_1 = \begin{cases} X_1^{1/3} & \text{if } X_1 > 0.008856 \\ 7.787 X_1 + 16/116 & \text{else} \end{cases}$$

$$Y_1 = \begin{cases} Y_1^{1/3} & \text{if } Y_1 > 0.008856 \\ 7.787 Y_1 + 16/116 & \text{else} \end{cases}$$

$$Z_1 = \begin{cases} Z_1^{1/3} & \text{if } Z_1 > 0.008856 \\ 7.787 Z_1 + 16/116 & \text{else} \end{cases}$$

$$L^* = 116 Y_1 - 16$$

$$a^* = 500 (X_1 - Y_1)$$

$$b^* = 200 (Y_1 - Z_1)$$

## 2.2 Formulas / From CIELab to RGB'

Please refer to explanations on the previous page.

A value RGB is out of gamut if any of the numbers is less than 0 or greater than 1 (normalized values).

L\*a\*b\* to XYZ

$$Y_1 = (L^* + 16) / 116$$

$$X_1 = a^* / 500 + Y_1$$

$$Z_1 = -b^* / 200 + Y_1$$

$$X_1 = \begin{cases} X_1^3 & \text{if } X_1 > 0.206893 \\ (X_1 - 16/116) / 7.787 & \text{else} \end{cases}$$

$$Y_1 = \begin{cases} Y_1^3 & \text{if } Y_1 > 0.206893 \\ (Y_1 - 16/116) / 7.787 & \text{else} \end{cases}$$

$$Z_1 = \begin{cases} Z_1^3 & \text{if } Z_1 > 0.206893 \\ (Z_1 - 16/116) / 7.787 & \text{else} \end{cases}$$

$$X = X_n X_1$$

$$Y = Y_n Y_1$$

$$Z = Z_n Z_1$$

XYZ to RGB (same white point D50)

$$\mathbf{R} = \mathbf{C}_{rx} \mathbf{X} = \mathbf{C}_{xr}^{-1} \mathbf{X}$$

XYZ to RGB (new white point D65, Bradford correction)

$$\mathbf{R} = (\mathbf{B} \mathbf{C}_{xr})^{-1} \mathbf{X} = \mathbf{C}_{rx} \mathbf{B}^{-1} \mathbf{X}$$

Generic gamma correction,  $G = 2.2$ ,  $C = R, G, B$

$$C' = C^{1/G}$$

sRGB gamma correction,  $C = R, G, B$

$$C' = \begin{cases} 12.92C & \text{if } C \leq 0.00304 \\ 1.055C^{1/2.4} - 0.055 & \text{else} \end{cases}$$

## 2.3 Formulas / From xyY to XYZ

The CIE chromaticity diagram is called xyY. The threedimensional space is called XYZ.

xyY to XYZ

$$z = 1 - x - y$$

$$X = Yx/y$$

$$Z = Yz/y$$

XYZ to xyY

$$D = X + Y + Z$$

$$x = X/D$$

$$y = Y/D$$

$$z = Z/D$$

Some important numbers

$$\sqrt[3]{0.008856} = 0.2068930$$

$$16/116 = 0.137931$$

$$116 \cdot 0.008856^{1/3} - 16 = 903.3 \cdot 0.008856 = 8.0$$

## 2.4 Formulas / Structure of the Conversion

The conversion from XYZ to  $L^*a^*b^*$  consists essentially of four steps:

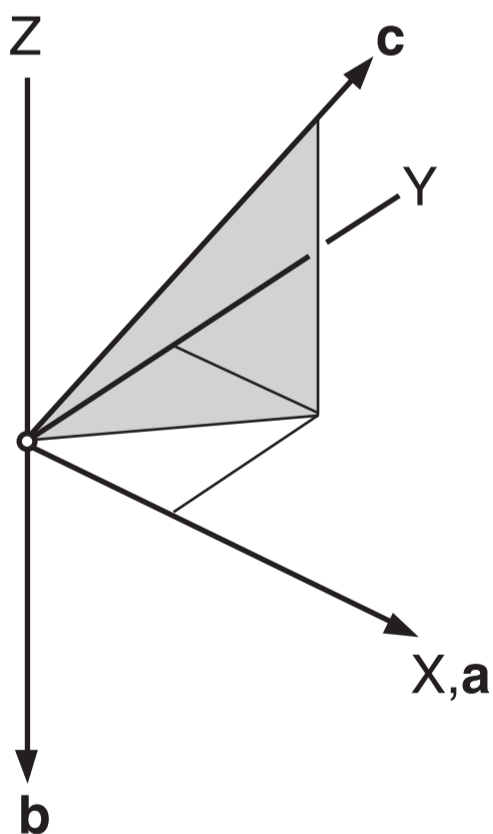
1. White point correction
2. Nonlinear distortion of the variables X,Y,Z by a cubic root
3. Linear transformation into a new vector basis **c**, **a**, **b**
4. Scaling

With some obvious simplifications the linear part can be written by matrices:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} L \\ A \\ B \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = L \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + B \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = L\mathbf{c} + A\mathbf{a} + B\mathbf{b}$$

$$\begin{bmatrix} L \\ A \\ B \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



Base vector **a** points into X-direction, **b** into negative Z-direction and **c** is along the diagonal in XYZ.

L, A and B are normalized substitutes for  $L^*$ ,  $a^*$  and  $b^*$ .

## 2.5 Formulas / Matrix Calculation

Matrices for Primaries and White Point [3]

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{C}_{xr} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \mathbf{C}_{rx} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$D = (x_r - x_b)(y_g - y_b) - (y_r - y_b)(x_g - x_b)$$

$$U = (x_w - x_b)(y_g - y_b) - (y_w - y_b)(x_g - x_b)$$

$$V = (x_r - x_b)(y_w - y_b) - (y_r - y_b)(x_w - x_b)$$

$$u = U/D$$

$$v = V/D$$

$$w = 1 - u - v$$

$$\mathbf{C}_{xr} = \begin{bmatrix} u x_r / y_w & v x_g / y_w & w x_b / y_w \\ u y_r / y_w & v y_g / y_w & w y_b / y_w \\ u z_r / y_w & v z_g / y_w & w z_b / y_w \end{bmatrix}$$

$$\mathbf{C}_{rx} = \mathbf{C}_{xr}^{-1}$$

In the next formulas use  $c_{xrik}$  for  $c_{ik}$

$$D = c_{11}(c_{22}c_{33} - c_{23}c_{32}) - c_{12}(c_{21}c_{33} - c_{23}c_{31}) + c_{13}(c_{21}c_{32} - c_{22}c_{31})$$

$$c_{rx11} = (c_{22}c_{33} - c_{23}c_{32}) / D$$

$$c_{rx12} = (c_{12}c_{33} - c_{13}c_{32}) / D$$

$$c_{rx13} = (c_{12}c_{23} - c_{13}c_{22}) / D$$

$$c_{rx21} = (c_{21}c_{33} - c_{23}c_{31}) / D$$

$$c_{rx22} = (c_{11}c_{33} - c_{13}c_{31}) / D$$

$$c_{rx23} = (c_{11}c_{23} - c_{13}c_{21}) / D$$

$$c_{rx31} = (c_{21}c_{32} - c_{22}c_{31}) / D$$

$$c_{rx32} = (c_{11}c_{32} - c_{12}c_{31}) / D$$

$$c_{rx33} = (c_{11}c_{22} - c_{12}c_{21}) / D$$



## 2.6 Formulas / Bradford Matrix Calculation

Bradford matrix calculation, based on [8]

$$\begin{aligned} Z_n &= 1 - x_n - y_n \\ Y_n &= 1 \\ X_n &= Y_n x_n / y_n \\ Z_n &= Y_n z_n / y_n \end{aligned}$$

$$\begin{aligned} Z_w &= 1 - x_w - y_w \\ Y_w &= 1 \\ X_w &= Y_w x_w / y_w \\ Z_w &= Y_w z_w / y_w \end{aligned}$$

Cone response

$$\mathbf{M}_{cx} = \begin{bmatrix} +0.8951 & +0.2664 & -0.1614 \\ -0.7502 & +1.7135 & +0.0367 \\ +0.0389 & -0.0685 & +1.0296 \end{bmatrix}$$

$$\mathbf{C}_n = \begin{bmatrix} \rho_n \\ \gamma_n \\ \beta_n \end{bmatrix} = \mathbf{M}_{cx} \mathbf{X}_n$$

$$\mathbf{C}_w = \begin{bmatrix} \rho_w \\ \gamma_w \\ \beta_w \end{bmatrix} = \mathbf{M}_{cx} \mathbf{X}_w$$

$$\mathbf{D} = \begin{bmatrix} \rho_n / \rho_w & 0 & 0 \\ 0 & \gamma_n / \gamma_w & 0 \\ 0 & 0 & \beta_n / \beta_w \end{bmatrix}$$

Bradford matrix

$$\mathbf{B} = \mathbf{M}_{cx}^{-1} \mathbf{D} \mathbf{M}_{cx}$$

The matrices  $\mathbf{C}_{xr}$  on the next page contain the XYZ coordinates of the RGB primaries. The first column delivers the base vector  $\mathbf{R}$  in XYZ, the second  $\mathbf{G}$  and the third  $\mathbf{B}$ .

For ICC profiles the matrices are multiplied by the Bradford matrix  $\mathbf{B}$ . Then the base vectors mean 'adapted primaries'. The original primaries under D65 (sRGB, AdobeRGB(98)) should look alike the adapted primaries under D50.

This can easily lead to confusions. We use strictly 'not-adapted' primaries and consider the Bradford transformation as an additional feature which *can* be applied. This should not change physical primaries or working space primaries.

# 3. Primaries and Matrices

For sRGB (709 primaries) the exponent gamma is 2.4, but because of a linear slope the effective gamma is 2.2 . The white point for NTSC is Illuminant C, 6774K.

<b>CIE RGB</b>			WP=Eq.Energy			G=2.2			<b>ProPhoto RGB</b>			WP=D50			G=1.8					
	x	y	z	X	Y	Z		x	y	z	X	Y	Z		x	y	z	X	Y	Z
R	0.7347	0.2653	0.0000	0.4899	0.1769	0.0000	R	0.7347	0.2653	0.0000	0.7978	0.2881	0.0000	R	0.7347	0.2653	0.0000	0.7978	0.2881	0.0000
G	0.2738	0.7174	0.0088	0.3100	0.8124	0.0100	G	0.1596	0.8404	0.0000	0.1352	0.7118	0.0000	G	0.1596	0.8404	0.0000	0.1352	0.7118	0.0000
B	0.1666	0.0089	0.8246	0.2001	0.0106	0.9903	B	0.0366	0.0001	0.9633	0.0313	0.0001	0.8251	B	0.0366	0.0001	0.9633	0.0313	0.0001	0.8251
W	0.3333	0.3333	0.3334	1.0000	1.0000	1.0003	W	0.3457	0.3585	0.2958	0.9643	1.0000	0.8251	W	0.3457	0.3585	0.2958	0.9643	1.0000	0.8251
N	0.3457	0.3585	0.2958	0.9643	1.0000	0.8251	N	0.3457	0.3585	0.2958	0.9643	1.0000	0.8251	N	0.3457	0.3585	0.2958	0.9643	1.0000	0.8251
Matrix Crx			Matrix Cxr			Matrix Crx			Matrix Cxr			Matrix Crx			Matrix Cxr					
	2.3650	-0.8967	-0.4681		0.4899	0.3100	0.2001		1.3458	-0.2556	-0.0511		0.7978	0.1352	0.0313		0.7978	0.1352	0.0313	
	-0.5151	1.4264	0.0887		0.1769	0.8124	0.0106		-0.5446	1.5083	0.0205		0.2881	0.7118	0.0001		0.2881	0.7118	0.0001	
	0.0052	-0.0144	1.0089		0.0000	0.0100	0.9903		0.0000	0.0000	1.2120		0.0000	0.0000	0.8251		0.0000	0.0000	0.8251	
Matrix B			Matrix B-inv			Matrix B			Matrix B-inv			Matrix B			Matrix B-inv					
	0.9979	-0.0041	-0.0295		1.0024	0.0036	0.0361		1.0000	-0.0000	0.0000		1.0000	0.0000	-0.0000		1.0000	0.0000	-0.0000	
	-0.0097	1.0183	-0.0086		0.0096	0.9820	0.0106		0.0000	1.0000	-0.0000		-0.0000	1.0000	0.0000		-0.0000	1.0000	0.0000	
	-0.0074	0.0135	0.8188		0.0089	-0.0161	1.2214		-0.0000	0.0000	1.0000		0.0000	0.0000	1.0000		0.0000	0.0000	1.0000	
<b>NTSC RGB</b>			WP=NTSC			G=2.2			<b>WideGamut RGB</b>			WP=D50			G=2.2					
	x	y	z	X	Y	Z		x	y	z	X	Y	Z		x	y	z	X	Y	Z
R	0.6700	0.3300	-0.0000	0.6070	0.2990	-0.0000	R	0.7347	0.2653	0.0000	0.7165	0.2587	0.0000	R	0.7347	0.2653	0.0000	0.7165	0.2587	0.0000
G	0.2100	0.7100	0.0800	0.1734	0.5864	0.0661	G	0.1152	0.8264	0.0584	0.1010	0.7247	0.0512	G	0.1152	0.8264	0.0584	0.1010	0.7247	0.0512
B	0.1400	0.0800	0.7800	0.2006	0.1146	1.1175	B	0.1566	0.0177	0.8257	0.1468	0.0165	0.7739	B	0.1566	0.0177	0.8257	0.1468	0.0165	0.7739
W	0.3100	0.3160	0.3740	0.9810	1.0000	1.1835	W	0.3457	0.3585	0.2958	0.9643	1.0000	0.8251	W	0.3457	0.3585	0.2958	0.9643	1.0000	0.8251
N	0.3457	0.3585	0.2958	0.9643	1.0000	0.8251	N	0.3457	0.3585	0.2958	0.9643	1.0000	0.8251	N	0.3457	0.3585	0.2958	0.9643	1.0000	0.8251
Matrix Crx			Matrix Cxr			Matrix Crx			Matrix Cxr			Matrix Crx			Matrix Cxr					
	1.9097	-0.5324	-0.2882		0.6070	0.1734	0.2006		1.4622	-0.1845	-0.2734		0.7165	0.1010	0.1468		0.7165	0.1010	0.1468	
	-0.9850	1.9998	-0.0283		0.2990	0.5864	0.1146		-0.5228	1.4479	0.0682		0.2587	0.7247	0.0165		0.2587	0.7247	0.0165	
	0.0582	-0.1182	0.8966		-0.0000	0.0661	1.1175		0.0346	-0.0958	1.2877		0.0000	0.0512	0.7739		0.0000	0.0512	0.7739	
Matrix B			Matrix B-inv			Matrix B			Matrix B-inv			Matrix B			Matrix B-inv					
	1.0377	0.0154	-0.0584		0.9648	-0.0164	0.0812		1.0000	-0.0000	0.0000		1.0000	0.0000	-0.0000		1.0000	0.0000	-0.0000	
	0.0170	1.0057	-0.0189		-0.0160	0.9941	0.0259		0.0000	1.0000	-0.0000		-0.0000	1.0000	0.0000		-0.0000	1.0000	0.0000	
	-0.0120	0.0205	0.6898		0.0173	-0.0298	1.4503		-0.0000	0.0000	1.0000		0.0000	0.0000	1.0000		0.0000	0.0000	1.0000	
<b>sRGB 709 primaries</b>			WP=D65			G=2.4			<b>OptiRGB</b>			WP=D65			G=2.2					
	x	y	z	X	Y	Z		x	y	z	X	Y	Z		x	y	z	X	Y	Z
R	0.6400	0.3300	0.0300	0.4124	0.2126	0.0193	R	0.6658	0.3340	0.0002	0.6173	0.3097	0.0002	R	0.6658	0.3340	0.0002	0.6173	0.3097	0.0002
G	0.3000	0.6000	0.1000	0.3576	0.7152	0.1192	G	0.1929	0.7816	0.0255	0.1576	0.6386	0.0208	G	0.1929	0.7816	0.0255	0.1576	0.6386	0.0208
B	0.1500	0.0600	0.7900	0.1805	0.0722	0.9505	B	0.1355	0.0399	0.8246	0.1755	0.0517	1.0680	B	0.1355	0.0399	0.8246	0.1755	0.0517	1.0680
W	0.3127	0.3290	0.3583	0.9505	1.0000	1.0891	W	0.3127	0.3290	0.3583	0.9505	1.0000	1.0891	W	0.3127	0.3290	0.3583	0.9505	1.0000	1.0891
N	0.3457	0.3585	0.2958	0.9643	1.0000	0.8251	N	0.3457	0.3585	0.2958	0.9643	1.0000	0.8251	N	0.3457	0.3585	0.2958	0.9643	1.0000	0.8251
Matrix Crx			Matrix Cxr			Matrix Crx			Matrix Cxr			Matrix Crx			Matrix Cxr					
	3.2410	-1.5374	-0.4986		0.4124	0.3576	0.1805		1.8436	-0.4458	-0.2814		0.6173	0.1576	0.1755		0.6173	0.1576	0.1755	
	-0.9692	1.8760	0.0416		0.2126	0.7152	0.0722		-0.8954	1.7848	0.0608		0.3097	0.6386	0.0517		0.3097	0.6386	0.0517	
	0.0556	-0.2040	1.0570		0.0193	0.1192	0.9505		0.0171	-0.0347	0.9352		0.0002	0.0208	1.0680		0.0002	0.0208	1.0680	
Matrix B			Matrix B-inv			Matrix B			Matrix B-inv			Matrix B			Matrix B-inv					
	1.0479	0.0229	-0.0502		0.9555	-0.0231	0.0633		1.0479	0.0229	-0.0502		0.9555	-0.0231	0.0633		0.9555	-0.0231	0.0633	
	0.0296	0.9904	-0.0171		-0.0284	1.0100	0.0210		0.0296	0.9904	-0.0171		-0.0284	1.0100	0.0210		-0.0284	1.0100	0.0210	
	-0.0092	0.0151	0.7519		0.0123	-0.0205	1.3304		-0.0092	0.0151	0.7519		0.0123	-0.0205	1.3304		0.0123	-0.0205	1.3304	
<b>AdobeRGB(98)</b>			WP=D65			G=2.2														
	x	y	z	X	Y	Z														
R	0.6400	0.3300	0.0300	0.5767	0.2973	0.0270														
G	0.2100	0.7100	0.0800	0.1856	0.6274	0.0707														
B	0.1500	0.0600	0.7900	0.1882	0.0753	0.9913														
W	0.3127	0.3290	0.3583	0.9505	1.0000	1.0891														
N	0.3457	0.3585	0.2958	0.9643	1.0000	0.8251														
Matrix Crx			Matrix Cxr																	
	2.0416	-0.5650	-0.3447		0.5767	0.1856	0.1882													
	-0.9692	1.8760	0.0416		0.2973	0.6274	0.0753													
	0.0134	-0.1184	1.0152		0.0270	0.0707	0.9913													
Matrix B			Matrix B-inv																	
	1.0479	0.0229	-0.0502		0.9555	-0.0231	0.0633													
	0.0296	0.9904	-0.0171		-0.0284	1.0100	0.0210													
	-0.0092	0.0151	0.7519		0.0123	-0.0205	1.3304													

## 4. Gamut Restrictions and Tests

Mostly it is assumed that all colors inside the gamut triangle can be shown by the respective device. Actually the gamut in CIE Lab is not the triangle R-G-B but the hexagon R-Y-G-C-B-M-R. We have to choose an interpolation path which shows also fully saturated yellow, cyan and magenta.

In the next diagrams we can see color patches which are out of gamut though they are inside the hexagon. The gamut in the chromaticity diagram is the projection of all available colors in XYZ onto xyY, ignoring the spatial distribution in XYZ.

The available gamut depends strongly on the luminance Y.

A color is considered as out of gamut if at least one value of R,G,B is larger than one or smaller than zero. The values are clipped for these limits then.

Such a color is usually shown by the device reasonably, though slightly wrong.

The color charts were tested by Photoshop 7.

E.g. the chart sRGB-050 for  $L^*=50$ . Select sRGB as working space and Rendering Intent Relative Colorimetric. Open document in RGB mode. Place page of PDF. Indicate CIE Lab values by info palette.

Photoshop Relative Colorimetric means ICC Media-Relative Colorimetric.

## 5. Inverse Gamma Correction

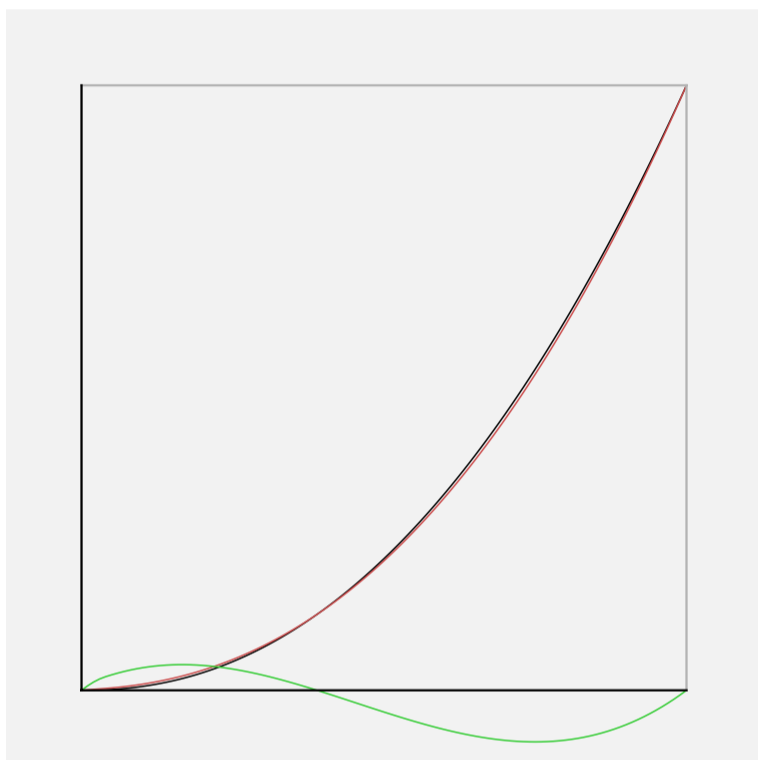
RGB values are transformed into nonlinear values  $R'G'B'$  by  $C' = C^{1/2.2}$  for CIE and NTSC primaries. This is a compensation for calibrated monitors.

sRGB differs slightly from Gamma=2.2 because the transfer function is composed of a linear part and a Gamma=2.4 part.

$$C' = \begin{cases} 1.055 C^{1/2.4} - 0.055 & \text{if } C > 0.00304 \\ 12.92 C & \text{else} \end{cases}$$

The maximal difference is less than 1% full scale.

The diagram shows the Gamma corrections instead of the inverse corrections.



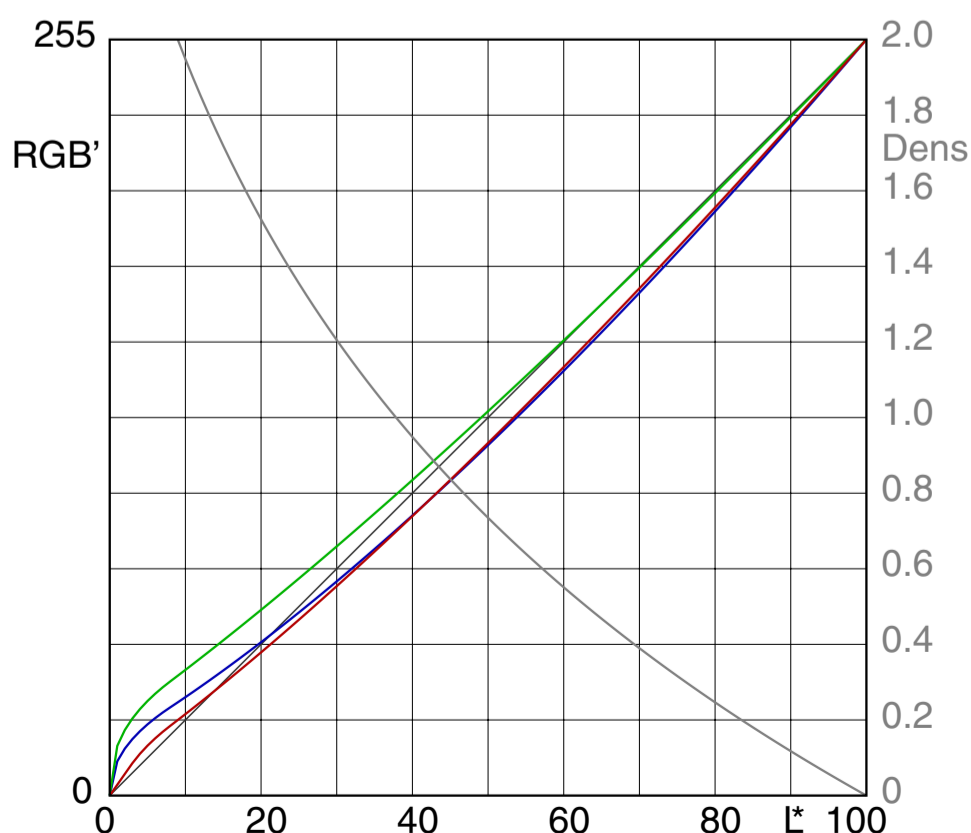
Black  $C = C'^{2.2}$

Red sRGB

Green Ten times the difference

According to profile informations, NTSC, CIE-RGB and AdobeRGB(98) use the simple Gamma correction (no linear part) with  $G=2.2$ . ProPhoto uses  $G=1.8$ .

Gamma encoded values  $C'$ , here for gray  $R'=G'=B'$  are nearly a linear function of  $L^*$ .



Black Linear

Green  $G=2.5$  / uncalibrated monitor

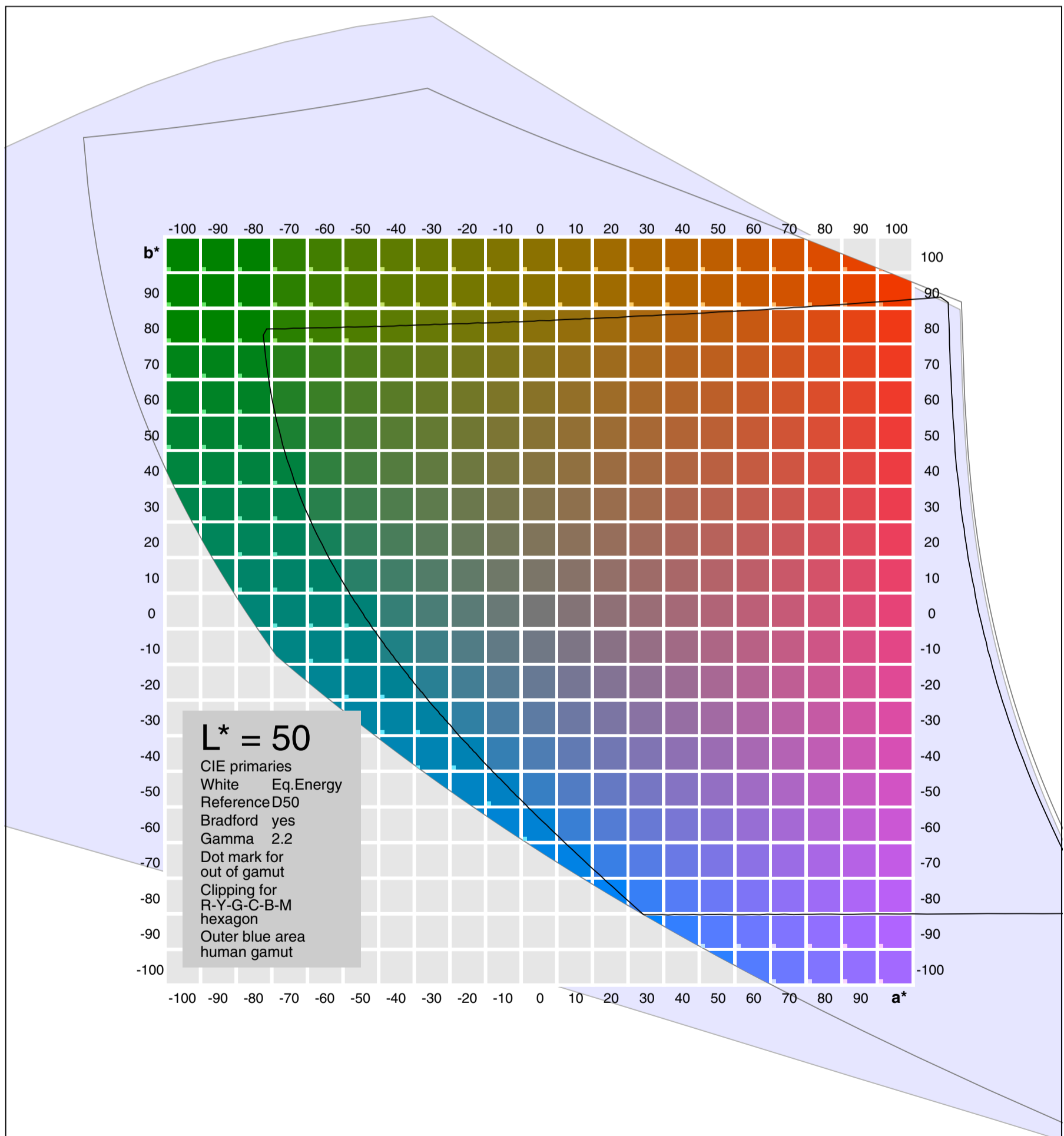
Blue  $G=2.2$  / calibrated monitor

Red  $G=2.4+\text{slope}$  / sRGB

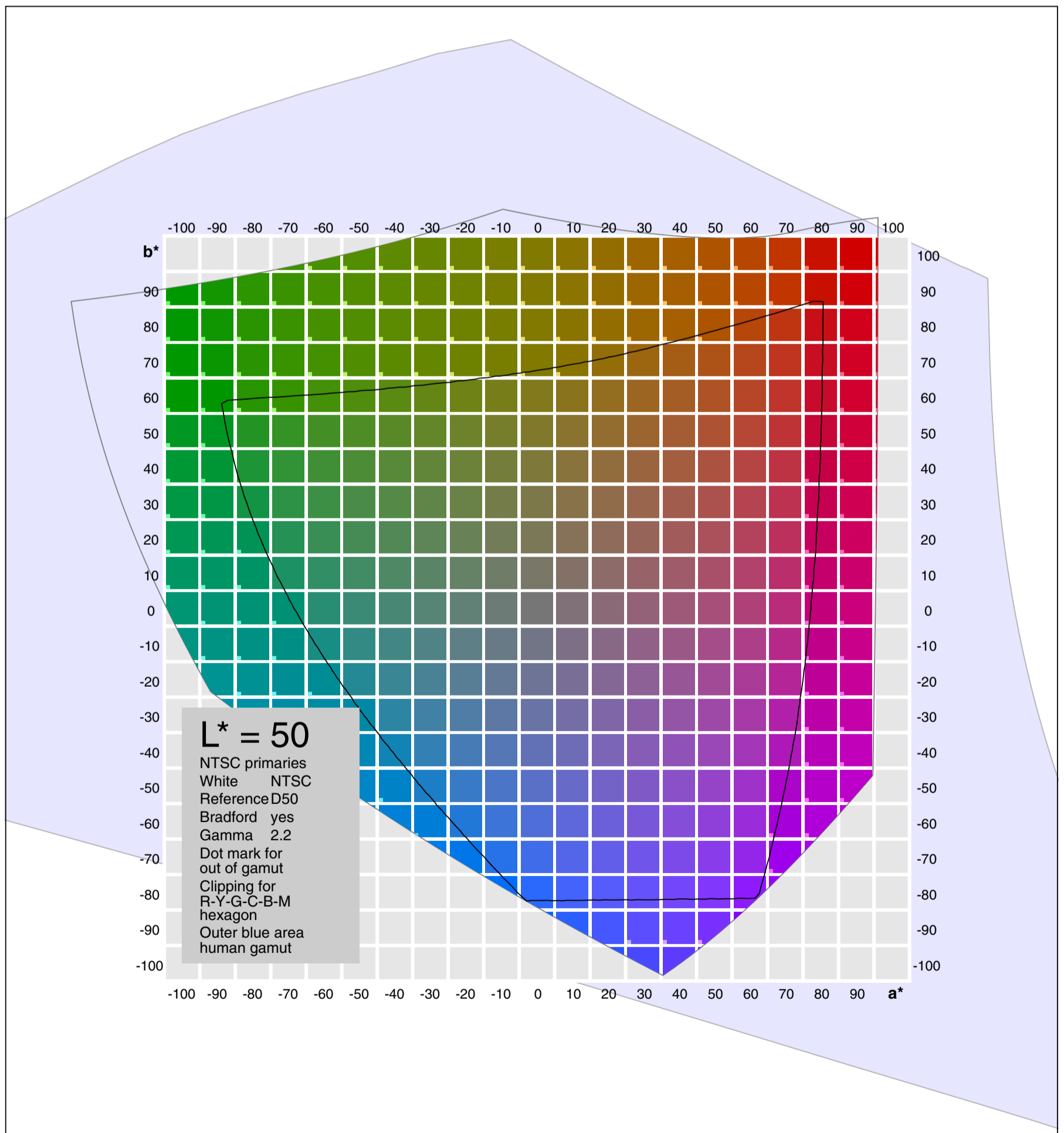
Gray Density

## 6. CIE $L^*=50$

The illustrations were made by PostScript EPS programs as vector graphics  
Converted to PDF by Acrobat Distiller they are still vector graphics  
Eventually the appearance can be improved by Smooth Line Art=Off



# 7. NTSC $L^* = 50$



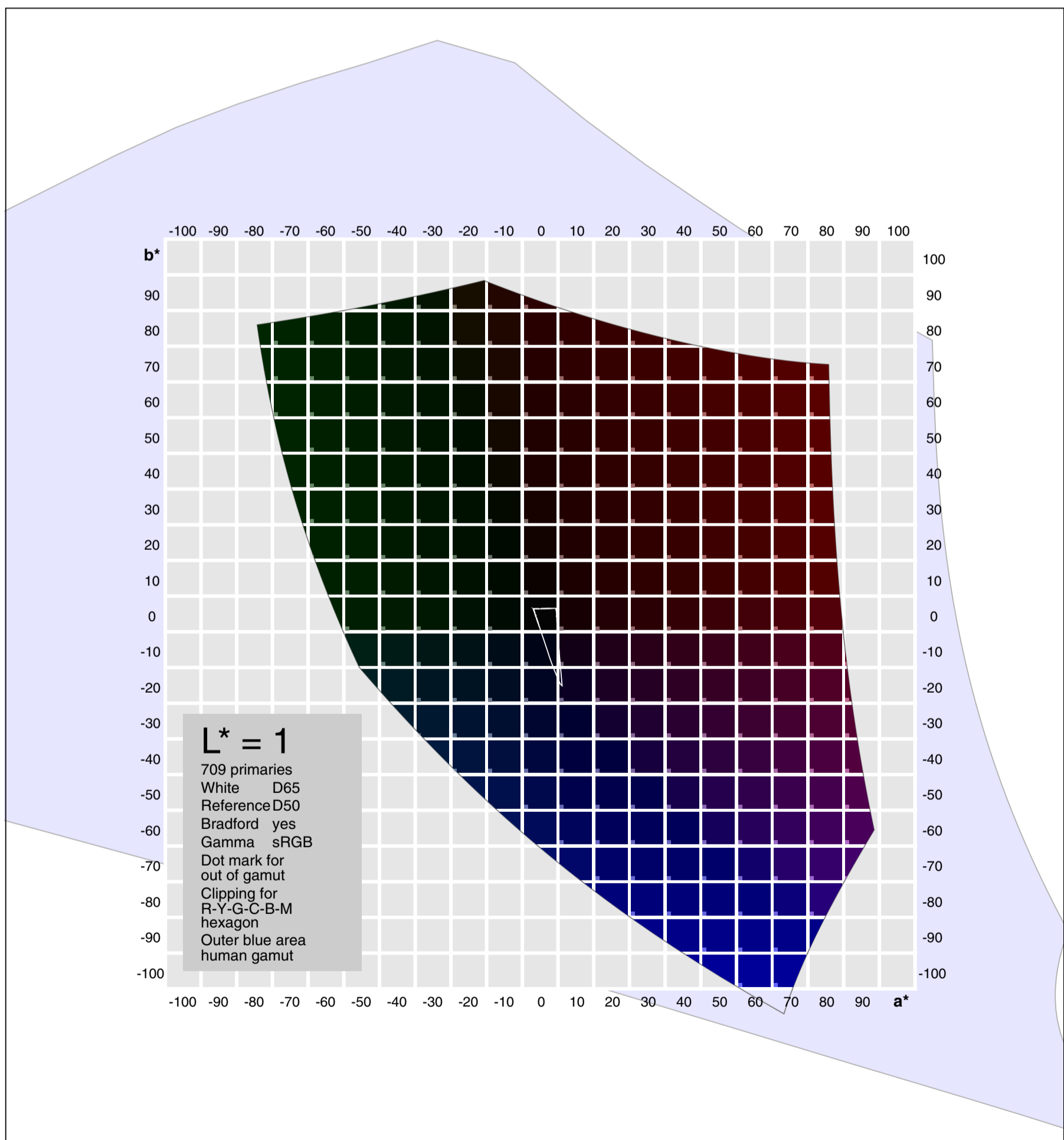
## 8.1 sRGB $L^*=1$

This chapter contains additionally to the RGB values for sRGB the CIELab values for the offset printing process ISOcoated\_v2\_eci.icc [13].

The graphic shows available Lab values as defined by the profile LUT AtoB1. These discrete values are shown as circles either black on light background or white on dark background. The profile has unity input and output curves and an identity matrix.

The printable values for  $L=L^*-1$  to  $L^*$  are shown as stroked round dots, those for  $L=L^*$  to  $L^*+1$  are shown as filled round dots.

It is obviously not a trivial task to define gamut boundaries formalistically by dominating convex shapes with locally weak concave appearance [12]. Continued next page.

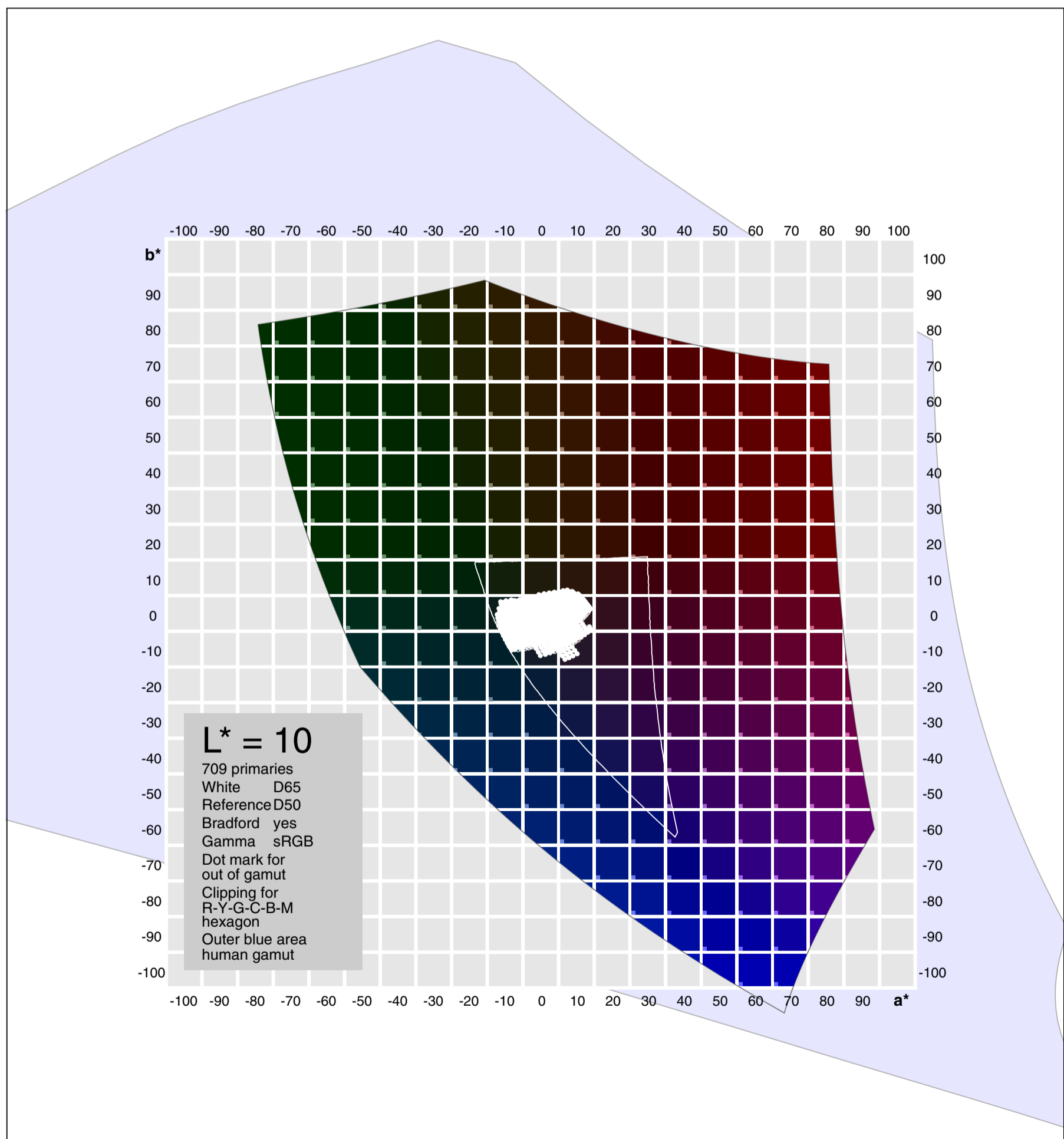


## 8.2 sRGB $L^*=10$

The CIE Lab values in the profile LUT AtoB1 are normalized for white at  $L^*=100$ ,  $a^*=0$  and  $b^*=0$ . For a better comparison with ProfileMaker5 (GamutView), the values were converted by mapping LUT white to media white.

Media white is defined in the profile by  $X_{mw}=0.84552$ ,  $Y_{mw}=0.87683$ ,  $Z_{mw}=0.74716$ . The sequence is algorithmically defined by LABtoXYZ,  $X=X \cdot (X_{mw}/X_n)$ ,  $Y=Y \cdot (Y_{mw}/Y_n)$ ,  $Z=Z \cdot (Z_{mw}/Z_n)$ , XYZtoLAB.

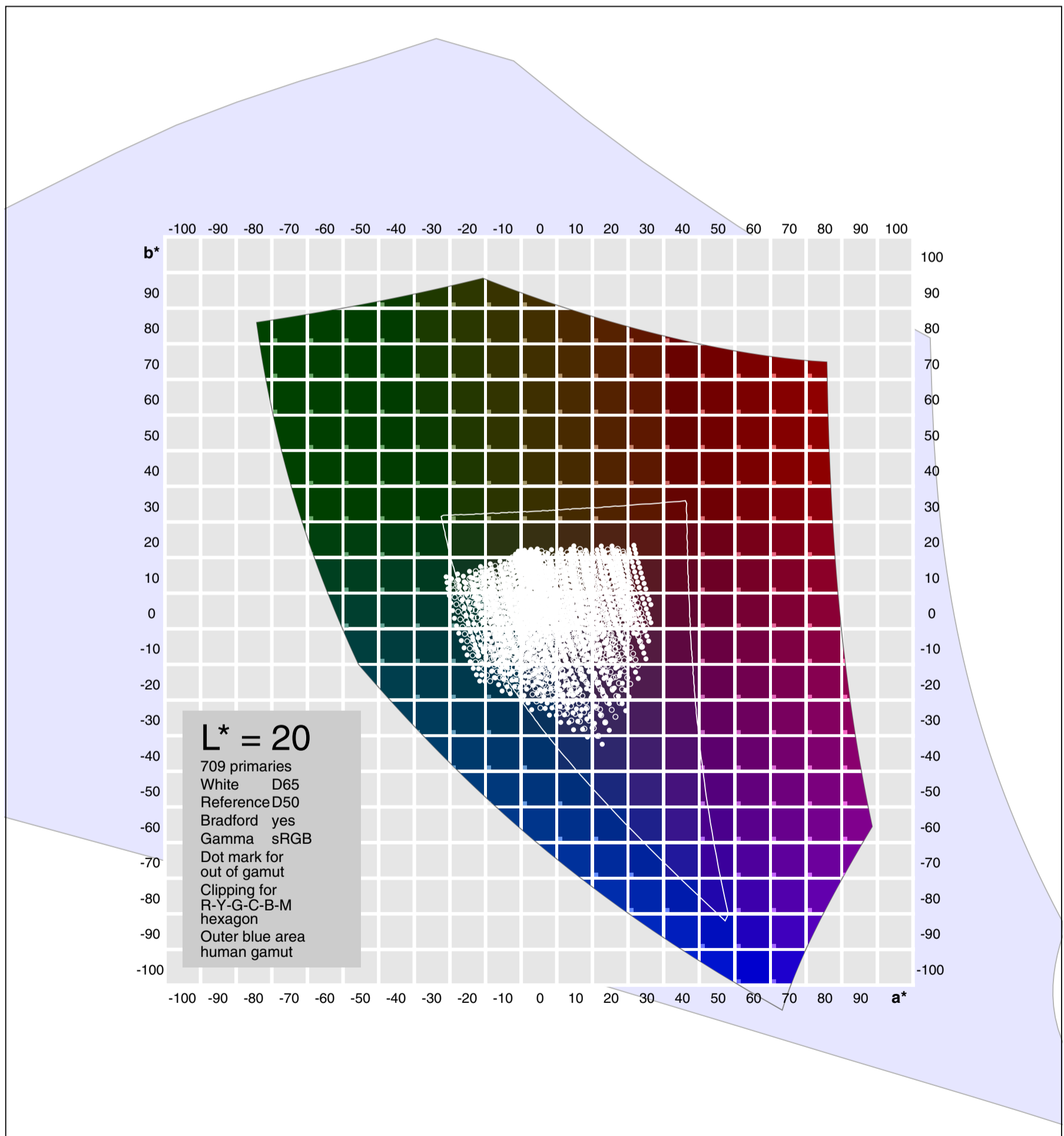
The graphic shows Lab values for sRGB and available values for the CMYK profile ISOcoated\_v2\_eci.icc as white round dots.





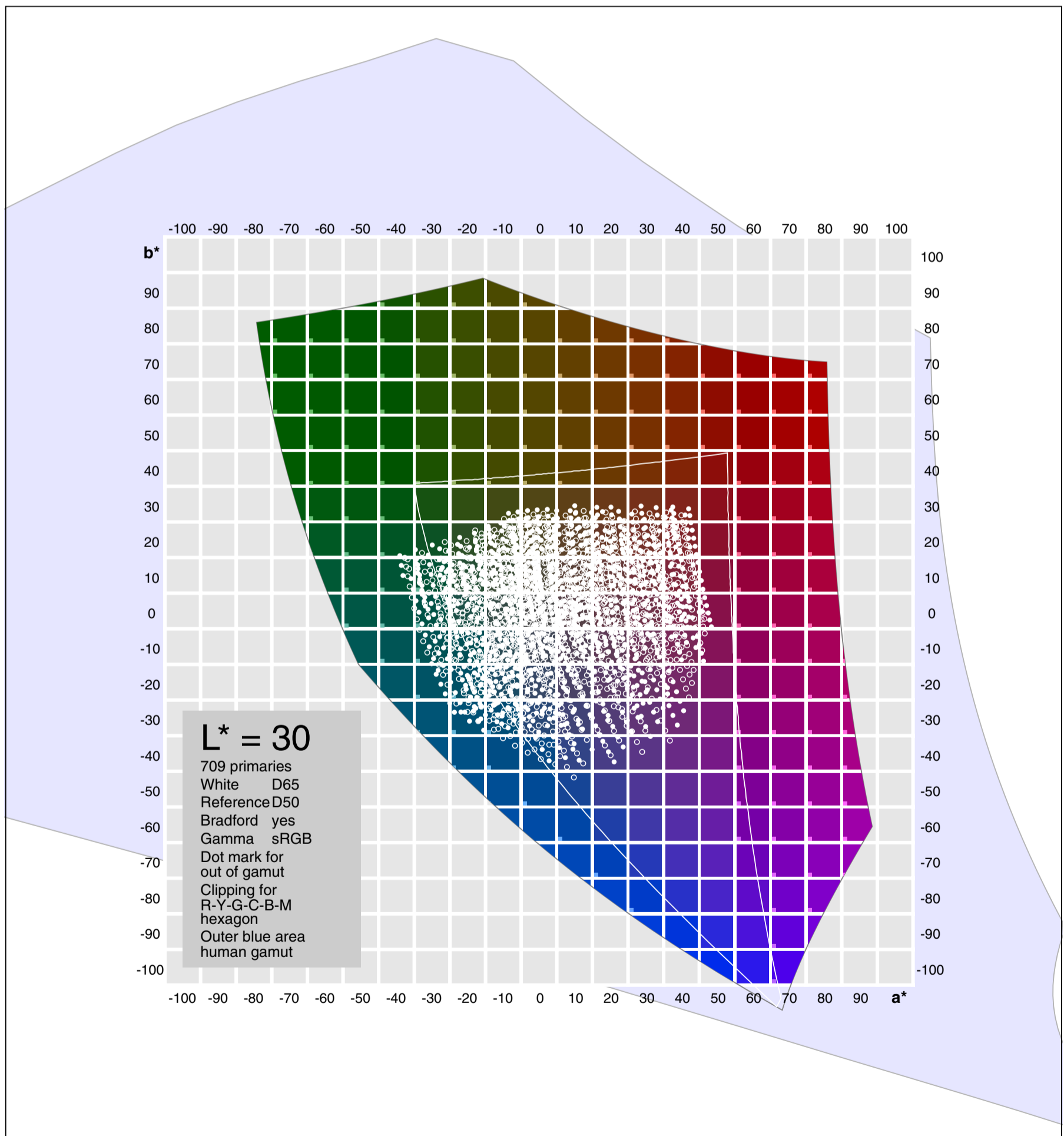
## 8.3 sRGB $L^*=20$

The graphic shows Lab values for sRGB and available values for the CMYK profile ISOcoated\_v2\_eci.icc as white round dots.



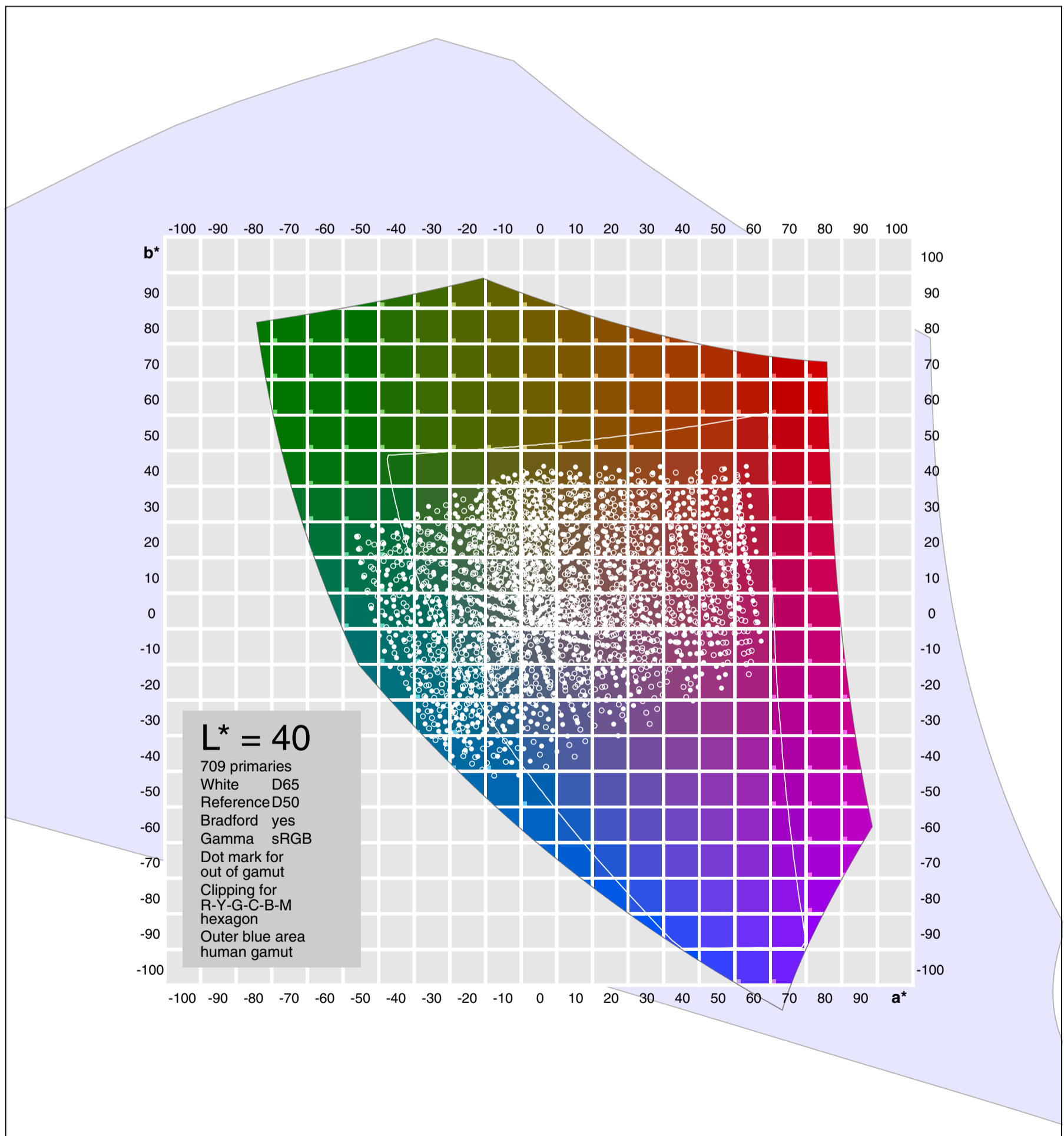
## 8.4 sRGB $L^*=30$

The graphic shows Lab values for sRGB and available values for the CMYK profile ISOcoated\_v2\_eci.icc as white round dots.



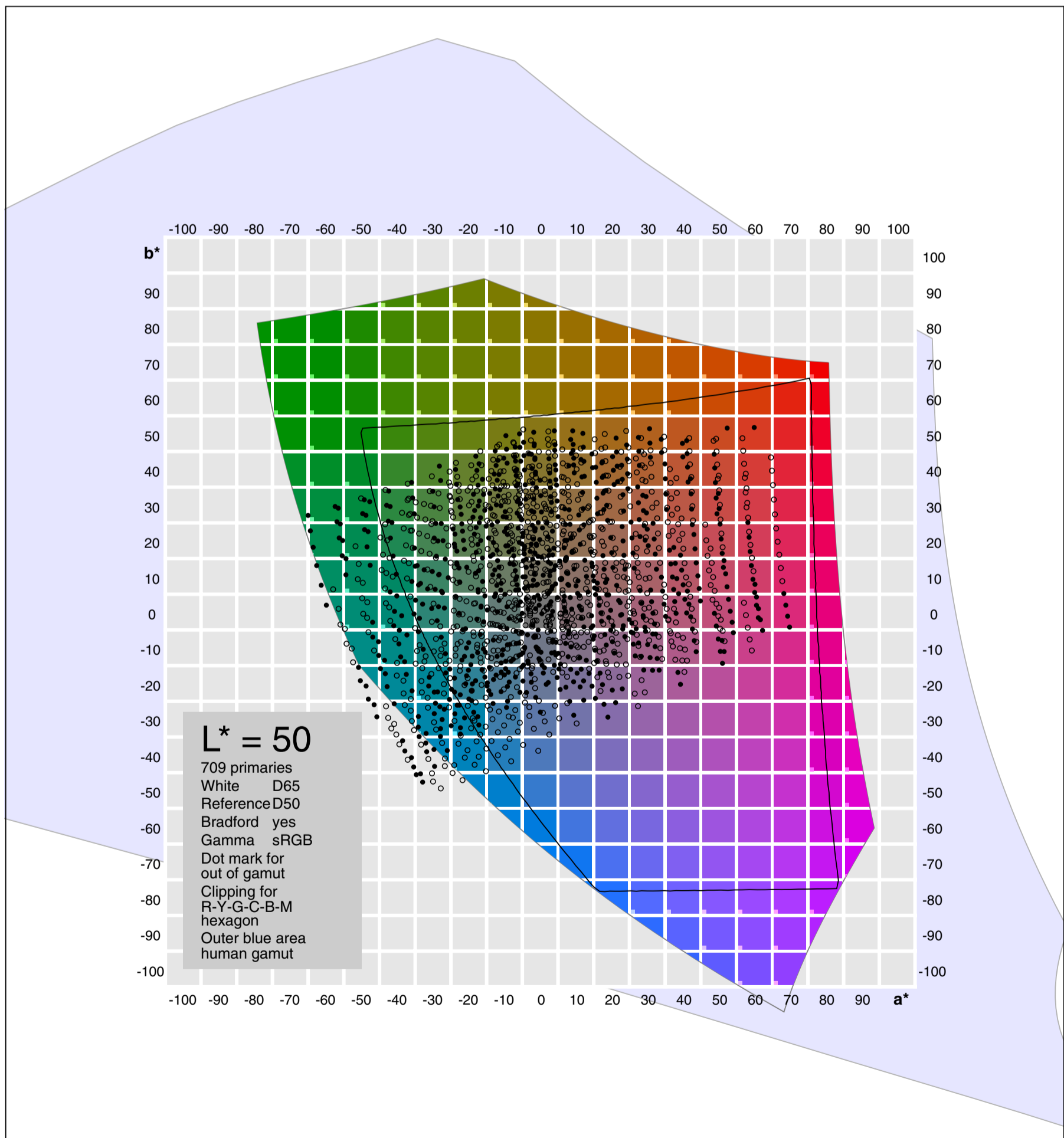
## 8.5 sRGB $L^*=40$

The graphic shows Lab values for sRGB and available values for the CMYK profile ISOcoated\_v2\_eci.icc as white round dots.



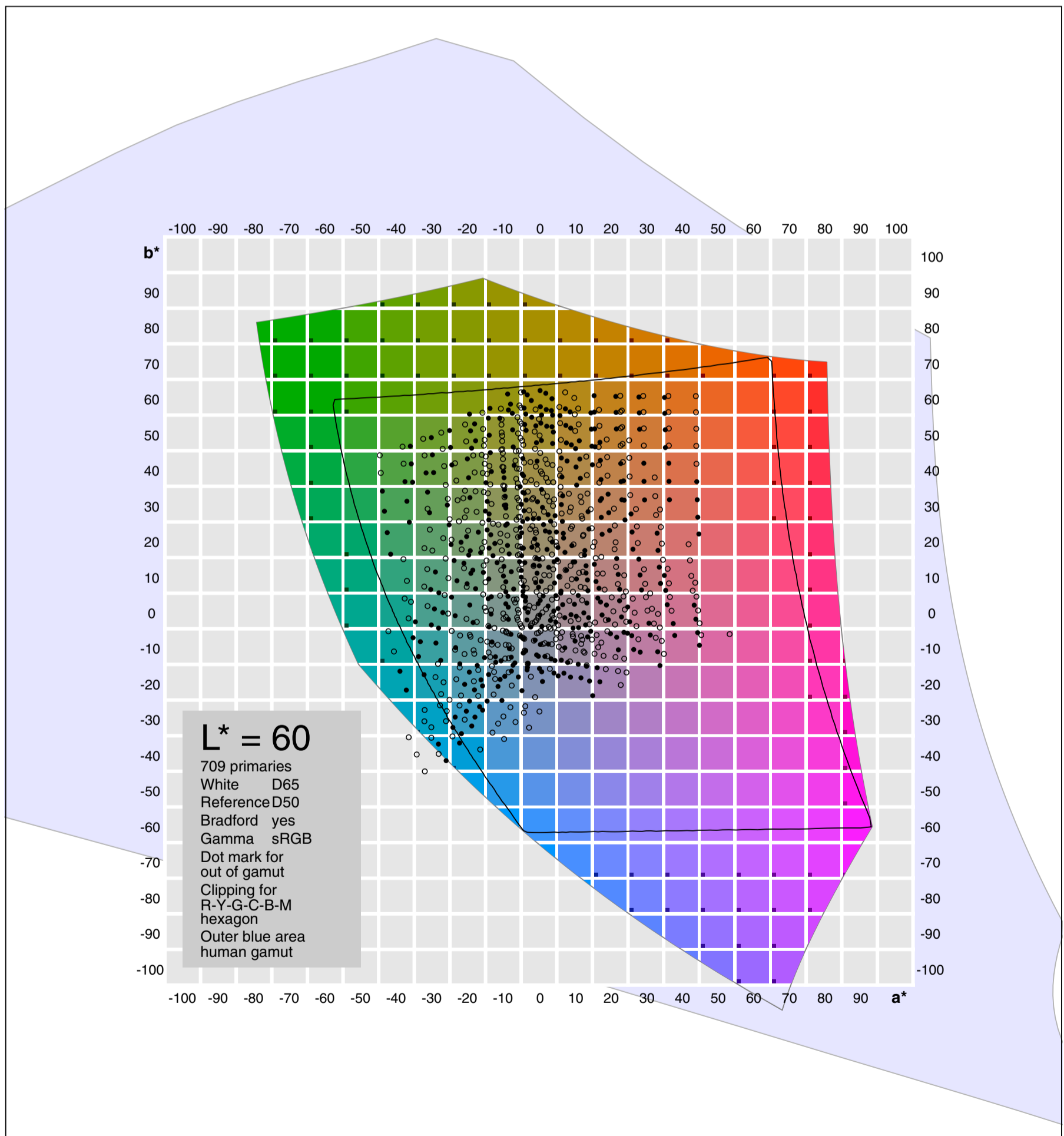
## 8.6 sRGB $L^*=50$

The graphic shows Lab values for sRGB and available values for the CMYK profile ISOcoated\_v2\_eci.icc as black round dots.



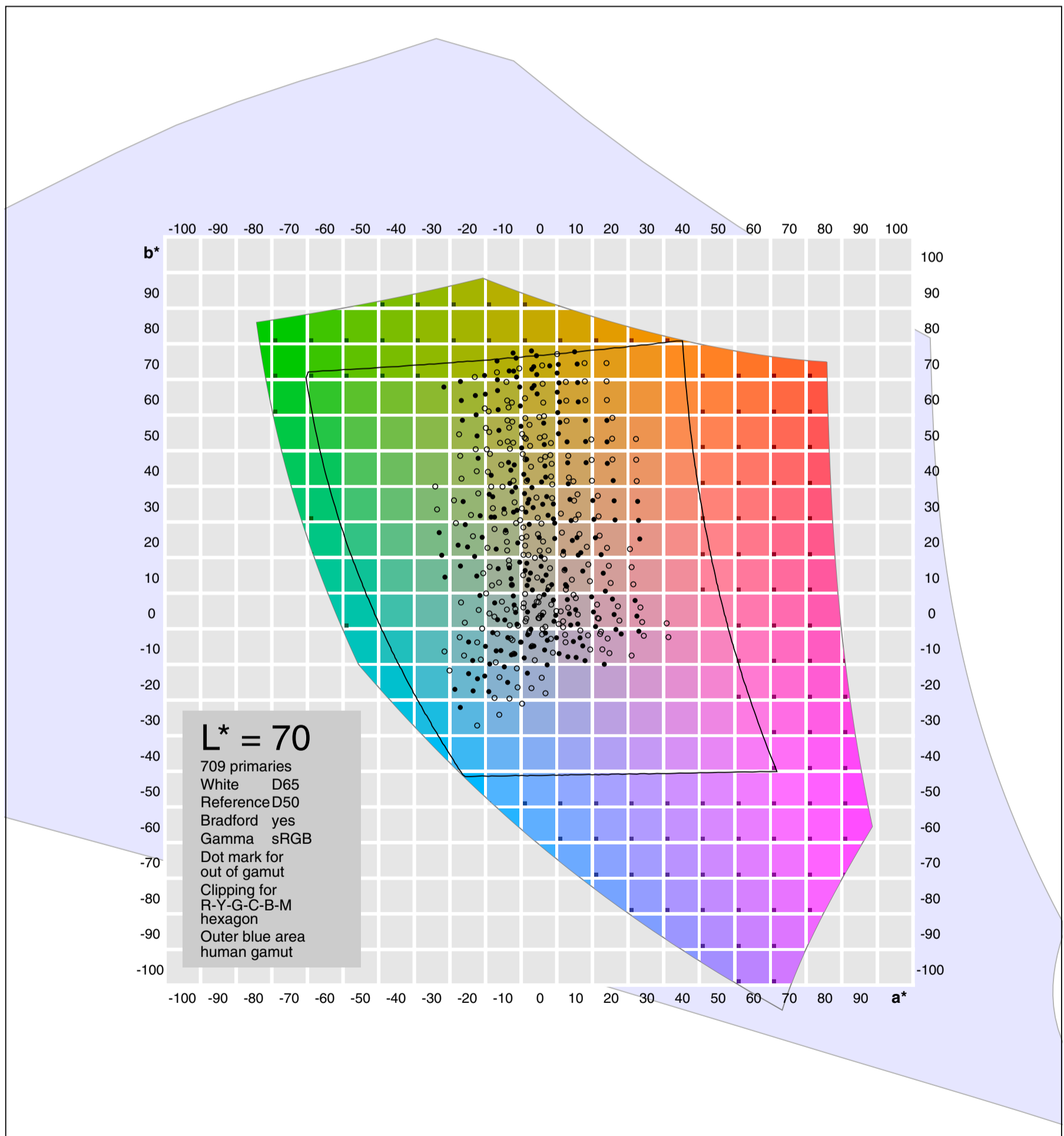
## 8.7 sRGB L\*=60

The graphic shows Lab values for sRGB and available values for the CMYK profile ISOcoated\_v2\_eci.icc as black round dots.



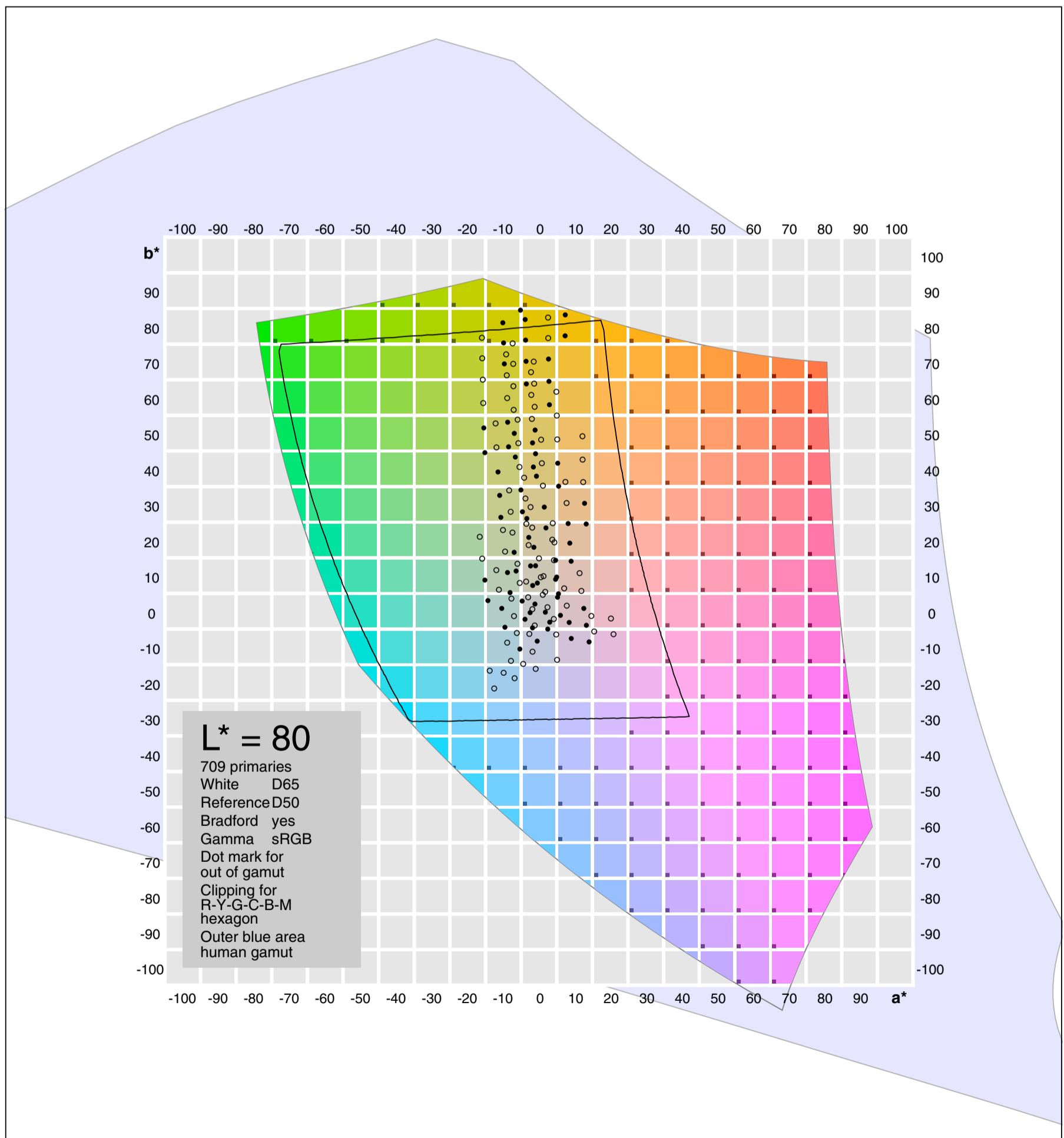
## 8.8 sRGB $L^*=70$

The graphic shows Lab values for sRGB and available values for the CMYK profile ISOcoated\_v2\_eci.icc as black round dots.



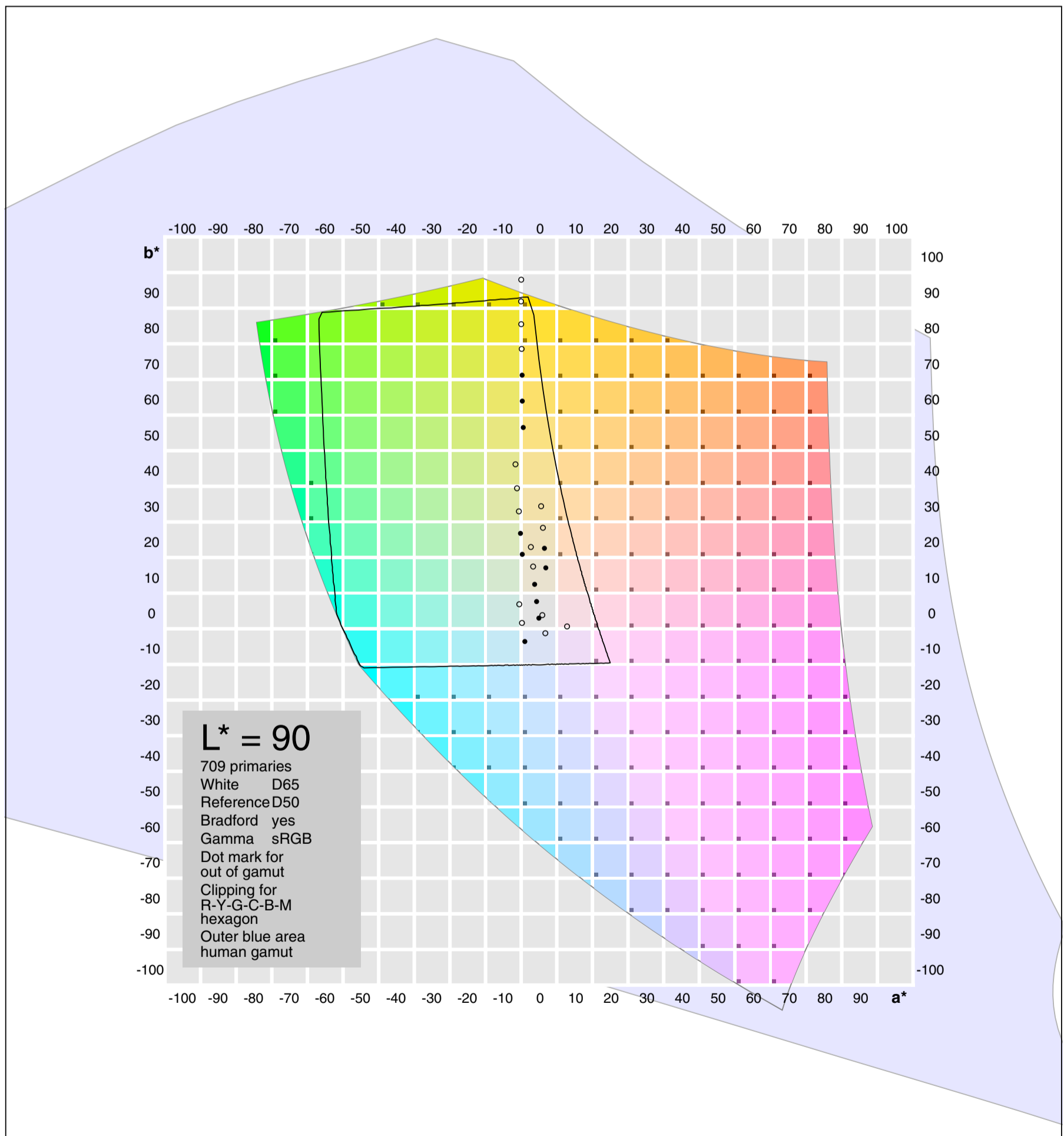
## 8.9 sRGB L\*=80

The graphic shows Lab values for sRGB and available values for the CMYK profile ISOcoated\_v2\_eci.icc as black round dots.



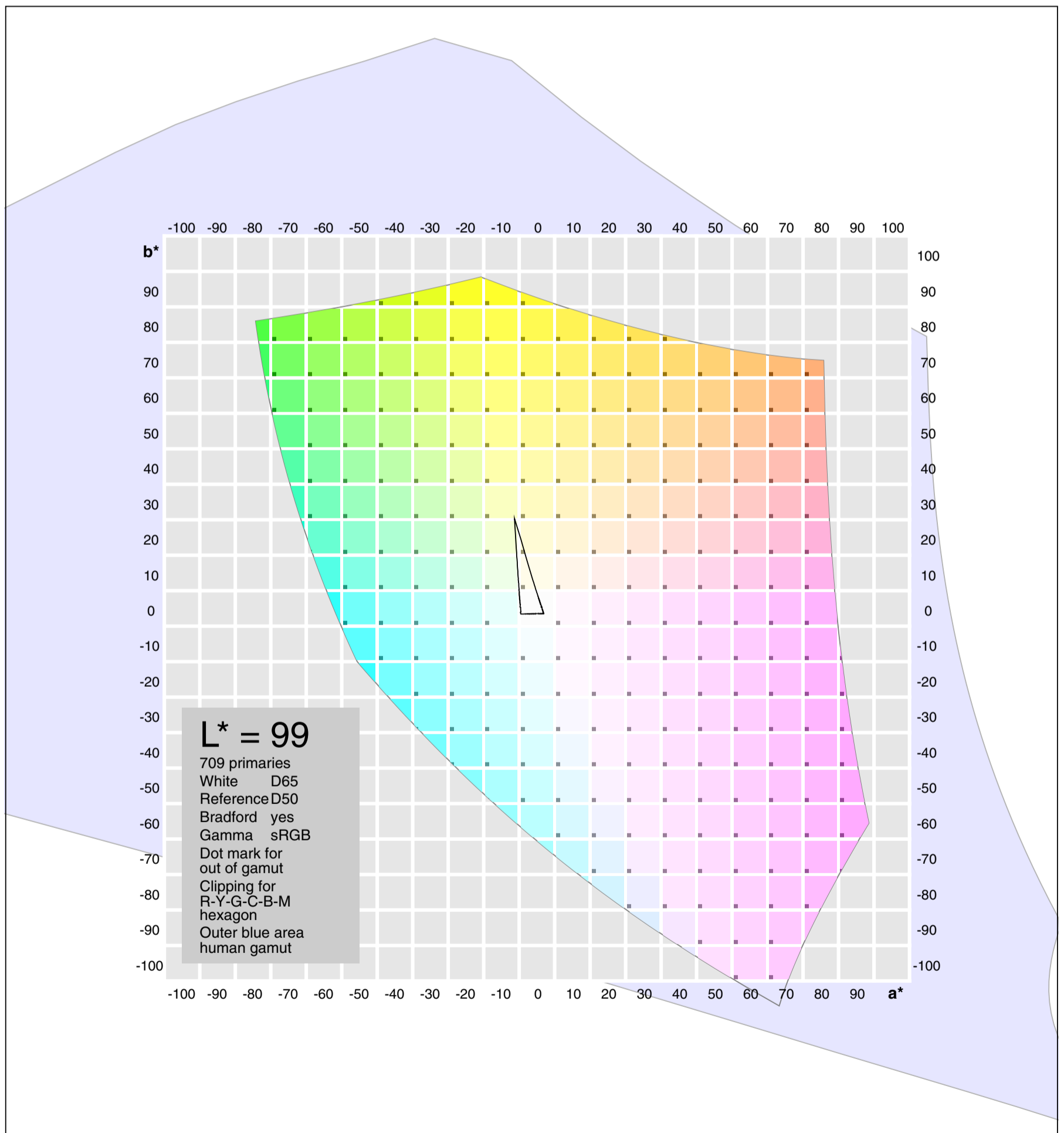
## 8.10 sRGB $L^*=90$

The graphic shows Lab values for sRGB and available values for the CMYK profile ISOcoated\_v2\_eci.icc as black round dots.



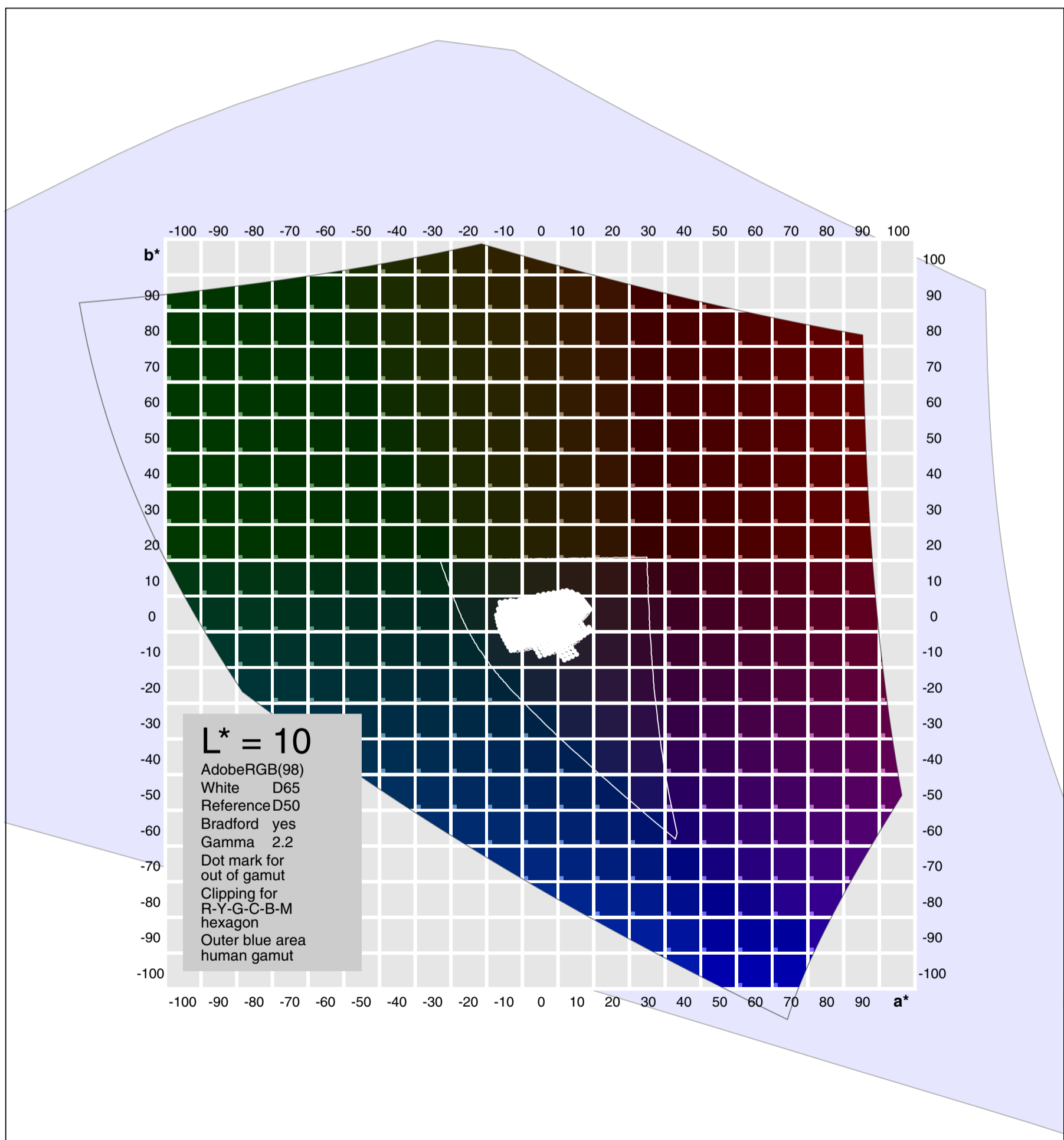


# 8.11 sRGB L\*=99



## 9.1 AdobeRGB $L^*=10$

This chapter contains additionally to the RGB values for AdobeRGB (1998 is omitted here) the CIELab values for the offset printing process ISOcoated\_v2\_eci.icc [13]. The graphic shows available Lab values as defined by the profile LUT AtoB1. These discrete values are shown as circles either black on light background or white on dark background. The profile has unity input and output curves and an identity matrix. The printable values for  $L=L^*-1$  to  $L^*$  are shown as stroked round dots, those for  $L=L^*$  to  $L^*+1$  are shown as filled round dots. It is obviously not a trivial task to define gamut boundaries formalistically by dominating convex shapes with locally weak concave appearance [12]. Continued next page.

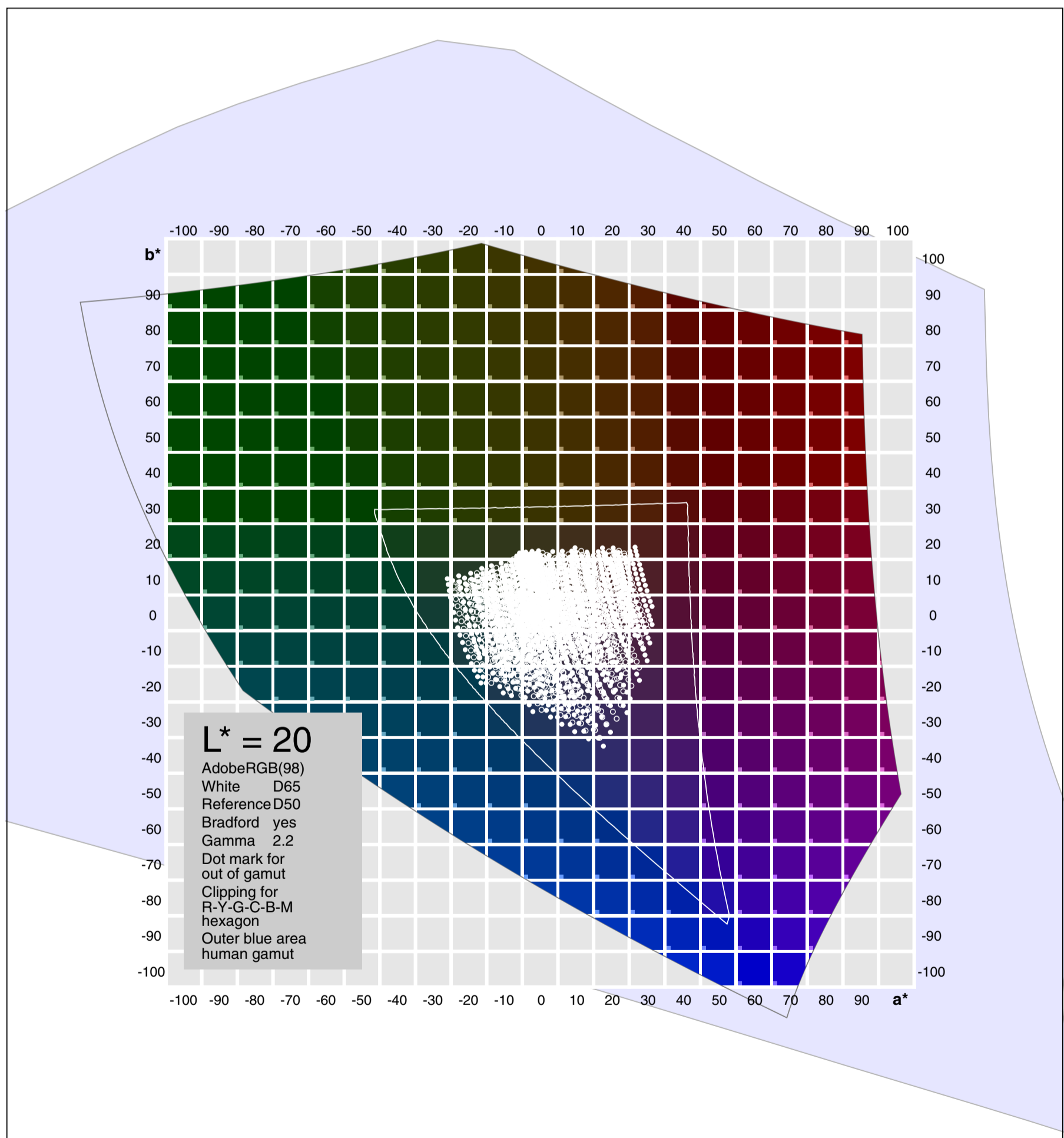


## 9.2 AdobeRGB L\*=20

The CIELab values in the profile LUT AtoB1 are normalized for white at  $L^*=100$ ,  $a^*=0$  and  $b^*=0$ . For a better comparison with ProfileMaker5 (GamutView), the values were converted by mapping LUT white to media white.

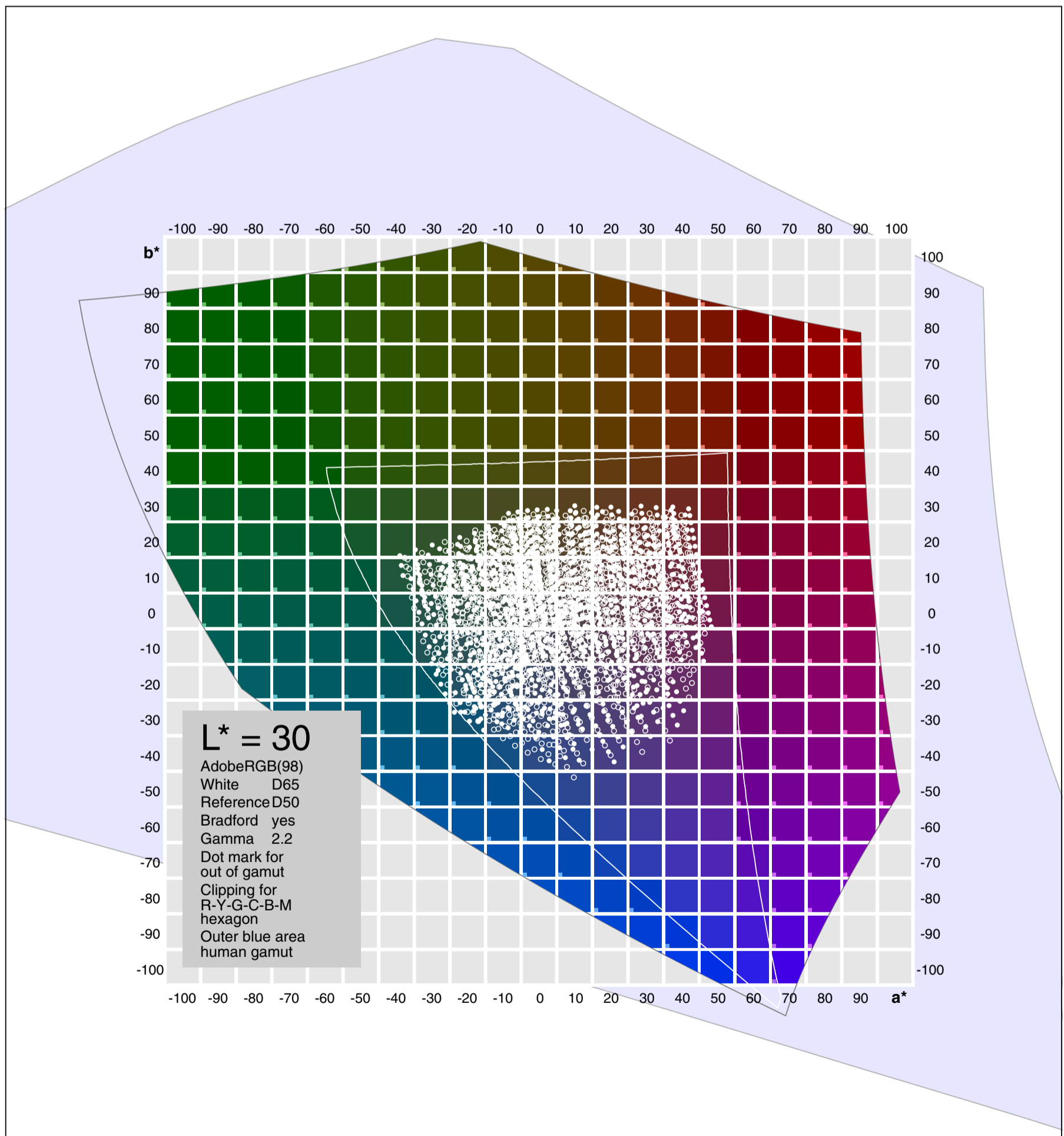
Media white is defined in the profile by  $X_{mw}=0.84552$ ,  $Y_{mw}=0.87683$ ,  $Z_{mw}=0.74716$ . The sequence is algorithmically defined by LABtoXYZ,  $X=X \cdot (X_{mw}/X_n)$ ,  $Y=Y \cdot (Y_{mw}/Y_n)$ ,  $Z=Z \cdot (Z_{mw}/Z_n)$ , XYZtoLAB.

The graphic shows Lab values for AdobeRGB and available values for the CMYK profile ISOcoated\_v2\_eci.icc as white round dots.



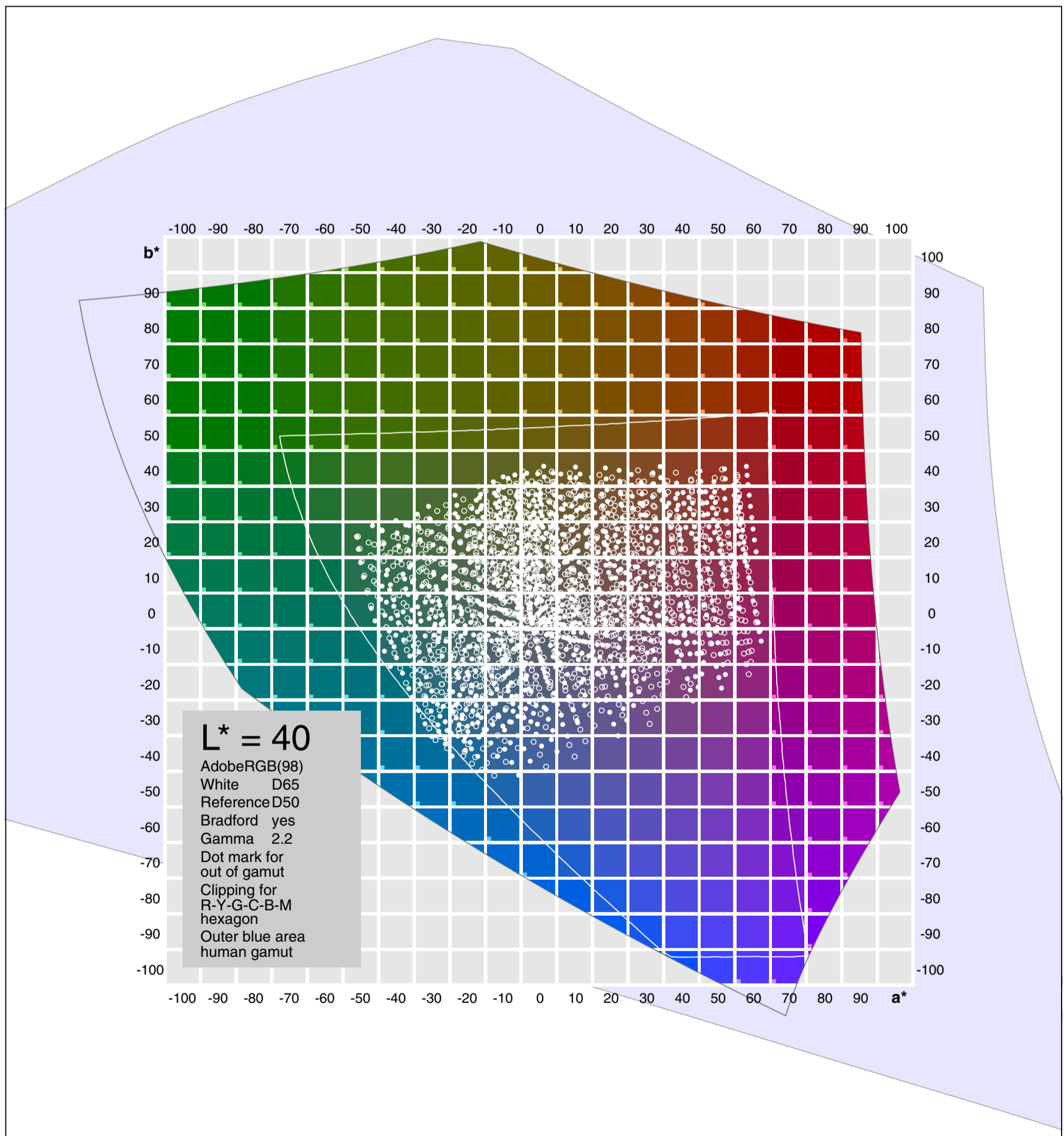
## 9.3 AdobeRGB L\*=30

The graphic shows Lab values for AdobeRGB and available values for the CMYK profile ISOcoated\_v2\_eci.icc as white round dots.



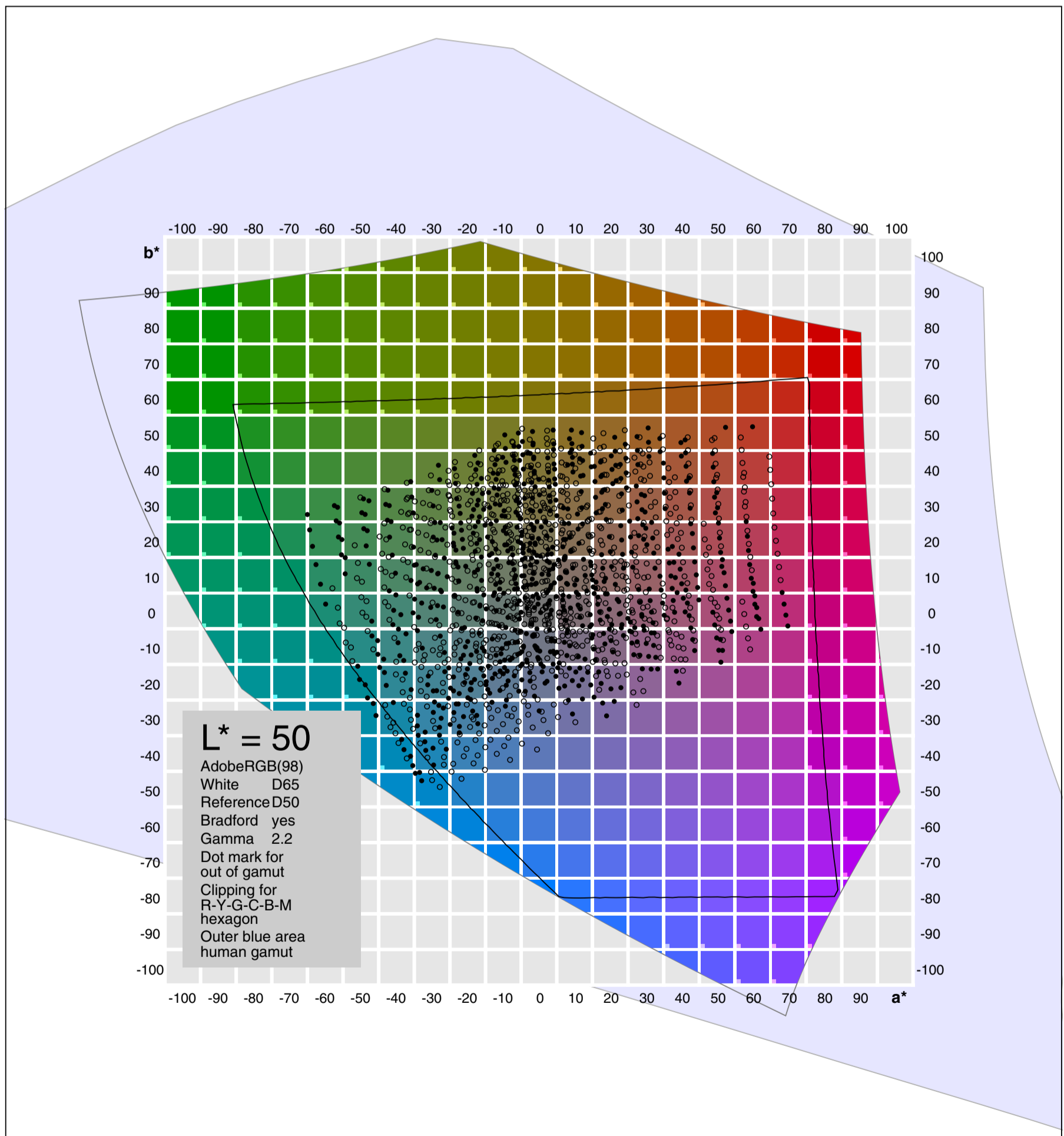
## 9.4 AdobeRGB $L^*=40$

The graphic shows Lab values for AdobeRGB and available values for the CMYK profile ISOcoated\_v2\_eci.icc as white round dots.



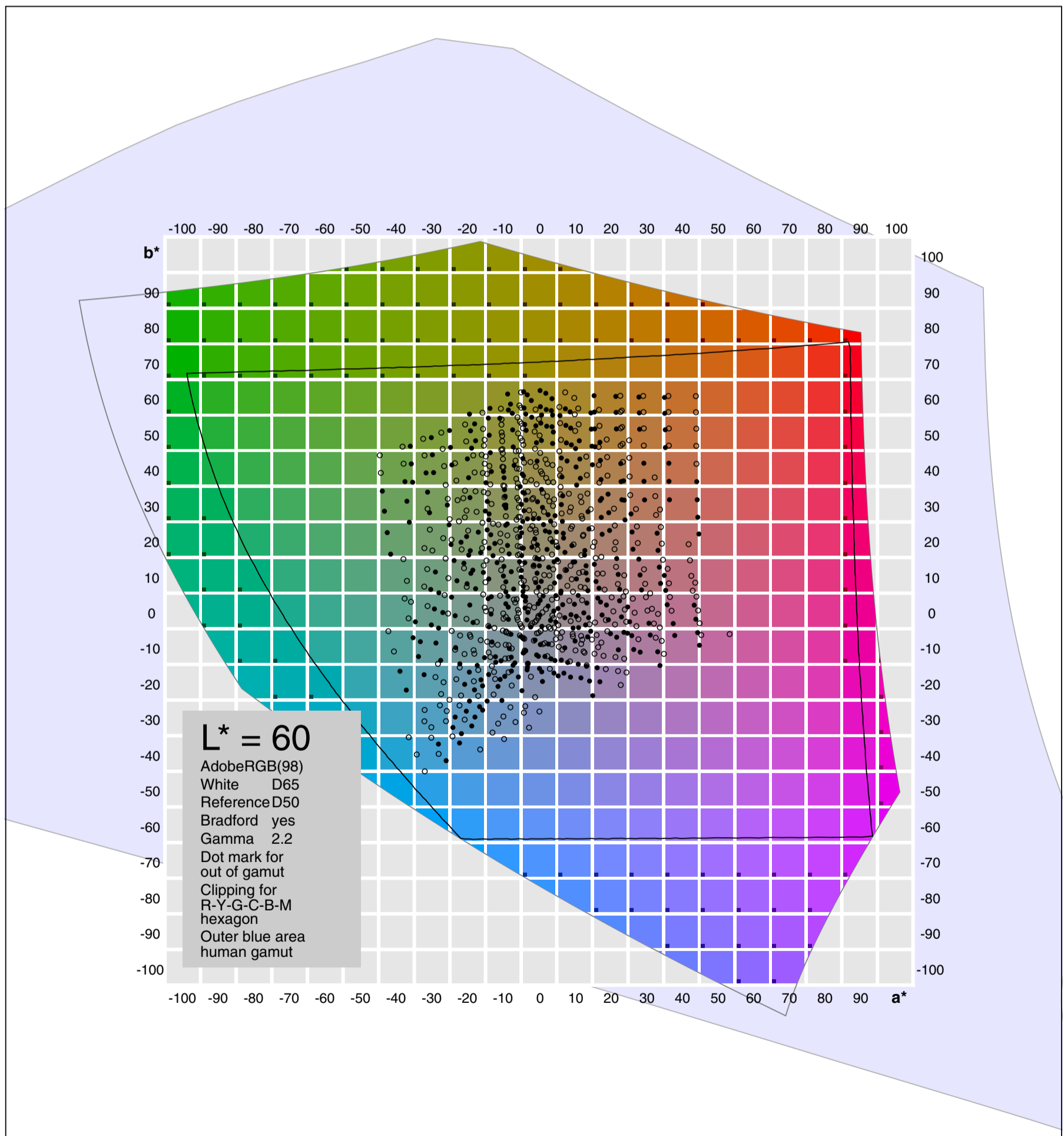
## 9.5 AdobeRGB L\*=50

The graphic shows Lab values for AdobeRGB and available values for the CMYK profile ISOcoated\_v2\_eci.icc as black round dots.



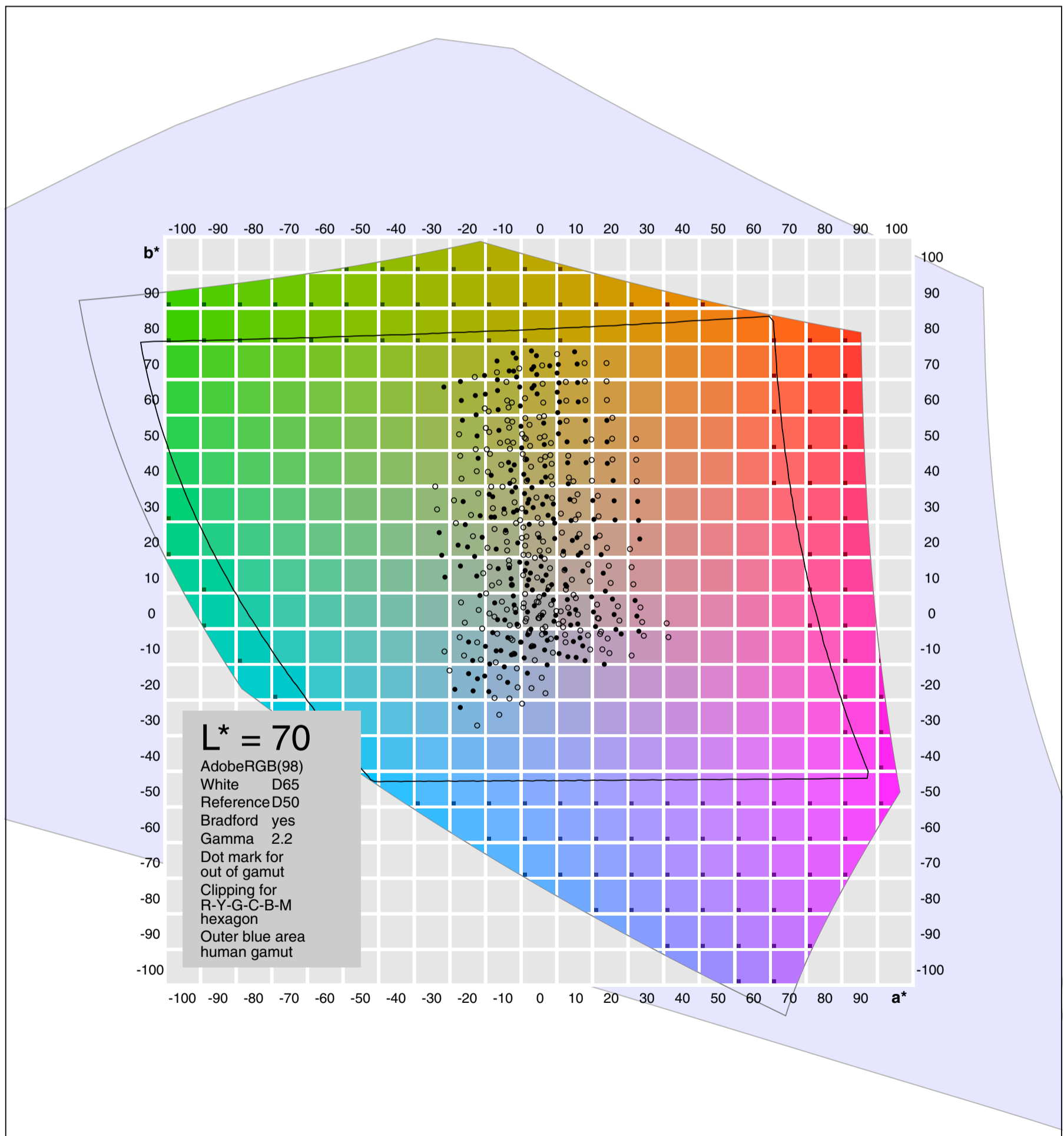
## 9.6 AdobeRGB L\*=60

The graphic shows Lab values for AdobeRGB and available values for the CMYK profile ISOcoated\_v2\_eci.icc as black round dots.



## 9.7 AdobeRGB L\*=70

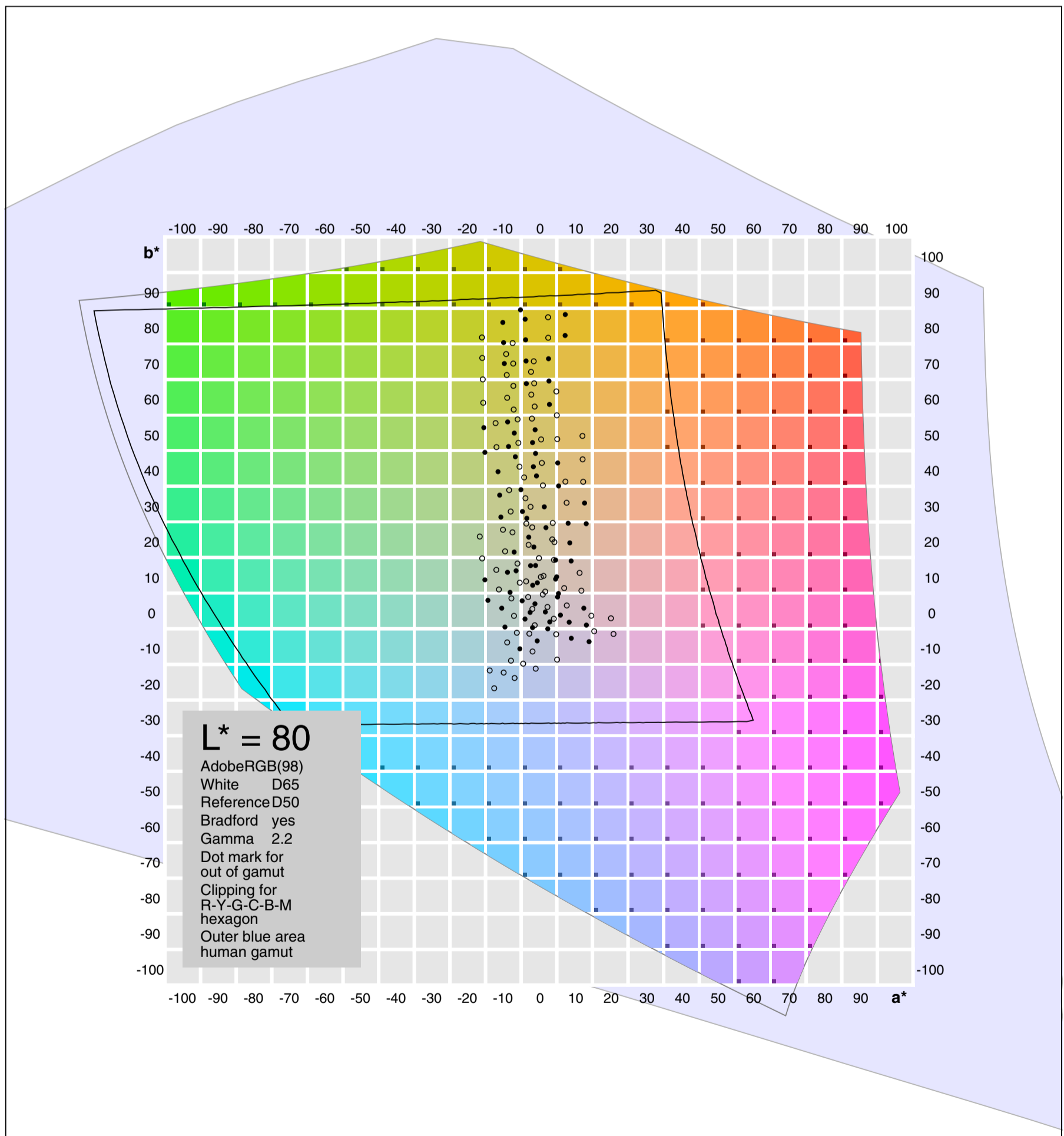
The graphic shows Lab values for AdobeRGB and available values for the CMYK profile ISOcoated\_v2\_eci.icc as black round dots.





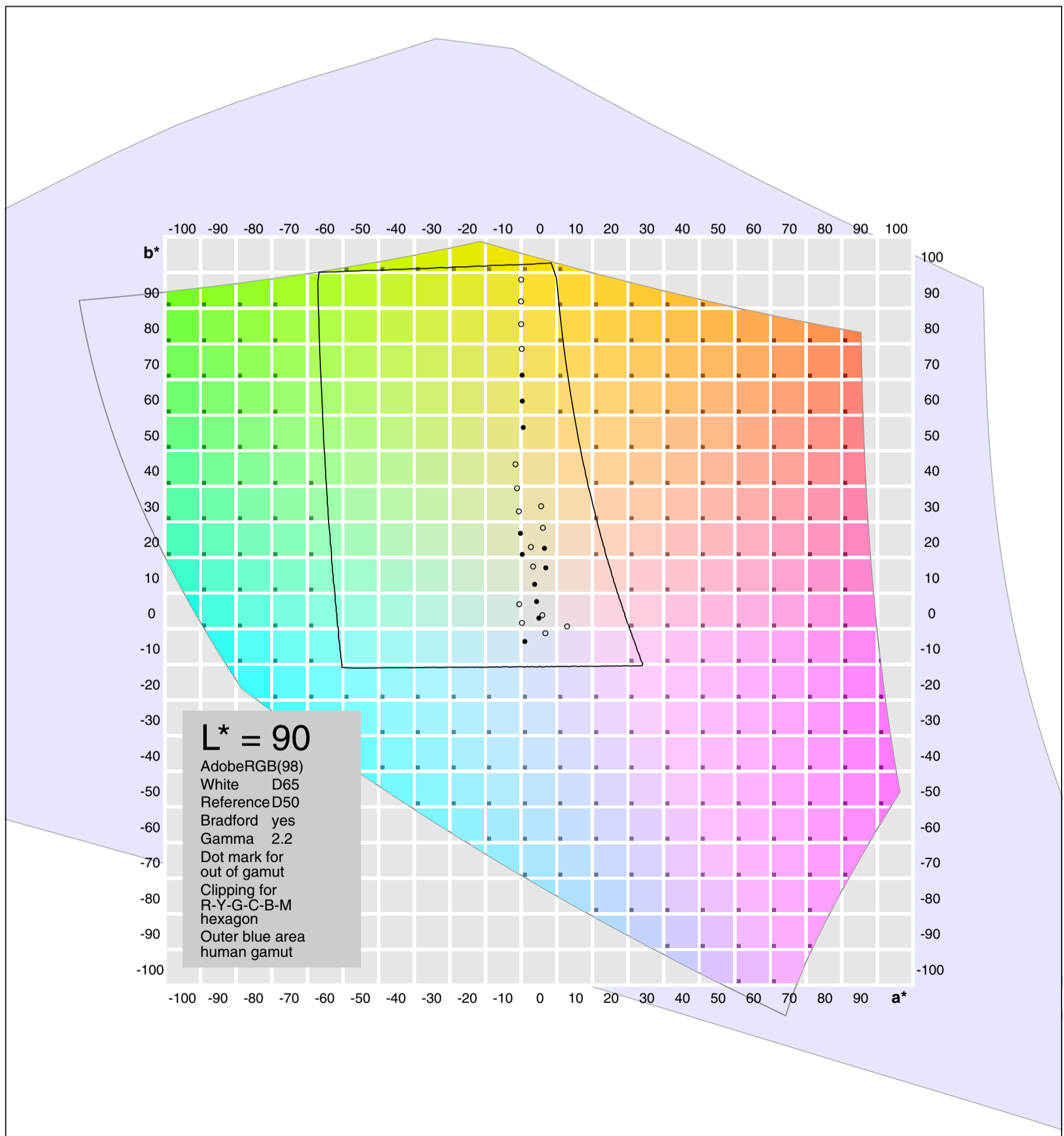
## 9.8 AdobeRGB L\*=80

The graphic shows Lab values for AdobeRGB and available values for the CMYK profile ISOcoated\_v2\_eci.icc as black round dots.



## 9.9 AdobeRGB L\*=90

The graphic shows Lab values for AdobeRGB and available values for the CMYK profile ISOcoated\_v2\_eci.icc as black round dots.



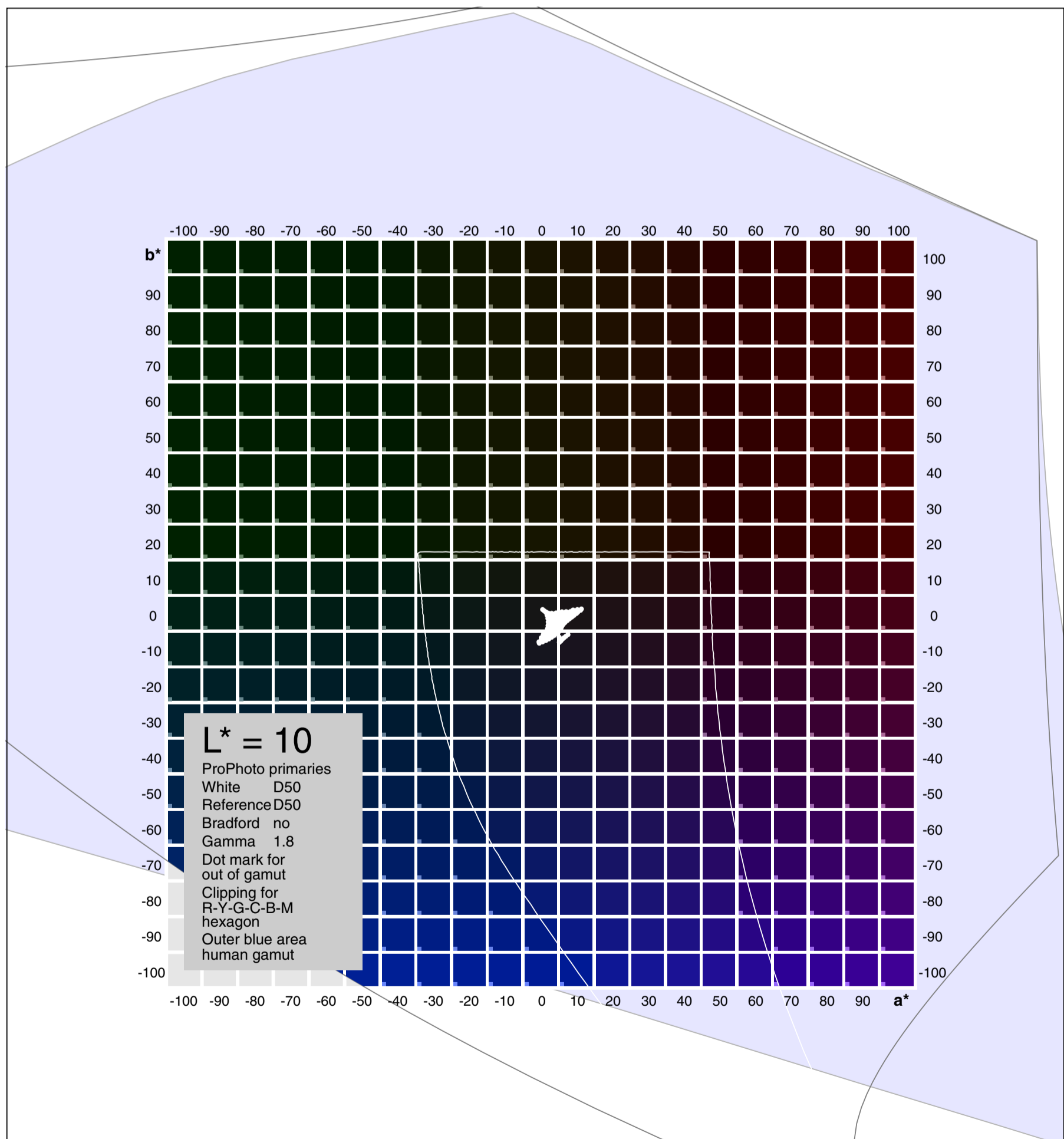
## 10.1 ProPhotoRGB $L^*=10$

The graphic shows Lab values for ProPhotoRGB and available values for the CMYK profile PSO\_Coated\_NPscreen\_ISO12647\_eci.icc as black round dots. This is a profile for FM printing.

See chapter 9.1 and 9.2 for some details how to extract the data from the profile.

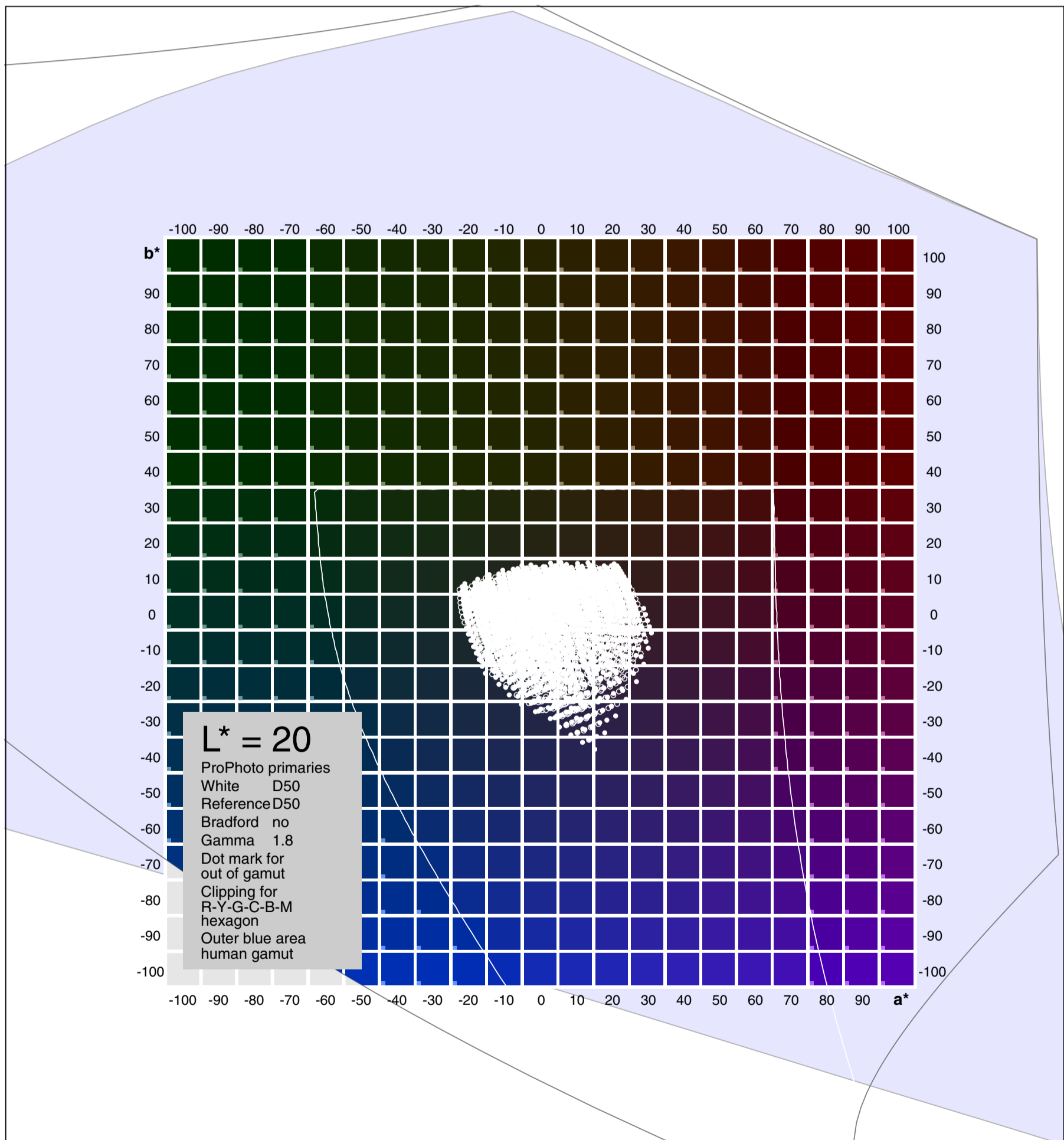
Media white is defined in the profile by  $X_{mw}=0.84386$ ,  $Y_{mw}=0.87556$ ,  $Z_{mw}=0.74486$ .

This profile is used for frequency-modulated printing, like inkjet printing. The gamut is a little larger than that of ISOcoated\_v2\_eci.icc (previous chapter), but it fits as well in the gamut of AdobeRGB.



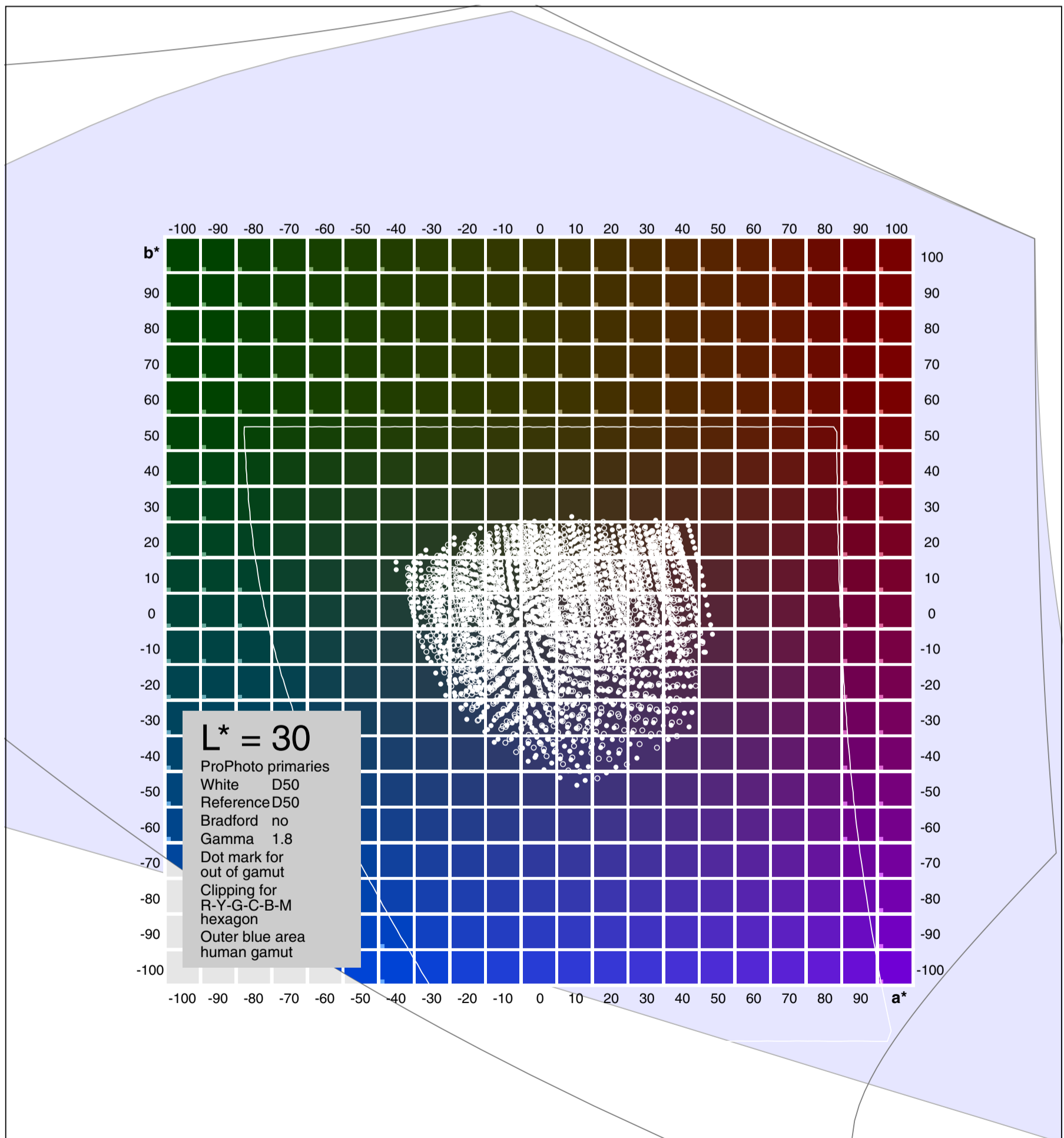
## 10.2 ProPhotoRGB $L^*=20$

The graphic shows Lab values for ProPhotoRGB and available values for the CMYK profile PSO\_Coated\_NPscreen\_ISO12647\_eci.icc as black round dots. This is a profile for FM printing.



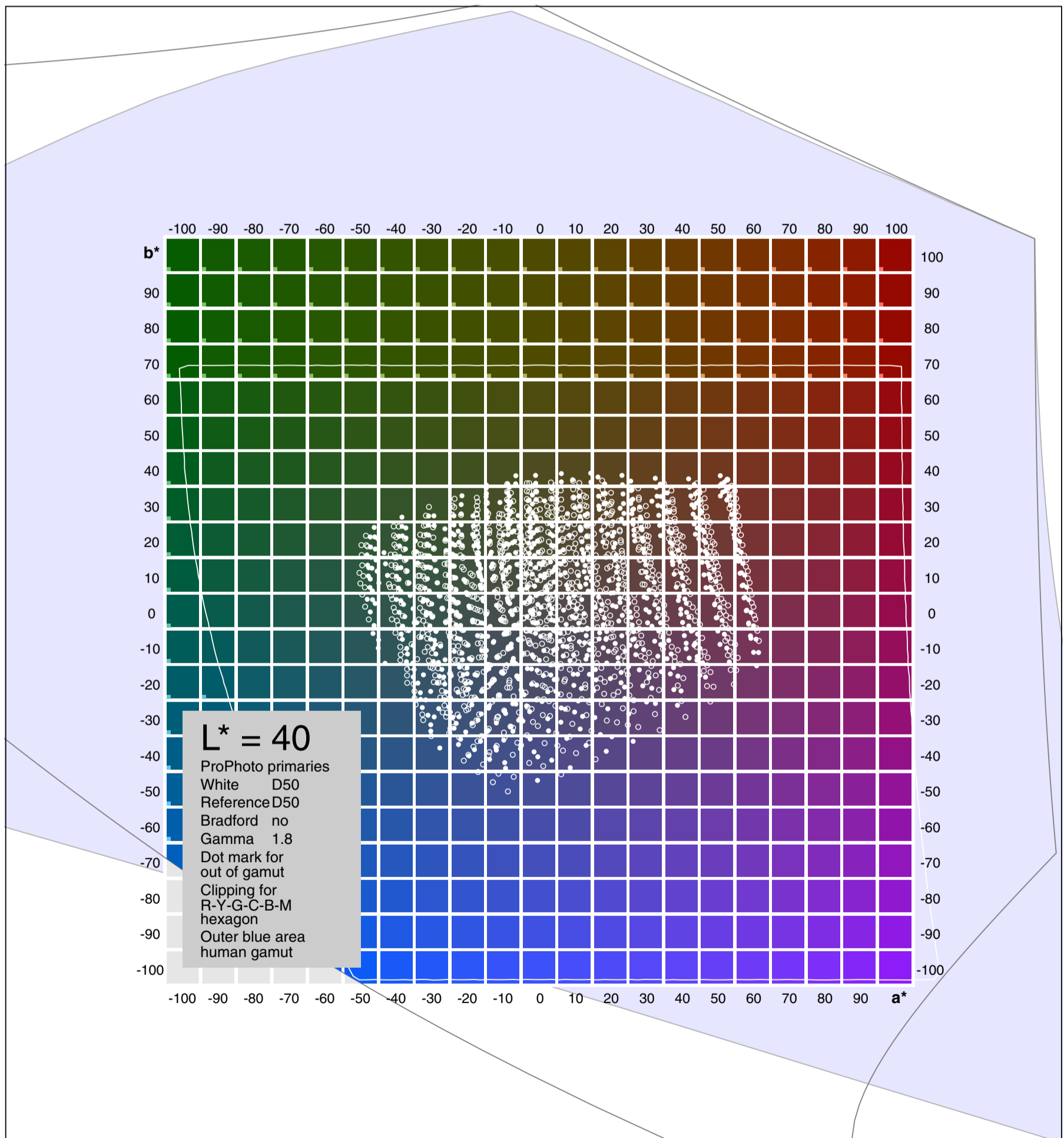
## 10.3 ProPhotoRGB $L^*=30$

The graphic shows Lab values for ProPhotoRGB and available values for the CMYK profile PSO\_Coated\_NPscreen\_ISO12647\_eci.icc as black round dots. This is a profile for FM printing.



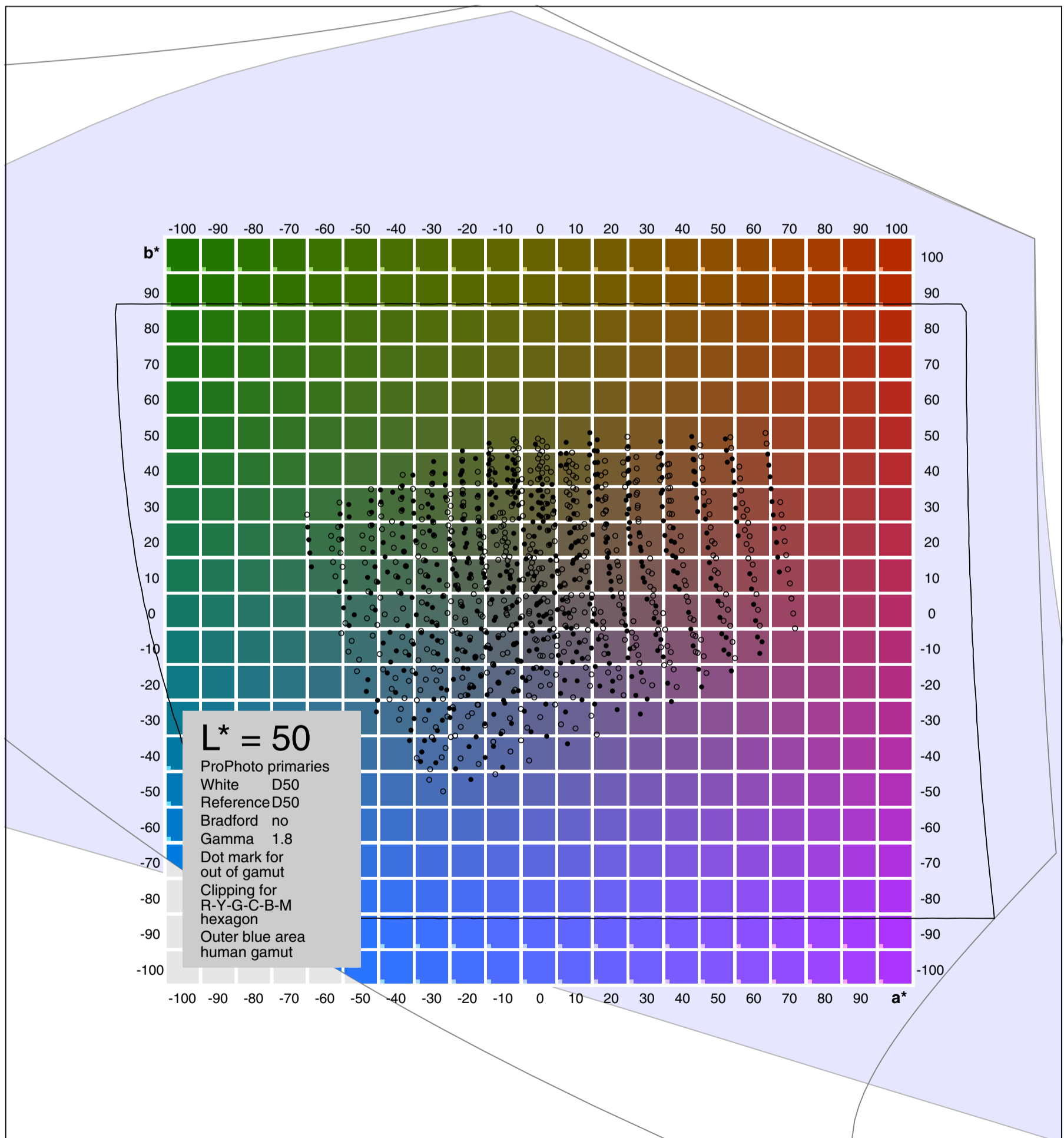
## 10.4 ProPhotoRGB $L^*=40$

The graphic shows Lab values for ProPhotoRGB and available values for the CMYK profile PSO\_Coated\_NPscreen\_ISO12647\_eci.icc as black round dots. This is a profile for FM printing.



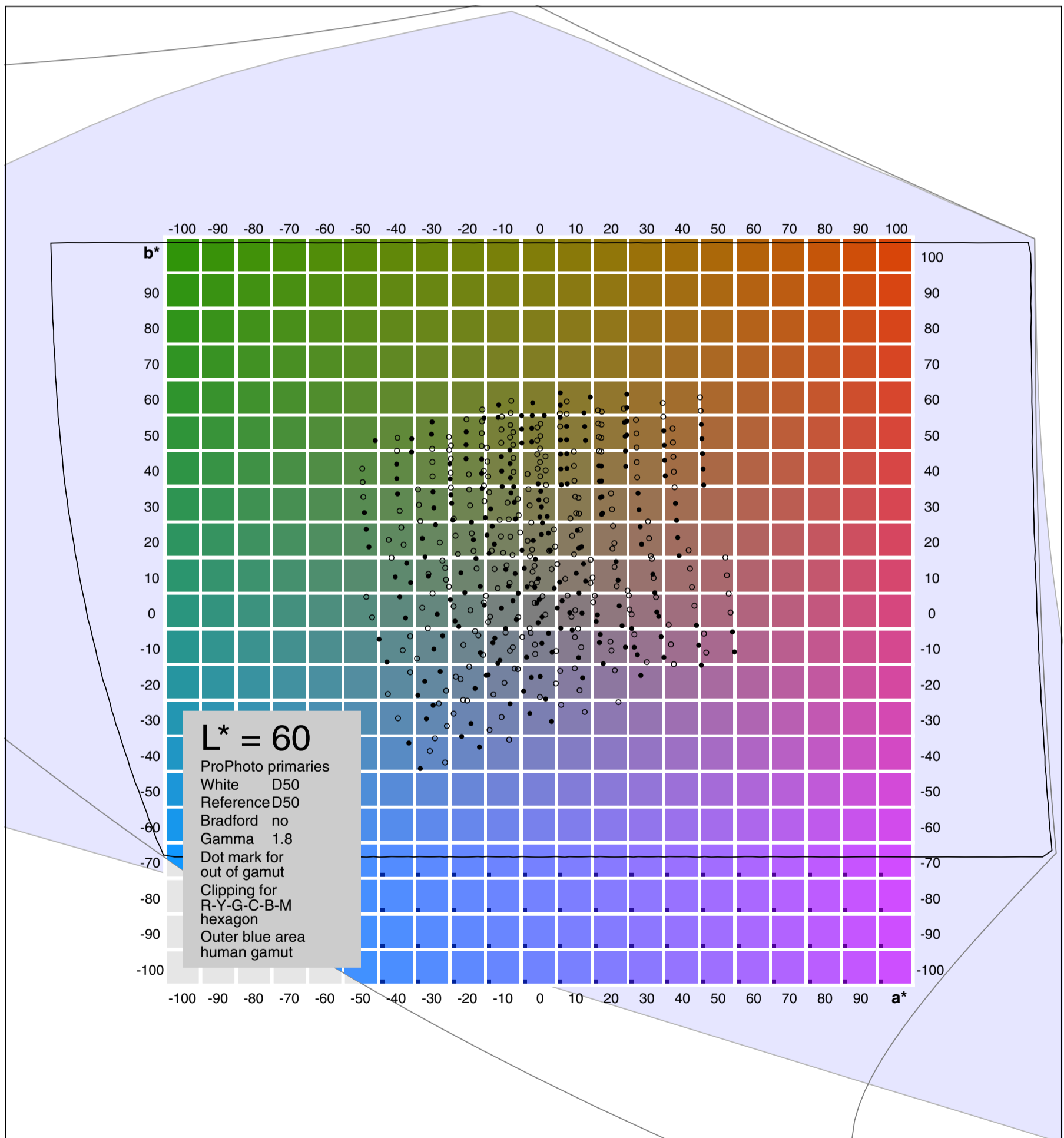
## 10.5 ProPhotoRGB $L^*=50$

The graphic shows Lab values for ProPhotoRGB and available values for the CMYK profile PSO\_Coated\_NPscreen\_ISO12647\_eci.icc as black round dots. This is a profile for FM printing.



## 10.6 ProPhotoRGB $L^*=60$

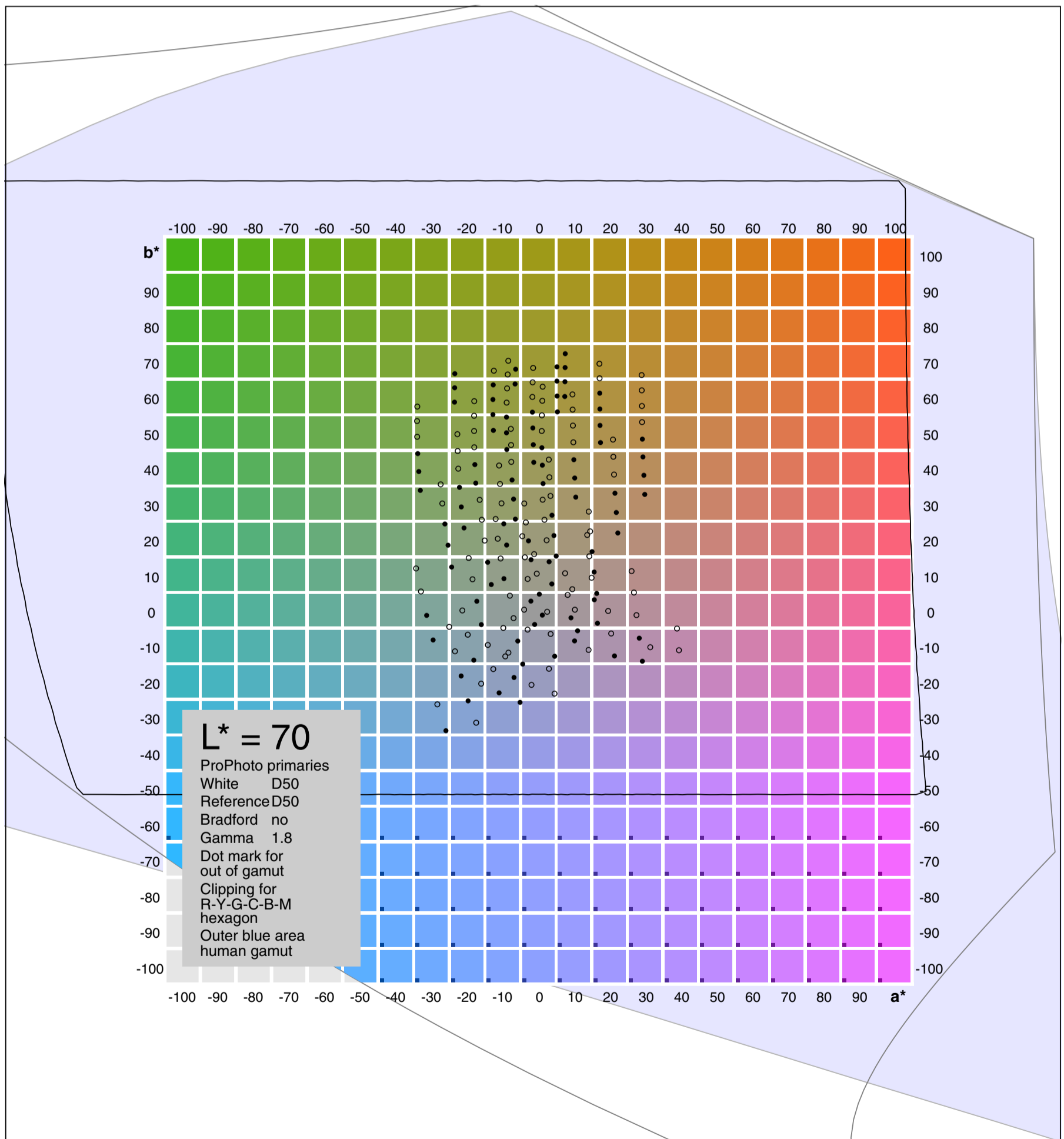
The graphic shows Lab values for ProPhotoRGB and available values for the CMYK profile PSO\_Coated\_NPscreen\_ISO12647\_eci.icc as black round dots. This is a profile for FM printing.





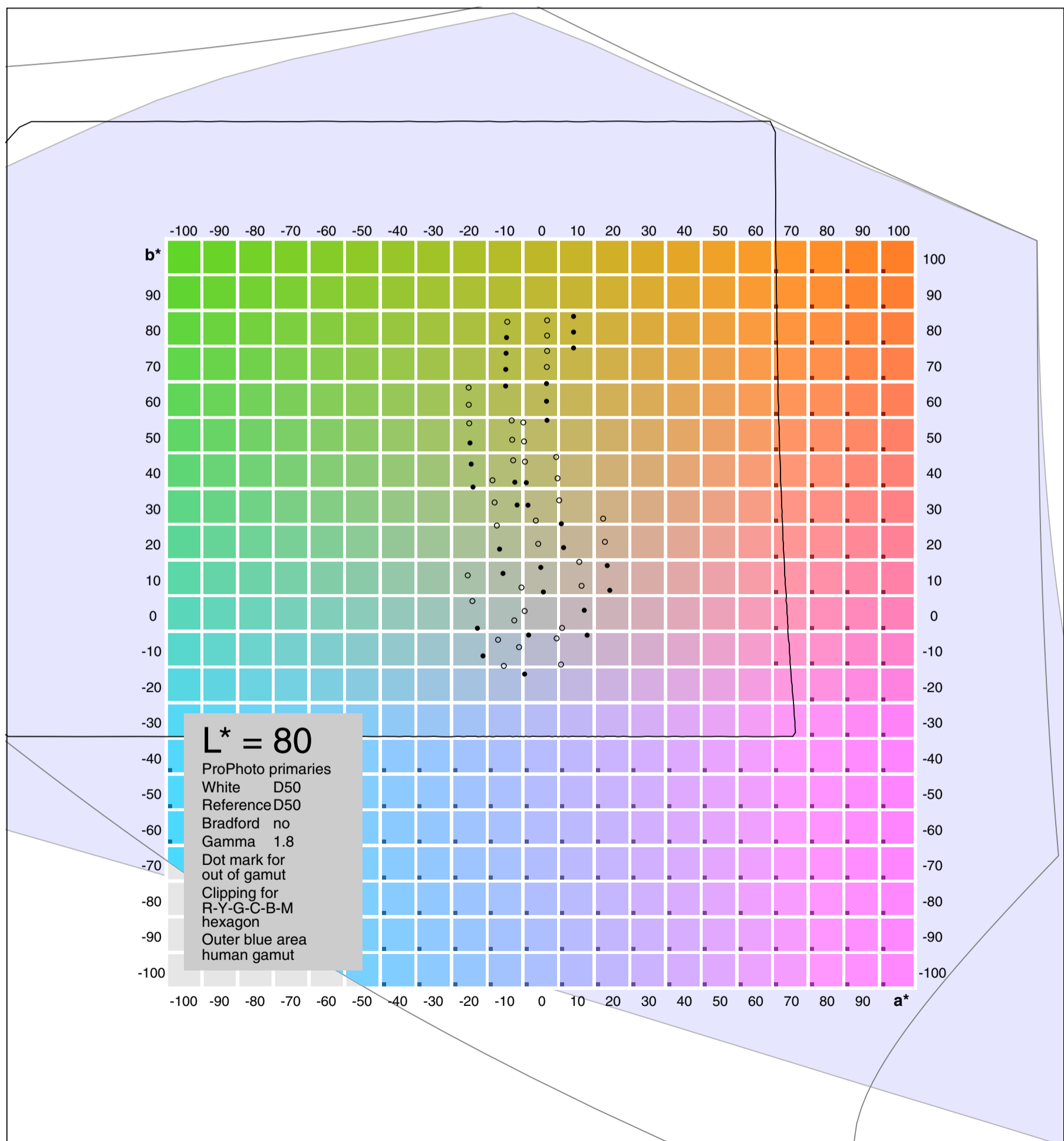
# 10.7 ProPhotoRGB L\*=70

The graphic shows Lab values for ProPhotoRGB and available values for the CMYK profile PSO\_Coated\_NPscreen\_ISO12647\_eci.icc as black round dots. This is a profile for FM printing.



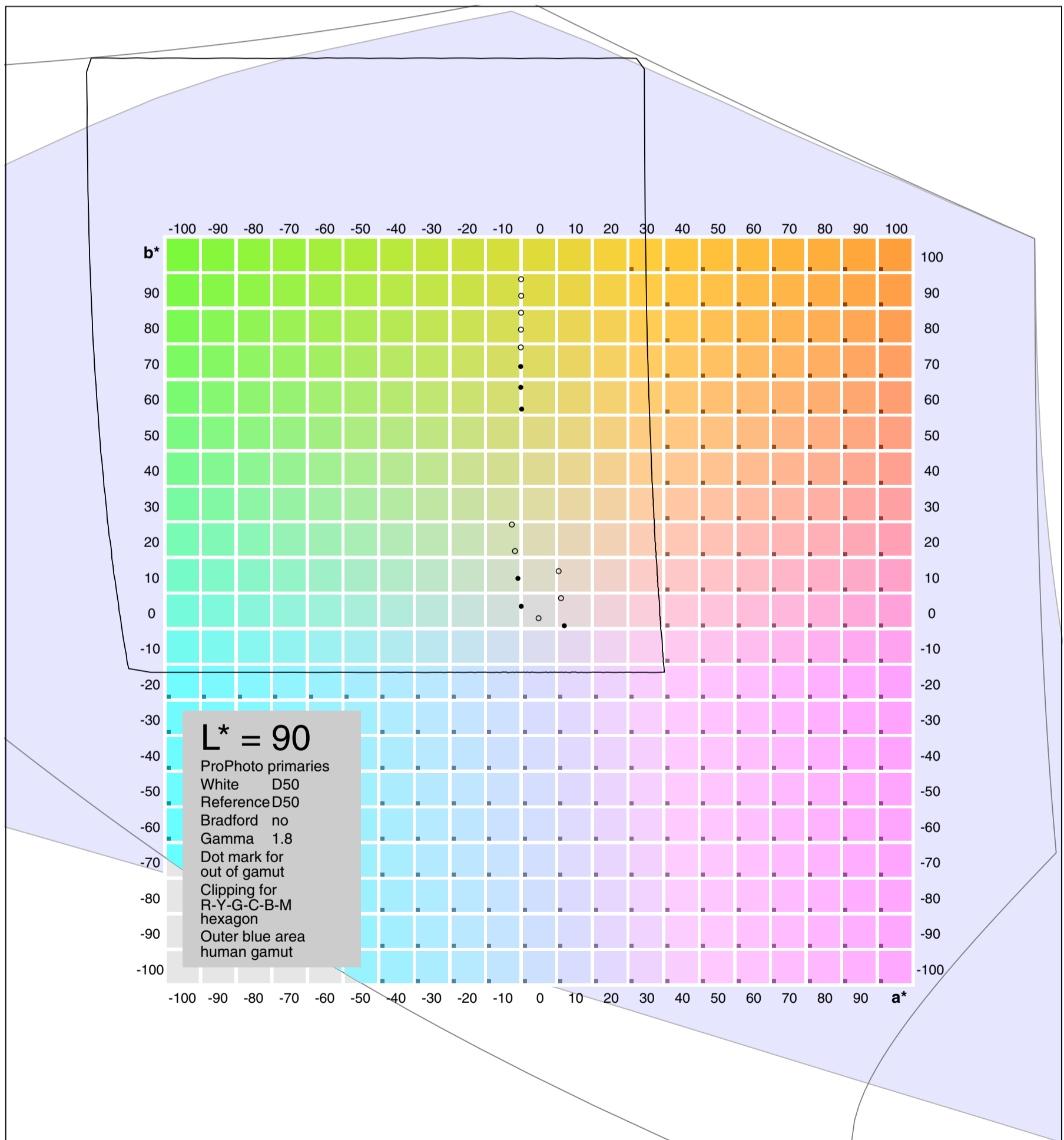
# 10.8 ProPhotoRGB L\*=80

The graphic shows Lab values for ProPhotoRGB and available values for the CMYK profile PSO\_Coated\_NPscreen\_ISO12647\_eci.icc as black round dots. This is a profile for FM printing.

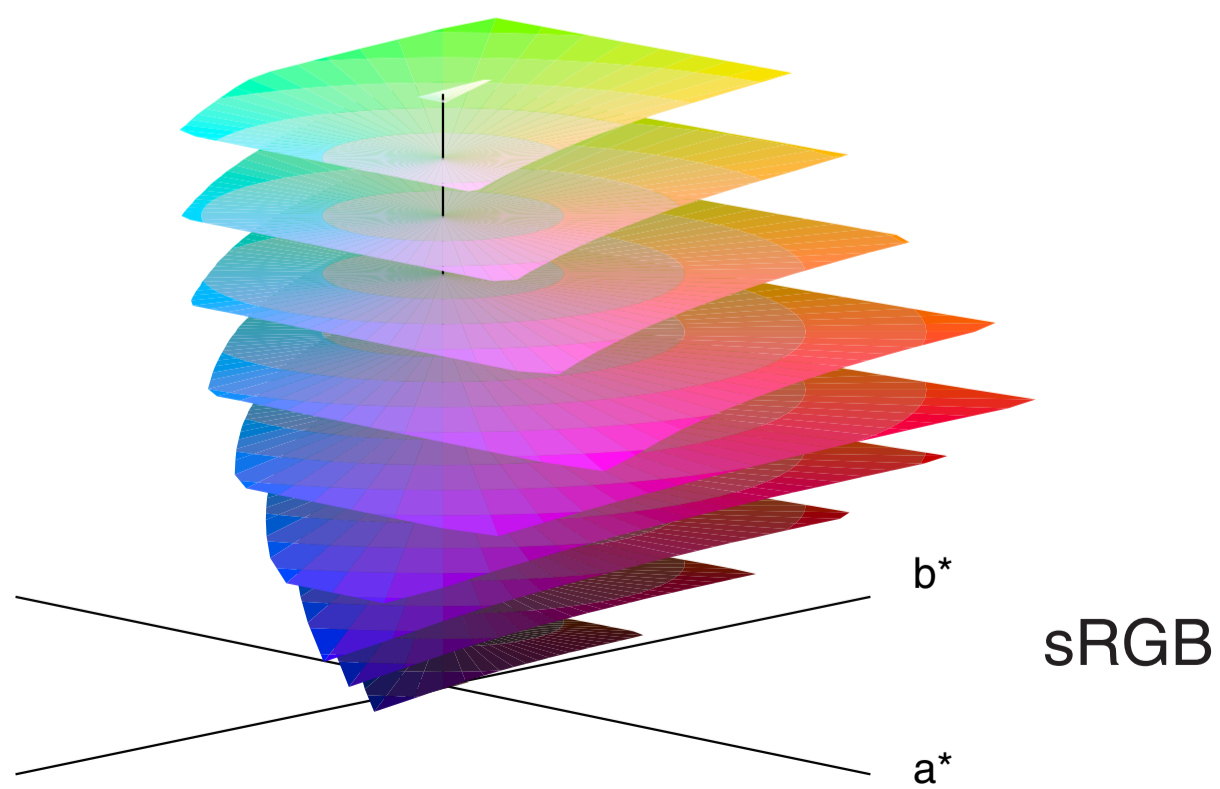
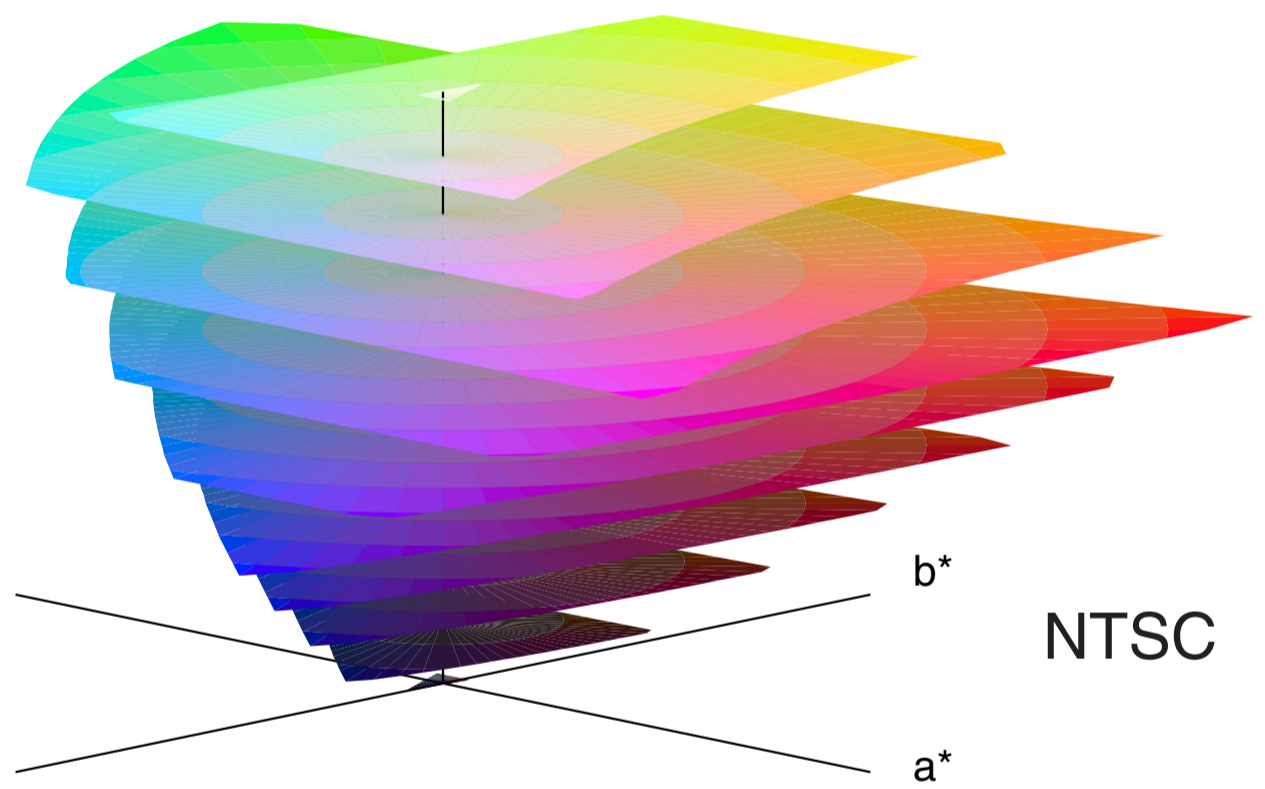
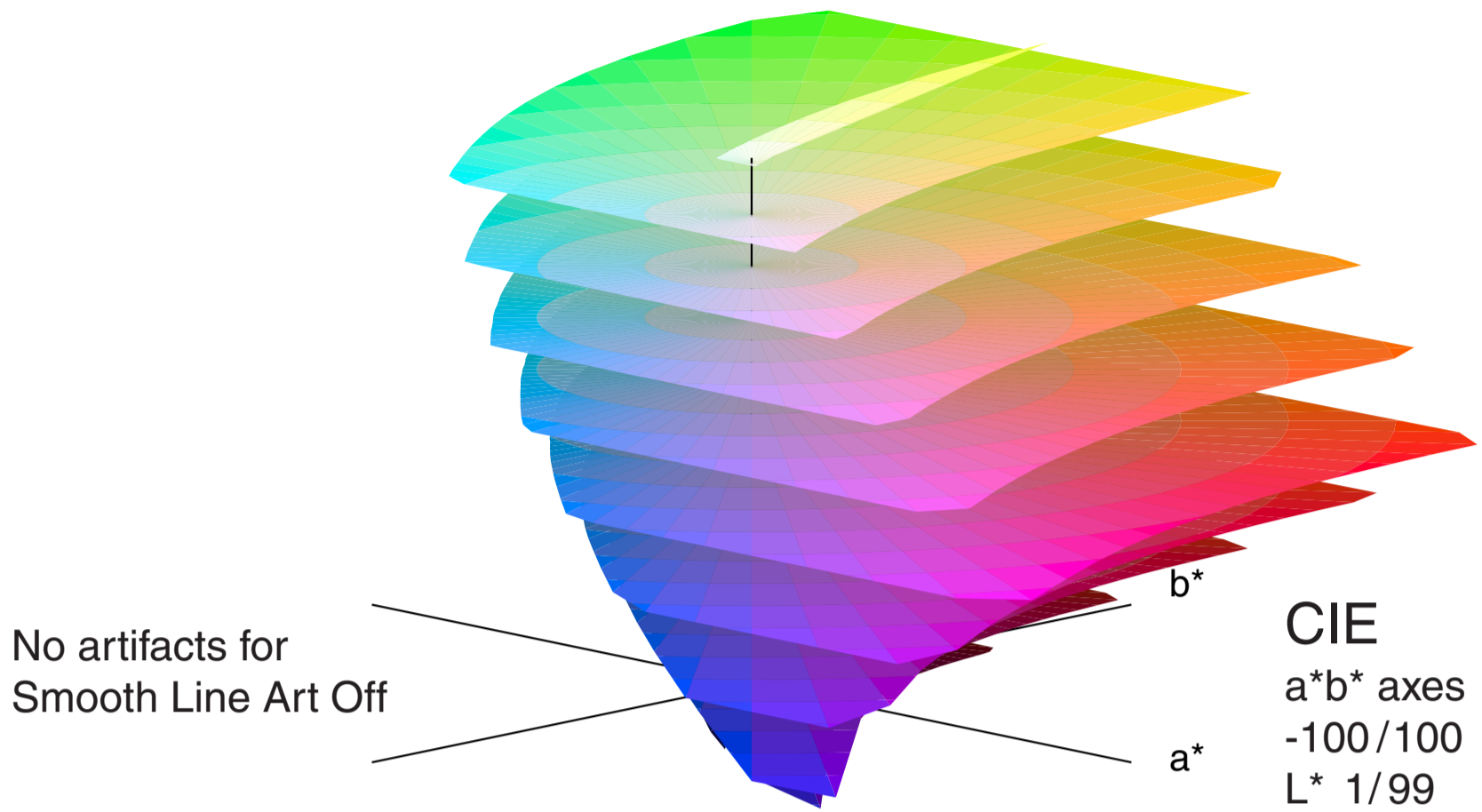


# 10.9 ProPhotoRGB L\*=90

The graphic shows Lab values for ProPhotoRGB and available values for the CMYK profile PSO\_Coated\_NPscreen\_ISO12647\_eci.icc as black round dots. This is a profile for FM printing.

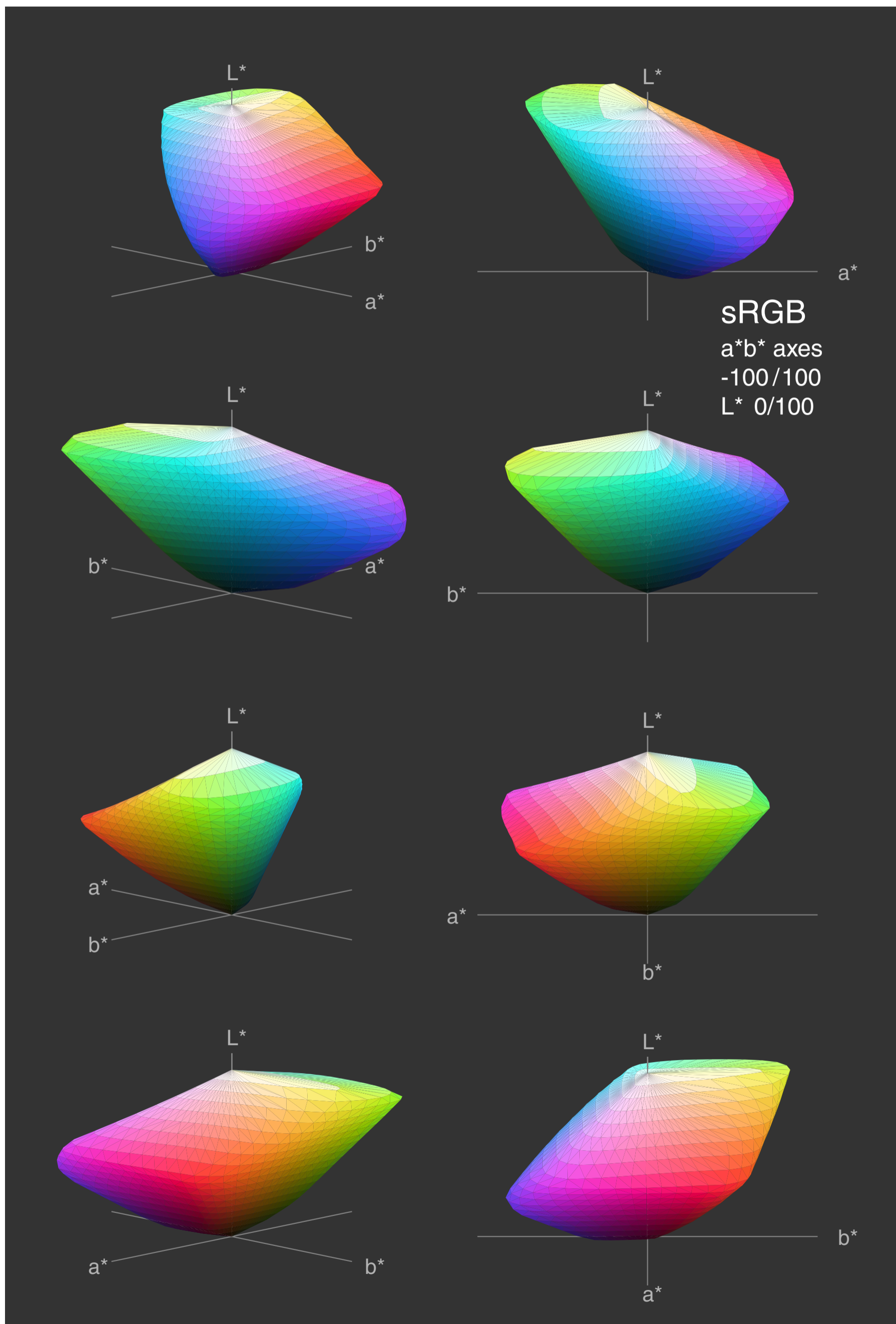


# 11.1 3D Views for three RGB-Spaces



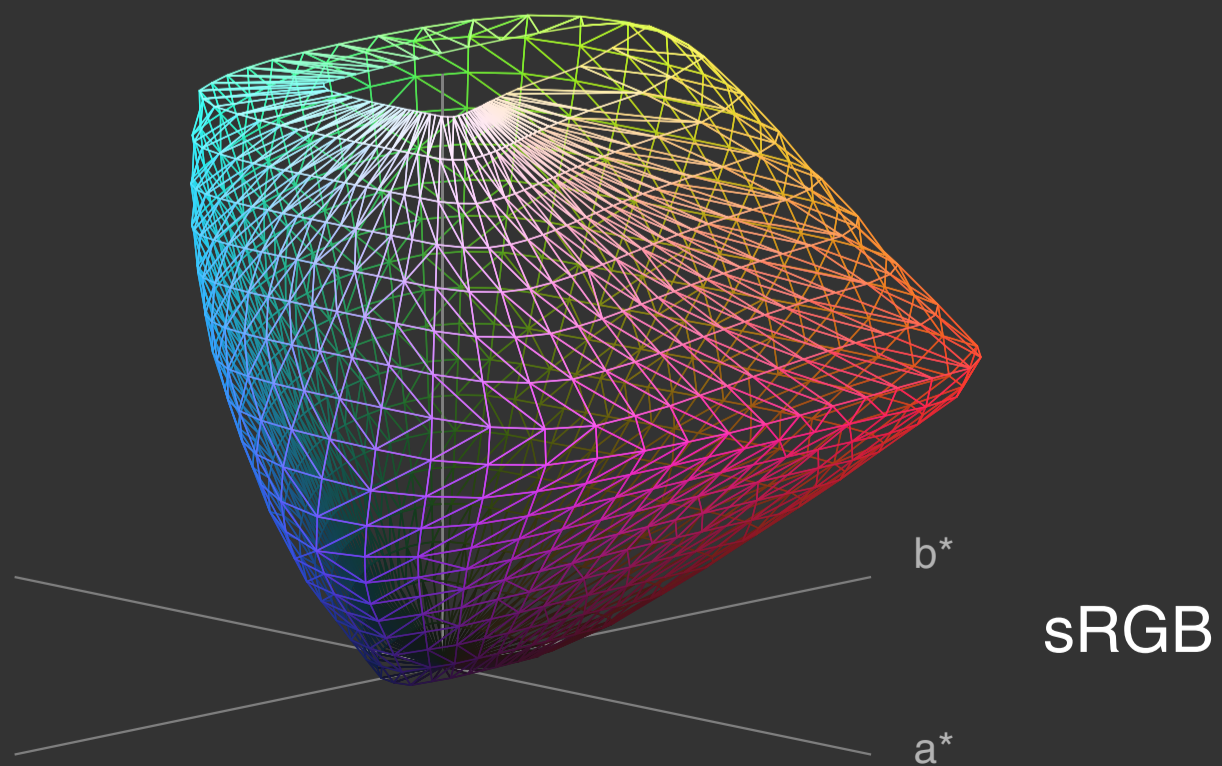
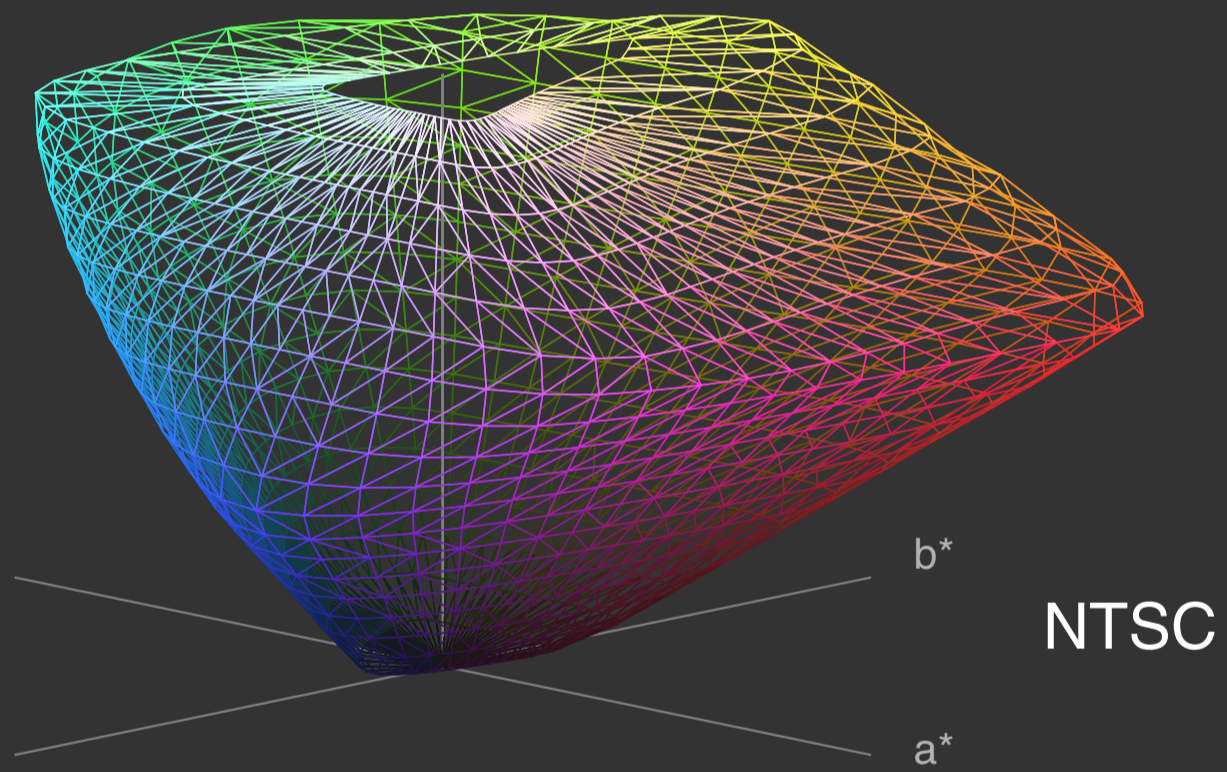
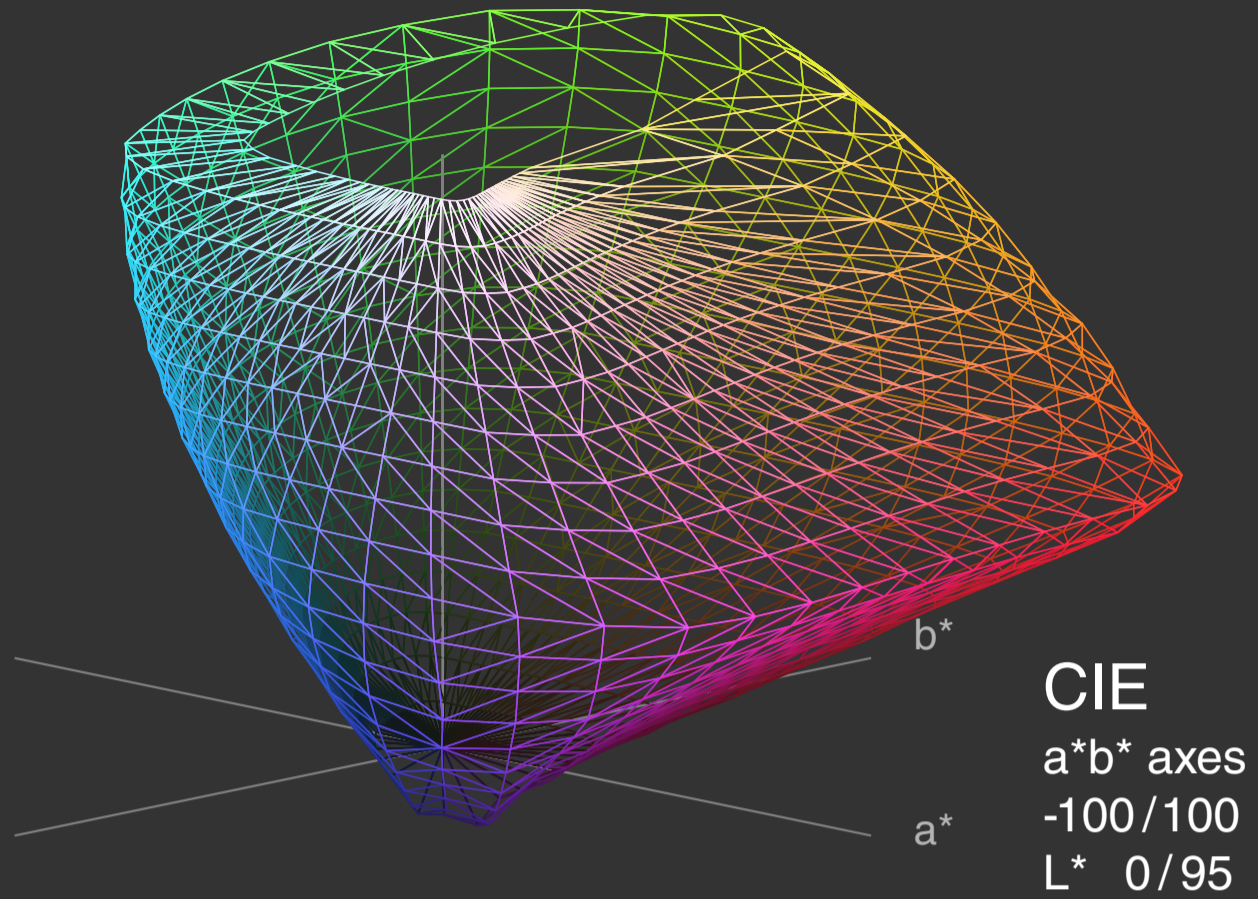
# 11.2 3D Views, Rotated, for sRGB

Volumes and wireframes use smoothed height contours





# 11.3 3D Views



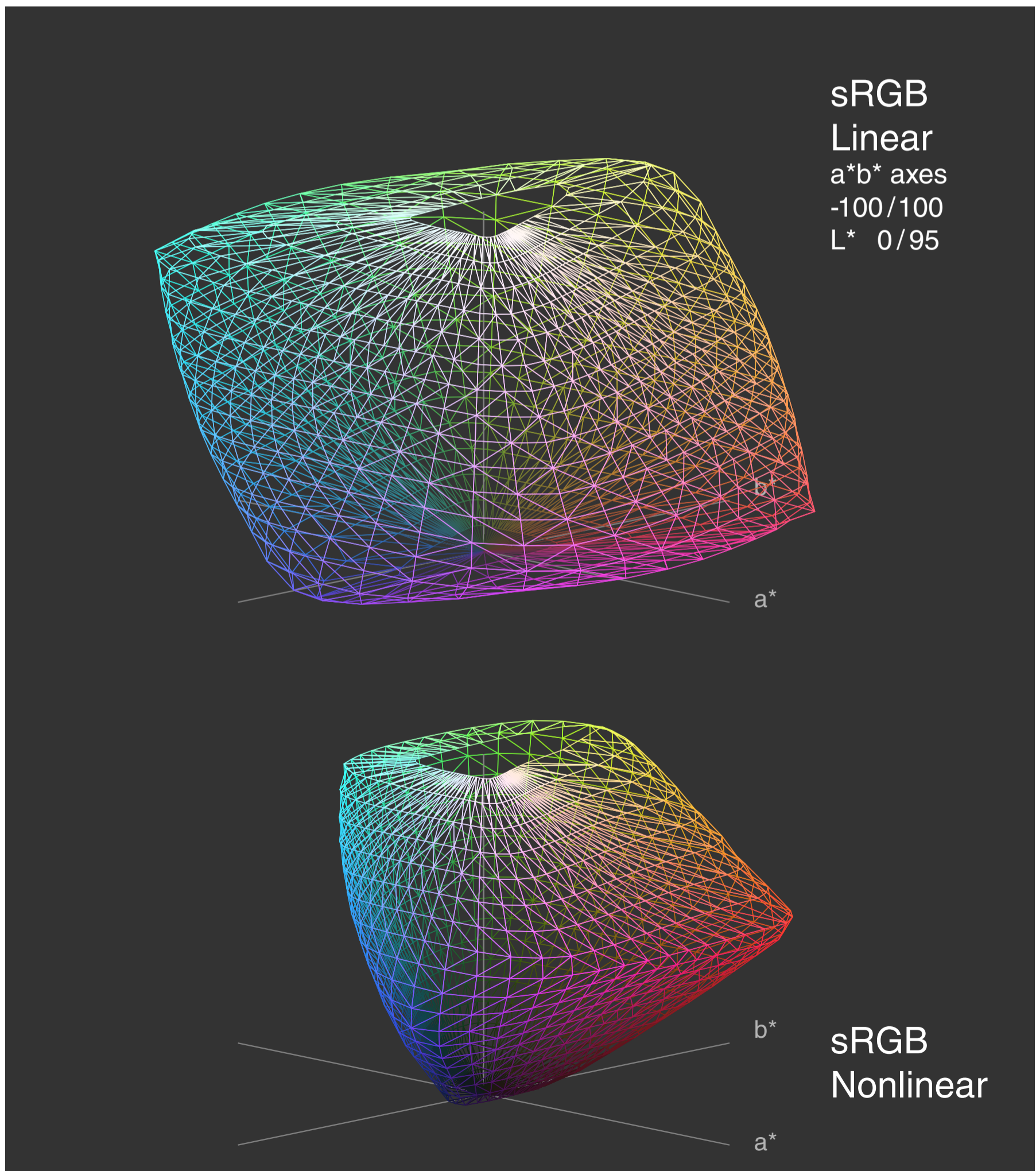


## 12. Linear and Standard Nonlinear CIELab

The bottom image shows the same 3D view for the sRGB color system as on the previous page. The top image shows a linear transformation, as in chapter 2.4:

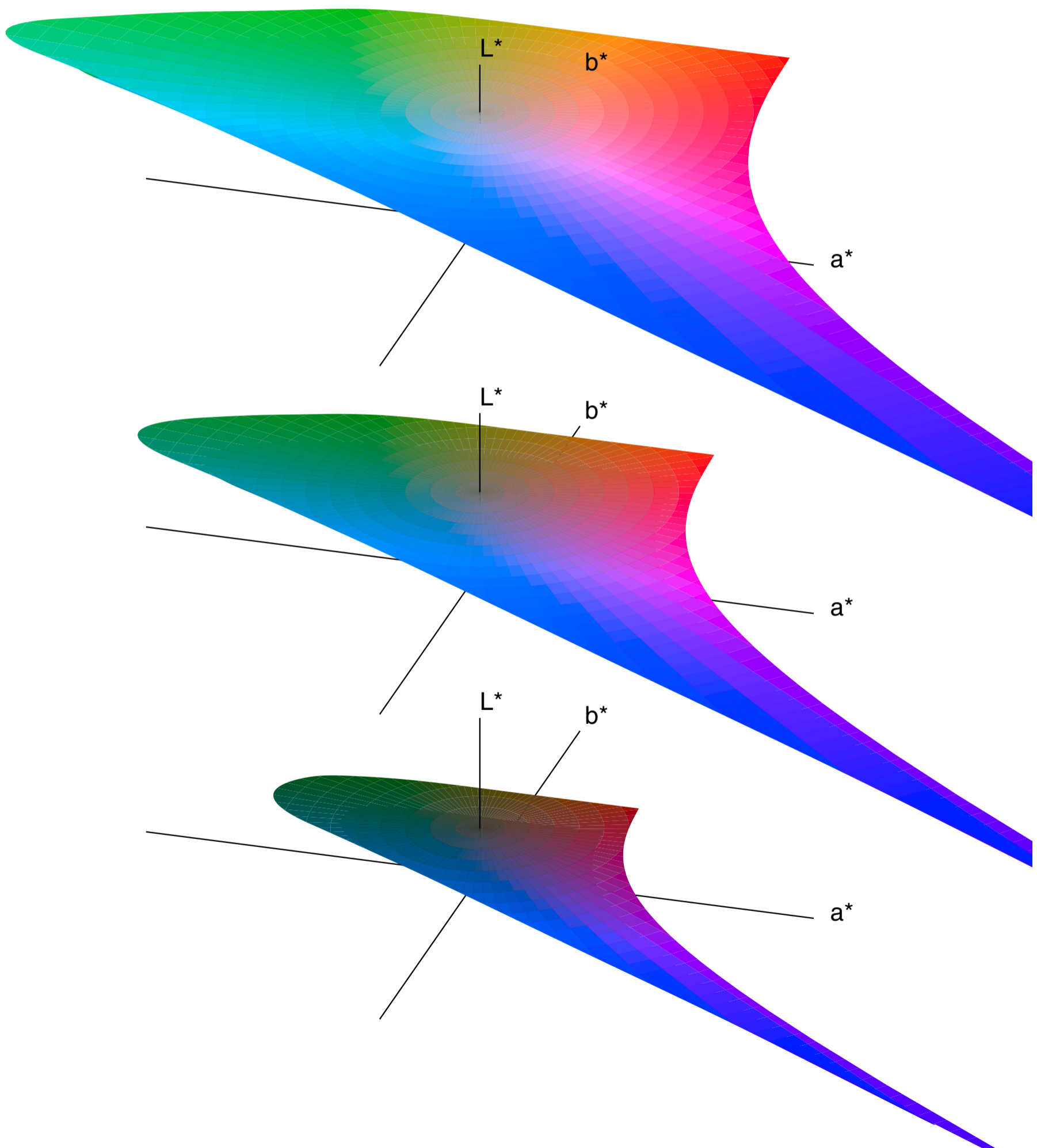
$$\begin{aligned}L^* &= 100 Y/Y_n \\ a^* &= 500(X/X_n - Y/Y_n) \\ b^* &= 200(Y/Y_n - Z/Z_n)\end{aligned}$$

Here we can see that the colors are too light for low luminance. This is the reason for the introduction of the nonlinear CIELab conversion, though the cubic root law is also not perfect.



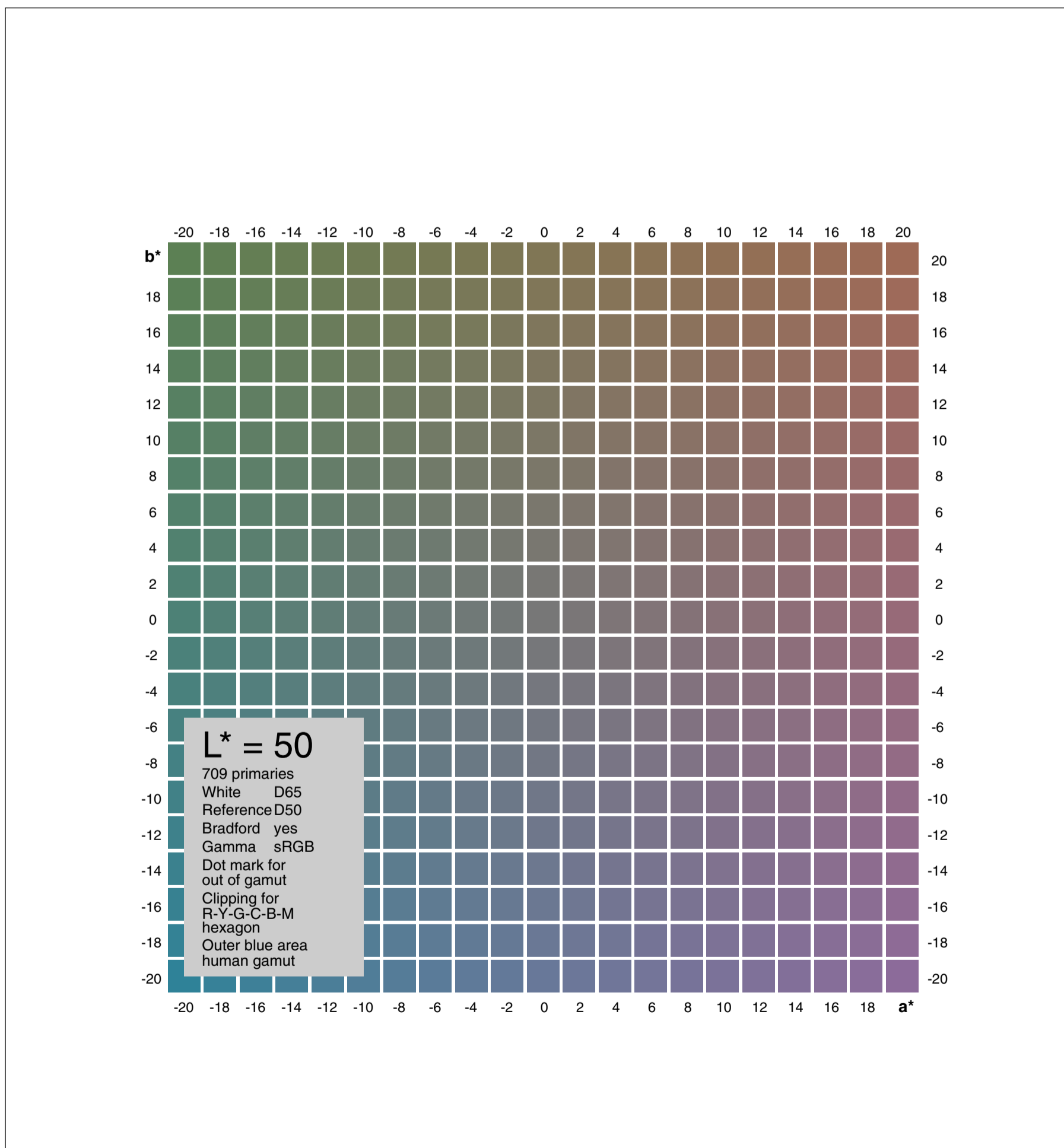
# 13. Human Gamut in CIELab

The human gamut in CIELab is a cone with the apex at the origin of the coordinate system. A three-dimensional illustration can be found in [2] as well. Here we see three cross sections at  $L^*=20, 40$  and  $60$ . The axes  $a^*$  and  $b^*$  are drawn with lengths  $\pm 200$ .

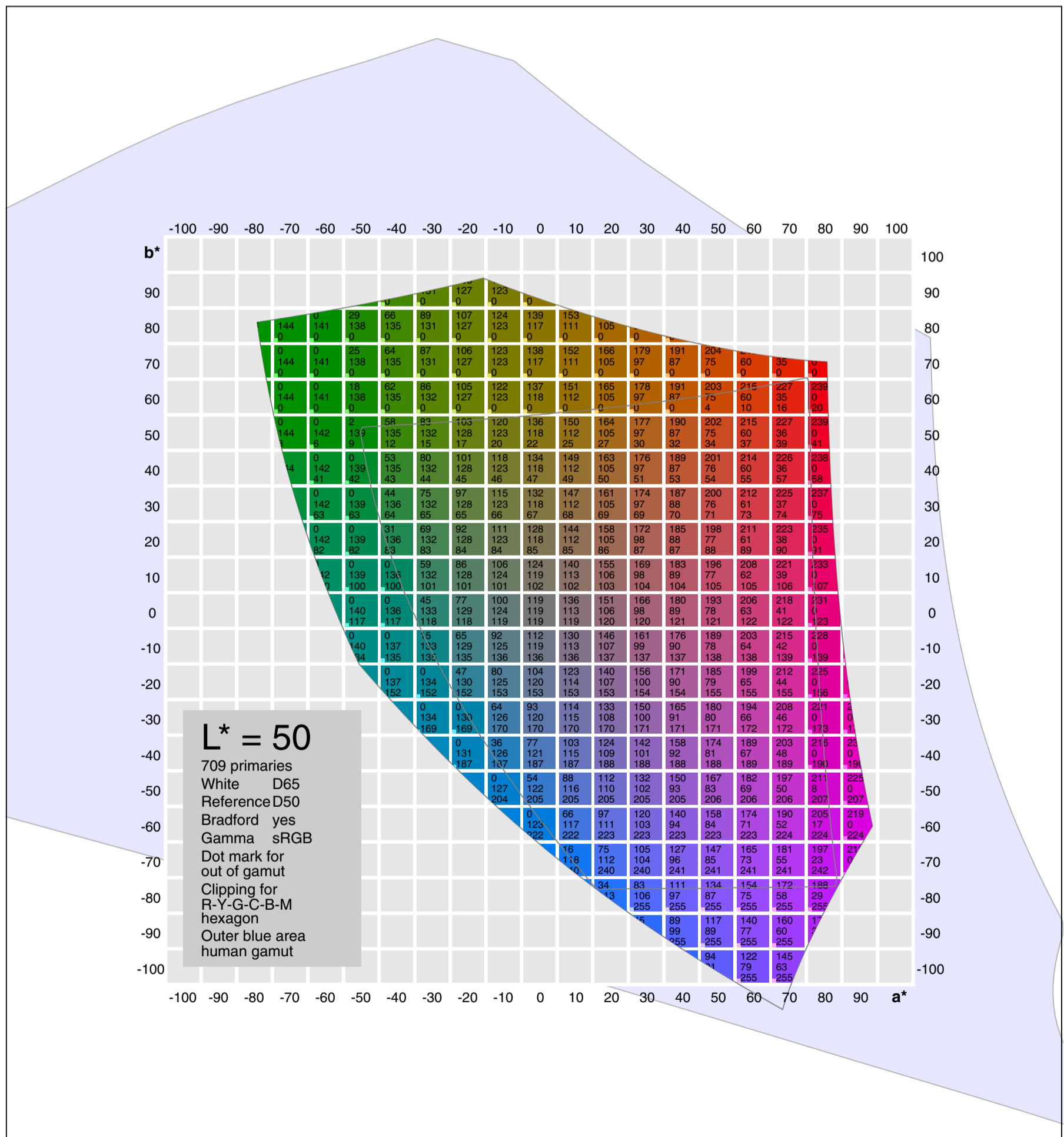




# 14. Low Chromaticity for sRGB $L^*=50$



# 15. sRGB L\*=50 with RGB Numbers



# 16.1 PostScript Kernels

```
/PrimsCie
{/txtP (CIE primaries) def
 /xr 0.73467 def /yr 0.26533 def
 /xg 0.27376 def /yg 0.71741 def
 /xb 0.16658 def /yb 0.00886 def
} def

/Prims709
{/txtP (709 primaries) def
 /xr 0.64000 def /yr 0.33000 def
 /xg 0.30000 def /yg 0.60000 def
 /xb 0.15000 def /yb 0.06000 def
} def

/PrimsNTSC
{/txtP (NTSC primaries) def
 /xr 0.67000 def /yr 0.33000 def
 /xg 0.21000 def /yg 0.71000 def
 /xb 0.14000 def /yb 0.08000 def
} def

/PrimsWide
{/txtP (WideGamut primaries) def
 /xr 0.734700 def /yr 0.265299 def
 /xg 0.115173 def /yg 0.826432 def
 /xb 0.156608 def /yb 0.017658 def
} def

/PrimsProP
{/txtP (ProPhoto primaries) def
 /xr 0.734698 def /yr 0.265302 def
 /xg 0.159599 def /yg 0.840401 def
 /xb 0.036600 def /yb 0.000107 def
} def

/PrimsAd98
{/txtP (AdobeRGB(98)) def
 /xr 0.64000 def /yr 0.33000 def
 /xg 0.21000 def /yg 0.71000 def
 /xb 0.15000 def /yb 0.06000 def
} def

/PrimsOpti
{/txtP (OptiRGB) def
 /xr 0.6658 def /yr 0.3340 def %610
 /xg 0.1929 def /yg 0.7816 def %535
 /xb 0.1355 def /yb 0.0399 def %465
} def

/WhiteEqE
{/xw 0.3333 def /yw 0.3333 def
 /txtW (Eq.Energy ) def
} def

/WhiteD65
{/xw 0.3127 def /yw 0.3290 def
 /txtW (D65) def
} def

/WhiteNTSC % Illuminant C
{/xw 0.3100 def /yw 0.3160 def
 /txtW (NTSC) def
} def

/White3000K % R.W.G.Hunt, Measuring Colours
{/xw 0.4368 def /yw 0.4041 def
 /txtW (3000K) def
} def

/White4000K
{/xw 0.3804 def /yw 0.3767 def
 /txtW (4000K) def
} def

/WhiteD50
{/xw 0.3457 def /yw 0.3585 def
 /txtW (D50) def
} def

/White6000K
```

## 16.2 PostScript Kernels

```
{/xw 0.3220 def /yw 0.3318 def
 /txtW (6000K) def
} def
/White7000K
{/xw 0.3063 def /yw 0.3165 def
 /txtW (7000K) def
} def
/White8000K
{/xw 0.2952 def /yw 0.3048 def
 /txtW (8000K) def
} def
/White9300K %
{/xw 0.2857 def /yw 0.2941 def % Wyszecki+Stiles interpolated
 /txtW (9300K) def
} def

/RefEqE
{/xn 0.3333 def /yn 0.3333 def
 /txtN (Eq.Energy ) def
} def
/RefD65
{/xn 0.3127 def /yn 0.3290 def
 /txtN (D65) def
} def
/RefNTSC
{/xn 0.3100 def /yn 0.3160 def
 /txtN (NTSC) def
} def
/RefD50
{/xn 0.3457 def /yn 0.3585 def
 /txtN (D50) def
} def

/MakeBrad
{% ICC-Spec p.110
 % Cone = CX*X; X = XC*Cone

% Cone Response
/CX11 0.8951 def /CX12 0.2664 def /CX13 -0.1614 def
/CX21 -0.7502 def /CX22 1.7135 def /CX23 0.0367 def
/CX31 0.0389 def /CX32 -0.0685 def /CX33 1.0296 def
% Inverse Cone Response
/D CX22 CX33 mul CX23 CX32 mul sub CX11 mul
  CX21 CX33 mul CX23 CX31 mul sub CX12 mul sub
  CX21 CX32 mul CX22 CX31 mul sub CX13 mul add def
/XC11 CX22 CX33 mul CX23 CX32 mul sub D div def
/XC12 CX12 CX33 mul CX13 CX32 mul sub D div neg def
/XC13 CX12 CX23 mul CX13 CX22 mul sub D div def
/XC21 CX21 CX33 mul CX23 CX31 mul sub D div neg def
/XC22 CX11 CX33 mul CX13 CX31 mul sub D div def
/XC23 CX11 CX23 mul CX13 CX21 mul sub D div neg def
/XC31 CX21 CX32 mul CX22 CX31 mul sub D div def
/XC32 CX11 CX32 mul CX12 CX31 mul sub D div neg def
/XC33 CX11 CX22 mul CX12 CX21 mul sub D div def

% Source
/Yws 1 def
/zw 1 xw sub yw sub def
/Xws xw yw div Yws mul def
/Zws zw yw div Yws mul def
% PCS
/Ywp 1 def
/zn 1 xn sub yn sub def
/Xwp xn yn div Ywp mul def
/Zwp zn yn div Ywp mul def

% Cone Source
/Rs CX11 Xws mul CX12 Yws mul add CX13 Zws mul add def
/Gs CX21 Xws mul CX22 Yws mul add CX23 Zws mul add def
/Bs CX31 Xws mul CX32 Yws mul add CX33 Zws mul add def
% Cone PCS
/Rp CX11 Xwp mul CX12 Ywp mul add CX13 Zwp mul add def
/Gp CX21 Xwp mul CX22 Ywp mul add CX23 Zwp mul add def
/Bp CX31 Xwp mul CX32 Ywp mul add CX33 Zwp mul add def
/all Rp Rs div def
```

## 16.3 PostScript Kernels

```
/a22 Gp Gs div def
/a33 Bp Bs div def

% Diag(aii)*CX
/X11 a11 CX11 mul def /X12 a11 CX12 mul def /X13 a11 CX13 mul def
/X21 a22 CX21 mul def /X22 a22 CX22 mul def /X23 a22 CX23 mul def
/X31 a33 CX31 mul def /X32 a33 CX32 mul def /X33 a33 CX33 mul def

% Bradford
% BM = XC*Diag(aii)*CX
/BM11 XC11 X11 mul XC12 X21 mul add XC13 X31 mul add def
/BM12 XC11 X12 mul XC12 X22 mul add XC13 X32 mul add def
/BM13 XC11 X13 mul XC12 X23 mul add XC13 X33 mul add def
/BM21 XC21 X11 mul XC22 X21 mul add XC23 X31 mul add def
/BM22 XC21 X12 mul XC22 X22 mul add XC23 X32 mul add def
/BM23 XC21 X13 mul XC22 X23 mul add XC23 X33 mul add def
/BM31 XC31 X11 mul XC32 X21 mul add XC33 X31 mul add def
/BM32 XC31 X12 mul XC32 X22 mul add XC33 X32 mul add def
/BM33 XC31 X13 mul XC32 X23 mul add XC33 X33 mul add def

% Inverse Bradford
/D BM22 BM33 mul BM23 BM32 mul sub BM11 mul
  BM21 BM33 mul BM23 BM31 mul sub BM12 mul sub
  BM21 BM32 mul BM22 BM31 mul sub BM13 mul add def
/MB11 BM22 BM33 mul BM23 BM32 mul sub D div def
/MB12 BM12 BM33 mul BM13 BM32 mul sub D div neg def
/MB13 BM12 BM23 mul BM13 BM22 mul sub D div def
/MB21 BM21 BM33 mul BM23 BM31 mul sub D div neg def
/MB22 BM11 BM33 mul BM13 BM31 mul sub D div def
/MB23 BM11 BM23 mul BM13 BM21 mul sub D div neg def
/MB31 BM21 BM32 mul BM22 BM31 mul sub D div def
/MB32 BM11 BM32 mul BM12 BM31 mul sub D div neg def
/MB33 BM11 BM22 mul BM12 BM21 mul sub D div def

% Bradford matrix XYZ(D50)=BM*XYZ(D65)
%/BM11 1.0479 def /BM12 0.0229 def /BM13 -0.0502 def
%/BM21 0.0296 def /BM22 0.9904 def /BM23 -0.0171 def
%/BM31 -0.0092 def /BM32 0.0151 def /BM33 0.7519 def
% XYZ(D50)=BM*XYZ(D65)
} def

/PrimToMat
{ % matrix CR for XYZ(WP)=CR*RGB(WP)
  /zr 1 xr sub yr sub def
  /zg 1 xg sub yg sub def
  /zb 1 xb sub yb sub def
  /zw 1 xw sub yw sub def
  /zn 1 xn sub yn sub def
  /D xr xb sub yg yb sub mul yr yb sub xg xb sub mul sub def
  /U xw xb sub yg yb sub mul yw yb sub xg xb sub mul sub def
  /V xr xb sub yw yb sub mul yr yb sub xw xb sub mul sub def
  /u U D div def
  /v V D div def
  /w 1 u sub v sub def
  % XYZ(WP)=CR(WP)*RGB(WP)
  /CR11 u xr mul yw div def /CR12 v xg mul yw div def /CR13 w xb mul yw div def
  /CR21 u yr mul yw div def /CR22 v yg mul yw div def /CR23 w yb mul yw div def
  /CR31 u zr mul yw div def /CR32 v zg mul yw div def /CR33 w zb mul yw div def

  Brad
  {% XR(D50)=BM*CR(D65)
  /XR11 BM11 CR11 mul BM12 CR21 mul add BM13 CR31 mul add def
  /XR12 BM11 CR12 mul BM12 CR22 mul add BM13 CR32 mul add def
  /XR13 BM11 CR13 mul BM12 CR23 mul add BM13 CR33 mul add def
  /XR21 BM21 CR11 mul BM22 CR21 mul add BM23 CR31 mul add def
  /XR22 BM21 CR12 mul BM22 CR22 mul add BM23 CR32 mul add def
  /XR23 BM21 CR13 mul BM22 CR23 mul add BM23 CR33 mul add def
  /XR31 BM31 CR11 mul BM32 CR21 mul add BM33 CR31 mul add def
  /XR32 BM31 CR12 mul BM32 CR22 mul add BM33 CR32 mul add def
  /XR33 BM31 CR13 mul BM32 CR23 mul add BM33 CR33 mul add def }
  {% XR(D65)=CR(D65)
```

## 16.4 PostScript Kernels

```
/XR11 CR11 def /XR12 CR12 def /XR13 CR13 def
/XR21 CR21 def /XR22 CR22 def /XR23 CR23 def
/XR31 CR31 def /XR32 CR32 def /XR33 CR33 def } ifelse
% Inverse of XR
/D XR22 XR33 mul XR23 XR32 mul sub XR11 mul
  XR21 XR33 mul XR23 XR31 mul sub XR12 mul sub
  XR21 XR32 mul XR22 XR31 mul sub XR13 mul add def
/RX11 XR22 XR33 mul XR23 XR32 mul sub D div def
/RX12 XR12 XR33 mul XR13 XR32 mul sub D div neg def
/RX13 XR12 XR23 mul XR13 XR22 mul sub D div def
/RX21 XR21 XR33 mul XR23 XR31 mul sub D div neg def
/RX22 XR11 XR33 mul XR13 XR31 mul sub D div def
/RX23 XR11 XR23 mul XR13 XR21 mul sub D div neg def
/RX31 XR21 XR32 mul XR22 XR31 mul sub D div def
/RX32 XR11 XR32 mul XR12 XR31 mul sub D div neg def
/RX33 XR11 XR22 mul XR12 XR21 mul sub D div def
/Yw 1 def
/Xw xw yw div Yw mul def
/Zw zw yw div Yw mul def
/Yn 1 def
/Xn xn yn div Yn mul def
/Zn zn yn div Yn mul def
} def

/LABtoRGB
{%/c0 1 3 div def Actual definition in main program
%/c1 0.008856 def
%/c2 7.787 def
%/c3 16 116 div def
%/c4 0.206893 def
/Y1 L* 0.16 add 1.16 div def
/X1 a* 5.0 div Y1 add def
/Z1 Y1 b* 2.0 div sub def
/X X1 c4 le {X1 c3 sub c2 div }{ X1 3 exp } ifelse Xn mul def
/Y Y1 c4 le {Y1 c3 sub c2 div }{ Y1 3 exp } ifelse Yn mul def
/Z Z1 c4 le {Z1 c3 sub c2 div }{ Z1 3 exp } ifelse Zn mul def
/R X RX11 mul Y RX12 mul add Z RX13 mul add def
/G X RX21 mul Y RX22 mul add Z RX23 mul add def
/B X RX31 mul Y RX32 mul add Z RX33 mul add def
} bind def

/RGBtoLAB
{%/c0 1 3 div def Actual definition in main program
%/c1 0.008856 def
%/c2 7.787 def
%/c3 16 116 div def
%/c4 0.206893 def
/X R XR11 mul G XR12 mul add B XR13 mul add def
/Y R XR21 mul G XR22 mul add B XR23 mul add def
/Z R XR31 mul G XR32 mul add B XR33 mul add def
/X1 X Xn div dup c1 le { c2 mul c3 add }{ c0 exp } ifelse def
/Y1 Y Yn div dup c1 le { c2 mul c3 add }{ c0 exp } ifelse def
/Z1 Z Zn div dup c1 le { c2 mul c3 add }{ c0 exp } ifelse def
/a* X1 Y1 sub 5.0 mul def
/b* Y1 Z1 sub 2.0 mul def
/ga* a* 1.05 add 100 mm mul def
/gb* b* 1.05 add 100 mm mul def
} bind def

/IGamma
{ /iga 1 gam div def
/og false def % out of gamut
R 0 lt { /R 0 def /og true def } if
G 0 lt { /G 0 def /og true def } if
B 0 lt { /B 0 def /og true def } if
R 1 gt { /R 1 def /og true def } if
G 1 gt { /G 1 def /og true def } if
B 1 gt { /B 1 def /og true def } if
gam 2.4 ne
{/R R iga exp def /G G iga exp def /B B iga exp def }
{/R R dup 0.00304 lt {12.92 mul }{iga exp 1.055 mul 0.055 sub } ifelse def
/G G dup 0.00304 lt {12.92 mul }{iga exp 1.055 mul 0.055 sub } ifelse def
/B B dup 0.00304 lt {12.92 mul }{iga exp 1.055 mul 0.055 sub } ifelse def } ifelse
R G B setrgbcolor
} bind def
/xyYtoLAB
```

## 16.5 PostScript Kernels

```
% input x,y,Y
{ /z 1 x sub y sub def
  /X x y div Y mul def
  /Z z y div Y mul def
  /R X RX11 mul Y RX12 mul add Z RX13 mul add def
  /G X RX21 mul Y RX22 mul add Z RX23 mul add def
  /B X RX31 mul Y RX32 mul add Z RX33 mul add def
  /max R def
  G max gt {/max G def } if
  B max gt {/max B def } if
  /R R max div def
  /G G max div def
  /B B max div def
  RGBtoLAB
} bind def

% -----

%--Choose Gamma
/G 1 def
G 0 eq {/gam 1.8 def} if % ProPhoto
G 1 eq {/gam 2.2 def} if
G 2 eq {/gam 2.4 def} if % sRGB

%--Choose one
/P 0 def
P 0 eq { PrimsCie } if
P 1 eq { Prims709 } if
P 2 eq { PrimsNTSC } if
P 3 eq { PrimsWide } if
P 4 eq { PrimsAd98 } if
P 5 eq { PrimsProP } if

%--Choose one
/W 0 def
W 0 eq { WhiteEqE } if
W 1 eq { WhiteD65 } if
W 2 eq { WhiteNTSC } if
W 3 eq { White3000K } if
W 4 eq { White4000K } if
W 5 eq { WhiteD50 } if % also Wide Gamut and ProPhoto
W 6 eq { White6000K } if
W 7 eq { White7000K } if
W 8 eq { White8000K } if
W 9 eq { White9300K } if

%--Choose one
/N 5 def
N 0 eq { RefEqE } if
N 1 eq { RefD65 } if
N 2 eq { RefNTSC } if
N 5 eq { RefD50 } if

%--Choose one
/B 1 def
B 0 eq { /Brad false def } if
B 1 eq { /Brad true def } if

%--Choose one
/L 5 def
L 0 eq {/L* 0.01 def} if
L 1 eq {/L* 0.1 def} if
L 2 eq {/L* 0.2 def} if
L 3 eq {/L* 0.3 def} if
L 4 eq {/L* 0.4 def} if
L 5 eq {/L* 0.5 def} if
L 6 eq {/L* 0.6 def} if
L 7 eq {/L* 0.7 def} if
L 8 eq {/L* 0.8 def} if
L 9 eq {/L* 0.9 def} if
L 10 eq {/L* 0.99 def} if

MakeBrad

PrimToMat
```



# 17.1 Mapping CIELab to xyY

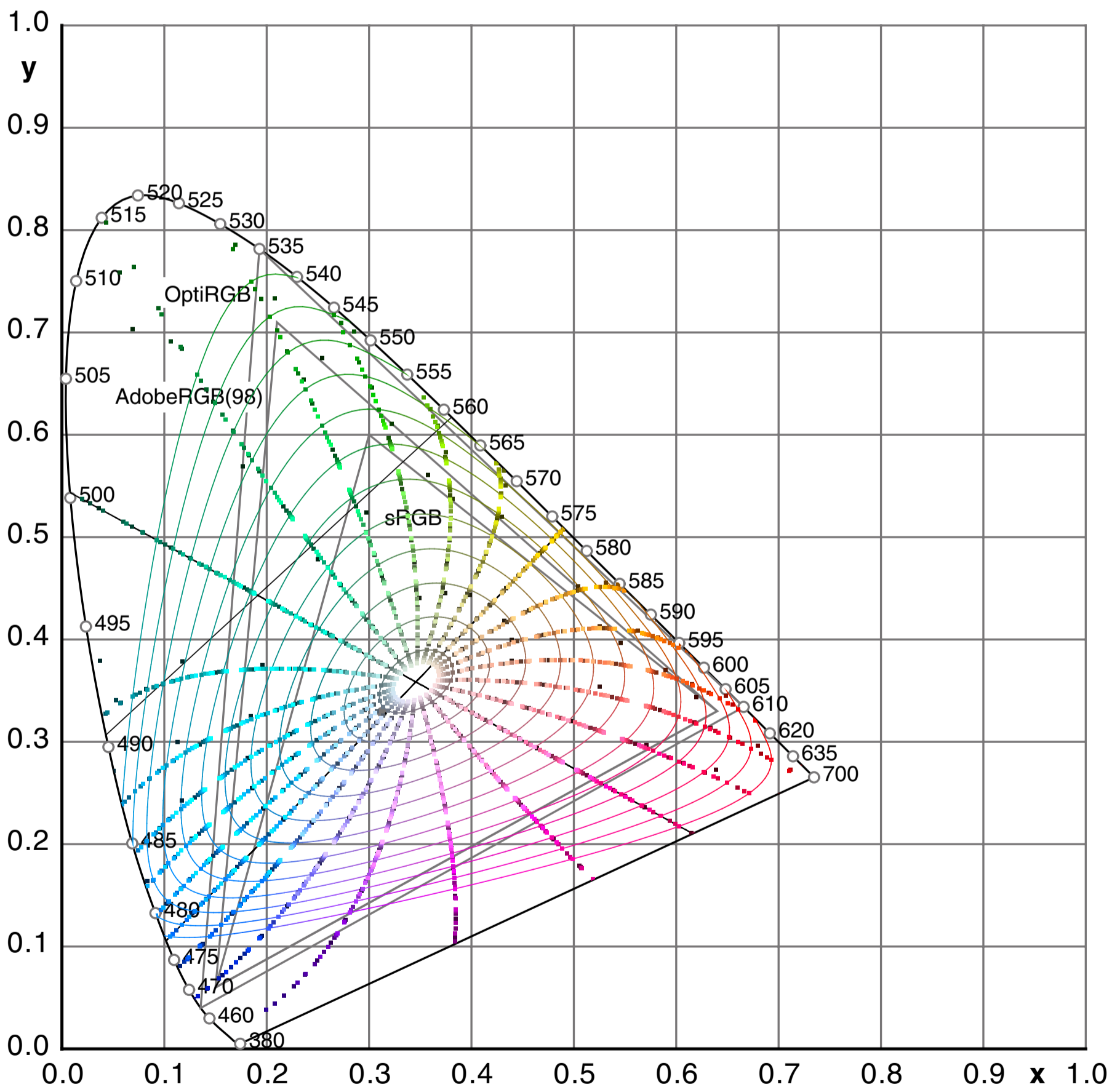
Clipped for human gamut

L = 10, 20 .. 90  
 Radius = 10, 20 ..120  
 Angle = 0, 15 .. 345°

Lines of constant radius for L=50  
 (circular lines, lines of constant ,chroma')  
 Lines of constant ,hue' (radial lines)  
 On straight lines we have either a=0 or b=0

A new working space is introduced:

OptiRGB  
 xr = 0.6658 yr = 0.3340 610nm  
 xg = 0.1929 yg= 0.7816 535nm  
 xb = 0.1355 yb= 0.0399 465nm  
 Gamma=2.2  
 D65



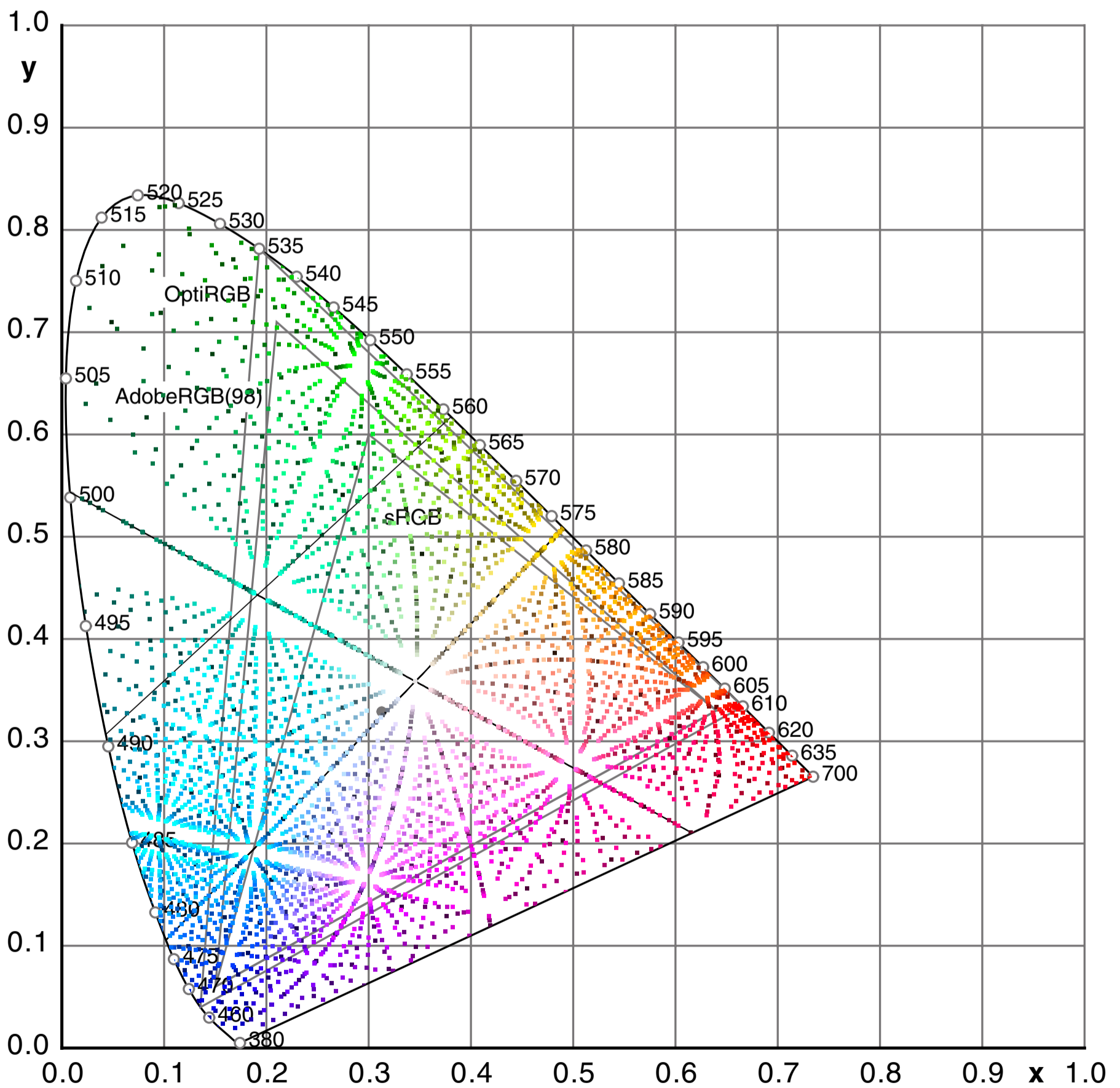


# 17.2 Mapping CIE Lab to xyY

Clipped for human gamut

L = 10, 20 .. 90  
a = -120, -110 .. +120  
b = -120, -110 .. +120

On straight lines we have either  
a=0 or b=0



## 17.3 Mapping CIE Lab to xyY

Clipped for the xy-unit-square. Many colors are out of human gamut (chapter 13).

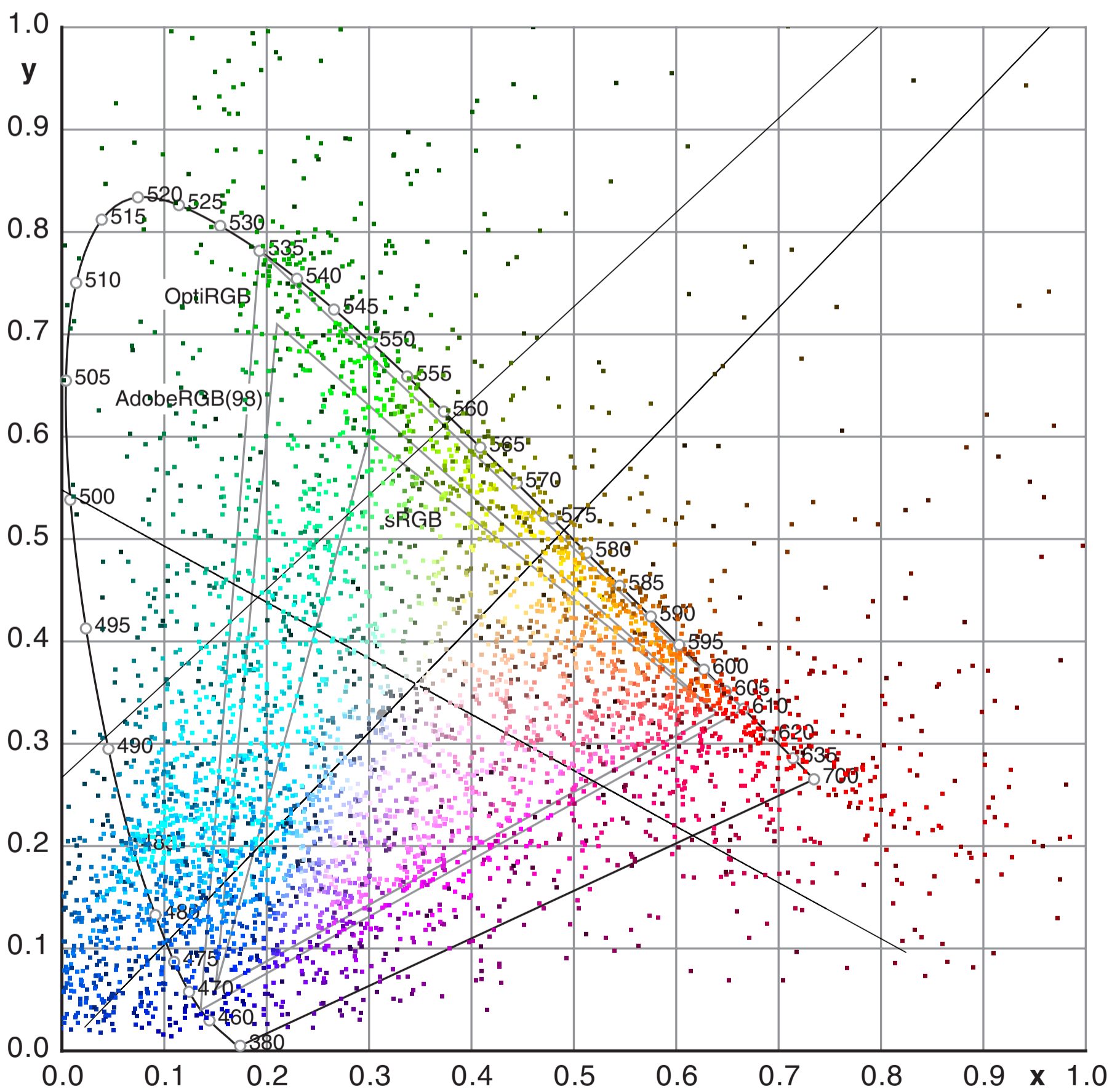
Random values

L = 5 .. 95

a = -120 .. +120

b = -120 .. +120

On straight lines we have either  
a=0 or b=0



# 18. Number of Different Colors

Test images sRGB with 1 million pixels each are converted to CIE Lab. The numbers in CIE Lab are rounded for 256/256/256 or 100/256/256 or 50/128/128 different levels. The CIE Lab values are converted back to sRGB, everything with double precision. The number of different colors in sRGB is counted before and after the conversion.

The first test image (not shown) contains random values for sRGB. The second is this portrait, cropped for height and width 1000 pixels.



## Test results

Mode	Random	Portrait
Pixels	1.000.000	1.000.000
Colors Source sRGB	969.864	182.629
Colors 256/256/256	795.538	89.451
Colors 100/256/256	611.904	47.006
Colors 50/128/128	146.712	9.485

The random image uses the whole sRGB space, the portrait only a small part. Therefore a loss of levels by percentage because of quantization is more likely for the portrait.

### Number N of colors in CIE Lab for sRGB, aRGB = AdobeRGB(98) and pRGB = ProPhotoRGB:

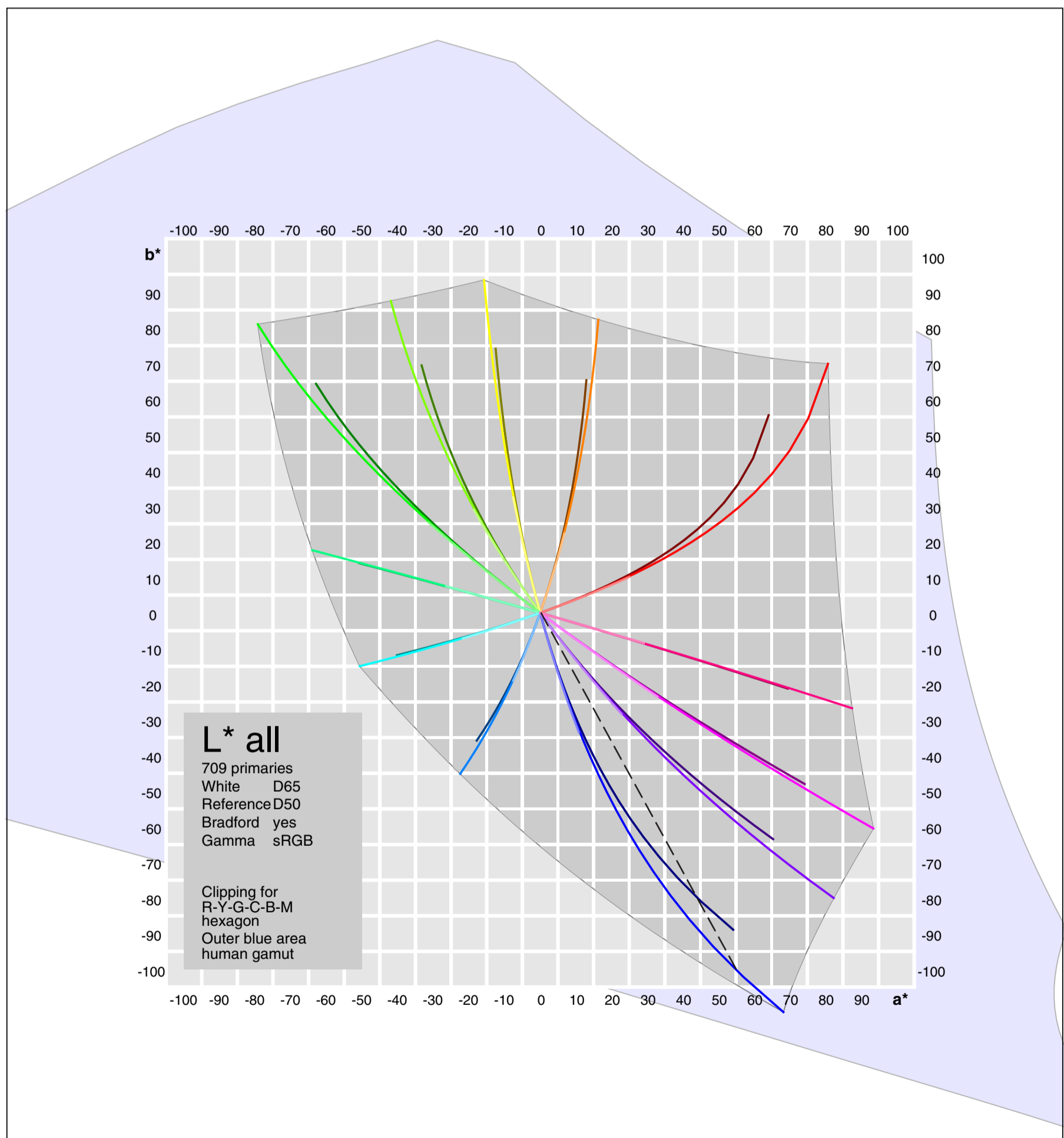
```

N=0
Define RGB space
For L*=0 To 100 Step 2 Do
For a*=-128 To +128 Step 2 Do
For b*=-128 To +128 Step 2 Do
Begin
Convert L*,a*,b* into R,G,B
If R,G,B in [0...255] Then N=N+1
End
N=8*N
    
```

Number N	
sRGB	832.752
aRGB	1.208.912
pRGB	2.659.728

## 19.1 HLS-Hue for sRGB in CIELab

The graphic shows lines with constant HLS-Hue  $H=0^\circ, 30^\circ, \dots, 330^\circ$  with varying HLS-Saturation  $S=0 \dots 1$  for three HLS-Lightnesses  $L=0.25, 0.50, 0.75$  in one plane  $a^*, b^*$ . Lines for Hues  $k \cdot 60^\circ$  hit the primaries RGB and the secondaries CMY at  $S=1$ . The inputs  $H, L, S$  are *here* converted by Foley-HLS [9] into *linear* RGB, whereas HLS to RGB delivers traditionally gamma encoded *nonlinear* values  $R'G'B'$ . Lines with constant HLS-Hue appear considerably curved in a CIELab plane  $a^*, b^*$ . Perceived Hue can be defined by Munsell colors. Lines with constant Munsell hue appear curved in CIELab as well. The dotted line is an interpolation path, see next page.



## 19.2 HLS-Hue for sRGB in CIELab

It has been often observed, that some blues which are out of gamut for a printer were mapped to in-gamut blues with a magenta tint [10].

The graphic shows two gradients with endpoints Lab1=35/55/-100 RGB1=47/47/250 (blue) and Lab2=35/0/0 RGB2=82/82/82 (gray). The upper gradient was made in sRGB, the lower in Lab.

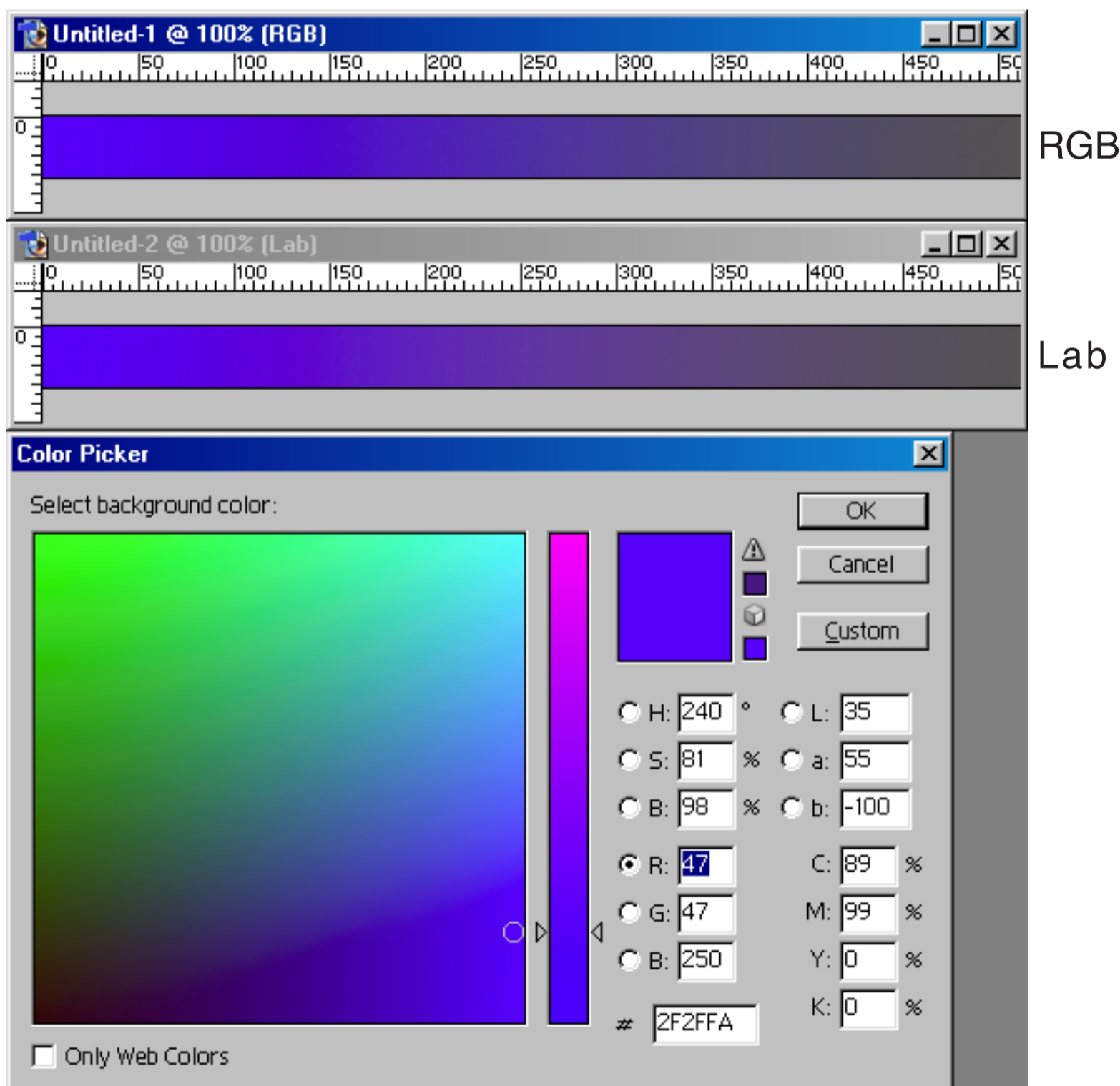
The sRGB gradient shows already a magenta tint though all interpolated colors have equal values  $R=G$ , which should result in less saturated blue without a tint.

According to [11] one has to take into account optical illusions:

Abney effect: hue changes with colorimetric purity (here).

Bezold-Brücke effect: hue changes with luminance.

The Lab gradient shows an even stronger magenta tint. On the previous page one can see a linear interpolation (dotted line) from Lab1 to Lab2. This interpolation path goes through magenta area. Therefore a gamut compression in planes of constant Lab hue is not ideal, but the other effects should not be ignored.



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This doc:

<http://docs-hoffmann.de/cielab03022003.pdf>

Gernot Hoffmann  
February 03 / 2003 + September 10 / 2013 + July 11 / 2019  
Website  
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