9

Multistate Systems: The Behavior of Simple Chains

Making simple 'chain' models more elaborate by adding more states, allowing greater connectivity among the states, and building 'smarter' rules governing flows allows us to model theories of quite complicated social processes. In the current chapter we will develop models of three commonly studied processes that illustrate some of the possibilities for models of this type: a model of population movements, a model of promotions in a hierarchical organization, and a model of movements among the social statuses in a "mobility" matrix.[1] The applicability of these models, however, goes well beyond the particular substantive contexts in which we will develop them. The kinds of processes that can be usefully conceptualized as simple chains are extremely numerous and central to all of the social science disciplines.

Population-age structures are generated by a very obvious process. Individuals are born and move in one direction through a series of states (age categories) with fixed "delays" or "waiting times" until they reach an "absorbing state." Such a system has many states, but relatively low connectivity, and relatively simple rules governing most transition rates. Many social processes have similar structures. Individual's changes in status within groups, tribes, formal organizations, and professions, for example, display such unidirectional change and more or less fixed waiting times. And virtually all models of any complexity in any of the social sciences must contain one or more "demographic" subsystems to account for the movements of people, data, or things over time.

Promotion regimes in hierarchical organizations (at least as idealized in most formal models) are similar to population-age structures but involve more complex rules governing transition rates. Generally, promotion rates are thought to depend on vacancies—so that such models are characterized by control by "goal oriented" (i.e. the elimination of vacancies) feedback. "Demand-driven" changes—such as vacancy chains of occupational positions—are important parts of the dynamics of most social structures. The "queuing" and delay of information, material goods, and people in all sorts of systems with fixed numbers of positions are found in applications in all of the social sciences.

Transition processes with multiple origins and multiple destinations, such as those analyzed with the "mobility matrix," are another variation on the theme. Such processes tend to be represented as governed by rather simple rules (e.g. Markov or semi-Markov processes), but involve movements among many closely connected states. Less restrictive models, like the "mobility matrix," that model movements between multiple-origin states and multiple destinations are extremely general in their applications: Individuals change religious affiliations, nations become parts of (or disengage from) alliances, and firms move across market niches, to suggest some of the possible applications of such models.

The three models considered here by no means exhaust the possible variations on simple chains. They do serve as useful starting points for further elaboration for the development of similar models for other such social processes, and to illustrate the behavioral possibilities of what are still relatively simple systems.

Population Age Structures

Among the most important social processes are those that generate age structures within populations. The distribution of populations by age is an important conditioning factor that limits other forms of social action as well as directly affecting the reproduction of the population itself. Age structures are often studied in and of themselves, as in demography, or as "subsystems" of other models, as in studies of economic development. Not only are age structures studied at the level of nation-states, but they are also very important in understanding the behavior of large-scale and small-scale social organizations such as bureaucracies or families.

There is broad consensus about the most useful ways to represent age structures and on the basic nature of the processes that govern their dynamics. Though by no means are all of the factors that affect the relevant rates (for example, "birth" and "death") fully understood.[2] Since our purpose here is largely to illustrate the general dynamic behavioral characteristics of systems composed of long but simple chains, we will focus on a highly idealized but familiar type of demographic process: the age structure of the general population. Models with finer descriptive character (e.g., differentiating gender groups, or narrow age categories) or for differing contexts (e.g., the age structure or distribution of "time in grade" of employees within an organization) can be seen as slight variations on the same theme.

Developing the Baseline Model

For simplicity, we will divide the population into six age categories: ages 0-1, 1-4, 5-14, 15-44, 45-64, and 65+. This particular classification is used because it corresponds to convention and best reflects some of the major nonlinearities in fertility and mortality rates. The particular groupings chosen are for convenience in capturing particular effects (and for reducing the tedium and computer cost of treating each year of age as a separate level). For example, the rather fine distinctions at the earlier ages are necessary to reflect the dramatic differences of mortality rates between infants (ages 0-1) and young children (ages 1-4); the age group 15-44 is convenient for use in modeling the birth rate. As always, the levels or states of a system are defined pragmatically and with an eye to the causal connections of the model.

The connectivity among these states is rather obvious, and is shown in Figure 9.1. The model is driven from a "source" by transitions that occur at a rate (the birth rate, BR), and by two kinds of transitions that occur for each of the levels in the model (save the last): Individuals make transitions from their current age category to the next one in the sequence (R12, R23, R34, R45), or to the "sink" (DR1, DR2, DR3, DR4, DR5). The basic structure of this chain is a quite common one of a single "normal" sequence of moves originating at the same status for all individuals, and having multiple exit points.

The control structures, or "rates" in this model are likewise quite straightforward. Each transition in the process (except the very first one of birth) is governed by a self-referencing "feed forward" at constant rates. For example, the number of persons moving from the "state" of infancy to the "state" of early childhood (R12) depends on the number of infants "at risk" (SS1) and a probability of such a transition (P3). In our baseline model we assume that this probability of survival is constant across all persons in the category. The probability that an infant makes a transition to the sink rather than to early childhood is also a function of the number at risk and a constant probability. Since a transition of one or the other sort must occur within one year, these two "transition probabilities" (PARM2 and PARM3) must sum to unity. There is a single (positive) "feedback" loop in the model that connects

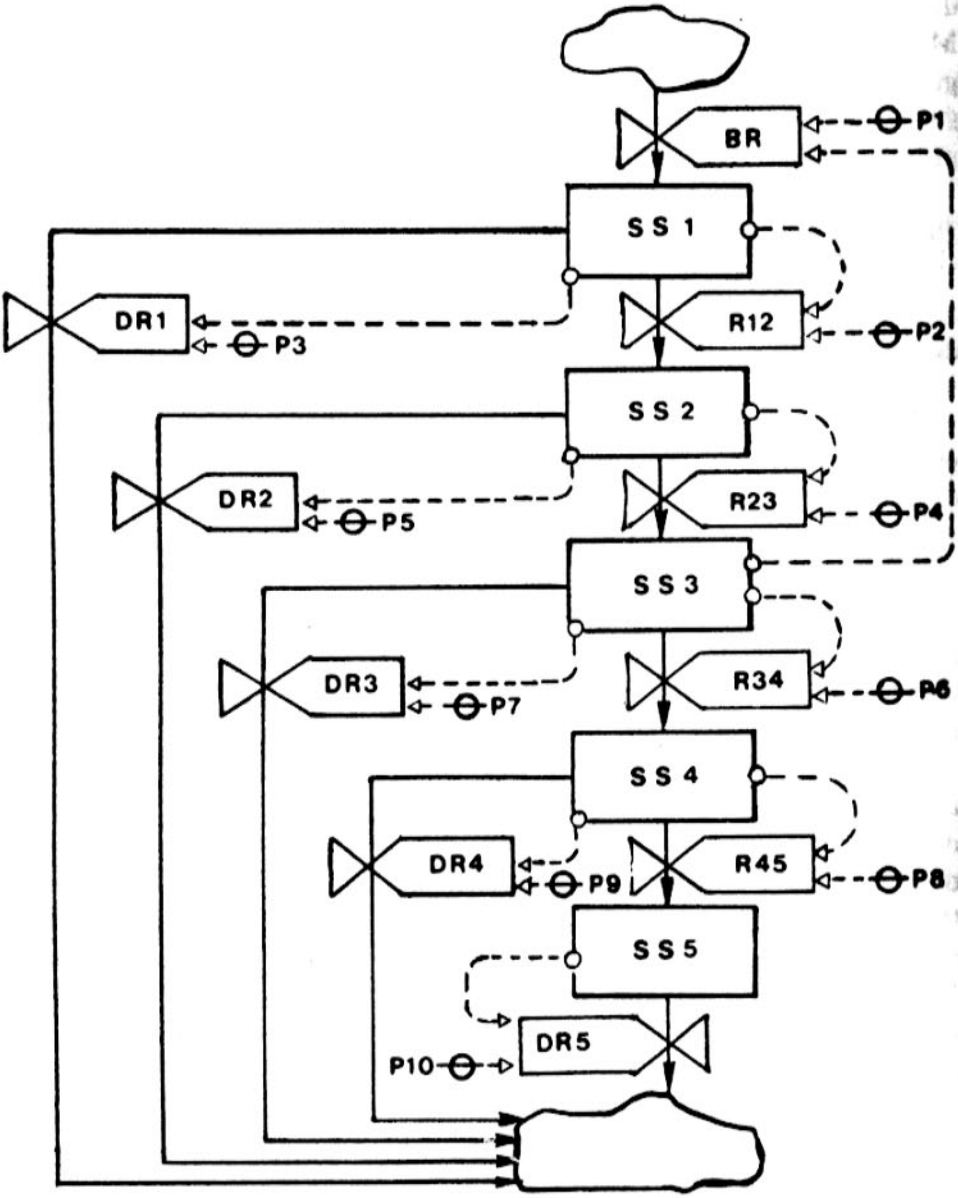

Figure 9.1: Population age structure.

the birth rate (BR) to the size of the population in the age category between 15 and 44 (SS3) and a "risk" probability (PARM1) of a birth that applies to this age group. This part of the process can be represented in DYNAMO code as:

```
L    P01.K = P01.J+DT(BR.JK-ISR.JK-IDR.JK)
R    ISR.KL = P01.K*PARM1
```

```
R    IDR.KL = P01.K*PARM2
R    BR.KL = P1544*FERT
C    FERT = PARM3
```

where PARM1 and PARM2 sum to unity and represent the probability of survival and death, respectively, among persons ages 0-1. The birth rate (BR) is represented as a constant function (FERT, for "fertility") of the number of persons in the age category of 15 to 44.

Each of the other levels in the model is similarly defined, having as a source the transitions out of the preceding level in the sequence, and having as outflows either "survivals" or "deaths." Because we have aggregated the age categories to save space and time, it has been necessary to impose the assumption that the age-specific mortality rates are homogeneous within each category and that there are equal numbers of persons in each of the specific ages within categories (the baseline model is provided as Appendix 9.1). These assumptions do introduce some inaccuracy into the model, particularly in studying responses to transient shocks. They do not, however, distort the general behavioral tendencies of the chain that are our major concern.[3]

Behavior of the Baseline Model

Because the connections in the simple demographic chain are so straightforward, the general dynamic tendencies of the model are well understood. Nonetheless, the first step toward understanding more complex problems is to get a firm grasp on the baseline condition. Let us then conduct a set of experiments to get comfortable with the dynamic implications of the rather simple theory we have specified so far. To do this, we need to provide starting values for the age structure of the population, for the fertility rate, and for age specific mortalities. To make things interesting, we have selected values that represent the situation of the United States in about 1980.[4]

While any number of useful baselines might be thought of for answering various "what if" questions about population age structure in the United States, let us focus first on the long-term or "equilibrium" tendencies of the situation prevailing at about 1980. If we substitute the observed fertility rates, age specific mortality rates, and sizes of population groups into our model and allow it to run for a long time (in our example, 200 years), we can observe the equilibrium tendencies of the basic model. The time traces of this simulation are shown in Figure 9.2 and the numeric results are given in Table 9.1.

The scenario that we have modeled in Figure 9.2 and Table 9.1 is

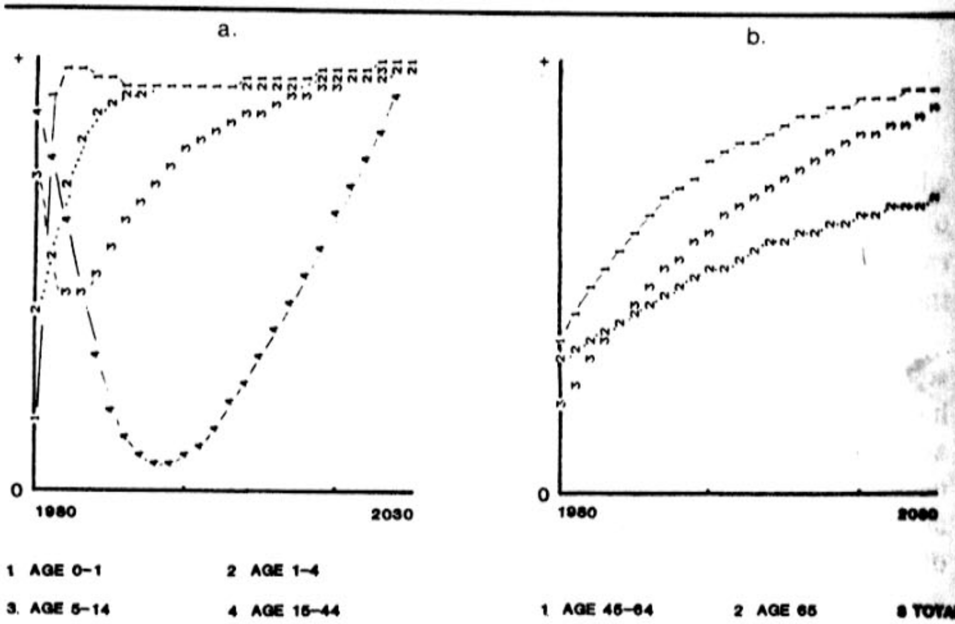

Figure 9.2: Age structure baseline model.

clearly "unrealistic" in a number of ways. We have made no attempt to deal with the effects of immigration or emigration here for simplicity's sake (though such effects are likely to be very important in reality). More immediately relevant, our "baseline" assumes that fertility and mortality rates remain fixed over the entire period. Clearly this will not be the case, but we need to first grasp the behavioral tendencies of the model when as many factors are "held constant," before we can really understand the consequences of change.

There are several things of interest in these results. First and foremost is that the "normal" or "long-run" or "equilibrium" tendency of the conditions prevailing in 1980 is to produce continued growth of the population, but at quite slow rates. Second, as must be the case when no change in fertility or mortality rates is allowed, the age structure of the population eventually reaches stability. It is important to note, however, that this "eventually" is quite a long time. Changes in the relative sizes of the younger population groups occur relatively quickly, taking about 50 years to complete most of their change. The adjustment of the size of the population over age 65, however, takes a bit longer to stabilize (as it is the receiver of all changes in other groups, with a lag). Third, the "equilibrium" age structure of the population implied by the conditions (age, specific mortality, and fertility) that existed at 1980 is quite different from the age structure of the population that existed at 1980.

TABLE 9.1
Population Age Structure Baseline (U.S., 1980)

	Population in Millions and Percentage of Population						
Year	*0-1*	*1-4*	*5-14*	*15-44*	*45-64*	*65+*	*Total*
1980	3.3 (1.4)	13.1 (5.8)	34.9 (15.4)	105.2 (46.4)	44.5 (19.6)	25.6 (11.3)	226.6
2030	3.6 (1.3)	14.2 (5.1)	35.5 (12.6)	105.3 (37.5)	67.8 (24.1)	54.6 (19.4)	280.9
2080	3.6 (1.3)	14.4 (4.9)	35.9 (12.3)	106.7 (36.5)	70.5 (24.1)	60.8 (20.8)	291.9
2180	3.7 (1.3)	14.7 (4.9)	36.9 (12.3)	109.5 (36.4)	72.5 (24.1)	63.1 (21.0)	300.5

Most striking is the projected increase in the proportion of the population over age 65, which nearly doubles from 11 to 21%.

So long as mortality and fertility rates are fixed at 1980 levels (and ignoring immigration and emigration), there is a tendency for the American population to continue to increase in size at a slow rate. This is a natural consequence of the "positive feedback" loop between the size of the fertile population and the number of births. This tendency is actually toward exponential growth in population size, though the slope is so slight that our plots look essentially linear with respect to time. More critically, there is a very strong tendency inherent in the model at 1980 toward rapid and extensive change in the age structure of the population. The total size of the population is projected to increase by roughly a third over the next 200 years; the number of persons over age 65 is projected to increase by 150%.

A Delicate Balance: Sensitivity to Fertility Change

In our baseline example, we assumed that the fertility rate remained constant at its 1980 level of 3.42 births per 100 persons ages 15-44 per year. With existing age specific mortality rates, we saw that this birth rate was above "zero population growth." The next step in assessing the behavior of this model might be to explore the sensitivity of the results to our assumption about the fertility rate.[5] We can easily do so by conducting two experiments: First we will decrease the fertility rate from 3.42 to 3.00 and observe the long-run effects on the trend in total population and in the age structure; then we will increase the fertility rate from 3.42 to 3.8. These are relatively mild manipulations, involving

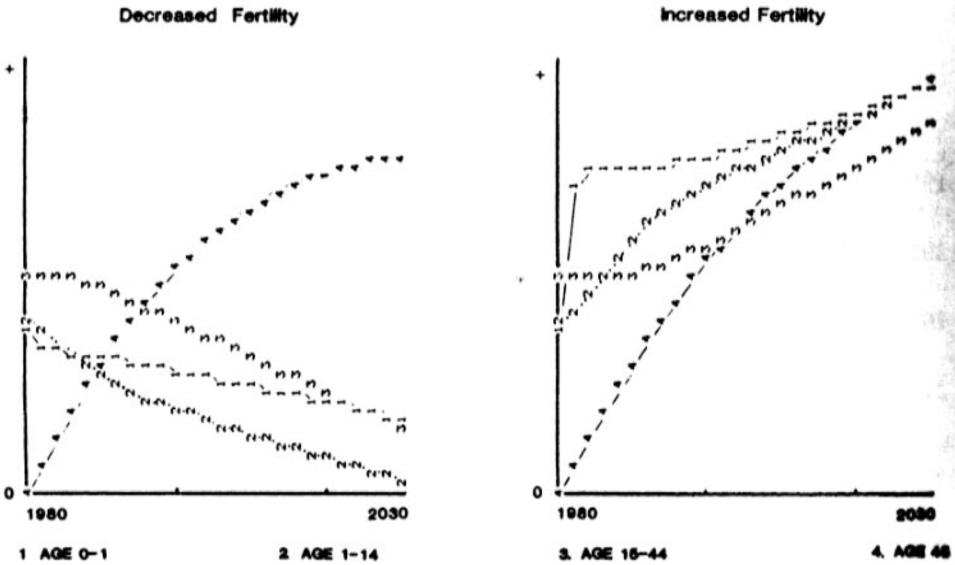

Figure 9.3: Age structure fertility experiment.

increases or decreases of about 10% in fertility. The results of these two experiments are shown in Figure 9.3 and Table 9.2.

The results of these experiments are rather dramatic, and suggest the delicate balances that are necessary to maintain stability in population structures. A roughly ten percent decrease in the fertility rate is sufficient to lead to a long-term pattern of decline in total population size (after the consequences of current disequilibria are fully realized), coupled with steady increases in the average age of the population. A rough ten percent increase in fertility, in contrast, is sufficient to lead to rapid growth in total population and a marked shift toward a younger age structure.

The Consequences of Differential Mortality

In addition to fertility, the other forces shaping the age structure of the population are the age-specific mortality rates. To gain a sensitivity to the impact of these factors, let us conduct another set of experiments.

Suppose, on one hand, that we focused all of our research and treatment efforts on the reduction of mortality among infants, and succeeded in reducing the rate of deaths in the first year by 25% from its 1980 level. Or, alternatively, suppose that we focused all of our resources and energies on the prolongation of life, and succeeded in reducing the mortality rate among persons of sixty-five or more by a similar proportion. What would the consequences be for the total size of the

TABLE 9.2
Population Age Structures
Under Alternative Fertility Rates

Year	*0-1*	*1-4*	*5-14*	*15-44*	*45-64*	*65+*	*Total*
Scenario I: Baseline (U.S., 1980)							
1980	3.3	13.1	34.9	105.2	44.5	25.6	226.6
	(1.4)	(5.8)	(15.4)	(46.4)	(19.6)	(11.3)	(100)
2030	3.6	14.2	35.5	105.3	67.8	54.6	280.9
	(1.3)	(5.1)	(12.6)	(37.5)	(24.1)	(19.4)	(100)
2080	3.6	14.4	35.9	106.7	70.5	60.8	291.9
	(1.3)	(4.9)	(12.3)	(36.5)	(24.1)	(20.8)	(100)
Scenario II: Decreased Fertility							
1980	3.3	13.1	34.9	105.2	44.5	25.6	226.6
	(1.4)	(5.8)	(15.4)	(46.4)	(19.6)	(11.3)	(100)
2030	2.8	11.3	29.1	94.4	63.7	52.7	254.1
	(1.1)	(4.5)	(11.5)	(37.1)	(25.1)	(20.7)	(100)
2080	2.5	10.0	25.6	83.0	58.0	52.4	231.5
	(1.1)	(4.3)	(11.1)	(35.9)	(25.1)	(22.6)	(100)
Scenario III: Increased Fertility							
1980	3.3	13.1	34.9	105.2	44.5	25.6	226.6
	(1.4)	(5.8)	(15.4)	(46.4)	(19.6)	(11.3)	(100)
2030	4.4	17.1	41.8	115.7	71.7	56.3	307.0
	(1.4)	(5.6)	(13.6)	(37.7)	(23.3)	(18.3)	(100)
2080	5.0	19.6	47.8	132.3	83.5	69.2	357.4
	(1.4)	(5.5)	(13.4)	(37.0)	(23.4)	(19.4)	(100)

population and its age structure? In Figure 9.4 and Table 9.3 the results of these experiments are displayed.

The results may seem, at first, a bit surprising. A substantial increase in the survival rate in the first year of life has little effect on either the total size of the population or on its age structure (compare the middle panel of Table 9.3 to the top panel). On the other hand, a substantial increase in the survival chances of the aged population leads to both rather rapid increases in total population and to a dramatic shift toward an older population.

This result is surprising because, in the abstract, changes in infant mortality might be expected to have greater long run effects—because they "multiply" through the system by increasing the size of the fertile population sometime later. Increasing the survival rates of the aged, in contrast, has only its first-round impacts, and is not multiplied by feedback. There is, however, no mystery here. The multiplicative impact

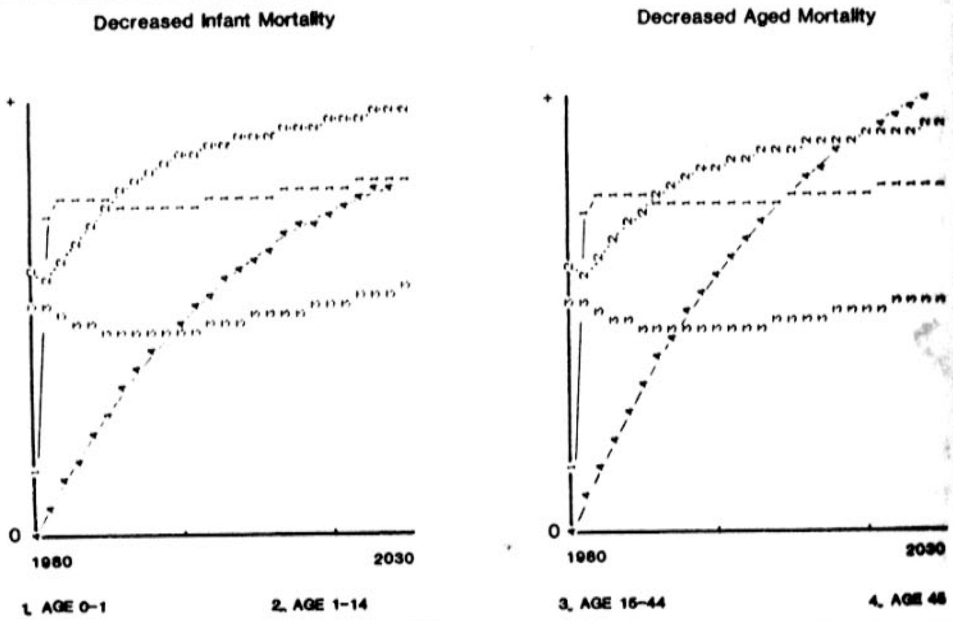

Figure 9.4: Age structure mortality experiment

of changes in the infant mortality rate are, in fact, present in these results; in a population with such a low birth rate, however, the numerical impact of even a quite dramatic change in infant survival chances is small. Because the aged population is much larger than the infant population, and because the reduction in mortality simulated applies continuously across many years of age (from age 65 onward, as opposed to the single year in the first scenario), the numerical impact is much larger.

There is an object lesson in this example. It is that changes that are more structurally important do not necessarily have observable results that are more dramatic than changes that are less structurally important in all realizations of a model. In systems terms, a modification of infant mortality is much more significant than a modification of mortality among the aged because it precedes the feedback loop from the size of the fertile population to the birth rate, and hence is multiplied by the feedback. Changing mortality among the aged has only its direct first-order effects that merely accumulate. Because of the sizes of the populations to which these effects apply in any realistic model, however, the latter change is of much greater numerical consequence.

Summary

The age-structure model is a very simple and straightforward elaboration of the simple chain system. The dynamic behavior of the

TABLE 9.3
Population Age Structures
Under Alternative Mortality Rates

Year	*0-1*	*1-4*	*5-14*	*15-44*	*45-64*	*65+*	*Total*
Scenario I: Baseline (U.S., 1980)							
1980	3.3	13.1	34.9	105.2	44.5	25.6	226.6
	(1.4)	(5.8)	(15.4)	(46.4)	(19.6)	(11.3)	(100)
2030	3.6	14.2	35.5	105.3	67.8	54.6	280.9
	(1.3)	(5.1)	(12.6)	(37.5)	(24.1)	(19.4)	(100)
2080	3.6	14.4	35.9	106.7	70.5	60.8	291.9
	(1.3)	(4.9)	(12.3)	(36.5)	(24.1)	(20.8)	(100)
Scenario II: Decreased Infant Mortality							
1980	3.3	13.1	34.9	105.2	44.5	25.6	226.6
	(1.4)	(5.8)	(15.4)	(46.4)	(19.6)	(11.3)	(100)
2030	3.6	14.3	35.7	105.6	67.9	54.6	281.7
	(1.3)	(5.7)	(12.7)	(37.5)	(24.1)	(19.4)	(100)
2080	3.7	14.5	36.3	107.4	70.8	61.0	293.7
	(1.2)	(4.9)	(12.3)	(36.6)	(24.1)	(20.8)	(100)
Scenario III: Decreased Aged Mortality							
1980	3.3	13.1	34.9	105.2	44.5	25.6	226.6
	(1.4)	(5.8)	(15.4)	(46.4)	(19.6)	(11.3)	(100)
2030	3.6	14.2	35.5	105.3	67.8	68.5	294.8
	(1.2)	(4.8)	(12.0)	(35.7)	(23.0)	(23.2)	(100)
2080	3.6	14.4	35.9	106.7	70.5	79.9	311.0
	(1.2)	(4.6)	(11.6)	(34.3)	(22.7)	(25.7)	(100)

system is, in the abstract, quite easy to understand from its structure as a simple chain with primarily "feed forward" linkages creating delay, and a single positive feedback loop. The forward linkages produce simple linear trends that occur at rates dependent upon the relative sizes of the transition probabilities (age-specific mortality rates). The model, however, also displays a good deal of sensitivity to changes in the birth rate that produces long-run tendencies away from stable equilibrium. This sensitivity and "instability" of population is a natural consequence of the simplicity of the system structure. While the model has many levels and rates, it has only one feedback loop, and this loop is a positive one (for example, the larger the size of the fertile population, the larger the number of births). Just as in the very simplest of positive feedback systems, population is inherently unstable and seeks either to collapse or explode. The speed and shape of the realization of this tendency is much more complex than the simple feedback systems we considered in earlier chapters, owing to the delays and differential rates of transitions among the many states. The basic dynamic tendencies of the system, however, are determined by the nature of the control structure, and not by the elaboration of the model into a more complex chain.

Vacancy Chains

The control structures governing the dynamics of population age structures are, in the terms that we have been using, a combination of "dumb" and self-referencing feedback. That is, the rates of change in age groups depend only on their current size and on constants (transition probabilities). The system is also quite simple in its connectivity in that only "one-way" transitions occur, and each state is connected only to one origin and two destinations.

In recent years economists, sociologists, and human resource managers have focused a good deal of attention on another demographic phenomenon that appears to be analyzable as a system with a very similar structure to general population movements: rates of promotion within hierarchies.[6] The primary tool used for analysis is a slightly smarter feedback system called the "vacancy chain." These models are not only of considerable interest in themselves as representation of dynamics of personnel movements; they also provide a useful contrast to our age structure models. And as with the simple demographic model, vacancy chains can be widely applied in other contexts.

In the population age-structure system, dynamics are governed primarily by "feed-forward." That is, the rates of transition of individuals from origins to destinations depend on the numbers in the origin state and transition probabilities. In a sense, this is a model in which "push" or "supply" is central. In contrast, most models of mobility processes in organizations are based on "pull" or "demand" factors; the number of persons who are promoted from one level to the next higher one is seen as a consequence of number of persons in the destination status (or, more correctly, the number of vacancies at the destination status). As the number of persons in the destination status departs from some goal, vacancies are created which pull individuals from lower to higher levels. When a vacancy occurs in a high level, demand is created at all lower levels; hence overall mobility rates depend on where vacancies occur and the relative sizes of the strata in the hierarchy.

Developing the Baseline Model

The levels and the degree and forms of connectivity in a model of a vacancy chain are very similar to those of the age-structure model. In our simple hierarchical or sequential system, we will imagine that there are three ranked subpopulations (we will call them "entry level," "mid level" and "executive"). All individuals enter the organization at the lowest level, and cannot be promoted to executive status without passing through the middle levels. In our simplest model, all individuals are seen as remaining in the organization until they attain executive status, from which they ultimately retire. While these assumptions are obviously far too simple, they will serve for the moment. A diagram of the basic model is shown as Figure 9.5.

The primary and important difference between the system shown in Figure 9.5 and the earlier age-structure model (see Figure 9.1) is the nature of the control system. In the current model, rates of transition—the hiring rate(HR), the promotion rate from entry to mid level (PR1), and the promotion rate from mid level to executive (PR2)—are governed by a comparison of the number of persons at the destination level to some goal state (the desired level). This comparison results in the perception of a vacancy (ELVAC, MLVAC, and EXVAC), which, in turn drives promotion rates. The DYNAMO program for this model is similar to the population model, and is provided in Appendix 9.2.

Behavior of the Baseline Model

The behavioral tendencies of the vacancy chain model should be quite easy to anticipate. The system is governed by goal-directed feedback, and hence tends toward a stable equilibrium in the number of persons at each level and in the rates of transitions among levels. Since we have not provided for misperception or delay in the baseline specification of the model, adjustment of the executive level to retirements occurs completely within one time period; adjustment of the middle level to a retirement at the executive level takes two time periods; and adjustment of the entry level to an executive retirement takes three periods as the vacancy "trickles down." The overall rates of mobility, then, depend entirely on retirements from the executive level. The probability that an executive retirement results in promotion for an individual at a lower level depends on the number of persons in the lower level—that is, on the shape of the pyramid. Where the numbers of persons in the levels are similar, a retirement improves everyone's chances of promotion equally. Where there are many more people at lower levels than at higher ones, retirements at the top improve the prospects of middle-level persons more than those of entry-level persons.

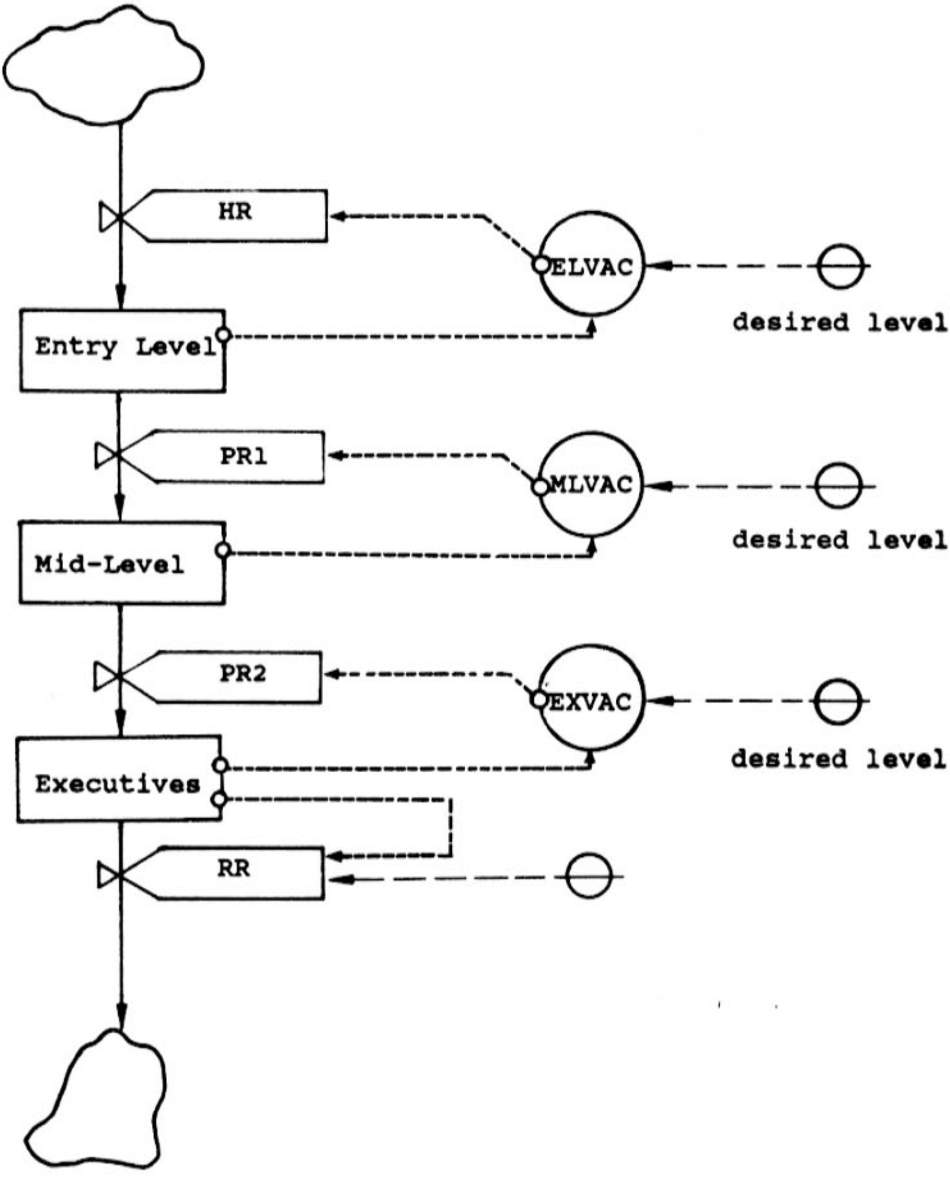

Figure 9.5: Vacancy chain.

The basic vacancy chain, then, has a structure much like that of the population age-structure model we examined above. But because its dynamics are governed by goal-oriented feedback rather than by self-referencing and dumb feedforward its behavioral tendencies are quite different. Where the population model has a tendency toward unconstrained growth or decline, the vacancy chain has a strong tendency toward stability. In the population model, dynamics are driven by supply (births); in the vacancy chain, dynamics are driven by demand (retirements). Let's explore some of the implications of these dynamics.

Growth, Structural Change, and Mobility

Research on structure of mobility in organizations has focused on the effects of size and technology (or, more properly, the division of labor resulting from a given technology). From our baseline model it is easy to understand why these two factors are critical in understanding mobility rates and patterns in demand-driven simple sequence models. Increases in the size of an organization (if the new vacancies are filled by hiring into entry-level positions and the relative sizes of the strata are preserved) improve the prospects of all organizational members and result in higher general rates of mobility. Reductions in size, if they are accomplished by increasing the rate of executive retirement (which they seldom are, in real world cases) also improve mobility chances.

Changes in the division of labor within an organization are also important in determining the structure of opportunity, but do not have exactly the same kinds of effects as changes in organizational size. Whereas general increases or decreases in size affect people in each strata proportionately, changes in the relative numbers of persons in levels of the hierarchy have disproportionate effects on mobility chances. Shifts in the shape of the pyramid toward a taller and thinner hierarchy (that is, one in which the "grade ratios" are closer to unity) result in a period of rapid promotion but limited hiring as new vacancies are filled at the top. Shifts in the opposite direction, toward an organization with larger numbers of persons at lower levels, conversely, improves hiring chances, but slows rates of mobility within the organization.

By way of illustration of these principles we can perform simulation experiments on our simple hierarchy. In our first experiment we will explore the implications of changes in organizational size. After setting the system in equilibrium (with 600 entry-level persons, 300 middle-level persons, and 100 executives), we will increase the goal states for total size of the organization by 100 persons per time period for 10 time periods, then decrease it for 10 time periods. In keeping with the assumption of constant grade ratios, these changes will be distributed proportionately across the grades. For our second experiment, we will alter the shape of the pyramid, first increasing the relative sizes of the higher levels at the expense of the lower level for 10 time periods, then reversing the process.

Experiment: Growth and Decline

The basic results of the simulation experiment with growth (from time 1 to time 10) and decline (from time 10 to time 20), are shown in Figure 9.6. In the first panel the number of personnel at each of the three levels is shown; in the second, the rates of transition (hiring, promotion, retirement) are presented.

The number of personnel at each level of the hierarchy, quite predictably, increases and decreases with changes in demand. The responses also show a certain degree of smoothing because of the delay inherent in positions "trickling down." And responses to decline are less than responses to growth because of the constraint that all "reductions in force" be accomplished by retirements at the senior level in this model. Since the senior level is relatively small (10% of the population) and since personnel retire at fixed rates, there is a limitation on the responsiveness of this model to decline.

The costs and benefits of growth and decline are not equally distributed across the hierarchical levels in the simple vacancy chain. In the period of rapid growth, hiring and promotion expand, increasing the mobility chances at all levels (see the second panel of Figure 9.6). As the size of the executive stratum expands, the chances that a middle-level manager will be promoted in a given period of time increase from about 6% to slightly over 8% by the fifth time period. Chances for promotion into the executive, however, stagnate and begin to decline as early as the fifth time period, despite continuing increases in the numbers promoted. Similarly, the chances that a given entry level person will be promoted to middle-level status roughly double in the first several time periods of growth (from about 4% to about 8%), but then decline toward their new equilibrium level, despite continuing increases in the numbers of personnel being promoted.

The consequences of decline in the simple hierarchy are even more dramatically unequal than responses to growth (time points 10 to 20 in Figure 9.6). Because some vacancies continue to occur at the executive level due to retirements, the impact of shrinking demand on the mobility chances of middle-level personnel is somewhat buffered. The number of retirements, however, does not create enough demand to absorb all of the surplus middle-level personnel. Consequently, while the chance of promotion from middle level to executive status declines very rapidly, promotion to the middle level and hiring cease altogether.

Changes in size, then, have differential impacts on mobility chances of personnel in the several strata under the constraint that vacancies are created only by retirement and proportional increases in stratum size.[7]

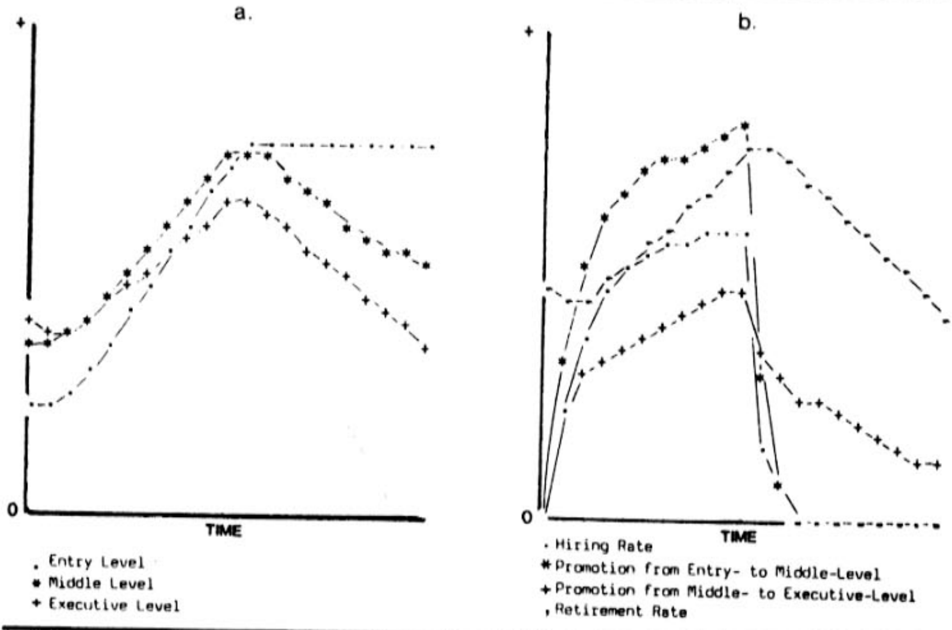

Figure 9.6: Growth and decline experiment.

While rapid growth improves the mobility chances of individuals at both the entry and middle levels in the current model, the negative consequences of rapid decline fall more heavily on the lower strata.

Experiments: The Shape of the Hierarchy

In a second experiment we manipulate the structure of the pyramid, while holding constant the total number of personnel. In the first 10 periods we induce a shift toward a "narrow and tall" hierarchy, that is, one with more equally sized strata. Over the second 10 periods we shift the shape of the organization's pyramid back to its original shape. The results of this manipulation are shown in Figure 9.7, with the numbers in each strata shown in the first panel and the rates of transition in the second panel.

The changes in strata size are accomplished rather smoothly, with slight delays due to the time taken in positions "trickling down." Unlike the previous example, however, the system is able to absorb the changes induced without stress. That is, at no time are there substantial gaps between the goal state of the system and the actual levels, as witnessed by the trace lines in the first panel of Figure 9.7 returning to their original levels by the twentieth time period. This result is in contrast to the size experiment, where substantial gaps exist, and the system does not reach its goal by the end of the experiment. This result is, of course, a

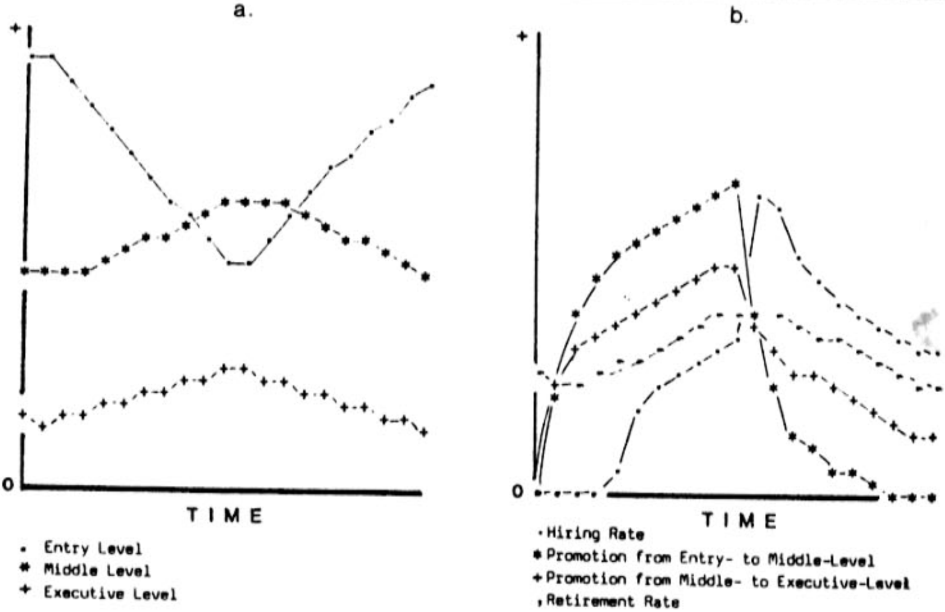

Figure 9.7: Structural change experiment.

consequence of the way that the experiment was designed. During the first 10 time periods increases in the size of the middle and upper strata were induced—creating little stress, as such vacancies are rapidly filled by promotion and hiring. In the second half of the experiment, where the sizes of the two upper strata are being reduced, retirements (which are proportional to the number of persons in the upper stratum in this experiment) occur at rates sufficient to accomplish the reductions in force at the upper levels without large departures from organizational goals. Had the rates of retirement been less, the rate of decline in the size goals for the upper strata higher, some stress would have been created. Had we run the experiment in the opposite direction, first decreasing the desired size of the upper strata and then increasing it, some stress would also have been induced.

Changes in the structure of the pyramid have differential impacts on the mobility chances of personnel at different levels (see panel b of Figure 9.7). As in the previous experiment, the consequences of change for the mobility chances of individuals at the middle levels are somewhat buffered by the presence of continuing vacancies above (due to retirement), while hiring and promotion to middle levels are more affected by structural changes. The number of upper-level personnel increases over the first half of the time period, and this buffers the impacts of later structural change because the retirement rates depend on the numbers in the upper strata. The shapes of the curves of numbers of transitions in the second half of the experiment clearly show these buffering effects. While the declines in all of the transition rates are exponential in the face of routinizing change, the declines are steeper the further down the hierarchy that one goes.

Summary

While our vacancy chain models are obviously unrealistic in many regards, they are suggestive of the complexity of behavior that can be produced by quite simple structures. Perhaps most important, the vacancy chain models suggest that changes induced by size and the division of labor can be complex and unequal—depending on the shape of the hierarchy and the rules governing the creation of vacancies. The experiments suggest that, in systems of this type, the consequences of growth and decline are not mirror images of one another: The consequences of increasing the number of upper-level positions and decreasing the number of upper-level positions are not mirror images of one another, and the degrees of organizational stress and the expansion or constriction of individual mobility chances induced by changes in size and structure can be quite complex even in quite simple systems.

Some Directions for Extensions

The vacancy chain model developed and briefly examined here is the simplest possible version. It is quite adequate for understanding the much greater complexity of possible behavior of simple chains governed by feedback, relative to those governed by simple feedforward. The model, however, is rather too simple to be a good representation of mobility processes in real hierarchies (though some very restricted cases, such as movements of lawyers in law firms or professors in academic departments can be captured to a degree).

There are a number of simple changes that might make it more useful as a research tool. We will mention only a few of the more important possibilities in passing.

First, the number of levels in the hierarchy might be modified—and even made a function of other variables. Organizations with different numbers of levels (say GS1 to GS18, for example) can be captured by simple extensions of the current three levels. If we wished to examine mobility processes over time in real organizations, it might be necessary to provide some external or internal mechanism that causes the numbers of levels to increase or decrease dynamically.

Second, individual's chances of promotion are dependent upon their duration in their current state. The elaboration of the simple hierarchy to include cohort groups within each hierarchical level would make it possible to capture this aspect of individual mobility more accurately than in the current model. It would be possible in this way to include both organizational factors (for example, the total number of vacancies) and individual factors (for example, the effect of time in state on the probability that an individual is selected to fill a vacancy) in the model of the mobility structure.

A third line of development of the current model would modify the connectivity among states rather than simply the number of states. We presently provide that the only way that individuals leave the organization is by way of retirement from the most senior level. Clearly this is unrealistic for most organizations. More commonly, individuals may leave the organization from any level, and some may never achieve the highest level prior to leaving—even if they remain in the organization for a very long time. The vacancies created, and hence mobility patterns, would be dramatically modified from the current model if individuals could be fired from each level, or perhaps left the organization at increasing rates voluntarily if they remained too long in a level without promotion.

Fourth, we have also supposed that an organization can be represented as a single hierarchy, that promotions occur only one step at a time, and that all vacancies are filled from within. Clearly the connectedness of states in most real organizations is far more complex. Organizations differ, through opportunity or policy, in the extent to which they are likely to fill vacancies at various levels from within or without. Obviously, such differences can have dramatic impacts on the distribution of opportunity. Most organizations are better characterized as a series of parallel "ladders" of opportunity (with the ladders being of different heights) rather than a single hierarchy. Some ladders, say within the sales department, never lead to the executive boardroom; other ladders, say within the finance department, do provide the possibility. And organizations differ in the degree to which it is possible for an individual to change from one ladder to another. These kinds of internal segments and differential closedness to outsiders are all relatively simple elaborations of the current model. While the modifications are logically simple, our current results suggest that the modification might have quite complicated and substantial consequences for both organization and individual.

A fifth major direction for making the current model more realistic would be to relax the assumption of population homogeneity. We have, in the current case, treated all individuals at a given level in the hierarchy as having the same probabilities of promotion. The notion that promotion possibilities for individuals differ according to "time in grade," and according to internal ladders and segments may go a long way toward more realistic representation of individual's chances. In addition, however, one might well suppose that channels of opportunity are differentially open, depending on characteristics of individuals (race, gender, education, language, presentational style, etc.). While full exploration of such effects would be more effectively accomplished with a mixed continuous and discrete state language, we could represent multiple subpopulations by race, age, gender, etc. in the same fashion as time-in-grade or ladder differences.

Sixth and finally, we might suppose that the processes of mobility are a good deal less "rational" than our model represents. Organizations may be slow to perceive the existence of vacancies, may perceive vacancies where none exist, or may create vacancies prospectively (rather than as delayed responses). Once a vacancy exists there may be substantial delay in filling it and there may be considerable error and noise in the process of selecting individuals. As we have suggested on numerous occasions, the dynamic behavior of many systems can be dramatically altered by the presence of noise, delay, and bias. "Intendedly rational" systems often become unwieldy and irrational in the presence of such informational problems.[8] There is little reason to expect that patterns of mobility within organizations are immune to such effects.

The Mobility Matrix

One of the most commonly used analytic tools in the study of patterns of social stratification is a cross-tabulation that describes the frequencies of movements between a set of origin statuses and a set of destination statuses. The densities of cases in regions of such a "mobility matrix" can be seen to represent a map of the degree and form of mobility chances in a population, and hence are a telling summary of the overall rates of upward and downward movement, short and long distance movement, propensity to "status inheritance," and permeability and impermeability of the several strata.[9]

The notion of a process with multiple origin statuses, multiple destination statuses, and reciprocal movements back and forth among origins and destinations, of course, is far more general than its application to "social mobility." Voters may move back and forth among the states of being Democrats, Republicans, and Independents;

nation states may move from "peripheral" to "semiperipheral" to "core" positions in the world political economy; workers may move back and forth between employed and unemployed status; families may move from one geographical location to another and (sometimes) back again. All of these problems (and, of course, many others) can be thought of as involving the movements of individuals back and forth between "origins" and "destinations."

The process that the mobility matrix summarizes is a relatively straightforward extension of the the simple chains that we have been considering in this chapter. The "origin" and "destination" statuses in a mobility matrix can be thought of as a single set of system states, observed at two points in time. In our dynamic formulation of the same process, we see the number of persons at each status level as varying continuously over time. The frequencies of movements between each origin and each destination in the mobility matrix are interpretable as transition probabilities. In our dynamic formulation, these transition probabilities become rates of "flow" of individuals among statuses. The basic structure of such a system is shown as Figure 9.8.

The main difference between the type of mobility process captured in the mobility matrix and those that we have discussed so far in this chapter is in the connectivity among the states. In both of our previous examples, transitions occurred in a single direction—age increased but never decreased, persons were promoted but not demoted. In the mobility matrix, movements are possible in both directions between each pair of statuses. As a result of this increased connectivity, the range of possible careers is much greater in mobility matrix than in the other simple chains. Nonetheless, the basic structure of the system of multistate origin to destination mobility is an easy extension of the models that we have considered previously.

Developing the Baseline Model

A basic dynamic model for the kinds of multistate transitions described by mobility matrices is quite easy to construct. The system has as many states as there are origins and destinations, the quantities in each state are "conserved," and changes in system levels are the simple sum of movements into the state from other states and movements out of the focal state to other states. If we were describing a system of two strata (say "white collar" and "blue collar"), we could write the basic equations as follows:

```
L     WC.K = WC.J+(DT)(BCWC.JK-WCBC.JK)
```

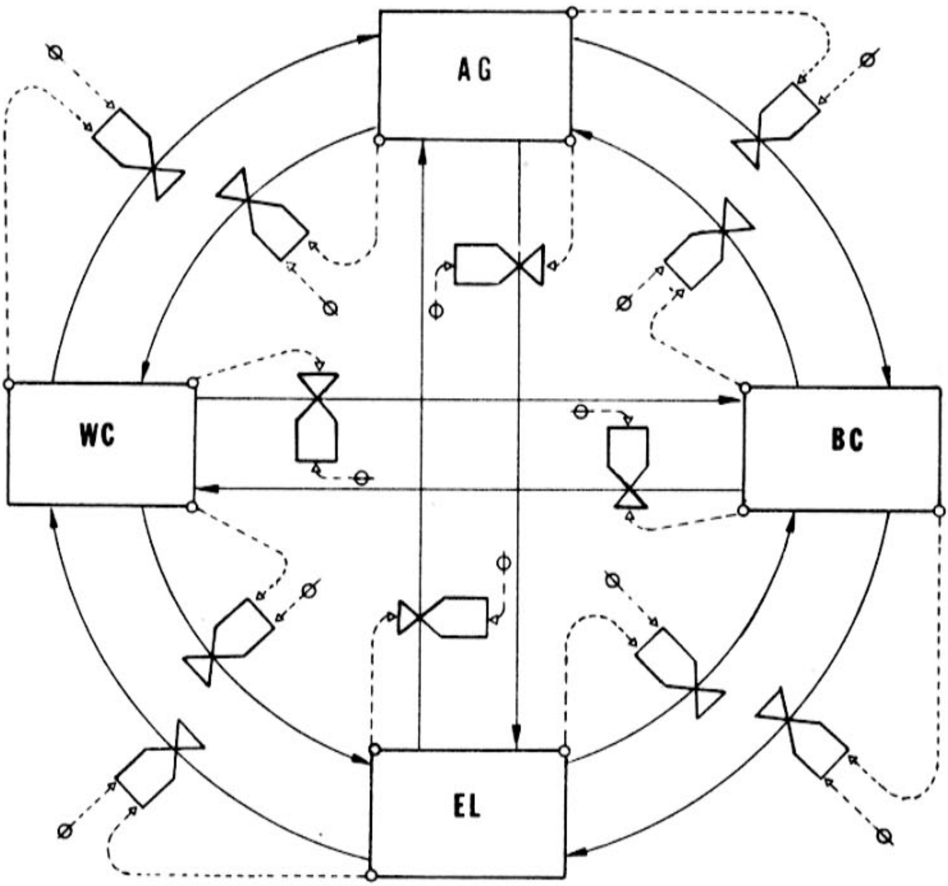

Figure 9.8: Mobility matrix.

L BC.K = BC.J+(DT)(WCBC.JK–BCWC.JK)

These two statements are straightforward accounting: The number of white-collar persons at time K (WC.K) is equal to the number of such persons at the previous time point plus the number moving from blue collar to white collar over the time period JK (BCWC.JK), less the number moving from white collar to blue collar over the same period (WCBC.JK). The same two flows also describe the changes in the blue-collar population, but take the opposite signs.

Change in the number of white-collar persons between the two time points depends on the rates of movement from blue collar into white collar and rates of movement from white collar to blue collar. In the most basic of mobility models, these rates are governed by "transition probabilities" that express the odds that an individual in a given origin status will move out of that status to a given destination in a fixed time period. The number of actual movements from an origin to a destination, then, depends on the transition probability of such a movement, the number of persons "at risk" (i.e., the number in the origin status), and the length of time that the people are exposed to the risk. For our two-state model, we could specify these rates with the following statements:

```
R    WCBC.KL = PWCBC*WC.K
R    BCWC.KL = PBCWC*BC.K
```

These statements say simply that the rate of movement from white collar to blue collar over the period from K to L (WCBC.KL) is equal to the constant probability of such a move (PWCBC) times the number at risk (WC.K); the rate of movement from blue collar to white collar over the period (BCWC.KL) is equal to the constant probability of such a move (PBCWC) times the number at risk (BC.K). There is one constraint to be noted here. The quantities PWCBC and PBCWC are probabilities, and hence can logically vary only between 0 and 1. In a model with larger numbers of origins and destinations, the persons in any origin status are constantly "at risk" of making a transition to any of the destination statuses. The sum of these "risks" or transition probabilities is also constrained to be less than or equal to unity. These constraints are perhaps more clearly seen in a model with more statuses, such as the five-state model that we will use as our baseline here. The DYNAMO code for such a five-state model is given in the Appendix.

Behavior of the Baseline Model

The dynamic behavior of first-order Markov processes such as the one we have created here is well understood, and is the same whether there are two, five, or any number of states.[10] Such systems have stable equilibria that are approached asymptotically; the values of the states of the system at equilibrium depend only on the transition probabilities—not on initial conditions, and the value of any state at any point in time is a function of its value at the previous point in time times the relevant transition probabilities (or, alternatively, is a function of the initial value, the length of time that the process has been operating, and the transition probabilities).

These properties can easily be deduced by simulation, as well as by direct solution of the equations of the system (though the latter is more elegant and general). If we simulate the five-state system as shown in the Appendix (with 100 persons in each stratum initially, and all transition probabilities set constant at .2) the system remains in a stable equilibrium. That is, the number of persons in each stratum does not change from period to period, though they are not the same people, as individuals are constantly changing statuses.

If we leave the transition probabilities constant, but change the initial conditions, the system ultimately returns to the same state as in the baseline, but does so after periods of time that vary with the difference between the initial conditions and the equilibrium. This property is illustrated by the results in the top panel of Table 9.4. In this panel we report the number of persons in the fifth stratum after various elapsed times for three scenarios: (a) the system has its equilibrium distribution at the start, (b) there are 50 rather than 100 persons in the fifth stratum at the start, and c) there is only a single person in the fifth stratum at the start.

The first column of the top panel shows that the system remains in equilibrium once equilibrium is established. This statistical equilibrium exists despite the fact that 10% of the people in stratum five leave the stratum each time period, because they are replaced by outflows from other strata. The second and third columns of the top panel report scenarios where numbers of persons smaller than the equilibrium number are initially in stratum five. The behavior of these two series demonstrates the tendency of the system to asymtopically approach its equilibrium. The speed with which this approach occurs, of course, depends on the magnitude of the transition rates—the more people are moving around, the faster the system approaches its steady state.

The first experiment suggests that this system has a a stable equilibrium, and that this equilibrium condition does not depend on the numbers of persons in each status initially. The second panel of Figure 9.4 shows the results of another series of experiments that explore the consequences of varying transition rates rather than system levels. In the first series we define stratum one as an "absorbing state." That is, persons flow into this stratum, but do not leave it. In the second column we set the outflow rate for the first stratum to 5%, or one-half of the outflow rates of the other strata; the third column sets the outflow rates of stratum one equal to those of the other strata (that is, 10%); the last column shows outflow rates from stratum one that are twice the outflow rates of the other strata.

This second series of experiments demonstrates the other basic property of first-order Markov processes—that the equilibrium distribution of cases across statuses does depend on the transition rates. Where the first stratum is an absorbing state (column one), eventually all of the 500 individuals end up in it. This occurs rather slowly because

TABLE 9.4
Baseline Mobility Matrix Experiments

Differing Initial Conditions	*Numbers of Persons in Stratum Five Initial*			
Elapsed Time	*100*	*50*	*1*	
2	100	82	65	
4	100	94	87	
6	100	98	95	
8	100	99	98	
10	100	100	99	
Differing Transition Rates	*Numbers of Persons in Stratum One Outflow Rate*			
Elapsed Time	*0%*	*5%*	*10%*	*20%*
0	100	100	100	100
2	173	130	100	62
4	232	147	100	57
6	281	156	100	56
8	321	161	100	56
10	354	164	100	56
15	411	166	100	56
20	446	167	100	56

of the large number of sequences of moves that are possible in each point in time. Where the outflow rates of a stratum are less than those of another, the stratum will asymptotically approach a level that is larger than the other. Where the rates are equal, equilibrium numbers are equal, where rates are less, final numbers are less. In all cases, the paths that levels follow toward their equilibrium levels are exponential.

These results are hardly original, but they are important. Formally, systems of this type have stable equilibria, these equilibria depend upon the transition rates and not on the initial conditions, and the levels tend to approach their equilibrium conditions by smooth exponential time-paths.

We can also state these conclusions somewhat less formally. There are many systems that approximate the ideal type of movements occurring back and forth among many states, where the chances individuals making transitions from their current status to another status is constant over time and the same for all individuals in the status. Systems of this type tend toward stable numbers of cases being in each status (even though lots of cases may be making transitions), and the relative numbers of cases in the various statuses depend solely on the odds of the various changes occurring.

Extensions of the "Mobility Matrix"

The baseline mobility matrix model is relatively simple in the number of levels, their connectivity, and the control structures governing rates of flow among the states. There are, however, several ways in which the baseline model could easily be modified to make it more useful for analyzing social mobility processes and for other systems of bidirectional flows among multiple origins and destinations.

As a model of processes of status mobility, the baseline is probably most seriously deficient in that it assumes that all of the people composing the flows are homogeneous. This follows from the specification that the chances of a given individual's undergoing mobility are the same as those of every other individual with the same origin status. There are many cases in which we might want to distinguish among the persons in the various statuses according to other characteristics, both because we can presume that they are not really homogeneous with regard to mobility chances, and because our interest may focus on the composition of the population of persons occupying statuses.

We might suppose that the mobility chances of males and females in entry-level positions are not identical in a large organization; our interest might well be in the ratio of males and females in executive positions that are the outcome of the dynamics of mobility. Similarly, we might suppose that people in a given status are at decreasing (or, more rarely) or increasing risk of undergoing transition the longer that they have been in the status—that is, the "population" is "heterogeneous" with respect to duration in the state. There is considerable theory and evidence suggesting, for example, that, after a time, the chances of further promotion decline with tenure in a position in many large organizations (that is, those who are going to be promoted are promoted early, those who remain behind are less likely to ever be promoted).

The baseline mobility model can be extended to deal with heterogeneity of transition probabilities by both traits and duration rather easily—though we will not do so here. The method for doing so is tedious, but relatively straightforward. If one is interested in taking into account gender differences in mobility in a three-state "matrix," six rather than three system states are created. Some of the logically possible flows, of course, cannot exist: Low-status female persons have zero probability of undergoing a transition to become middle-status males; their chances of attaining middle-level status, however, may now be modeled as different from those of low-status males. Duration dependence of transition rates can similarly be captured by creating chains of transitions within each status that represent heterogeneity by duration (for example, new low-status persons, low-status persons of average tenure, low-status persons of longer than mean duration). In some cases the first- and third-order delay functions provided by DYNAMO (or others designed by the theorist using macros) can capture this form of heterogeneity quite compactly.

As a model for theorizing about social mobility, the baseline developed above is deficient in a second way. In the current model we follow the spirit of Markov models in assuming that transition probabilities for individuals are fixed functions with respect to time (though, as we have just seen, they may differ across individuals). In real systems, the number of positions (or vacancies) at various levels may well change over time—increasing or decreasing all rates of movement into or out of statuses. Indeed, most studies of social mobility at the societal level suggest that the largest part of all status changes can be accounted for by changes in the "structure" of opportunity. Again, there would be little difficulty in adapting the baseline-mobility matrix model to take into account such structural change, though we will not do so here. In the current model, rates are modeled as dependent upon constant transition probabilities and the numbers of persons in the origin statuses. To take into account opportunity structures, more complex rules could be written that take into account the number of vacancies as well as the number of candidates.

Chains as Building Blocks

The models that we have examined in this chapter are extremely general and important. We have developed our examples, perhaps somewhat self-indulgently, from the fields of stratification and demography in sociology. Simple chains coupled by differing kinds of control systems, however, are central in the concerns of all of the social sciences. Indeed, from the abstract systems perspective that we have taken, many of the problems about which very diverse social scientists theorize have much the same structure.

"Chain" models involve the rates of movements of conserved quantities among networks of states. A moment's reflection suggests that many central social science problems can be (and, in fact, have been) usefully thought of in this way.

The dynamics of economic relations involve the flows of human and physical capital, money, and natural resources among "statuses." The "circulation of capital," the production "process," and exchanges between buyers and sellers can all be readily conceived as flows of quantities (be these quantities money, people, capital, or resources) among states (where the states may reflect qualitative characteristics or ownership or both). Indeed, the starting point for both macro and micro economic theory is with these basic "accounting systems" of flows of money, people, and things: Raw materials are extracted, transported, transformed, sold, and eventually discarded; human resources are employed, endowed with training and experience, and retired; money is acquired through exchange with customers, and transferred to workers and suppliers. The chains needed to usefully capture the dynamics of economic systems can be considerably more complicated (in terms of numbers of states, connectivity among them, and in control systems) than the basic models that we have examined in this chapter. These more complicated systems, however, are built up from the same kinds of simple chains that we have considered here.[11]

Many of the central theoretical concerns of political science are also built upon "accounting" of conserved flows. Most obviously, the shifts of voters and legislators from support to opposition for policies and candidates can be usefully thought of as a "chain" of states. Political phenomena are often thought of as systems composed of flows of information, influence, and resources among individuals, parties, and governments. Again, the mobility models developed in this chapter are probably a bit too simple to capture the dynamics of most political systems without substantial modification. Simple chains of conserved flows of persons, votes, and resources, however, lie at the core of theorizing about political behavior.[12]

While it may at first seem more natural to think of the application of simple chain models to "macro" phenomena such as population movements, economic production, or voter support; phenomena at the "micro" level can also be usefully conceived of as composed of "chains" of states. One very obvious and important application is in studying the microstructure of social relations. Many social structures can be thought of as networks of individuals (or firms, families, nations, etc.) connected by "relations" involving conserved flows (individuals exchange emotional support, firms exchange personnel and money, and so on). The dynamics of such networks can be captured rather nicely using relatively simple models.

With relatively little modification, then, "simple" chain models of various numbers of states, forms and degrees of connectivity, and complexity of control can be widely applied to basic and important problems across the social sciences. As we have seen throughout the chapters in this section, a wide array of important forms of social dynamics can be modeled with these "simple" models. Simple chains are of great importance for another reason as well. All models of greater complexity are built up out of simple chains. It is a necessary, but not a sufficient condition for understanding the dynamics of the more complex systems that we understand the dynamics of simple chains.

Models composed of single chains, regardless of their complexity, however, are not sufficient to capture the dynamics of many systems. In the next section we will consider how theories of greater complexity are built up out of coupling together "simple" chains with more complex control structures; just as the current chain models are composed of single states coupled with increasing complex control structures. The behavior of "chains" requires the understanding of single states and rates, but is not wholly reducible to the component parts; the behavior of more complex systems requires an understanding of each of the chains of which they are composed, but they also have unique possibilities that are not wholly predictable from their parts.

Notes

1. The three kinds of systems that we will examine here are well known and widely studied by statistical and mathematical approaches, as well as by simulation methods. Our treatment of these systems is not intended to make original contributions to the substantial bodies of theory and analysis that exist with regard to each; rather, the intent is to see these important dynamic processes as the outcomes of relatively simple chain models.

2. Basic demographic processes are presented in any number of excellent texts, including Keyfitz (1977), Shryock, Siegel, and Associates (1976), Bogue (1969), Cox (1959), and Hauser and Duncan (1959).

3. For purposes of accurate projections over lengthy periods of time, such simplifying assumptions and the accuracy of the integration algorithms are of great consequence. The population predictions presented in this chapter should be taken only as illustrative of general patterns and dynamics. They should not be taken as serious population projections.

4. Data are calculated from tables presented in United States Bureau of the Census (1985).

5. For some interesting and more sophisticated modeling of fertility dynamics and their consequences, see Keyfitz (1971, 1975).

6. The classic work on vacancy chains is that of Harrison White (1970). Recent elaborations have extended the model in interesting ways; see particularly Stewman (1975), Rosenbaum (1979), and Stewman and Konda (1983).

7. Explorations into the effects of changes in system size and shape have been extensively explored by many analysts, among them Anderson and Warkov (1961), Blau (1970), Hummon (1971), Kassarda (1974), Kennedy (1962), and Land (1970, 1975).

8. Problems of information delay, distribution, and distortion have been a major topic of interest to theorists of organizations. See, for some particularly interesting examples, Ackoff (1959), Bavelas (1950), Bonini (1963), Cohen and Cyert (1965), Cyert et al. (1971), Cyert and March (1963), Emery and Trist (1960), Katz and Kahn (1966), Kochen and Deutsch (1980), Marshall (1967), and Simon (1947).

9. There is a large literature on such processes in sociology, particularly with regard to the intra-and intergenerational changes in individual's occupational prestige. A flavor of some of the mathematical and statistical approaches to problem can be found in Blumen (1966), Boudon (1975), Ginsberg (1971), Mayer (1972), McFarland (1970), McGinnis (1968), Singer and Spilerman (1974, 1976), and Spilerman (1972a, 1972b).

10. See, for detailed discussions of Markov processes, Bartholomew (1983) (for a mathematical treatment) or Leik and Meeker (1975) (for an applied treatment).

11. See particularly Forrester's (1961) model of the firm, and Meadows and Robinson's (1985) discussion of macroeconomic models.

12. Two very useful macropolitical models are offered by Brunner and Brewer (1971) and Ilchman and Uphoff (1969).

APPENDIX 9.1. Population Age Structure Model

```
* SIMPLIFIED DEMOGRAPHIC MODEL
NOTE BASELINE DATA ARE USA 1980
NOTE                        THE NUMBER AGES ZERO TO ONE IS
NOTE                        DETERMINED BY THE BIRTH RATE, WHICH IS A
NOTE                        FUNCTION OF THE NUMBER AGES 15-44.
L           P01.K = P01.J+(DT)(BR.JK-ISR.JK-IDR.JK)
R           BR.KL = FERT*P1544.K
C           FERT = .0342
R           ISR.KL = P01.K*.98711
R           IDR.KL = P01.K*.01289
NOTE ISR IS INFANT SURVIVAL, IDR IS INFANT DEATHS
L           P14.K = P14.J+(DT)(ISR.JK-CSR.JK-CDR.JK)
R           CSR.KL = (P14.K/4)(.99936)
R           CDR.KL = (P14.K/4)(.00064)
NOTE CSR IS CHILD SURVIVAL, CDR IS CHILD DEATH RATE
NOTE                        LEAVING RATE IS SIMPLIFIED TO BE 1/4 OF
NOTE                        NUMBER IN THE LEVEL AT EACH POINT IN
NOTE                        TIME.
L           P514.K = P514.J+(DT)(CSR.JK-ASR.JK-ADR.JK)
R           ASR.KL = (P514.K/10)(.9969)
R           ADR.KL = (P514.K/10)(.00031)
NOTE ADOLESCENT SURVIVAL AND DEATH RATES ASR, ADR
L           P1544.K = P1544.J+(DT)(ASR.JK-PSR.JK-PDR.JK)
R           PSR.KL = (P1544.K/30)(.99854)
R           PDR.KL = (P1544.K/30)(.00146)
NOTE PRIME-AGE SURVIVAL AND DEATH RATES PSR, PDR
L           P4564.K = P4564.J+(DT)(PSR.JK-MSR.JK-MDR.JK)
R           MSR.KL = (P4564.K/20)(.99044)
R           MDR.KL = (P4564.K/20)(.00956)
NOTE MIDDLE-AGED SURVIVAL AND DEATH RATES MSR, MDR
L           P65.K = P65.J+(DT)(MSR.JK-AGDR.JK)
R           AGDR.KL = P65.K*.05668
NOTE DEATH RATES FOR AGE 65+, AGDR
S           TPOP.K = P01.K+P14.K+P514.K+P1544.K+P4564.K+P65.K
NOTE TOTAL POPULATION IS COMPUTED
```

```
N          P01 = 3269600
N          P14 = 13078400
N          P514 = 34942000
N          P1544 = 105203000
N          P4564 = 44503000
N          P65 = 25550000
NOTE INITIALS ARE SET AT USA 1980 VALUES
SPEC       DT = .25/LENGTH = 100/PRTPER = 5/PLTPER = 1
PRINT      P01,P14,P514,P1544,P4564,P65,TPOP
PLOT       P01/P14/P514/P1544
PLOT       P4564/P65/TPOP
RUN
```

APPENDIX 9.2. Vacancy Chain Model

```
*          SIMPLE VACANCY CHAIN MODEL
NOTE                This model is an example of a simple vacancy chain
NOTE                with three levels: "Entry" level (EL), "Middle"
NOTE                level (ML) and "Senior" level (SL).
L          EL.K = EL.J+(DT)(HIRE.JK-PREM.JK)
NOTE                HIRE IS THE NUMBER HIRED INTO ENTRY
NOTE                POSITIONS
NOTE                PREM IS THE NUMBER PROMOTED TO MIDDLE
NOTE                LEVEL
R          HIRE.KL = MAX(DISC1.K,0)
NOTE DISC1 IS THE DISCREPANCY BETWEEN THE NUMBER OF
NOTE                ENTRY LEVEL PERSONS AND THE "GOAL" OR
NOTE                DESIRED NUMBER.
A          DISC1.K = EGOAL-EL.K
NOTE                THE GOAL FOR ENTRY LEVEL NUMBERS IS 600
NOTE                PERSONS
C          EGOAL = 600
R          PREM.KL = MAX(DISC2.K,0)
NOTE                THE RATE OF PROMOTION FROM ENTRY TO
NOTE                MID LEVEL IS SET EQUAL TO THE DISCREPANCY
NOTE                BETWEEN THE NUMBER OF PERSONS AT THE
NOTE                MIDDLE LEVEL, AND THE GOAL FOR THAT
NOTE                LEVEL.
A          DISC2.K = MGOAL-ML.K
NOTE                THE GOAL FOR MID LEVEL IS SET TO 300.
C          MGOAL = 300
NOTE                THE MIDDLE LEVEL IS NOW DEFINED.
L          ML.K = ML.J+(DT)(PREM.JK-PRES.JK)
NOTE                THE NUMBER OF PERSONS AT THE MID
NOTE                LEVEL IS AUGMENTED BY PROMOTIONS FROM
NOTE                BELOW (PREM) AND DECREMENTED BY
NOTE                PROMOTIONS TO THE SENIOR LEVEL (PRES).
R          PRES.KL = MAX(DISC3.K,0)
```

```
A       DISC3.K = SGOAL-SL.K
NOTE            PROMOTIONS FROM MID TO SENIOR LEVEL
NOTE            DEPEND ON THE DISCREPANCY (DISC3)
NOTE            BETWEEN THE NUMBER AT THE SENIOR LEVEL
NOTE            (SL) AND THE GOAL FOR THAT LEVEL (SGOAL).
C       SGOAL = 100
NOTE            THE SENIOR LEVEL IS NOW DEFINED
L       SL.K = SL.J+(DT)(PRES.JK-RR.JK)
NOTE            SENIORS ARE AUGMENTED BY PROMOTIONS
NOTE            FROM MID LEVEL (PRES), AND DECREMENTED
NOTE            BY THE RETIREMENT RATE (RR).
R       RR.KL = SL.K/5
NOTE            RETIREMENTS ARE A CONSTANT 20% OF THE
NOTE            SENIORS    INITIALIZATION OF LEVELS
N       EL = ELI
C       ELI = 600
N       ML = MLI
C       MLI = 300
N       SL = SLI
C       SLI =100
NOTE            SUPPLEMENTAL INFO: PROMOTION CHANCES.
S       PROMO1.K = PREM.KL/EL.K
S       PROMO2.K = PRES.KL/ML.K
NOTE            OUTPUT SPECIFICATION
SPEC       DT = .1/LENGTH = 25/PRTPER = 1/PLTPER = 1
PRINT      EL,ML,SL,RR,PROMO1,PROMO2
PLOT       EL/ML/SL
PLOT       PROMO1/PROMO2/RR
RUN
```

APPENDIX 9.3. Five-State Mobility Matrix

```
*       FIVE-STATE MOBILITY MATRIX WITH BIDIRECTIONAL FLOWS
NOTE            THE NUMBERS IN THE FIVE STATES ARE DEFINED
L       S1.K = S1.J+(DT)(R21.JK+R31.JK+R41.JK+R51.JK-R12.JK-R13.JK
X         -R14.JK-R15.JK)
L       S2.K = S2.J+(DT)(R12.JK+R32.JK+R42.JK+R52.JK-R21.JK-R23.JK
X         -R24.JK-R25.JK)
L       S3.K = S3.J+(DT)(R13.JK+R23.JK+R43.JK+R53.JK-R31.JK-R32.JK
X         -R34.JK-R35.JK)
L       S4.K = S4.J+(DT)(R14.JK+R24.JK+R34.JK+R54.JK-R41.JK-R42.JK
X         -R43.JK-R45.JK)
L       S5.K = S5.J+(DT)(R15.JK+R25.JK+R25.JK+R45.JK-R51.JK-R52.JK
X         -R53.JK-R54.JK)
NOTE
NOTE            LEVELS ARE INITIALIZED WITH CONSTANTS
NOTE            SI1 . . . SI5
```

```
NOTE
N         S1 = SI1
C         SI1 = 100
N         S2 = SI2
C         SI2 = 100
N         S3 = SI3
C         SI3 = 100
N         S4 = SI4
C         SI4 = 100
N         S5 = SI5
C         SI5 = 100
NOTE
NOTE                    OUTFLOW TRANSITION RATES ARE DEFINED
NOTE                    AS CONSTANT PARAMETERS TIMES THE
NOTE                    NUMBERS IN THE ORIGIN STATES
R         R12.KL = PARM12*S1.K
R         R13.KL = PARM13*S1.K
R         R14.KL = PARM14*S1.K
R         R15.KL = PARM15*S1.K
NOTE
R         R21.KL = PARM21*S2.K
R         R23.KL = PARM23*S2.K
R         R24.KL = PARM24*S2.K
R         R25.KL = PARM25*S2.K
NOTE
R         R31.KL = PARM31*S3.K
R         R32.KL = PARM32*S3.K
R         R34.KL = PARM34*S3.K
R         R35.KL = PARM35*S3.K
NOTE
R         R41.KL = PARM41*S4.K
R         R42.KL = PARM42*S4.K
R         R43.KL = PARM43*S4.K
R         R45.KL = PARM45*S4.K
NOTE
R         R51.KL = PARM51*S5.K
R         R52.KL = PARM52*S5.K
R         R53.KL = PARM53*S5.K
R         R54.KL = PARM54*S5.K
NOTE
NOTE                    THE PARAMETERS ARE NOW SET. NOTE THE
NOTE                    CONSTRAINT THAT THE SUM OF THE OUTFLOW
NOTE                    RATES FROM A GIVEN STATE MUST BE LESS
NOTE                    THAN OR EQUAL TO UNITY. FOR EXAMPLE
NOTE                    PARM12+PARM13+PARM14+PARM15 MUST BE
NOTE                    LESS THAN OR EQUAL TO UNITY.
C         PARM12 = .2
C         PARM13 = .2
C         PARM14 = .2
C         PARM15 = .2
NOTE
```

```
C          PARM21 = .2
C          PARM23 = .2
C          PARM24 = .2
C          PARM25 = .2
NOTE
C          PARM31 = .2
C          PARM32 = .2
C          PARM34 = .2
C          PARM35 = .2
NOTE
C          PARM41 = .2
C          PARM42 = .2
C          PARM43 = .2
C          PARM45 = .2
NOTE
C          PARM51 = .2
C          PARM52 = .2
C          PARM53 = .2
C          PARM54 = .2
NOTE
NOTE               OUTPUT SPECIFICATION
NOTE
SPEC          LENGTH = 18/DT = .10/PRTPER = 1/PLTPER = 1
PRINT         S1,S2,S3,S4,S5
PRINT         R12,R13,R14,R15
PRINT         R21,R23,R24,R25
PRINT         R31,R32,R34,R35
PRINT         R41,R42,R43,R45
PRINT         R51,R52,R53,R54
PLOT          S1,S2,S3,S4,S5(0,100)
RUN
```

Part III

Complex Action and Interaction

In the second part of this volume we considered processes that could be represented by systems with a single state, or by systems of relatively small numbers of states formed into "chains." The range of dynamic phenomena that can be represented by such "simple" systems is surprisingly broad, and includes most of the kinds of processes normally studied by mathematical and statistical means. Nonetheless, there are many phenomena that are not representable by systems of the types that we've discussed so far, and we must now turn to more complex models.

The systems that we've considered so far have been used to represent the dynamics of single individuals or single populations. This is obviously not good enough for many applications in the social sciences that involve multiple actors: persons, business firms, clans, nations, political parties, etc. The dynamics that we have considered have represented actors (individuals, variables, or aggregates) responding to "internal" stimuli, or to their environments. We have not, however, considered dynamics of *interaction* among multiple "smart" actors.

The elements of systems thinking that we discussed in part one of this volume are the building blocks of the single-state systems that we discussed in the first portion of part two. These "single-state" systems, in turn, are the building blocks of the somewhat more complex "chain" models that we've just considered. Not surprisingly, the more complex models that we will discuss in this section use "chains" as their basic building blocks, and achieve their greater complexity by coupling chains together with control systems.

The kinds of systems that we will consider in this third section are useful for representing patterns of social interaction among multiple actors, or among the "parts" of differentiated systems. In Chapter 10 we will examine two nations competing in an arms race; in Chapter 11 we will examine the relationships between an individual and their network of social support. Each of these dynamics call for systems of multiple actors in dynamic interaction. In Chapter 12 we will examine two alternative views of the political economy of capitalist nations. The models developed to represent these systems describe a single actor (a society) that is composed of multiple institutional subsystems in dynamic interaction (economy, state, and cultural sectors). These models also serve as examples of the range of phenomena that can be approached with increasingly complex dynamic systems models.

Social Action and Interaction

The models examined so far might be termed "closed-system" models, in that they deal with the behavior of single aggregates or single individuals (persons, organizations, societies, etc.). Social action does often resemble such "closed-system" situations in which individuals act independently, or simply respond to environmental stimuli. Where such an assumption is reasonable, each individual is the same as every other one, and we can understand the behavior of each actor and all actors by modeling one. A good deal of social action, however, cannot be represented in this fashion. In many cases individual's actions are not merely responsive to a stable environment and to their own states. In many cases individual actors interact with the environment (which may be composed of many other actors). The models we've examined so far are concerned with social action (i.e., action that is based on learned meanings and goals and takes others into account), but have not really considered social interaction (i.e., where actors are mutually responsive to each others acts through systems of shared meanings).

Each of the models discussed so far has a second limitation for describing many forms of social behavior. In each of the cases we've considered, only the simplest forms of "coupling" and "feedback-control" have been considered. In trying to create useful theories of human behavior, we must confront the fact that another part of what we mean by "social" behavior is often based on extremely complex systems of monitoring, calculation, and goal setting—that is, social interaction is often quite "smart" in the sense that we have used the term in this volume. Most of the models that we've examined so far have been quite simple in this regard, involving either direct and straightforward "dumb" control or relatively simple "goal seeking." Many patterns of social action cannot be adequately represented with such simple tools.

Models of "social" action, by definition, are based on actors' responses to stimuli as they perceive them and attach meaning to them. In the models we've examined so far, we have (implicitly) assumed that actor's perceptions, assigning of meanings, and choices among strategies were identical and not problematic. For example, we ignored the process by which individuals perceive messages (or fail to, or distort), the calculations of costs and benefits that they may make according to their own values and preferences in deciding to adopt an innovation or not, and the problems and delays that they may encounter in implementing change. For many purposes, of course, it is perfectly fine to make assumptions about these processes and treat them as "black boxes" that generate an expected distribution of outcomes (the probability of adoption in a period of time). But sometimes we might prefer to focus our attention on theorizing about variance in these processes of social cognition and decision making at the individual level.

Systems Complexity

The limitations of the kinds of models that we have been considering so far can also be seen from a "general systems" perspective. The kinds of processes that we have considered are, in the terms used here, not highly complex. That is, they involve relatively few states, these states are coupled in simple patterns (usually single chains), and the mechanisms of control have been relatively simple—often being easily describable by very simple linear equations.

The models that we have considered thus far are predominantly of "closed" systems. That is, the models reflect the working out of the consequences of the initial conditions where only the levels of the states of the focal system have effects on the realizations of the processes. Of course, as we have pointed out, such models could be made into "open" systems by allowing for exogenous changes of various sorts. However, we have not attempted to model interaction between the focal system and others—that is, processes in which the actions of each actor become the environment to which the other actors respond.

The linkages among states in the models we have examined so far are also relatively simple. For the most part, the states of the models that we have been considering are governed by direct material flows (as in people moving from one age group to another) and simple laws describing information effects (e.g., the flow of people from one level of a hierarchy to another is governed by the "information" of the number of vacancies at the higher level). In most social interaction, we might imagine that the linkages are more complex, more contingent, and more filled with error, selective perception, and distortion than are the "flows through chains" types of models.

The models that we will consider in the next several chapters are of considerable complexity in that they couple multiple "chains" together by means of (often quite complicated) flows of information. The range of dynamic behaviors that such models can produce is virtually unlimited. And the complexity of phenomena that can be modeled by putting together simple chains is limited only by imagination and resources. With the consideration of the "linked chain" models in this section, we will have in hand all of the "templates" of system types that one may need in order to undertake the building of theories about phenomena of any degree of complexity.