Water Resour Manage (2008) 22:239–258
DOI 10.1007/s11269-007-9155-2

Optimal Water Resource Allocation in Arid and Semi-Arid Areas

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Received: 18 January 2006 / Accepted: 14 January 2007 /
Published online: 9 February 2007

Abstract Areas of water shortage comprise many smaller sub-areas into which water is transported from external sources. Fairness and efficiency of distribution are overriding principles. Each local area requires adequate water for community and ecological purposes as well as a supply sufficient to maximise economic growth. Within arid and semi-arid areas, there are conflicts between the sub-areas and between these three types of water use, which can erupt into violent confrontations between different user groups. This study has developed a dynamic model for equitable distribution of water in water-shortage areas and aims to optimally satisfy the requirements of each locality, given limited supplies, and to maximise the total economic benefit of the entire area. The Heihe River Basin in northwest China was chosen as the area for the pilot study.

Key words water allocation · fairness and efficiency · community · ecology and economy

1 Introduction

1.1 Objectives

Approximately 30% of total global land area comprises populated arid and semi-arid areas and water shortages are a major obstacle to social and economic development in these

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areas. The basic principles for the allocation of water resources are efficiency, equity, and sustainability, with the aims of pursuing the maximum benefit for society, the environment and the economy, whilst maintaining fair allocation among various areas and people. Sustainable economic development in arid and semi-arid areas depends heavily on sustainable water resource management, defined by Loucks (2000) as "...water resource systems designed and managed to fully contribute to the objectives of society, now and in the future, while maintaining their ecological, environmental, and hydrological integrity."

The rational distribution of water resources requires a complex balance between demand and supply, and over time (annual, inter-annual, or even generational), in various economic sectors (Yamout and El-Fadel 2005), especially across sub-areas in arid and semi-arid climate zones (Just and Netanyahu 1998). Regional development planning requires the incorporation of economic objectives with many factors, including historical, technological, natural and resource constraints and potentials. Together, these factors dictate population distribution, economic structure and patterns of ecology and, consequently, the proportions of water distributed for these purposes in arid and semi-arid areas. Regional community, economic and ecological differences require different sustainable development strategies in regions of water scarcity.

This study develops an integrated solution for the distribution of scarce water resources for community, ecological and economic uses among different sub-areas in arid and semi-arid climate zones, using the principles of efficiency and equity.

1.2 A Review of Water Allocation

Here, the temporal, spatial, and sectoral dimensions of water allocation and the general mechanism for water distribution are briefly reviewed.

The theories and practices of optimal allocation of water resources across seasons and years have been studied thoroughly; examples being the strategy of diversion and storage of reservoir water (Alaya et al. 2003; Chang et al. 2005), and optimal use of atmospheric, surface, and ground water (Liu 1992).

Various approaches to spatial allocation of water have been developed and can be classified into six categories:

International river mode (Wolf 1999) Water resources are divided along the lines of equality and sovereignty, which is usually accomplished through negotiation (Mimi and Sawalhi 2003) with little consideration of the potentials for combined economic benefit.

Quota mode (Office for Yangtze River Planning, unpublished report, 1987) Water is rationed according to the size of industry, agriculture and population in each sub-area, as in the cases of the Yellow River and the South-North Water Transfer Project in progress in China.

Single-purpose mode Water is diverted only for ecological restoration, military bases or other specific purposes; examples of this include water transfer projects in America, Russia and the Middle East, and some recent projects in northwest China.

Macroeconomic mode (United Nations Development Program (UNDP) and the Chinese Ministry of Science and Technology (MOST), et al. 1994; Xu et al. 1997; Duarte et al. 2002; Jin et al. 2003) In 1986, the United Nations Development Project (UNDP) collaborated with Chinese scientists and the local government of Hebei Province in north China. By linking economic input and output tables and a technique for linear programming, they designed an optimal water resource coordination scheme for different economic sectors. This model is sophisticated but difficult to use because of the many variables and parameters needed. After reorganisation of demand and supply across two basins, Ballestero (2004) developed an inter-region bargaining model to achieve equilibrium in quantity and price.

Supplier profit mode (Fisher et al. 2002) Private water firms use the dynamics of demand in different areas to maximise profits from water-diversion projects.

Inter-year mode (Cai et al. 2002) Annual water benefits are maximised, and between-year variations are minimised to promote sustainable development.

The multi-objective programming technique reduces multiple objectives into one (Haimes and Allee 1982; Steuer 1986; United Nations Development Program (UNDP) and the Chinese Ministry of Science and Technology (MOST), et al. 1994; Xu et al. 1997), and then maximises the single objective.

Table 1 summarises the six modes used to spatially allocate water compared to the method developed in this paper.

Among sectors, community, ecological and economic uses of water are co-dependent, and have to be coordinated within the total available water resource. One approach to this is to define economic values for each type of use, and coordinate them using mathematical programming. Alternatively, constraints can be added before modelling. Some studies have established input-output relationships between economic and ecological systems in areas where the ecological system has significant economic value (Jin et al. 2003; Elzen and Moor 2002). In practice in arid and semi-arid areas, the local government usually prioritises water use in the order of human consumption (community), ecology, and industry (economic). The elasticity of water demand among these three sectors and the total water available are implicit in decision-making.

From the institutional point of view, pricing or marketing is one of the instruments used to allocate water resources (Fisher et al. 2002; Haddad 2000; Zylicz 2003), while tariffs form a complicated sub-system to be further optimised (Molden and Sakthivadivel 1999; Dalhuisen and Nijkamp 2002). The requirements for (and economics of) water distribution are much studied. Saving and recycling water for ecological purposes (Chen 1995; Baird and Wilby 1999; Cheng 2002a, 2002b), water technology (Loschel 2002), water-related macro-and microeconomics (Roger et al. 1993; Wang et al. 2000), ecological protection with least economic cost (Yang et al. 2003) and policy games (Slobadan and Hussan 1999; Hamilton et al. 2002), have all been investigated in recent years. Although pricing and marketing are currently the main focus in water distribution, an efficient economic system can result from pricing policy only when there is sufficient free competition. However, there are problems with this approach, especially with cross-boundary issues. Water resources are legally owned by the state in China, therefore each local district expects fair allocation of public resources in terms of its population, gross domestic product (GDP), or length of river across its territory. After meeting the basic demands of human consumption, ecology and production, the state is expected to pursue the maximum benefit from economic use of the resource.

1.3 Outline

The model proposed here distributes water fairly among different localities following the principle of optimality in the order of community, ecological, and the economic use.

Table 1 Summary of modes of spatial water allocation

Mode	Formula/mechanism	Input variables	Output variables	Parameters
1. International River (Wolf 1999)	Equality and sovereignty	History, land size and river length	Demarcation of river water usage	International constitution bank
2. Domestic Ration (Office for the Yangtze River Planning 1987)	Water supply=quota×size	Size: population, land, or GDP	Water allocation into sub-areas	Quota by per capita, area, GDP, etc
3. Exclusive use	Supply=demand	Demand of user	Supply to the user	Basic demand
4. GDP/Input-Output (Xu et al. 1997; Hebei Water Research Center 1994)	Maximise: total GDP constraints: input-output table	Population, industry, agriculture, ecological	Water for different requirements	Input/output table
5. Supplier Profit (Fisher et al., 2002)	Maximise: supplier net profit constraints: demand elasticity	1. Water price	1. Net benefit of supplier	1. Demand elasticity ($*^{\$/m3}$∟m3)
		2. maximum values limit	2. water supply/demand	2. waste water charge (*$\$/m^{-3}$)
			3. Water transported	3. Recycling cost ($\$/m^{-3}$)
6. Interyear Mode (Cai, et al., 2002)	Maximise: water benefit minimising: interyear spillover	Weighting: (1) agro/eco	4. Water recycled	4. Water transfer cost ($\$.m^{-3}$)
		(2) Criteria	1. Water benefit and loss	1. Crop: price and cost
			2. Crop area	
			3. Irrigation withdrawal	
7. Fairness and Efficiency (this paper)	Aim: (1) Community and ecology →relative fairness+optimal	Assumed or available water for coordination in space	4. Ecological water use	2. Power: price and investment
	(2) Economic benefit→maximum		1. Water economic benefit	1. Relative fairness (m^3/capita, m^3/ha)
	Mechanism: (1) multi-objectives preference		2. Water for community	2. Optimal (m^3/capita, m^3/ha)
	(2) Marginal benefit		3. Ecological water use	3. Marginal benefit ($*^{\$/m-3}$∟$_{m3}$)
			4. Industrial water use	4. Local water resources
			5. Water spatial allocation	

Optimal, in this instance, refers to the inevitable difference between the water that can realistically be expected to be available in areas of water scarcity and the volume ideally required by users if water supplies were plentiful. Priority of distribution is flexible to allow for policy preferences. Economic demands are met according to the marginal revenue for water in each local sub-area.

The next section explains the system of water allocation, and the mathematical basis for it, before describing a pilot study in the Heihe River Basin in northwest China. The concept of balance between supply and demand is used to identify arid and semi-arid regions, and distinguish them from other areas. The elements of the system are introduced in turn before a discussion of the flexibility of the system.

2 System

2.1 Assumptions

The analysis is based on the following assumptions, which are principles accepted by regional planners, managers, local residents and academics.

External sources The system is concerned only with the distribution of water into sub-areas. All sub-areas suffer water scarcity but have the capacity to adapt to changing rates of supply.

Local sources The local water resources of each sub-area are used locally and are not exported.

Fairness Using the principles of fairness among districts and optimal allocations for community and ecological uses, the model achieves a compromise between the historical patterns of water use in any sub-area and the right for equality across all sub-areas. In this study, water-allocation models capture both the major features of mechanisms and the impacts of control strategies, assuming that the per capita quotas for community and ecological uses are fairly distributed in each sub-area.

Agriculture and industry Despite severe water shortages in the area, flood irrigation is still a common practice, therefore, there is capacity for improvement in agricultural productivity by adoption of water-saving techniques and the selection of appropriate crop varieties. In addition, careful use of water fosters cumulative efficiency. Industry is chosen as the favoured sector for receipt of water from external sources because its marginal revenue is about ten times that of agriculture (Long et al. 2002; Lange 1998).

2.2 Elements

The superscript k denotes the type of water use. l=community life, e=ecology, p=economic production. The subscript i denotes the sub-area.

In supply side,

W total input water from external sources, to be distributed rationally into different sub-areas in the study area.

${}_iW$ volume of river water being allocated to sub-area i, $\Sigma\, {}_iW=W$.

^k_iW volume of river water used by requirement k (community life l, ecology e, and economic production p) and allocated to water-shortage sub-area i

${}_iQ_0$ local water resources in sub-area i

$^k_iQ_0$ local water resources used by requirement k (community life l, ecology e, and economic production p) in sub-area i

^k_iQ total water used by requirement k (community life l, ecology e, and economic production p) in sub-area i

and, the total water supply is:

$$S = \sum_{ik} \left(^k_iQ \right) = \sum_{ik} \left(^k_iQ_0 +^k_i W \right) \tag{1}$$

In demand side,

${}_iwl$ water quota per capita for community use per year in sub-area i (m^3/person/year).

${}_iwe$ water quota for ecological use per year in sub-area i (mm/year).

${}_iwp$ water quota for economic production per year in sub-area i (tonne/10,000 Chinese Yuan).

${}_iN$, ${}_iA$, ${}_iGDP$ population, territorial area and GDP of sub-area i.

Then,

$$\begin{aligned}
&\text{Domestic water demand in sub} - \text{area } i \text{ is} : {}_iD_l =_i wl \times_i N \\
&\text{Ecological water demand in sub} - \text{area } i \text{ is} : {}_iD_e =_i we \times_i A \\
&\text{Economic water demand in sub} - \text{area } i \text{ is} : {}_iD_p =_i wp \times_i GDP \\
&\text{Total water demand is} : D = \sum_i \left({}_iD_l + {}_iD_e + {}_iD_p \right) = \sum_i \left({}_iwl \times {}_iN + {}_iwe \times {}_iA + {}_iwp \times {}_iGDP \right)
\end{aligned} \tag{2}$$

2.3 States

Sufficient water state Supply>>Demand: For those regions with sufficient water, any modest change in the balance of water use by sector does not affect the existing relationships between water supply and demand, or does not lead to *Supply<Demand*; the interaction between *Supply* and *Demand* is irrelevant.

Absolute drought state Supply<<Demand: In a desert, any change in sectoral water demand is seriously constrained owing to lack of water. A modest reduction in demand would not relieve the other elements. Responsiveness to supply rate change cannot occur.

Critical balance state Supply≈Demand or *Supply>Demand* or *Supply<Demand*. In this state there can be interactions among water users with room to manoeuvre, which enables the system to adjust the balance between demand sectors, irrespective of whether the present state becomes degraded, remains unchanged or is upgraded. In arid and semi-arid areas, water allocation among use sectors always adapts in response to external stimuli in order to keep *S=D*. In other words, *S≡D* is a persistent state but incorporates flexibility in the different combinations of demand allocations. *S≠D* is a transient state, and any change in demand from one sector interrupts the current balance of supply and demand. A new balanced state will be reached quickly by adaptation. This situation is examined below.

2.4 Mechanism

Figure 1 shows the relationships between regional development aims, interactions between water supply and demand for community, ecological and economic production uses, and the decision variables.

Water distribution is prioritised in the order of community, ecological, and economic demands. First, water is distributed fairly between community and ecological demands in the various sub-areas, then an optimal distribution is implemented for the remainder. When optimal quotas are fulfilled for community and ecological demands, the total economic benefit from water is maximised for the entire area.

Strategic decisions about water supply and demand are made by both local and regional governments. The local government has responsibility for local water resources and supply, and the regional government for imported water resources. Options to increase the total water supply include storage (such as dams, groundwater aquifers, and small-scale water harvesting), inter-basin transfers and novel water sources (such as desalination and wastewater recycling). Measures controlling demand management include water pricing (and markets), rationing, and regulation. The balance between supply and demand can be adjusted by changing the relative allocations of water to different sectors, which feed back into the system.

The relationship described by Fig. 1 can be modelled as follows. The combination of the system elements is defined as $C = C({}_iQ_0, {}_iW|{}_iwl, {}_iwe, {}_iwp|{}_iN, {}_iA, {}_iGDP)$, where ${}_iQ_0$, ${}_iW$, ${}_iwe$ and ${}_iA$ are physical variables, and ${}_iwl$, ${}_iwp$, ${}_iN$ and ${}_iGDP$ are human variables. The supply

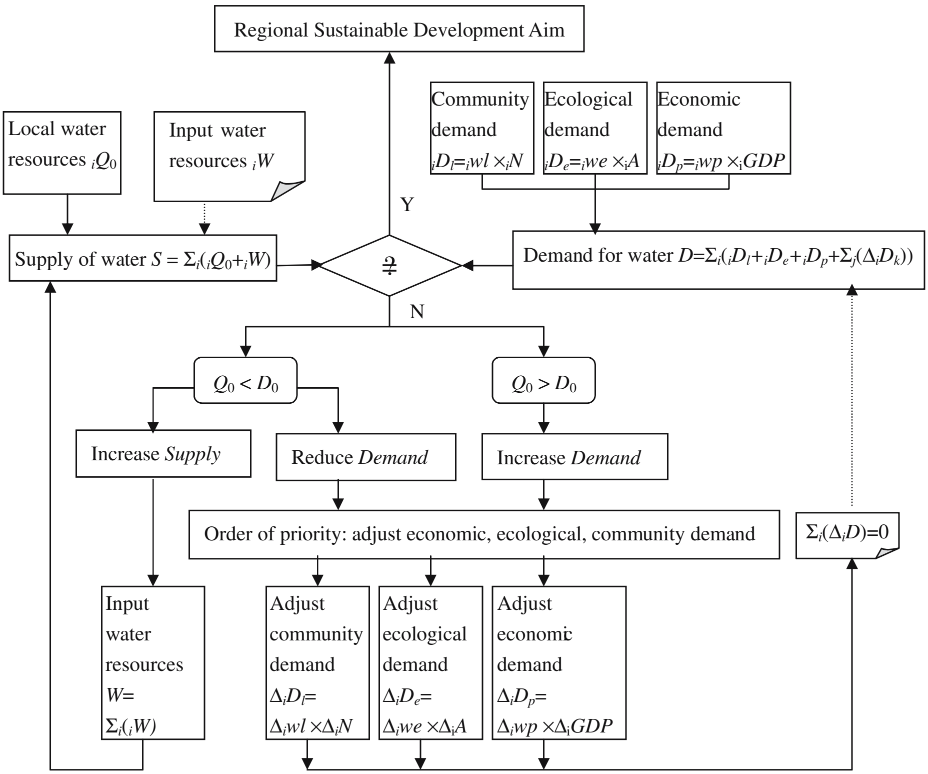


Fig. 1 Water allocation balancing and decision system

combination (${}_iQ_0$, ${}_iW$) is composed of supply elements. The local government in area j can adjust the local water resources ${}_iQ_0$ and the regional government can adjust the input water resources ${}_iW$. The demand combination (${}_iwl$, ${}_iwe$, ${}_iwp$, ${}_iN$, ${}_iA$ and ${}_iGDP$) comprises demand elements. Both local and regional decision makers can adjust the demand combinations.

2.5 Adjustment

The adjustment or disturbances of system $C({}_iQ_0, {}_iW|{}_iwl, {}_iwe, {}_iwp|{}_iN, {}_iA, {}_iGDP)$ could be presented as: $\Delta = \Delta C(\Delta_i Q_0, \Delta_i W, \Delta_i wl, \Delta_i we, \Delta_i wp, \Delta_i N, \Delta_i A, \Delta_i GDP)$, where Δ denotes the change of amount. ΔC represents the continuous functions of policy, technical, biotechnological and management systems. The following are examples of specific measures for the adjustment of the system elements:

$\Delta_i Q_0$/tonne, water storage, wastewater recycling, sea water desalination
${}_iW$/tonne, increasing/reducing imported water
$\Delta_i wl$/tonne·person^{-1}, raising/reducing the price of water, water saving technology
${}_iwe$/mm, improvement/degradation of ecological standards, sprinkler or drip irrigation, new crops and varieties of crops
$\Delta_i wp$/tonne·RMB^{-1}, economic structure adjustment, water saving technology, efficiency requirements
$\Delta_i N$/number of persons, births and deaths, migration
$\Delta_i A$/km^2, urbanisation, desertification, reclamation
$\Delta_i GDP$/RMB, increasing input of production materials

3 Mathematical Equations

The model consists of two sets of equations. A governing set lists general relationships that should always be maintained in water planning, while a set of complementary equations can be employed to specify various policy preferences. The governing and complementary equations together form a definite mathematical problem. In this study, a set of complementary equations are provided as default; others can be developed according to the specific region or question at hand.

3.1 Governing Equations

Equations The proposed model maintains equilibrium, always keeping a balance between water supply and demand. Its self-adaptive nature means that any imbalance will be transient and soon corrected by within-system feedback. The constraints equation Water Supply=Water Demand has the following form according to Eqs. (1) and (2):

$$Q = \sum_{ik} \left({}_i^k Q_0 + {}_i^k W\right) = \sum_i \left({}_iwl \times {}_iN + {}_iwe \times {}_iA + {}_iwp \times {}_iGDP\right) = D \tag{3}$$

Under the constraints of the balance between supply and demand, upon satisfying optimal quotas for community and ecological water demand, the total economic benefit brought by water is maximised for the entire area:$GDP = \sum_i ({}_iGDP) = \sum_{i=1}^{I} \int_{{}_i^pQ_0}^{{}_i^pQ} {}_if({}_i^pQ) d_i^pQ = \sum_{i=1}^{I} \int_0^{{}_i^pS} {}_if({}_i^pQ_0 + {}_i^pW) d_i^pW$ ${}_if({}_i^pQ)$ refers to the economic marginal revenue production of water in sub-area i, which is equal to the reciprocal of the water quota in economic production, i.e., ${}_if\left({}_i^kQ\right) = 1/{}_iwp(GDP)$.

Initial condition The initial balance $Q_0 = \sum_i ({}_iQ_0) = \sum_i ({}_i\mathrm{wl}_0 \times {}_iN_0 + {}_i\mathrm{we}_0 \times_i \mathrm{A}_0 + {}_i\mathrm{wp}_0 \times {}_iGDP_0) = D_0$; Initial combination of elements $C_0\left({}_i^kQ_0, {}_i^kW_0|{}_iwl_0, {}_iwe_0, {}_iwp_0|{}_iN_0, {}_iA_0, {}_iGDP_0.\right)$;

Balance A new condition can be achieved after adjusting the elements: New balance $S_1 = \sum_i ({}_iQ + {}_iW + \Delta_iW) = \sum_i [({}_iwl + \Delta_iwl) \times ({}_iN + \Delta_iN) + ({}_iwe + \Delta_iwe) \times ({}_iA + \Delta_iA) + ({}_iwp + \Delta_iwp) \times ({}_iGDP + \Delta_iGDP)] = D_1$; New combination $C_1 = C_1({}_iQ_0 + \Delta_iQ_0, {}_i W + \Delta_iW, {}_iwl + \Delta_iwl, {}_i we + \Delta_iwe, {}_i wp + \Delta_iwp, {}_i N + \Delta_iN, {}_i A + \Delta_iA, {}_i GDP + \Delta_iGDP)$.

3.2 Complementary Equations

After establishing the general equations, local conditions may be specified by a set of complementary equations. A default set is provided here, based on generally accepted principles, relationships and policy. The conditions can be modified, removed or expanded if necessary.

Community uses, ecological preservation and economic development form a rational order of priority for water distribution in an area facing critical water scarcity. Geographical equanimity is a major concern in water distribution for community and ecological uses, and efficiencies should be pursued for economic development. The decision preference is summarised in Table 2.

Case 1 Optimal quotas for community and ecological uses are met, and production marginal revenues are equal when:

(1) all the sub-areas use water for community use with equal and optimal quotas;
(2) all the sub-areas use ecological water with equal and optimal quotas; and
(3) all the sub-areas reach equal marginal production revenue.

The optimal quotas for community and ecological water use are first satisfied, and equal marginal revenue is attained in all sub-areas. Then more water is distributed among the sub-areas to maximise economic growth. In order to achieve the maximum economic benefit, the marginal revenue from each unit of water must remain equal in all sub-areas.

Case 2 Optimal quotas for community and ecological uses are met when:

(1) all the areas have equal and optimal water quotas for community;
(2) all the areas have equal and optimal water quotas for ecological use; and
(3) the marginal production revenues vary geographically.

Table 2 Order of priority for water distribution*

Aim	Community fair	Community optimal	Ecological fair	Ecological optimal	Production efficiency	Production growth
Input order						
Case 6	↑					
Case 5	X	↑				
Case 4	x	x	↑			
Case 3	x	x	x	↑		
Case 2	x	x	x	x	↑	
Case 1	x	x	x	x	x	↑

*“x” denotes that the aim was reached, “↑” denotes that water is being distributed to reach the aim

Under this scenario, fair and optimal water supplies for community and ecological use have already been achieved. Marginal revenues are still unequal geographically and the economic benefit for the whole area is yet to be maximised. Water is first allocated to the sub-area that has the maximum initial marginal revenue until the marginal revenue of the first sub-area is reduced to that of the second largest initial marginal revenue. This second sub-area is then qualified to receive water. In this way, water is always distributed to the sub-area that has the maximum marginal revenue, and this guarantees the maximum total economic benefit.

Case 3 Optimal quotas for community use are met when:

(1) all the sub-areas have equal and optimal water quotas for community use; and
(2) all the sub-areas have equal water quotas for ecological use, but these are sub-optimal in that all quotas equal that for the sub-area that demands the most water for ecological uses. Reallocations of supplies are required according to their specific ecological needs.

Case 4 Water quotas for community use are identical when:

(1) all the sub-areas have equal and optimal water quotas for community use; and
(2) the quotas for ecological use vary geographically, and require optimisation for specific ecological needs.

Fairness of distribution requires that water is first allocated to the sub-area with the lowest initial ecological water quota until this quota is equal to that of the second lowest initial ecological water quota. This second sub-area is then qualified to receive water. In this way, water is always distributed to the sub-area that has the lowest ecological water quota, which guarantees relative fairness.

Case 5 Water quotas for community use are not identical: all the sub-areas have equal water quotas for community use, but they are sub-optimal in that all quotas are equal to that of the sub-area that demands the most water for community use. In this case, the objective is to optimise water demands for community uses.

Case 6 Water quotas for community use are not identical: water quotas for community use vary geographically. To achieve fairness, water is first allocated to the sub-area with the lowest initial water quota for community use until this quota is equal to that of the second lowest initial water quota for community use, the second sub-area is then qualified to receive water. In this way, water is always distributed to the sub-area that has the lowest water quota for community use, guaranteeing relative fairness among sub-areas.

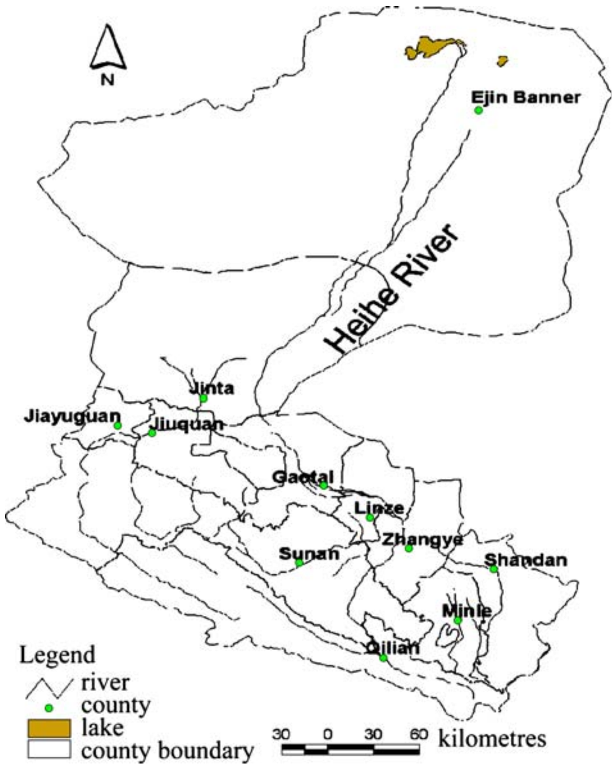
The solutions to each case are presented in Appendix 2.

4 Empirical Study

4.1 Introduction of the Study Area

The Heihe River Basin was chosen as a pilot study area (Fig. 2). It is located in the arid northwest region of China and is the second largest interior territorial river of the country, with a total length of 821 km. The river originates from a glacier in the Qilian Mountains, at the northeast edge of the Tibetan plateau, then flows north-east across the Zhangye Basin in Gansu Province before ending in the Ejina Banner of the Inner Mongolia Autonomous

Fig. 2 Study area



Region. The Zhangye Basin, located in the middle reaches of the river, is a commodity grain production base with developed irrigation farming. The Ejina oasis irrigated by the river is a natural screen, which blocks the influx of wind and sand from Mongolia. Climatic changes and excessive abstraction over the past 30 years have caused surface runoff to decrease sharply in the river's lower reaches which has resulted in the disappearance of large areas of wetland vegetation, including rose willow, poplar, and other plant species, and has led to worsening desertification and the intensification of sandstorms in the area.

The Zhangye Basin comprises five counties: Shandan, Minle, Zhangye, Linze and Gaotai. The Heihe River is the major water source for all of the five sub-areas, and each of them is in a state of critical water balance. Conflicts over water use between the sub-areas are becoming much more severe and enduring, thus impeding their sustainable development. The above model was used to allocate water resources to the sub-areas.

4.2 Data Sources

The parameters and corresponding data sources are listed in Table 3.

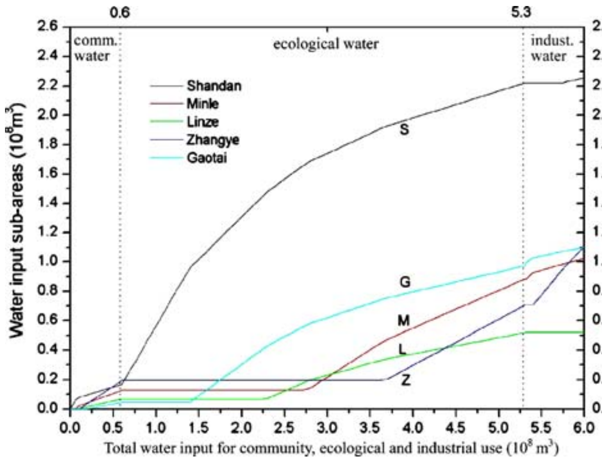
4.3 Results and Discussion

Figure 3 shows the water distribution plan with increased water input. The X-axis represents total water input and the Y-axis denotes the volumes distributed to the various sub-areas. The first phase was to distribute water for community use until all sub-areas reached their optimum quotas. This consumed 6×10^{7} m^{3} of water. Phase two then

Table 3 Parameters

Item	Parameters	1 Shandan	2 Minle	3 Zhangye	4 Linze	5 Gaotai	Sources
(1)	Population: ${}_iN$ (×10,000 persons)	19.49	23.24	43.21	14.50	12.56	GIWRSP 2001, p 3
(2)	Water for community use: ${}_iQ_0$ (×10,000 m^3)	606.3	886.34	2,102.01	592	1040	GIWRSP 2001, p 15
(3)	Established water quota for community wl_0=(2)/(1) (m^3/capita)	31.09	38.14	48.65	40.83	82.80	(2)/(1)
(4)	Optimal water quota for community $wl_z=\max\{{}_iwl_0\}$ (m^3/capita)	82.80	82.80	82.80	82.80	82.80	sup(3)
(5)	Area: ${}_iA$ (×10^4 ha)	5.85	8.13	9.96	3.53	4.29	Chen 1995, p 124
(6)	Initial ecological water quota we_0 (m^3/ha)	1,353.7	2,998.5	3534.7	2581.3	2,237.3	Chen 1995, p 124
(7)	Optimal ecological water quota $we_z=\max\{{}_iwe_0\}$ (m^3/ha)	3,534.7	3,534.7	3,534.7	3,534.7	3,534.7	sup(6)
(8)	Industrial ${}_iGDP$ (×10,000 Yuan RMB)	44,578	14,985	92,879	62,115	24,968	GIWRSP 2001, p 3
(9)	Industrial water use: ${}_iQ_0$ (×$10^8 m^3$)	0.0981	0.031567	0.280937	0.2363	0.0332	GIWRSP 2001, p 15
(10)	Initial marginal revenue (Yuan/m^3)	29.11	113.85	79.54	−5.11	139.987	Using (9) to check corresponding MRs, Long, et al. 2002, p 409
(11)	wp_0=1/(10) (m^3/Yuan)	0.03435	0.00878	0.01257		0.00714	1/(10)

Fig. 3 Optimal water distribution for community, ecological and industrial use



distributed 4.7×10^8 m^3 of water to eliminate geographical disparities in supply, allowing sub-areas to meet their ecological needs. Upon the completion of the first two stages, all sub-areas had reached their optimal community and ecological quotas. The final phase distributed sufficient water to maximise the aggregate economic benefit from water use, terminating when the aggregate marginal revenue decreased to zero.

Figures 4, 5 and 6 show the components of Fig. 3 in detail. Figure 4 illustrates the process of optimal water distribution for community use in all sub-areas, and Fig. 5 shows the subsequent distribution of water for ecological purposes. Differences in the initial quotas for community and ecological water uses account for the different starting points of the curves on the X-axis, and the different curve shapes are due to other differences such as population sizes and extent of oases among sub-areas. The starting order for increased allocations for community use was prioritised as Shandan, Minle, Linze, Zhangye, and Gaotai (Fig. 4), which is the opposite of the established order for community use. The total water input for each sub-area is determined by its initial water resource and size of population. When the total volume of water imported reached 6×10^7 m^3, quotas in all sub-areas were optimal for community use, and water distribution for ecological purposes could begin, as shown in Fig. 5. The starting order for ecological water distribution was

Fig. 4 Optimal water distribution for community use

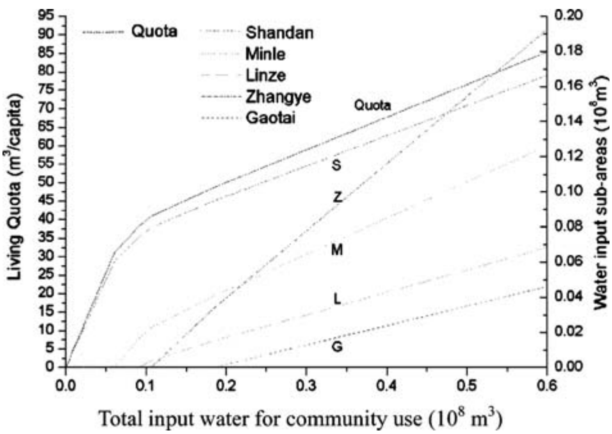
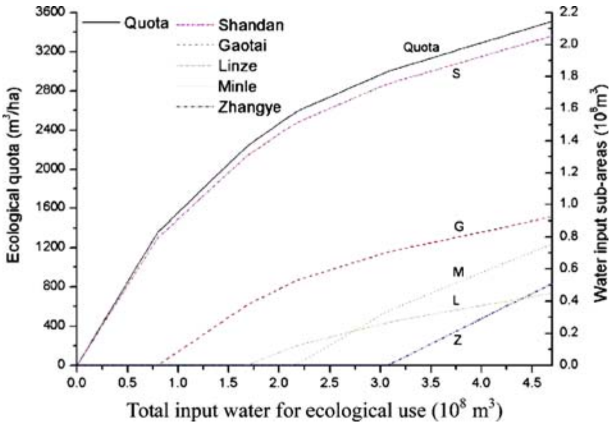


Fig. 5 Optimal water distribution for ecological use



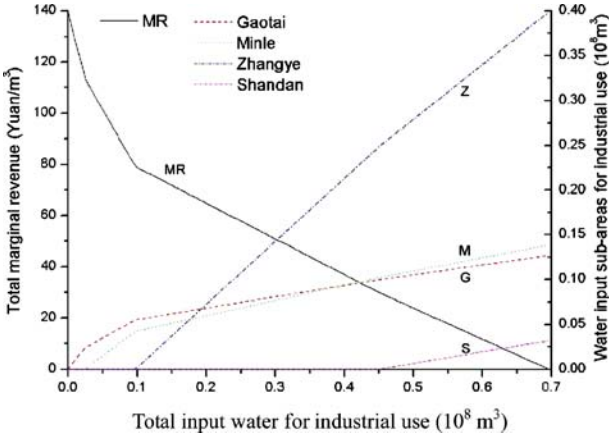
determined by the order of the original ecological water quota. Water was distributed first to the sub-area that initially had the lowest ecological water quota (in this case Shandan).

Figures 6 and 7 show the process of optimal distribution of water to different sub-areas for maximisation of economic benefits. The order and quantity of water distribution is shown in Fig. 6 and the total revenue in Fig. 7. The aggregated marginal revenue of the entire area decreases and the curves tend to flatten as input water increases. The sub-area Linze was not included in this phase because its initial marginal revenue was negative, which means that local water resources were greater than the maximum requirement in recent years, so that additional supplies were unnecessary. Allocation of additional water resources for the other four sub-areas was in the order Gaotai, Minle, Zhangye, and Shandan (Fig. 6).

5 Conclusions

Water resources play a crucial role in social and economic development and ecological protection in arid and semi-arid areas. In order to meet the water demands of a sub-area, local water resources can be supplemented by water from external sources such as inter-regional rivers and long-distance water transfer canals. No single area can unilaterally

Fig. 6 Optimal water distribution for industrial use



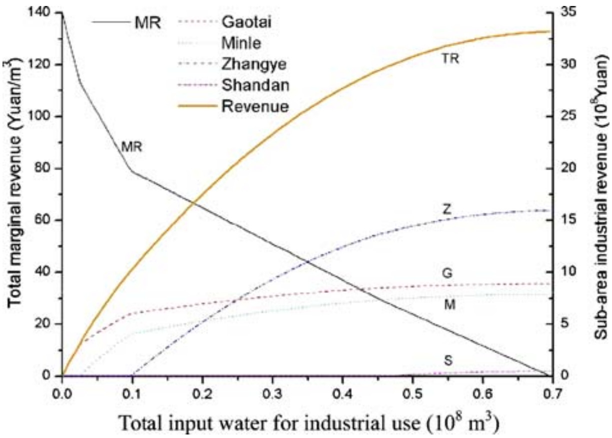


Fig. 7 Optimal water distribution for industrial use and revenue

determine the fate of extraterritorial water supplies without causing inter-regional conflict. This model distributes water for community, ecological, and economic needs according to the principles of efficiency and equity. After optimal quotas for community and ecological water demands have been fulfilled, the economic benefit is maximised by equalising marginal revenues among the sub-areas.

There is a critical balance between supply and demand for water resources in arid and semi-arid areas, which means that the competing requirements of community, ecological and economic uses within a region are responsive to supply. Any addition of water can satisfy demands to some extent. This is in contrast to situations where water is plentiful or where there is extreme drought. In these cases, there will be no response to additional supplies, and input water will fail to produce measurable benefits.

Given the known input water volumes, the model described above can fulfil an optimal allocation plan that includes appropriate timings and volumes of water distribution in each sub-area, and maximises the economic benefits to be gained. In addition, where detailed information about specific sectors in industry or agriculture is provided, the model can be used to allocate water among sectors. This study has mainly discussed the spatial aspects of water allocation, but the model framework could be adapted to allocate water resources using a combination of spatial, temporal and sectoral elements.

Acknowledgments This work was supported by grants from the Natural Science Foundation of China (No. 70571076) and the Ministry of Science and Technology of China. The authors would also like to thank Ian Macleod for his help in preparing the manuscript.

Appendix 1

Variables Used in Solutions

Q	amount of water used.
${}_if({}_i^pQ)$	economic marginal revenue of water in sub-area i, where p denotes economic production.
${}_iy_0 = {}_if({}_i^pQ_0)$	initial economic marginal revenue of water in sub-area I
$y(Q)$	equilibrium marginal revenue of water corresponding to Q.

${}_{ij}Q_0 = {}_if^{-1}({}_iy_0) = {}_ig({}_iy_0)$	water in sub-area i corresponding to initial marginal revenue ${}_jy_0$ in sub-area j. $i < j$.
0	right subscript, denotes the initial state; without 0 denotes current or cleanup state.
z	right subscript, denotes the optimal state. The word "optimal" denotes that there always exist differences between the ideal water demand and realistic available water supply in areas of water shortage.
b	right subscript, denotes the relative fair state for community and ecological water uses and efficiency for economic use.

therefore,

$$ {}^lW_z \equiv \sum_{i=1}^{I} (wl_z - {}_iwl_0) \times {}_iN; \quad {}^eW_z \equiv \sum_{i=1}^{I} (we_z - {}_iwe_0) \times {}_iA $$

$$ {}^lW_b \equiv \sum_{i=1}^{I} (wl_b - {}_iwl_0) \times {}_iN; \quad {}^eW_b \equiv \sum_{i=1}^{I} (we_b - {}_iwe_0) \times {}_iA; \quad {}^pW_b \equiv \sum_{i=1}^{I} (wp_b - {}_iwp_0) \times {}_iG $$

$$ {}^l_{lI}W \equiv \sum_{i=1}^{I} ({}_{lI}wl - {}_iwl_0) \times {}_iN; \quad {}^e_{eI}W \equiv \sum_{i=1}^{I} ({}_{eI}we - {}_iwe_0) \times {}_iA; \quad {}^p_{pI}W \equiv \sum_{i=1}^{I} \left({}_{pI}wp - {}_iwp_0\right) \times {}_iG $$

then ordering:

i of (1, 2, ..., I) into ($l1$, $l2$, ..., lI) by $({}_{l1}wl < {}_{l2}wl < \ldots < {}_{lI}wl)$, in community dimension

i of (1, 2, ..., I) into ($e1$, $e2$, ..., eI) by $({}_{e1}we < {}_{e2}we < \ldots < {}_{eI}we)$, in ecological dimension

i of (1, 2, ..., I) into ($p1$, $p2$, ..., pI) by $\left({}_{p1}wp_0 < {}_{p2}wp_0 < \ldots < {}_{pI}wp_0\right)$, in economic dimension

Appendix 2

Solutions to Equation

Case 1 Ideal quotas for community and ecology are met, production marginal revenues are equal

(1) Equal and optimal community quota: ${}_iwl = wl_z, \forall i.$
(2) Equal and optimal ecological quota: ${}_iwe = we_z, \forall i$
(3) Equal production marginal revenue: ${}_{pI}w \le {}_iwp = wp_b, \forall i$; that is, $W \ge^l W_z +^e W_z + {}_{pI}{}^pW$.

How can more water be distributed ${}^pW = W - \left({}^lW_z +^e W_z + {}^pW_b\right)$ to maximise GDP? Given ${}^pW$, the algorithm of Lagrange conditional maximum is applied:

$$ \frac{d\left[GDP + \lambda\left({}^pW - \sum_{i=1}^{I} {}^p_iW\right)\right]}{d\,{}^p_iW} = {}_if({}^p_iQ_0 + {}^p_iW) - \lambda = 0 $$

and

$$\sum_{i=1}^{I} {}_i^pW = {}^pW, \forall i$$

$$\sum_{i=1}^{I} \left[{}_if^{-1}(\lambda) - {}_i^pQ_0\right] = {}^pW = \sum_{i=1}^{I} {}_i^pW$$

$$\lambda = \phi({}^pW)$$

.

Further, ${}_if$ is a decreasing function (Samuelson and Nordhaus 1992, pp 171–172)

$$\frac{d^2\left[S + \lambda\left(W - \sum_{i=1}^{I} {}_iW\right)\right]}{d_iW^2} = \frac{d_if({}_iQ_0 +_i W)}{d_iW} < 0;$$

then GDP reaches its maximum when $\lambda = {}_if({}_i^pW)$ or ${}_i^pW = {}_if^{-1}(\lambda) -_i^p Q_0$, where λ, and then the ${}_i^pW's$ can be numerically calculated given various W's. They can also be found analytically if we fit the empirical marginal data by some simple continuous functions.

Case 2 Ideal quotas for community and ecology are met

(1) Equal and optimal community quota: ${}_iwl = wl_z, \forall i$
(2) Equal and optimal ecological quota: ${}_iwe = we_z, \forall i$
(3) Marginal production revenues vary geographically: ${}_iwp \neq {}_jwp \leq {}_{pI}wp, i \neq j, \exists i,j = 1, 2, \ldots, I$.

that is, ${}^lW_z + {}^eW_z \leq W < {}^lW_z + {}^eW_z + {}_{pI}^pW$.

How can we distribute more water ${}^pW = W - \left({}^lW_z + {}^eW_z\right)$ to realise ${}_iwp = wp_b, \forall i$?

Assume J sub-areas participate in water allocation, with $J<I$. The model for water allocation is the same as in Case 1, but we need to replace I with J. The crucial problem for incomplete sub-area water allocation is to find J with knowing the value of W. Actually, we may first list $\left\{{}_{ij}^pQ_0, i,j = 1, 2, 3 \ldots I, i \leq j\right\}$, and calculate ${}_j^pW_0 = \sum_{i=1}^{j} {}_{ij}^pW_0 = \sum_{i=1}^{j} \left({}_{ij}^pQ_0\left({}_jy_0\right) - {}_i^pQ_0\left({}_jy_0\right)\right)$, then order $\left\{{}_j^pW_0, j = 1, 2, 3, \ldots, I\right\}$, which expresses the water cumulatively supplied to sub-areas i until j corresponds to the marginal revenue ${}_jy_0$ of initial water ${}_j^pQ_0$. For the different ${}^pW$'s, we may determine J by $\left\{J \middle| {}_j^pW_0 \leq {}^pW <_{j+1}^p W_0, J = 1, 2, 3, \ldots, I\right\}$.

Case 3 Ideal quotas for community are met

(1) Equal and optimal community quota: ${}_iwl = wl_z, \forall i$.
(2) Equal ecological quota: ${}_{eI}we \leq {}_iwe = we_b < we_z, \forall i$.

that is, ${}^lW_z +_{el}^e W \leq W <^l W_z +^e W_z$

How can we distribute water ${}^{e}W = W - \left({}^{l}W_z + {}^{e}_{eI}W_0\right)$ to realise ${}_{i}we = we_z, \forall i$?

$$
\begin{aligned}
&{}^{e}W = \sum_{i=1}^{I} {}^{e}_{i}W = \sum_{i=1}^{I} ({}_{eI}we - we_b) \times {}_{i}A \\
&we_b = {}_{el}W + {}^{e}W / \sum_{i=1}^{I} {}_{i}A \\
&{}^{e}_{i}W = (we_b - {}_{eI}we) \times {}_{i}A = {}^{e}W \times {}_{i}A / \sum_{i=1}^{I} {}_{i}A
\end{aligned}
$$

Case 4 Quotas for community are identical

(1) Equal community quota: ${}_{i}wl = wl_z, \forall i$.
(2) Unequal ecological quota: ${}_{i}we \neq {}_{j}we \leq {}_{eI}we, i \neq j, \ \exists\ i,j = 1, 2, \ldots, I$.

that is, ${}^{l}W_z \leq W < {}^{l}W_z + {}^{e}_{eI}W$.

How can we distribute water ${}^{e}W = W - {}^{l}W_z$ to realise ${}_{i}we = {}_{eI}we, \forall i$?

Assume J sub-areas participate in water allocation, with $J<I$. The crucial problem for incomplete sub-area water allocation is to find J with knowing the value of W. Let ${}^{e}_{e1}W_0 = {}_{e1}N \times {}_{e1}we$ and ${}^{e}_{eJ}W_0 = {}_{e(J-1)}{}^{e}W_0 + \sum_{li=l1}^{lJ} {}_{li}N \times \left({}_{lJ}wl - {}_{l(J-1)}wl\right)$, then order $\left\{{}^{e}_{eJ}W_0, eJ = e1, e2, \ldots, eI\right\}$, which expresses the water cumulatively supplied to sub-areas $e1$ to eJ, with $eJ < eI$.

For the different ${}^{e}W$'s, we can determine eJ by $\left\{eJ \middle| {}^{e}_{eJ}W_0 \leq {}^{e}W < {}_{e(J-1)}{}^{e}W_0, eJ = e1, e2, \ldots, eI.\right\}$:

$$
\begin{aligned}
&{}^{e}_{eJ}W = {}_{e(J-1)}{}^{e}W_0 + \sum_{li=l1}^{lJ} {}_{li}N \times \left(wl_b - {}_{l(J-1)}wl\right) \text{ with } {}_{e0}we = 0 \text{ and } {}^{e}_{0}we_0 = 0 \text{ and} \\
&we_b = {}_{e(J-1)}we + \left({}^{e}_{eJ}W_0 - {}_{e(J-1)}{}^{e}W_0\right) \Big/ \sum_{li=1}^{lJ} {}_{li}N \\
&{}^{e}_{ei}W = {}^{e}_{e(J-1)}W_0 + \sum_{li=l1}^{lJ} {}_{li}N \times \left(wl_b - {}_{l(J-1)}wl\right) \text{ with } e(J-1) < ei < eJ.
\end{aligned}
$$

Case 5 Quotas for community are not all identical

Equal community quota: ${}_{lI}wl < {}_{i}wl = wl_b < wl_z, \forall I$, that is, ${}^{l}_{lI}W_0 \leq W < {}^{l}W_z$.

How can we distribute water ${}^{l}W = W - {}^{I}_{lI}W_0$ to realise ${}_{i}wl = wl_z \forall i$?

$$
\begin{aligned}
&{}^{l}W = \sum_{i=1}^{I} {}^{l}_{i}W = \sum_{i=1}^{I} (wl_b - {}_{lI}wl_0) \times {}_{i}N \\
&wl_b = {}_{lI}wl_0 + {}^{l}W / \sum_{i=1}^{I} {}_{i}N \\
&{}^{l}_{i}W = (wl_b - {}_{lI}wl) \times {}_{i}N = {}^{l}W \times {}_{i}N / \sum_{i=1}^{I} {}_{i}N = \left(W - {}^{I}_{lI}W_0\right) \times {}_{i}N / \sum_{i=1}^{I} {}_{i}N
\end{aligned}
$$

Case 6 All quotas for community are not identical

Unequal community quota: ${}_iwl \neq {}_jwl < {}_{lI}wl, \exists i,j = 1,2,\ldots,I$, that is, $W < {}^{I}_{lI}W$.

How can we distribute water ${}^{l}W$=W to realise ${}_iwl = wl_b, \forall i = 1,2,\ldots,I$?

Assume J sub-areas participate in water allocation, with J<I. The crucial problem for incomplete sub-area water allocation is to find J with knowing W. Let ${}^{l}_{l1}W_0 = {}_{l1}N \times {}_{l1}wl_0$ and ${}^{l}_{li}W_0 = {}_{l(i-1)}{}^{l}W_0 + \sum_{lk=l1}^{li} {}_{lk}N \times \left({}_{lk}wl_0 - {}_{l(k-1)}wl_0\right), li = l1, l2, \ldots lI\}$, and then order $\left\{{}^{l}_{li}W_0, li = l1, l2, \ldots, lI\right\}$, which expresses the water cumulatively supplied to sub-areas $l1$ to li, with li<lI.

For different $W = {}^{l}W$, we may determine lJ by $\left\{lJ \middle| {}^{l}_{lJ}W_0 \leq {}^{l}W < {}^{l}_{l(J-1)W_0}, lJ = l1, l2, \ldots, lI\right\}$; ${}^{l}_{lJ}W_b = W - {}^{l}_{l(J-1)}W_0$ is distributed only in sub-areas li=$l1$,...,lJ in the range of $\left({}_{l(J-1)}wl_0, {}_{lJ}wl_0\right)$. ${}^{l}_{li}W = W$ is distributed only in sub-areas li=$l1$,...,lJ in the range of $\left({}_{l(i-1)}wl_0, {}_{lJ}wl_0\right)$:

$$
{}^{l}_{lI}W_b = \sum_{li=l1}^{lJ} \left({}_{lJ}wl_b - {}_{l(J-1)}wl_0\right) \times {}_{li}N
$$

$$
{}_{iJ}wl_b = {}_{l(J-1)}wl_0 + {}^{l}_{lJ}W_b / \sum_{li=l1}^{lJ} {}_{li}N = {}_{i(J-1)}wl_0 + \left(W - {}_{i(J-1)}{}^{l}W_0\right) / \sum_{li=l1}^{lJ} {}_{li}N
$$

$$
{}^{l}_{l1}W = {}_{lJ}wl_b \times {}_{l1}N
$$

$$
{}^{l}_{li}W = \left({}_{lJ}wl_b - {}_{l(i-1)}wl_0\right) \times {}_{li}N = {}^{l}_{lJ}W \times {}_{li}N / \sum_{li=l1}^{lJ} {}_{li}N = \left(W - {}^{l}_{l(i-1)}{}^{l}W_0\right) \times {}_{li}N / \sum_{li=l1}^{lJ} {}_{li}N.
$$

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