Conflict of Interests, (Implicit) Coalitions and Nash Policy Games.*

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May 2009

Abstract

By introducing the concepts of implicit coalitions and conflict of interests in a multiple-player context, this paper generalizes some theorems on policy invariance and equilibrium existence and uniqueness for LQ policy games.

JEL: C72, E52, E61

Keywords: Conflict of interest, coalition, policy invariance, Nash equilibrium existence, controllability

1 Introduction

Acocella and Di Bartolomeo (2006) analyze conditions for policy invariance and equilibrium existence in a LQ Nash game, using the concept of controllability of an economic system introduced by Tinbergen (1952, 1956). Their equilibrium conditions generalize the well-known sufficient conditions stated by Dasgupta and Maskin (1986).

Their analysis, however, does not consider the possibility of a convergence of interests between different players, i.e., the formation of "implicit" coalitions among players sharing the same target values of some variables. When we take account of the possible absence of a conflict among some players (with a residual opposition among groups of them), issues of existence of an equilibrium (or multiplicity of equilibria) and policy invariance can be dealt with in more general terms by means of the same theoretical tools introduced in the classical theory of economic policy.

*We are grateful to Maria Chiarolla and Andrew Hughes Hallett for helpful comments on a previous draft.

2 The model

We consider an economy where agents interact strategically. This economy is described by the following linear system:

$$Ax = Bu = \sum_{i=1}^{M} B_i u_i \tag{1}$$

where x is a vector of N target variables; u is a vector of M instruments set by the agents; A, B are appropriate matrices (respectively of order $N \times N$ and $N \times M$). We assume that A and B are of full rank, i.e. targets and instruments are both linearly independent (e.g., Petit, 1990: 33). We also assume that the total number of instruments is not greater than the total number of target variables (i.e. $M \leq N$).[1]

Note that we indicate matrices with uppercase letters, vectors with lowercase and the i-th element (column) of a vector (matrix) with a subscript.

Each agent j aims at minimizing a quadratic criterion defined on the deviation from a desired target vector $(\bar{x}(j))$:[2]

$$L(j) = (x - \bar{x}(j))' \, Q(j) \, (x - \bar{x}(j)) \tag{2}$$

where $Q(j)$ is a positive semi-definite diagonal matrix, which represents the weights that the agent places on deviations from the desired targets. We do not require that $Q(j)$ is of full rank as one agent may not be interested in one or more target variables. The agent's j first best is obtained for $x = \bar{x}(j)$; as a consequence $L(j) = 0$.

The weights that player j associates to deviations from his targets are indicated by vector $q(j)$, i.e. the diagonal of $Q(j)$. Thus $q_k(j)$ is the weight associated by player j to the deviation from his k-th target.

We assume that each agent is endowed with only one instrument; then there are M agents.[3] This assumption is not restrictive at all; the case of an agent who is endowed with more than one instrument can be simply introduced by assuming more agents minimizing the same criterion. We also assume that the instrument set by agent i is effective on his target variables; this formally requires $\left(A^{-1}B_i\right)' Q(i) \neq 0$, for all i. The assumption simply implies that each player is active in the game; otherwise his reaction function would not exist.

3 Coalition controllability

Before analyzing the issues of existence, multiple equilibria and policy invariance, we need to introduce some preliminary concepts.

[1] This assumption is introduced to rule out the trivial case that would lead to either infinite Nash equilibria or non existence (for a formal discussion see Acocella *et al.*, 2008).

[2] For the sake of brevity, without loss of generality, the criterion is assumed to be strictly quadratic. See Acocella and Di Bartolomeo (2004) for a discussion.

[3] Index i and j can then be used in an interchangeable manner.

Definition 1 (Implicit coalition) *We define an (implicit) coalition, $C(X)$, associated to a given set of values for some (or all) target variables, X, as the set of agents for whom X contains their first best.*

Given the $\bar{x}(i)$ for $i = 1...M$, there is a finite set of sets of values for target variables that individuate a coalition, we denote by Ω this set and indicate by X_Z a generic element of it, i.e. $X_Z \in \Omega$ for all Z. Each X_Z can contain from 1 to N elements. Since we have assumed that some agents may be interested only in subsets of target variables, we can have implicit coalitions formed by agents each with a different number of target variables.[4] Moreover, notice that implicit coalitions can be formed also by only one agent.

We must stress that our concept of implicit coalition does not imply any kind of cooperative solution. In particular, the concept is different from that arising from a coalition Nash equilibrium or those associated with other endogenous coalition models. In a coalition Nash equilibrium, agents usually play a two-stage game. In the first stage they negotiate the formation of coalitions and – in case – sign a binding agreement to play the second stage as a coalition against other players or coalitions.[5] By contrast, when we consider the idea of implicit coalitions, each agent always acts non-cooperatively and coalitions just naturally emerge from the absence of conflict among their members; no commitment mechanism is thus advocated.

Now, we can formally define the conflict of interests.

Definition 2 (Conflict of interests) *We say that two coalitions $C(X_1)$ and $C(X_2)$ have a conflict of interests if there exists at least a desired target value k such that $\bar{x}_k(i) \neq \bar{x}_k(j)$ with $q_k(i) \neq 0$, $q_k(j) \neq 0$ for $i \in C(X_1)$ and $j \in C(X_2)$.*

Less formally, a conflict of interests between two coalitions means that X_1 and X_2 contain at least one different target value for the same variable in which they are both interested.

It is useful to extract from the matrix of policy multipliers, $A^{-1}B$, only those relevant for the members of coalition $C(X_Z)$. We thus define matrix $M(X_Z)$ as the matrix obtained from $A^{-1}B$ by keeping only the rows corresponding to the target variables included in X_Z and the columns corresponding to the instruments of the coalition players. The order of matrix $M(X_Z)$ is given by the dimension of X_Z and the cardinality of $C(X_Z)$. We also define a matrix of members' weights for the coalition $C(X_Z)$ as a matrix where each row j is the vector $q(j)$ of member j of $C(X_Z)$. From this matrix we obtain the matrix $Q(X_Z)$ by deleting the columns corresponding to the target variables not included in X_Z. The order of matrix $Q(X_Z)$ is given by the cardinality of $C(X_Z)$ and the dimension of X_Z.

Finally, we define controllability.

[4] For instance, $X_Z = \{x_2 = 5,\, x_3 = 7\} \in \Omega$ is a the set that contains the first best of player i and j if they aim to minimize $L(i) = q_2(i)(x_2 - 5)^2$ and $L(j) = q_2(j)(x_2 - 5)^2 + q_3(j)(x_2 - 7)^2$. In this case they are both members of the implicit coalition $C(X_Z)$.

[5] Different equilibrium concepts can be defined according to the way players form expectations about the behavior of other players. See, among others, Chwe (1994) and Ray and Vohra (1999).

Definition 3 (Coalition controllability) *A coalition is said to satisfy coalition controllability if the strategies of coalition members always imply that they reach their first best outcomes for any given strategy of the other players.*

Coalition controllability implies that a combination of instruments of the coalition members (coalition policy) exists such that, for any given set of values for the instruments chosen by non-coalition members, the coalition members always achieve their first best outcomes. Coalition controllability is assured by the conditions stated in the following theorem.

Theorem 4 (Coalition controllability) *A coalition $C(X_Z)$ satisfies coalition controllability if 1) $M(X_Z)$ is of full column rank; 2) $M(X_Z)' \cdot Q(X_Z)$ is of full column rank, where "$\cdot$" denotes the element by element product (Hadamard product).*

Proof. Given the policies of non-coalition members, if condition 1) is satisfied the coalition X_Z faces a sub-system of the economy that is controllable in the Tinbergen terms (by a unique decision-maker). A policy assuring that $x_Z - \bar{x}_Z = 0$ thus exists, where $x_Z - \bar{x}_Z$ are the deviations from the target values included in X_Z. However, coalition members play in a non-cooperative manner, i.e. each member sets one instrument separately; then it is not obvious that they can implement this policy. Their first order conditions are $\left(A^{-1}B_i\right)' Q(i)(x - \bar{x}(i)) = 0$ for all $i \in C(X_Z)$ and can be written in a compact form as $\left[M(X_Z)' \cdot Q(X_Z)\right](x_Z - \bar{x}_Z) = 0$. This system has a unique solution equal to $x_Z = \bar{x}_Z$, if the matrix $\left[M(X_Z)' \cdot Q(X_Z)\right]$ is left-invertible. This occurs if condition 2) is satisfied. ■

It is worth noting that coalition controllability is obtained without any kind of cooperation among the (implicit) coalition members.

Coalition controllability clearly generalizes the traditional Tinbergen and Theil conditions to the case of multiple players, augmenting the traditional results with an implementation requirement. Condition 1) in fact simply means that the coalition has a number of independent instruments greater or equal to its independent targets, while condition 2) implies that the agents always independently implement the Tinbergen policy.

4 Existence, policy invariance and multiple equilibria

The concept of coalition controllability can be used to derive interesting properties about uniqueness of the equilibrium, indeed (sufficient) conditions for multiple equilibria.

Theorem 5 (Multiple equilibria) *In a game where at least an equilibrium exists, multiple equilibria arise if the cardinality of a controlling coalition $C(X_Z)$ is greater than the dimension of X_Z.*

Proof. The proof is trivial. Assume that an equilibrium exists; if the cardinality of $C(X_Z)$ is greater than the dimension of X_Z, there are other infinite combinations of the policies of coalition members supporting their first best outcomes, given the policy of non coalition members. All these combinations will also be Nash equilibria. ■

Multiple equilibria always emerge when there is a "too numerous" controlling coalition, i.e. a controlling coalition with more members (i.e.instruments for our assumption) than targets. This means that, even if no player has more instruments than targets, the controlling coalition faces an overdetermined problem and, since its members play in a non-cooperative manner, they have the problem of coordinating their action towards one of the first best equilibria.[6] This result complements that obtained by Acocella et al. (2008), who consider a game where $M > N$.[7]

The case of excess of instruments, which represents a richness in the Tinbergen world of a unique decision-maker (or a centralized solution), is thus cumbersome for a Nash equilibrium. This problem of multiple equilibria would find a natural accommodation in the focal point approach or with a centralization of policy and is likely to be more relevant if M is large. Alternative solutions can be related to the idea of correlated equilibrium (Aumann, 1974).

The concept of coalition controllability can be also used to derive necessary conditions for Nash equilibrium existence.

Theorem 6 (Existence) *A Nash equilibrium does not exist if there is a conflict of interests between two controlling coalitions.*

Proof. Assume that an equilibrium exists and there are two controlling coalitions, $C(X_1)$ and $C(X_2)$, with conflicting interests. Taking as given the equilibrium values for the agents who do not belong to the coalitions, by definition, coalition controllability implies that the equilibrium satisfies $x_k = \bar{x}_k(i)$ for any k such that $q_k(i) \neq 0$ for all the members of the two coalitions. However, the definition of the conflict also implies that there are at least two agents, j and h, for whom: $\bar{x}_k(j) \neq \bar{x}_k(h)$ with $q_k(j) \neq 0$, $q_k(h) \neq 0$ and $j \in C(X_1)$, $h \in C(X_2)$. Thus no equilibrium can exist. ■

The nature of the conditions stated by the theorem must be clear. It shows sufficient conditions for non existence and thus necessary conditions for existence of a Nash equilibrium. As for non existence, they are more general than those contained in Acocella and Di Bartolomeo (2006) and reduce to these for controlling coalitions formed by only one player. They extend the case of non existence to situations where some agents having conflicting target values do

[6] The same problem arises in the battle of sex, when both players only care of being together.

[7] Specifically, Acocella *et al.* (2008) show that if $M > N$, any given vector x^* could be obtained by an appropriate combination of policies, since the system would be controllable in Tinbergen's terms. But if x^* were consistent with a Nash equilibrium, there would be infinite many combinations of instruments supporting it, independently of the player who sets them (the system, in fact, would be globally over-determined), and therefore, infinite Nash equilibria would arise. Multiple equilibria, however, would emerge only in the case of equal target values for the players; in general no equilibrium would exist.

not control them individually, but through the coalitions they belong to. As for existence, they are more stringent than Acocella and Di Bartolomeo's (2006): for an equilibrium to exist, in fact, one needs to rule out the existence of a conflict not only between individual players but also between coalitions.

Finally, we can use the above discussion to derive some sufficient conditions for policy invariance as in Acocella and Di Bartolomeo (2004). Before, however, this must be defined, as in a game context the traditional definition cannot be used because policies are endogenous. The classical definition of policy neutrality in fact implies that autonomous changes in the policy instrument have no influence on some outcomes. In the realm of policy games, the following definition of neutrality can be accepted: When the equilibrium values of some outcomes do not depend on the preferences of the policymaker, policy is neutral with respect to such outcomes (Acocella and Di Bartolomeo, 2004).

If an equilibrium exists, policy invariance emerges when there is at least one controlling coalition, $C(X_Z)$, not including all players. In this case all the variables defined in X_Z will be fixed by the coalition and changes in the preference parameters of the non-coalition members cannot affect them. Hence their policies will be neutral with respect to the target variables defined in X_Z. If this was not the case, i.e., if players outside the coalition were able to reach (control) their targets, a contradiction would arise with the assumption of controllability by the coalition of different targets.

5 Conclusions

This paper has started from the notion of existence or absence of a conflict among the players in a policy game. If the latter is the case, implicit coalitions can be formed among the players sharing the same target values of some variables. The traditional tools of controllability introduced by Tinbergen with respect to a single decision-maker can be extended to implicit coalitions in a policy game and properties of Nash equilibrium derived. In a nutshell, the existence of a conflict implies an opposition among the (implicit) coalitions that leads to the non existence of an equilibrium when more than one coalition controls conflicting targets. If only one coalition controls the economic system, this implies policy invariance. Issues of multiple equilibria and coordination problems arise when a coalition has more instruments than targets available.

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