Equal Gain Combining Over Nakagami-n (Rice) and Nakagami-q (Hoyt) Generalized Fading Channels

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***Abstract*—Motivated by the importance of Nakagami-n (Rice) and Nakagami-q (Hoyt) statistical models to describe channel fading in land, mobile, terrestrial, and satellite telecommunications, we present an alternative moments-based approach to the performance analysis of equal-gain combining (EGC) receivers over independent, not necessarily identically distributed Rice- and Hoyt-fading channels. Exact closed-form expressions for the moments of the signal-to-noise ratio (SNR) at the output of the combiner are derived and significant performance criteria such as, the average output SNR, the amount of fading and the spectral efficiency at the low power regime, are studied. Moreover, using Padé rational approximation to the moment-generating function of the output SNR, the average symbol error probability and the outage probability are evaluated. We also study the suitability of modeling a Hoyt-fading environment by a properly chosen Nakagami-m model, as far as the error performance of the EGC is concerned.**

***Index Terms*—Amount of fading, average error probability, equal-gain combining (EGC), Nakagami-n (Rice) fading, Nakagami-q (Hoyt) fading.**

I. Introduction

Diversity at the receiver is a well-known promising avenue for improving mean signal strength and reducing signal level fluctuations in fading channels, where the multiple received copies can be combined intelligently to provide a higher average received signal-to-noise ratio (SNR). The main diversity techniques are equal-gain combining (EGC), maximal-ratio combining (MRC), selection combining (SC), and a combination of MRC and SC, called generalized-selection combining (GSC). Among them, EGC presents significant practical interest because it provides performance comparable to optimal MRC technique, but with greater simplicity. The performance evaluation of EGC receivers operating over Nakagami-n (Rice) and Nakagami-q (Hoyt) fading channels, has not yet received as much attention as the Rayleigh- and Nakagami-m fading channels, mainly due to the complex form of their probability density functions (pdfs), despite the fact that these models exhibit an excellent fit to experimental fading channel measurements for land, mobile, terrestrial, and satellite telecommunications. More specifically, Nakagami-n (Rice) distribution [1], [2], which contains the Rayleigh distribution as a special case, provides the optimum fits to collected data from indoor [3], [4], outdoor [5] and mobile satellite applications [6]. Nakagami-q (Hoyt) [2] distribution is normally observed on satellite links subject to strong ionospheric scintillation and ranges from one-sided Gaussian fading to Rayleigh fading [7], [8]. Next, we call Rice and Hoyt the two fading models under investigation.

Previous related work concerning predetection EGC diversity with Rice and Hoyt fading are included in [9]–[12] (and references therein). More specifically, Abu-Dayya and Beaulieu [9], approximating the cumulative distribution function (cdf) of the sum of L statistically independent Rician random variables (rvs), evaluated the error performance of coherent BPSK and noncoherent BFSK employing EGC diversity. However, as mentioned by Helstrom [13], the ingenious series of Beaulieu has the form of 0.5 plus or minus a sum. When the tails of the distribution are sought, that sum is close to ± 0.5 and many terms are needed to determine the sum accurately enough, so that significant figures are not lost by round-off error, when added to or subtracted from 0.5. Vitetta *et al.* [10] studied the error performance for noncoherent BFSK employing dual EGC in the presence of slow, correlated, Rician time-selective fading. Karagiannidis *et al.* [11] presented an alternative, semi-analytical approach for the evaluation of the cdf of the weighted sum of L independent Rician rvs with or without additive white Gaussian noise (AWGN), using Hermite numerical integration. This result is then used to evaluate error rates for coherent BPSK. Finally, Annamalai *et al.* [12] presented an alternative approach to evaluate average error rates for EGC receivers operating in several fading environments (Rayleigh, Nakagami-m, Rice, Hoyt), transforming the average error integral into the frequency domain, using the Parseval's theorem. However, the error rate expressions given for the Rice- and Hoyt-fading cases, include infinite range integrals and integrands composed of infinite sums of complex functions (confluent hypergeometrics), due to the complex form of the their characteristic functions (chfs).

In this letter, we overcome the difficulty of finding the pdf of the sum of fading rvs and the use of the complex forms of the chfs, by presenting an alternative moments-based approach to the performance analysis of EGC receivers, operating over independent and nonidentical Rician- and Hoyt-fading channels. We study important performance criteria using exact closed-form expressions for the moments of the combiner's output SNR and approximating the corresponding moment-generating function (mgf) with the Padé approximants theory [14]. More specifically: 1) the average output SNR, the amount of fading (AoF) and the spectral efficiency (SE) at the low power regime of the EGC, are expressed in simple closed-forms, for arbitrary

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number of input branches and arbitrary fading severity parameters; and 2) the average symbol error probability (ASEP) and the outage probability are accurately approximated using the well-known mgf approach [15]. Moreover, we investigate whether a Hoyt-fading environment can be represented by a properly chosen Nakagami-m model, as far as the error performance of the EGC is concerned. The proposed mathematical analysis is illustrated by various numerical results and graphs.

II. Moments of the Output SNR

We consider an L-branch EGC receiver with statistically independent but not necessarily identically distributed input branches, operating in a flat fading environment. Such a channel model covers the case of antenna diversity, where the input channels tend to be identically distributed as well as multipath diversity frequency-selective fading channels where the input power delay profile (pdp) tends to be nonuniform. The output SNR, γ_{out}, of the receiver is given by

$$\gamma_{\text{out}} = \frac{E_s}{L N_0}\left(\sum_{i=1}^{L} a_i\right)^2 \tag{1}$$

where a_i is the envelope of the ith input path, modeled as Rice or Hoyt rv, E_s is the symbol energy, and N_0 is the one-sided power spectral density of the AWGN.

By definition, the nth moment of the output SNR is

$$\begin{aligned} E\left\langle\gamma_{\text{out}}^n\right\rangle &= E\left\langle\left[\frac{E_s}{LN_0}\left(a_1+\cdots+a_L\right)^2\right]^n\right\rangle \\ &= \left(\frac{E_s}{LN_0}\right)^n E\langle(a_1+\cdots+a_L)^{2n}\rangle \end{aligned} \tag{2}$$

where $E\langle\cdot\rangle$ denotes expectation. Expanding the term $(a_1+\cdots+a_L)^{2n}$, using the multinomial identity [16, eq. (24.1.2)], (2) can be written as

$$E\left\langle\gamma_{\text{out}}^n\right\rangle = \left(\frac{E_s}{LN_0}\right)^n (2n)! \times \sum_{\substack{k_1,\ldots,k_L=0\\ k_1+\cdots+k_L=2n}}^{2n}\left[\frac{E\left\langle a_1^{k_1}\cdots a_L^{k_L}\right\rangle}{\prod_{j=1}^{L} k_j!}\right] \tag{3}$$

and in terms of the instantaneous SNR of each diversity path, $\gamma_i = a_i^2 E_s/N_0$, (3) can be rewritten as

$$E\left\langle\gamma_{\text{out}}^n\right\rangle = \frac{(2n)!}{L^n}\sum_{\substack{k_1,\ldots,k_L=0\\ k_1+\cdots+k_L=2n}}^{2n}\left[\frac{E\left\langle\gamma_1^{k_1/2}\cdots\gamma_L^{k_L/2}\right\rangle}{\prod_{j=1}^{L} k_j!}\right]. \tag{4}$$

We assume in this paper that the branches of the EGC are uncorrelated, thus (4) can be simplified to

$$E\left\langle\gamma_{\text{out}}^n\right\rangle = \frac{(2n)!}{L^n}\sum_{\substack{k_1,\ldots,k_L=0\\ k_1+\cdots+k_L=2n}}^{2n}\left[\prod_{j=1}^{L}\frac{E\left\langle\gamma_j^{k_j/2}\right\rangle}{k_j!}\right]. \tag{5}$$

When the receiver operates over Rice-fading channels, the SNR of each diversity path is distributed according to a noncentral chi-square distribution. Using the definition for the moments of a noncentral chi-square rv [15, eq. (2.18)] with (5), the moments of the EGC output SNR can be written in a simple and closed-form expression given by

$$E\left\langle\gamma_R^n\right\rangle = \frac{(2n)!}{L^n}\sum_{\substack{k_1,\ldots,k_L=0\\ k_1+\cdots+k_L=2n}}^{2n}\left[\prod_{j=1}^{L}\frac{\overline{\gamma}_j^{k_j/2}\Gamma(1+k_j/2)}{k_j!(1+K_j)^{k_j/2}} \times {}_1F_1\left(-\frac{k_j}{2},1;-K_j\right)\right] \tag{6}$$

where $\overline{\gamma}_j = \Omega_j E_s/N_0$ is the average SNR per symbol of the ith branch with $\Omega_j = \overline{a_j^2}$, $\Gamma(\cdot)$ is the Gamma function [16, eq. (6.1.1)], ${}_1F_1(\cdot,\cdot;\cdot)$ is the confluent hypergeometric function of the first kind [16, ch. (13)], and K_j is the Rice factor of the jth input path, defined as the ratio of the signal power in dominant component over the scattered power. For $K_j = -\infty$ (dB) the Rayleigh fading is described, while $K_j = \infty$ (dB) represents the no-fading situation. Values of Rice factor in land mobile terrestrial (outdoor and indoor) and satellite applications usually range from 0–12 dB [5]. Into the following and without loss of generality, we assume that the Rice factor takes the same value for all diversity paths, i.e., $K_1 = \cdots = K_L = K$.

When the receiver operates over Hoyt-fading channels, the moments of the output SNR can be obtained substituting the moments of the input paths SNR's [15, eq. (2.13)] in (5) resulting in

$$E\left\langle\gamma_H^n\right\rangle = \frac{(2n)!}{L^n}\sum_{\substack{k_1,\ldots,k_L=0\\ k_1+\cdots+k_L=2n}}^{2n}\left[\prod_{j=1}^{L}\frac{\overline{\gamma}_j^{k_j/2}\Gamma(1+k_j/2)}{k_j!} \times {}_2F_1\left(-\frac{k_j-2}{4},-\frac{k_j}{4},1;\left(\frac{1-q_j^2}{1+q_j^2}\right)^2\right)\right] \tag{7}$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gauss hypergeometric function [16, eq. (15.1.1)] and q_i is the Nakagami-q fading parameter of the ith branch, which ranges from 0 (one-sided Gaussian fading) to 1 (Rayleigh fading). Again, we will assume into the following, without loss of generality, that q_i takes the same value for all diversity paths, i.e., $q_1 = \cdots = q_L = q$.

The moments of the output SNR are used into the following to study significant performance criteria of the EGC, such as the average output SNR, the AoF and the SE at low SNR of the output. Also, using the moments, the outage, and the error performance are studied, approximating the mgf with the Padé approximants theory. It must be noted, that the higher order moments are also useful in signal processing algorithms for signal detection, classification, and estimation and they play a fundamental role in understanding the performance of wideband communication systems in the presence of fading [17].

A. Average Output SNR

The average output SNR of the L-branch EGC receiver operating over independent and nonidentical Rice-fading channels can be obtained setting $n = 1$ in (6), which after some algebraic manipulations results in

$$\overline{\gamma}_R = \frac{1}{L}\sum_{i=1}^{L}\overline{\gamma}_i + \frac{\pi}{4L(K+1)}\left[{}_1F_1\left(-\frac{1}{2},1;-K\right)\right]^2 \times \sum_{i=1}^{L}\sum_{\substack{j=1\\ j\neq i}}^{L}\sqrt{\overline{\gamma}_i\overline{\gamma}_j}. \tag{8}$$

For the independent and identically distributed (i.i.d.) case ($\overline{\gamma}_i = \overline{\gamma}$), (8) reduces to the simpler expression

$$\overline{\gamma}_R = \overline{\gamma}\left[1 + \frac{\pi(L-1)}{4(K+1)}\left[{}_1F_1\left(-\frac{1}{2}, 1; -K\right)\right]^2\right]. \quad (9)$$

Note, that for $K \to -\infty$ dB, it can be easily verified that (9) is reduced to a previous published expression for Rayleigh fading channels [15, eq. (9.49)].

In a Hoyt fading environment, the average output SNR of the L-branch EGC receiver in the case of independent and nonidentical paths, can be obtained setting $n = 1$ in (7), which after some algebraic manipulations results in

$$\overline{\gamma}_H = \frac{1}{L}\sum_{i=1}^{L}\overline{\gamma}_i + \left[{}_2F_1\left(\frac{1}{4}, -\frac{1}{4}; 1; \left(\frac{1-q^2}{1+q^2}\right)^2\right)\right]^2 \times \frac{\pi}{4L}\sum_{i=1}^{L}\sum_{\substack{j=1\\ j\neq i}}^{L}\sqrt{\overline{\gamma}_i\overline{\gamma}_j}. \quad (10)$$

For the i.i.d. case, (10) reduces to

$$\overline{\gamma}_H = \overline{\gamma} + (L-1)\frac{\overline{\gamma}\pi}{4}\left[{}_2F_1\left(\frac{1}{4}, -\frac{1}{4}; 1; \left(\frac{1-q^2}{1+q^2}\right)^2\right)\right]^2. \quad (11)$$

For $q = 1$ (Rayleigh fading), (11) is simplified to [15, eq. (9.49)].

B. AoF And SE

AoF was introduced by Charash in [18] as a unified measure of the severity of a fading channel. It is typically independent of the average fading power and is defined as

$$\mathrm{AoF} = \frac{\mathrm{var}(\gamma_{\mathrm{out}})}{\overline{\gamma}_{\mathrm{out}}^2} = \frac{E\left\langle\gamma_{\mathrm{out}}^2\right\rangle}{\overline{\gamma}_{\mathrm{out}}^2} - 1 \quad (12)$$

where $\mathrm{var}(\gamma_{\mathrm{out}})$ is the variance of the output SNR. Using (6), the AoF of the EGC receiver operating over Rice fading channels [see (13), located at the bottom of the page].

When the EGC receiver operates in Hoyt fading channels, the AoF can be expressed, using (7) [see (14), located at the bottom of the page].

AoF can be used to study the SE of a flat-fading channel in the very noise (low-power) region. In such a region the minimum bit energy per noise level, required for reliable communication is $(E_b/N_0)_{\min} = -1.59$ dB and the slope S_0 of the SE curve versus (E_b/N_0) in b/s/Hz per 3 dB at $(E_b/N_0)_{\min}$ is given by [19]

$$S_0 = \frac{2E^2\left\langle a_{\mathrm{out}}^2\right\rangle}{E\left\langle a_{\mathrm{out}}^4\right\rangle} = \frac{2\overline{\gamma}_{\mathrm{out}}^2}{E\left\langle\gamma_{\mathrm{out}}^2\right\rangle} \quad (15)$$

with a_{out} being the combiner's output envelope. Equation (15) can be written in terms of the combiner's output AoF, i.e.,

$$S_0 = \frac{2}{\mathrm{AoF}+1}. \quad (16)$$

Using (13) and (14) with (16), the SE in the low-power region can be evaluated for EGC receivers operating over Rice and Hoyt fading channels, respectively.

C. Numerical Results

Assuming that the receiver operates with an exponentially decaying pdp ($\overline{\gamma}_i = \overline{\gamma}_1 e^{[-\delta(i-1)]}$), Fig. 1 plots the first branch normalized average output SNR of EGC, as a function of L, for Rice ($K = 0$ and $K = 7$ dB) and Hoyt fading ($q = 0.3$ and $q = 0.6$) and several values of the power decay factor δ. In Fig. 2, the AoF for EGC operating over Rice and Hoyt fading is depicted versus the corresponding fading parameter, K or q, for several values of L. As it was expected, the combiner mitigates the fading more effectively as the number of branches increase, but this improvement does increase proportionally with L. Fig. 3 plots the SE in the high noise region for an EGC receiver operating in a Rician-fading environment, for several values of L and K. In Fig. 3, the SE of the AWGN channel is also plotted for comparison reasons.

III. Error Rates and Outage Probability

A. ASEP

The mgf-based approach presented in [15, ch. 1] is a unified method to calculate error rates for several modulation schemes. However, for the EGC receiver of interest, a useful expression for the mgf of the output SNR is not available. For this reason, we propose in this paper the Padé approximants theory as an alternative and simple way to approximate the mgf. For the reader's convenience we explain briefly how the Padé approximants theory can be applied, in order to find an accurate rational

$$\mathrm{AoF}_R = \frac{4!\sum_{\substack{k_1,\ldots,k_L=0\\ k_1+\cdots+k_L=4}}^{4}\left[\prod_{j=1}^{L}\frac{\overline{\gamma}_j^{k_j/2}\Gamma(1+k_j/2)}{k_j!(1+K)^{k_j/2}}{}_1F_1\left(-\frac{k_j}{2}, 1; -K\right)\right]}{\left[\sum_{i=1}^{L}\overline{\gamma}_i + \frac{\pi}{4(K+1)}\left[{}_1F_1\left(-\frac{1}{2}, 1; -K\right)\right]^2\sum_{i=1}^{L}\sum_{\substack{j=1\\ j\neq i}}^{L}\sqrt{\overline{\gamma}_i\overline{\gamma}_j}\right]^2} - 1 \quad (13)$$

$$\mathrm{AoF}_H = \frac{4!\sum_{\substack{k_1,\ldots,k_L=0\\ k_1+\cdots+k_L=4}}^{4}\left[\prod_{j=1}^{L}\frac{\overline{\gamma}_j^{k_j/2}\Gamma(1+k_j/2)}{k_j!}{}_2F_1\left(-\frac{k_j-2}{4}, -\frac{k_j}{4}, 1; \left(\frac{1-q^2}{1+q^2}\right)^2\right)\right]}{\left[\sum_{i=1}^{L}\overline{\gamma}_i + \frac{\pi}{4}\left[{}_2F_1\left(\frac{1}{4}, -\frac{1}{4}; 1; \left(\frac{1-q^2}{1+q^2}\right)^2\right)\right]^2\sum_{i=1}^{L}\sum_{\substack{j=1\\ j\neq i}}^{L}\sqrt{\overline{\gamma}_i\overline{\gamma}_j}\right]^2} - 1 \quad (14)$$

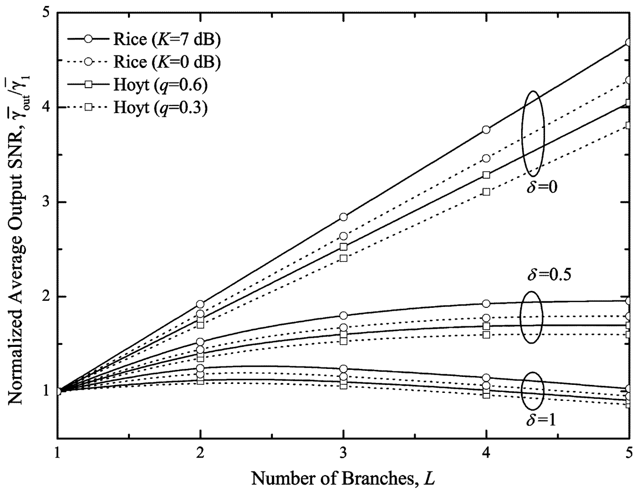

Fig. 1. First branch normalized average output SNR of EGC versus L, for Rician and Hoyt fading with exponentially decaying pdp.

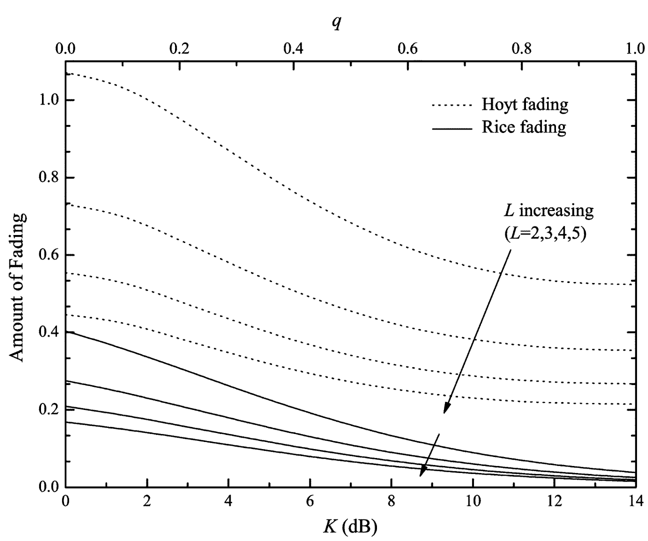

Fig. 2. AoF for the EGC receiver operating in a Rician or Hoyt fading environment versus the corresponding fading parameter.

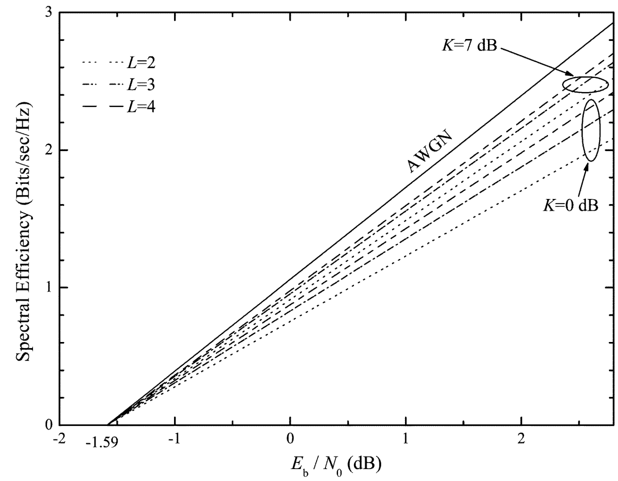

Fig. 3. SE in the low-power regime of the EGC receiver operating in a Rician fading environment.

approximation to the mgf of the output SNR, $\mathcal{M}_{\gamma_{\text{out}}}(s)$. By definition, the mgf is given by [20]

$$\mathcal{M}_{\gamma_{\text{out}}}(s) \triangleq E\langle \exp(s\gamma_{\text{out}})\rangle \tag{17}$$

and can be represented as a formal power series (e.g., Taylor) as

$$\mathcal{M}_{\gamma_{\text{out}}}(s) = \sum_{n=0}^{\infty} \frac{1}{n!} E\left\langle \gamma_{\text{out}}^n \right\rangle s^n. \tag{18}$$

We cannot conclude that the power series in (18) has a positive radius of convergence and where or whether it is convergent. To overcome this problem, the Padé approximants theory [14] is proposed, as a simple and alternative way to approximate the mgf. A Padé approximant, is that rational function approximation to $\mathcal{M}_{\gamma_{\text{out}}}(s)$ of a specified order B for the denominator and A for the nominator, whose power series expansion agrees with the $A+B$ order power expansion of $\mathcal{M}_{\gamma_{\text{out}}}(s)$, i.e.,

$$\begin{aligned} R_{[A/B]}(s) &\equiv \frac{\sum_{i=0}^{A} a_i s^i}{1+\sum_{i=1}^{B} b_i s^i} \\ &= \sum_{n=0}^{A+B} \frac{E\left\langle \gamma_{\text{out}}^n \right\rangle}{n!} s^n + O(s^{N+1}) \end{aligned} \tag{19}$$

with $O(s^{N+1})$ being the remainder after the truncation. Hence, the first $(A+B)$ moments need to be evaluated in order to construct the approximant $R_{[A/B]}(s)$. Next, $\mathcal{M}_{\gamma_{\text{out}}}(s)$ is approximated using subdiagonals Padé approximants $(R_{[A/A+1]}(s))$, since it is only for such approximants that the convergence rate and the uniqueness can be assured [14], [21]. With the aid of Padé approximants the error-rate expressions can be calculated directly for noncoherent and differential binary signaling [noncoherent binary phase shift keying (BPSK), differentially BPSK (DBPSK)], since for all other cases single integrals with finite limits and integrands composed of elementary functions have to be readily evaluated via numerical integration.

B. Outage Probability

In addition to the average error rate, outage probability is another standard performance criterion of communication systems operating over fading channels. It is defined as the probability that the combined SNR γ_{out} falls below a specified threshold γ_{th}, i.e., [15, ch. 1]

$$P_{\text{out}} = F_{\gamma_{\text{out}}}(\gamma_{\text{th}}) = \mathcal{L}^{-1}\left[\mathcal{M}_{\gamma_{\text{out}}}(s)/s\right]\Big|_{\gamma_{\text{th}}} \tag{20}$$

where $F_{\gamma_{\text{out}}}(\cdot)$ and $\mathcal{L}^{-1}(\cdot)$ denote in our case the cdf of the EGC output SNR and the inverse Laplace transform, respectively. Due to the Padé rational form of $\mathcal{M}_{\gamma_{\text{out}}}(s)$,

$$\mathcal{M}_{\gamma_{\text{out}}}(s) \cong \frac{\sum_{i=0}^{A} a_i s^i}{1+\sum_{i=1}^{B} b_i s^i} = \sum_{i=0}^{B} \frac{\lambda_i}{s+p_i} \tag{21}$$

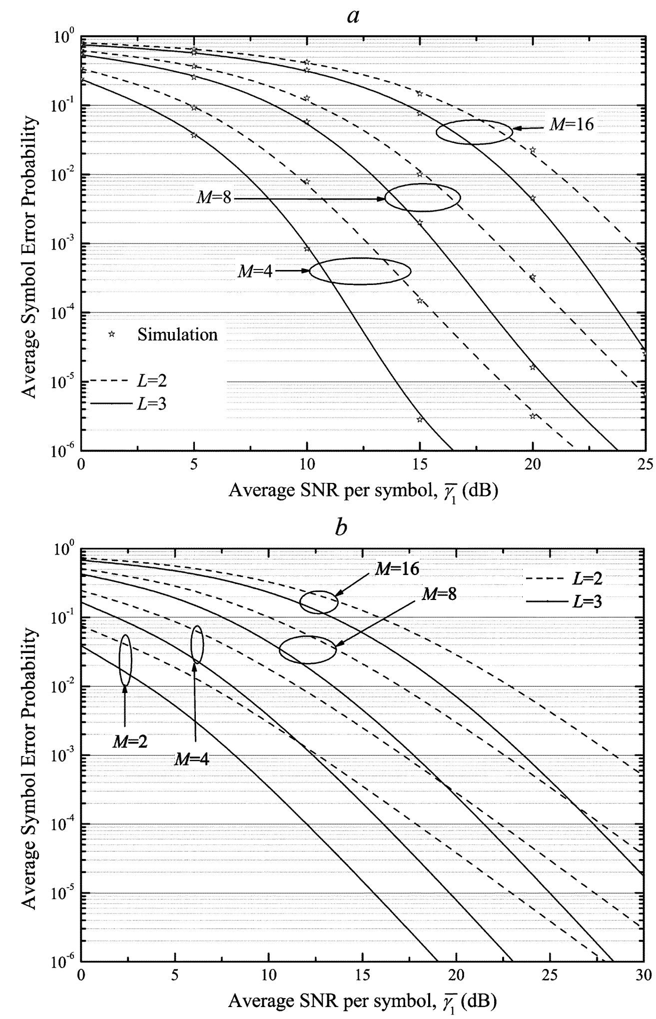

Fig. 4. Error performance of (a) M-DPSK in a Rician-fading environment with $K = 7$ dB and (b) M-PSK in a Hoyt fading environment with $q = 0.5$.

and using the residue inversion formula, the outage probability can be easily evaluated from (20) as

$$P_{\text{out}} = \sum_{i=1}^{B} \frac{\lambda_i}{p_i} \exp\left(-p_i \gamma_{\text{th}}\right) \tag{22}$$

where p_i and λ_i are the poles and the residues of the approximant, respectively. More about the approximation of pdfs and cdfs using approximants can be found in [21].

C. Numerical Results

Now, some numerical results are presented to illustrate the proposed mathematical analysis. Fig. 4(a) plots the ASEP of 4-DPSK, 8-DPSK, and 16-DPSK, employing EGC over Rice fading with $K = 7$ dB for $L = 2, 3$. In the Fig. 4(a), computer simulation results are also plotted in order to check the accuracy of the proposed Padé approximants approach. As it is clear, a very good match between computer simulation and analytical results is observed. Fig. 4(b) plots the error performance of BPSK, Q-PSK, 8-PSK, and 16-PSK, employing EGC in a Hoyt fading environment with $q = 0.5$, for $L = 2, 3$. In Fig. 5, the outage performance of the dual and triple EGC receiver operating over i.i.d. Rice- and Hoyt-fading channels, is plotted, versus the inverse normalized outage threshold $\bar{\gamma}_1/\gamma_{\text{th}}$ for several values of the fading severity parameters. Similar with the error performance of the EGC, the outage probability improves with an increase of the combiner's branches and with an improve of the channel's fading conditions (lower values of K and q).

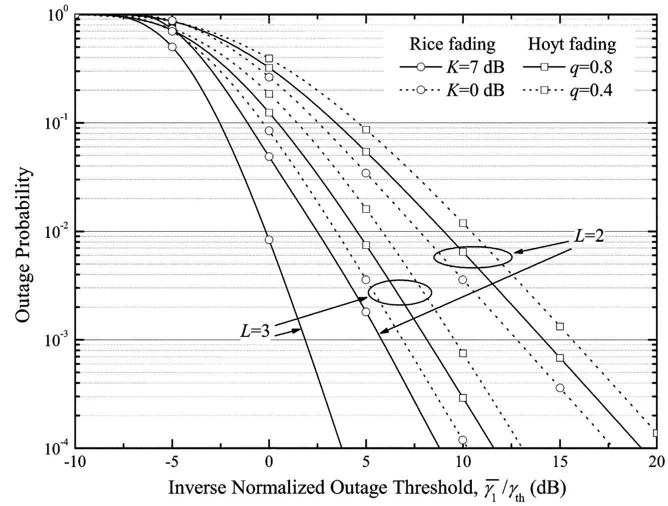

Fig. 5. Outage probability P_{out} versus $\bar{\gamma}_1/\gamma_{\text{th}}$, for a receiver employing EGC diversity in Rician- and Hoyt-fading environments.

IV. Approximation of the Hoyt Model by the Nakagami Model

In this section, we investigate whether a Hoyt-fading environment can be approximated by a properly chosen Nakagami-m model, as far as the error performance of the EGC is concerned. The corresponding suitability for the Rice-fading channel was investigated in [9], due to the simpler form of the Nakagami-m pdf, where the authors presented some ranges of the interrelation of the parameters where the equivalence is good and some where the equivalence is poor.

The approximation of the Hoyt model by a suitable Nakagami-m model was proposed by Nakagami in [2]. The relation between the fading parameter of the Nakagami-m distribution, m, and the q fading parameter was given by

$$m = \frac{(1+q^2)^2}{2(1+2q^4)}, \quad m \leq 1. \tag{23}$$

In order to examine the approximation of the Hoyt fading environment by a suitable Nakagami-m model, using (23), we give in Fig. 6 curves for the error performance of BPSK employing EGC both in Hoyt and Nakagami-m fading environments. A Nakagami-m model with $m = 0.75$, that approximates a Hoyt fading environment with $q = 0.7$, is used. The results obtained here show that the equivalence of the two models improves the average SNR decreases [average bit-error probability (ABEP) increases] and as the number of branches L increases. For example, at ABEP $= 10^{-3}$, the Hoyt model is 3, 1.68, 1.4 dB superior of the equivalent Nakagami-m model using $L = 2, 3, 4$, respectively. The corresponding values at ABEP $= 10^{-4}$ are 3.85, 2.76, and 2.02, respectively. For a more severe Hoyt fading environment with $q = 0.3$, it can be easily shown that for ABEP $= 10^{-3}$, the performance of the Hoyt

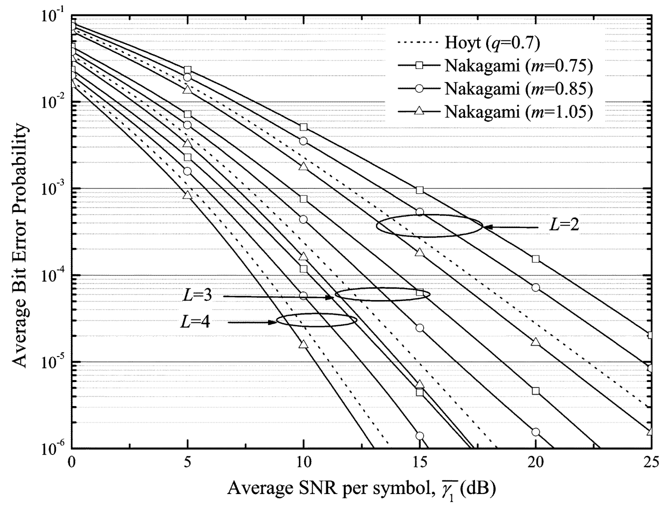

Fig. 6. Approximation of the error performance of BPSK employing EGC in a Hoyt fading environment with $q = 0.7$, with a Nakagami-m model.

model is 4.13, 2.48, 1.45 dB superior of the equivalent Nakagami-m model with $m = 0.585$, for $L = 2, 3, 4$ respectively, since for ABEP $= 10^{-4}$ the corresponding values are 5.51, 4.14, and 2.66 dB, respectively. By comparing the results for the approximations of the two Hoyt fading environments with the equivalent Nakagami-m model, we observe that the equivalence between the two models improves, as the fading gets less severe. We can also see form Fig. 6 that the approximation of the Hoyt fading environment with the equivalent Nakagami-m model, as suggested by (23), always underestimates the performance of the Hoyt model (gives an upper bound). We have determined empirically that a better upper bound for the Hoyt model would be by adding 0.1 to the equivalent m parameter. Moreover, adding 0.3 to the equivalent m parameter results in a lower bound for the Hoyt model.

V. Conclusion

We studied the performance of predetection EGC receivers over Rice- and Hoyt-fading channels. The analysis assumes independent and nonidentical fading paths. Deriving exact closed-form expressions for the moments of the output SNR, we evaluated important performance parameters, such as the average output SNR, AoF, and SE at the low-power regime. The ASEP and the outage probability were accurately approximated using the mgf approach and the Padé approximants theory. The suitability of modeling a Hoyt-fading environment by a properly chosen Nakagami-m model, as far as the error performance of the EGC is concerned, was also examined for first time in the literature. The results have shown that the approximation of the Hoyt-fading environment with the equivalent Nakagami-m model always underestimates the performance of the Hoyt model.

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