Autocatalyses

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Abstract

The notion of autocatalysis actually covers a large variety of mechanistic realisations of chemical systems. From the most general definition of autocatalysis, that is a process in which a chemical compound is able to catalyze its own formation, several different systems can be described. We detail the different categories of autocatalyses, and compare them on the basis of their mechanistic, kinetic, and dynamic properties. It is proposed that the key signature of autocatalysis is its kinetic pattern expressed in a mathematical form. It will be shown how such a pattern can be generated by different systems of chemical reactions.

Introduction

The notion of "autocatalysis" was introduced by Ostwald in 1890 for describing reactions showing a rate acceleration as a function of time. It is for example the case of esters hydrolysis, that is at the same time acid catalyzed and producing an organic acid (Laidler, 1986). Defined as a chemical reaction that is catalyzed by its own products, it has quickly been described on the basis of a characteristic differential equation (Ostwald, 1902, 1912). Typically used to describe complex behaviors of chemical systems, like oscillatory patterns (Lotka, 1910), it has immediately appeared to be essential for the description of biological systems: growth of individual living beings (Robertson, 1908), population evolution (Lotka, 1920) or gene evolution (Muller, 1922).

Extending this concept from a chemical description to a more open context was initially carefully described as an analogy, sometime qualified by the more general notion of "autocatakinesis" (Lotka, 1925; Witzemann, 1933). However, this eventually leads to an overgeneralization of the term of autocatalysis, tending to be assimilated to the notion of "positive feedback", for example in economy (Malcai et al., 2002).

The notion of autocatalysis is now actively being used for describing self-organizing systems, namely in the field of emergence of life. Autocatalytic processes are the core of the mechanisms leading to the symmetry breaking of chemical compounds towards homochirality (Frank, 1953; Plasson et al., 2007), and could be identified in several experimental systems (Kondepudi et al., 1990; Soai et al., 1995). However, how such autocatalytic processes shall manifest is still under heavy debate (Plasson, 2008; Blackmond, 2009).

The purpose of this article is thus to clarify the meaning of chemical autocatalysis and this effort will be undertaken by covering these following points:

- What is autocatalysis for a chemical system? On the basis of the general description of autocatalysis as a process allowing a chemical compound to enhance the rate of its own formation, it is defined by a kinetic signature, expressed in a mathematical form.
- How can an autocatalytic process be realized? As many mechanisms can reduce to the same macroscopic kinetic laws exhibiting autocatalysis, the focus is put on several mechanistic realisations of autocatalytic processes, on the basis of simple models further illustrated by concrete chemical examples.
- How can autocatalysis be observed and characterized? The focus is put on the dynamic properties, showing that this observable is the direct consequence of the kinetic pattern, rather than the underlying mechanism.
- What is the role of autocatalysis? Embedded in non-equilibrium reaction network, the competition between autocatalytic processes allows the onset of chemical selection, that is the existence of bifurcation phenomena allowing the extinction of some compounds in favor of others.

Autocatalysis: a Practical Definition

A Kinetic Signature

From its origin, the notion of autocatalysis has focused on the kinetic pattern of the chemical evolution (Ostwald, 1902). The general definition of autocatalysis as a chemical process in which one of the products catalyzes its own formation can be mathematically generalized as:

$$\frac{dx_i}{dt} = k(\boldsymbol{X}) \cdot x_i^n + f(\boldsymbol{X}), \quad k > 0;\ n > 0;\ |k| \gg |f| \quad (1)$$

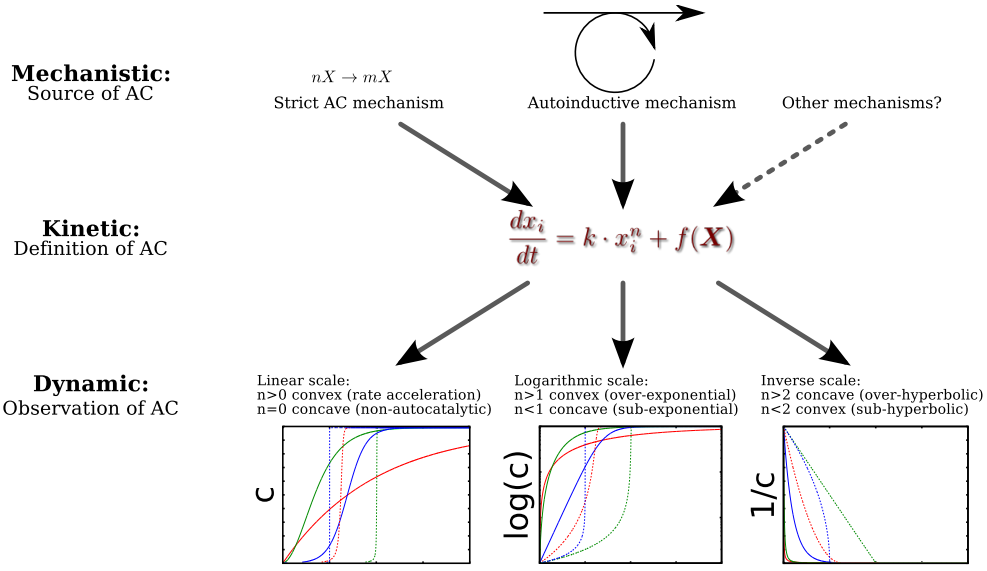


Figure 1: Classification of the concepts of autocatalysis (AC) depending on their descriptions (mechanistic, kinetic, and dynamic). The graphs represents the time evolution of a non-autocatalytic reaction (red), and of autocatalytic reaction of order $1/2$ (green), 1 (blue), $3/2$ (dotted red), 2 (dotted green), and 3 (dotted blue).

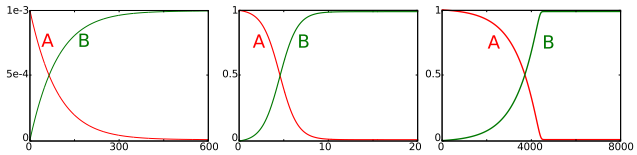
The term $k(\boldsymbol{X}) \cdot x_i^n$ describes the autocatalytic process itself, while $f(\boldsymbol{X})$ describes the sum of all other contributions coming from the rest of the chemical system.

We have an effective practical definition of the concept of autocatalysis, based on a precise mathematical formulation. The causes of this kinetic signature can be investigated, searching what mechanism is responsible for the autocatalytic term. This leads to the discovery of a series of different kinds of autocatalysis processes, and their respective effect, describing what observable behavior is generated by the autocatalytic term (see Fig. 1).

Potential vs Effective Autocatalysis

This kinetic definition is purely structural. As a matter of fact, a system may contain *potential* autocatalysis i.e. an autocatalytic core exists in the reaction network. However, in the absence of some specific conditions necessary for this autocatalysis to be *effective*, the potential autocatalysis may be hidden by other kinetic effects, thus turns out not to manifest its behavior in practice.

Possibly, in Eq. (1), the term $f(\boldsymbol{X})$ may simply overwhelm the autocatalytic process. This is typically the case when an autocatalysis is present together with the non-catalyzed version of the same reaction, that may not be negligible in all conditions. Imagine the simple example of a system simultaneously containing a direct autocatalysis $A + B \rightarrow 2B$, concurrent with the non autocatalytic reaction $A \rightarrow B$. The autocatalytic process follows a bimolecular kinetics, and will be more efficient in a concentrated than in a diluted solution. The dynamic profile of the reaction is thus sigmoidal for high initial concentration of A, but no more for low initial concentration (see Fig. 2(a-b)).



(a) Non-autocatalytic (b) Autocatalytic (c) Undamped AC

Figure 2: (a-b): First order autocatalytic process ($\Gamma_1 = 10^2$ M.s^{-1}) in presence of a non-autocatalytic reaction ($\Gamma_2 = 10^{-2}$ M.s^{-1}) of spontaneous transformation of A into B ($K_A = 1$ M, $K_B = 10^2$ M). (a) Diluted ($a_o = 10^{-3}$ M). (b) Concentrated ($a_0 = 1$ M). (c) Undamped autocatalysis (Indirect autocatalysis, described in Fig. 4(b), $\Gamma_4 = 0.1$ M.s^{-1})

It can also be seen that the term $k(\boldsymbol{X})$ may also vary during the reaction process. In a simple autocatalytic process as describe above, k is proportional to the concentration in A, and is thus more important at the beginning of the reaction (thus an initial exponential increase of the product B) that at the end (thus a damping of the autocatalysis) resulting in a global sigmoidal evolution. In systems were the influence of A on k is weaker, as detailed further, an undamped autocatalysis will be observed characterized by an exponential variation until the very end (see Fig. 2(c)).

Mechanistic Distinctions

How can this kinetic pattern be realized? Let us now detail several types of mechanisms. They can all be reduced,

in some conditions, to the autocatalysis kinetic pattern of Eq. (1). All of them will be equally defined in the paper as autocatalytic, while this status may have been disputed in the past on account of the distinct chemical realisations. In the following, we emphasize the major mechanistic pattern to eventually be reduced to an equivalent kinetic autocatalysis, and discuss where their difference comes from.

Template Autocatalysis

The simplest autocatalysis is obtained by the $X \rightarrow 2X$ pattern. It can be represented by:

$$A + B \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} B + B \tag{2}$$

The corresponding network is given in Fig. 3(a). It can further be decomposed through the introduction of an intermediate compound C:

$$A + B \overset{\Gamma_1}{\rightleftharpoons} C \tag{3}$$

$$C \overset{\Gamma_2}{\rightleftharpoons} B + B \tag{4}$$

The corresponding network is given in Fig. 3(b).

The first mechanism entails the following kinetic evolution:

$$\dot{b} = -\dot{a} = k_1 ab - k_{-1} b^2 \tag{5}$$

This can be expressed as a chemical flux φ, by relying on the Mikulecky formalism (Peusner et al., 1985; Mikulecky, 2001; Plasson and Bersini, 2009):

$$\varphi = \Gamma_1 (V_A V_B - V_B^2) = \Gamma_1 V_B (V_A - V_B) \tag{6}$$

$$V_A = \frac{a}{K_A} \tag{7}$$

$$V_B = \frac{b}{K_B} \tag{8}$$

$$\Gamma_1 = k_1 \cdot K_A K_B = k_{-1} \cdot K_B^2 \tag{9}$$

Formally there is a linear flux φ of transformation of A into B, coupled to a circular flux of same intensity from B back to B (see Fig. 3(a-b)). In presence of an intermediate compound, the equations becomes:

$$\varphi_1 = \Gamma_1 (V_A V_B - V_C) \tag{10}$$

$$\varphi_2 = \Gamma_2 (V_C - V_B^2) \tag{11}$$

Under the hypothesis that C is an unstable intermediate, (i.e. $K_C \ll K_B, K_A$), the variation of C can be neglected compared to the variations of A and B (quasi steady-state approximation, hereafter QSSA), so that:

$$\varphi_1 \simeq \varphi_2 \tag{12}$$

$$= \varphi \tag{13}$$

$$\Rightarrow \quad \varphi = \frac{\Gamma_1 \Gamma_2}{\Gamma_1 + \Gamma_2} (V_A V_B - V_B^2) \tag{14}$$

The system is strictly equivalent to the direct autocatalysis, with an apparent rate $\Gamma_1 \Gamma_2 / (\Gamma_1 + \Gamma_2)$. With these two systems, we are in presence of the perfect kinetic signature of an autocatalytic system i.e. following a sigmoidal evolution (see Fig. 4(a)). This equivalence is guaranteed as long as the compound C remains unstable. When it is not the case, the dimeric intermediate C hardly liberates the final compound B, which gives rise to an autocatalytic process of order $1/2$ rather than 1 (von Kiedrowski, 1993; Wills et al., 1998).

Template autocatalysis requires a direct association between the reactants and the products. This is typically the case of DNA replication, one double strand molecule giving birth to two identical double strand molecules, thanks to the very selective association of complementary nucleotides along each strand. More simple examples can be found in some biological mechanisms that requires autocatalytic processes, for example for the generation of chemical oscillation inducing circadian rhytmicity in cells. The system described by Mehra et al. (2006) is based on a non equilibrium system of association/dissociation of proteins forming a large chemical cycle $[C \rightarrow AC \rightarrow AC^* \rightarrow ABC^* \rightarrow BC^* \rightarrow C^* \rightarrow C]$, maintained by a flux of ATP consumption, one cycle consuming and freeing A and B. The oscillations are generated by coupling this chemical flux to an autocatalytic process of phosphorylation obeying to the reaction scheme: $A + C + AC^* \rightarrow 2AC^*$ (Wang and Wu, 2002).

Network Autocatalysis

The direct mechanism of template autocatalysis just seen is conceptually the simplest framework. It may actually not be the most representative class of autocatalysis, as a similar kinetic signature can appear as resulting from a complex reaction network.

Indirect Autocatalysis: The autocatalytic effect may be only indirect when reactant and products never directly interact:

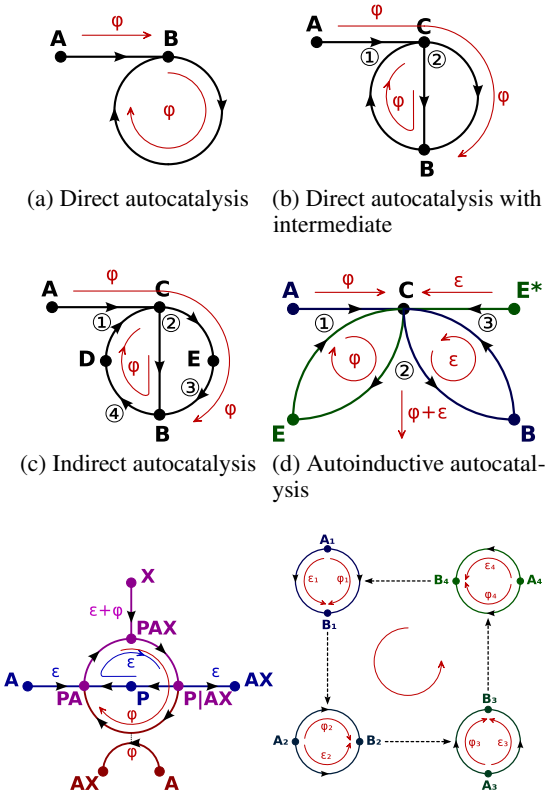
$$A + D \overset{\Gamma_1}{\rightleftharpoons} C \tag{15}$$

$$C \overset{\Gamma_2}{\rightleftharpoons} B + E \tag{16}$$

$$E \overset{\Gamma_3}{\rightleftharpoons} B \tag{17}$$

$$B \overset{\Gamma_4}{\rightleftharpoons} D \tag{18}$$

There is no direct A/B coupling, nor direct $2B$ formation, but the presence of a dimeric compound C. The network decomposition of this system (see Fig. 3(c)) implies once again a linear flux of transformation of A into B, linked to a large cycle of reaction transforming B back to B. Nevertheless, this system is still reducible to an $X \rightarrow 2X$ pattern.

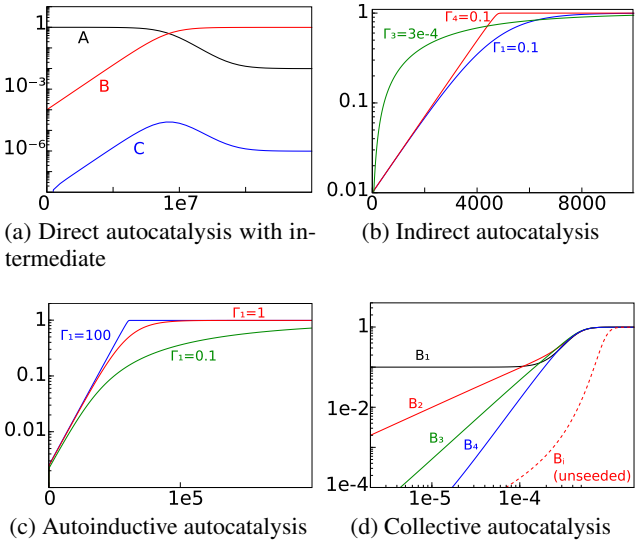


(a) Direct autocatalysis (b) Direct autocatalysis with intermediate

(c) Indirect autocatalysis (d) Autoinductive autocatalysis

(e) Iwamura et al. (2004) system (f) Collective autocatalysis

Figure 3: Reaction network of different autocatalytic processes of spontaneous transformation of A into B (a-d), of $A + X$ into AX (e), and of A_i into B_i (f). The indicated fluxes correspond to what is observed within the QSSA.



(a) Direct autocatalysis with intermediate (b) Indirect autocatalysis

(c) Autoinductive autocatalysis (d) Collective autocatalysis

Figure 4: Time evolution of compound concentrations for different autocatalytic processes of spontaneous transformation of A into B ($K_A = 1$ and $K_B = 100$) in a logarithmic scale for concentrations (a-c), or logarithmic scales for both time and concentrations (d). K and concentrations are in M, times in s, and Γ in M.s^{-1}. (a): Fig. 3(b), $\Gamma_1 = 1$, $\Gamma_2 = 10^{-4}$, $K_C = 0.01$; (b): Fig. 3(c), $\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = 10$ (except the values indicated on the graph), $K_C = K_D = K_E = 0.01$; (c): Fig. 3(d), $\Gamma_2 = \Gamma_3 = 100$, $K_C = K_E = 1$, $K_{E^*} = 10$; (d): Fig. 3(f), $\Gamma_1 = 100$, $\Gamma_2 = 1$.

The QSSA for compounds C, D, E allows to express the reaction flux as:

$$\varphi = \frac{\Gamma_1\Gamma_4}{\Gamma_1 V_A + \Gamma_4} V_A V_B - \epsilon \qquad (19)$$

ϵ express the back-reactions fluxes, and can be neglected as long as Γ_3 is large enough. If it is not the case, the autocatalytic effect is destroyed.

When $\Gamma_1 \ll \Gamma_4$, the system can behave like a simple autocatalytic system, with $\varphi \propto a \cdot b$ before the reaction completion, implying a progressive damping of the exponential growth as long as A is consumed. When $\Gamma_1 \gg \Gamma_4$, the flux is $\varphi \propto b$: the profile remains exponential up to the reaction completion, with no damping due to A consumption (see Fig. 4(b)).

Network autocatalysis is probably the most common kind of mechanisms. A typical biochemical example is the presence of autocatalysis in glycolysis (Ashkenazi and Othmer, 1977; Nielsen et al., 1997). In this system, there is a net balance following the $X \to 2X$ pattern. ATP must be consumed to initiate the degradation of glucose, but much more molecules of ATP are produced during the whole process. While these systems are effectively autocatalytic, there is obviously no possible "templating" effect of one molecule of ATP to generate another one.

Collective Autocatalysis: More general systems, reminiscent of the Eigen's hypercycles (Eigen and Schuster, 1977), are responsible of even more indirect autocatalysis. No compound influence its own formation rate, but rather influences the formation of other compounds, which in turn influence other reactions, in such a way that the whole set of compounds collectively catalyzes its own formation.

A simple framework can be built from the association of several systems of transformation $A_i \to B_i$, each B_i catalyzing the next reaction (see Fig. 3(f)):

$$A_i + B_{i-1} \underset{}{\overset{\Gamma_i}{\rightleftharpoons}} B_i + B_{i-1} \qquad (20)$$
$$i = \{1, 2, 3, 4\}$$

with $B_5 = B_0$ to close the cycle of reactions. There are four independent systems, only connected by catalytic activities.

If the system is totally symmetric, then all b_i are equal, and

all a_i are equal, so that the rates become:

$$\varphi_i = \Gamma_i V_{B_{i-1}}(V_{A_i} - V_{B_i}) \quad (21)$$
$$\varphi = \Gamma V_B(V_A - V_B) \quad (22)$$

This leads to a *collective* autocatalysis with all compounds present. They mutually favor their formation, which results in an exponential growth of each compound (see Fig. 4(d) dotted curve).

With symmetrical initial conditions (i.e. identical for the four systems), the system strictly behaves autocatalytically. If the symmetry is broken, e.g. by seeding only one of the B_i, the system acts with delays. The evolution laws are sub-exponential, of increasing order: At the very beginning of the reaction, considering that A_i do not significantly change and that B_i are in low concentration, we obtain $\varphi_i \propto t^{i-1}$. If seeding with B_1, the compound 2 evolves in t^2. Its impact on compound 3 induces an evolution in t^3. In its turn, the impact of compound 3 on compound 4 induces an evolution in t^4. The compound 1 at first remains constant, and it is only following a given delay that it gets catalyzed by B_4 (see Fig. 4(d)).

This system is actually not characterized by a direct cyclic flux, but by a cycle of fluxes influencing each other and resulting in a cooperative collective effect:

$$\begin{array}{c}(A_1 + A_2 + A_3 + A_4) + (B_1 + B_2 + B_3 + B_4) \\ \longrightarrow \quad 2(B_1 + B_2 + B_3 + B_4)\end{array} \quad (23)$$

The simultaneous presence of all different compounds is needed to observe a first order autocatalytic effect. Given asymmetric initial conditions, a transitory evolution of lower order is first observed, until the formation of the full set of compounds.

A typical example of collective autocatalysis is observed for the replication of viroids (Flores et al., 2004). Each opposite strand of cyclic RNAs can catalyze the formation of the other one, leading to the global growth of the viroid RNA in the infected cell.

Template vs Network Autocatalysis: Nevertheless, all these systems can still be reduced to a $X \rightarrow 2X$ pattern. This is characterized by a linear flux coupled to a loop flux, i.e. for each molecule (or set of molecules) A transformed into B, one B is transformed and goes back to B, following a more or less complex pathways. They can be considered as mechanistically equivalent: a seemingly direct autocatalysis may really be an indirect autocatalysis once its precise mechanism is known, decomposing the global reaction into several elementary reactions.

Practically, autocatalysis will be considered to be direct (or template) when a dimeric complex of the product is formed (i.e. allowing the "imprint" of the product onto the reactant). If such template complex is never formed, we preferentially speak of network autocatalysis, in which the $X \rightarrow 2X$ pattern only results from the reaction balance.

Autoinductive Autocatalysis

Some reactions are not characterized by an $X \rightarrow 2X$ pattern, but still exhibit a mechanism for the enhancement of the reaction rate through the products. This is typically the case for systems where the products increase the reactivity of the reaction catalyst rather than directly influencing their reaction production itself. These systems still possess the kinetic signature of Eq. (1), but are sometime referred as "autoinductive" instead of "autocatalytic" (Blackmond, 2009).

Let us take a simple reaction network of a tranformation $A \rightarrow B$ catalyzed by a compound that can exist under two forms E/E^*, E^* being the more stable one. These two forms of the catalyst interact differently with the product B (see Fig. 3(d)):

$$A + E \underset{}{\overset{\Gamma_1}{\rightleftharpoons}} C \quad (24)$$
$$C \overset{\Gamma_2}{\rightleftharpoons} B + E \quad (25)$$
$$C \overset{\Gamma_3}{\rightleftharpoons} B + E^* \quad (26)$$

There is no dimeric compound in the system, even indirectly formed.

Provided the catalyst, present in C, E, E^*, is in low total concentration, the QSSA implies the presence of two fluxes: the transformation of A into B catalyzed by E of intensity φ, and the transformation of E^* into E catalyzed by B of intensity ε, with $\varphi \gg \varepsilon$. This decomposition gives:

$$\varphi = \frac{\alpha V_A V_B}{\beta V_B + \gamma} - \delta V_B \quad (27)$$

with $\alpha = \delta(\Gamma_1 + \Gamma_2)$, $\beta = \Gamma_2 - \Gamma_1 \frac{K_B}{K_A}$, $\gamma = \frac{\Gamma_1}{V_A^{\text{tot}}}$ and $\delta = \frac{\Gamma_1}{V_{E^*}^{\text{tot}}}$.

The autoinduction is kinetically equivalent to the indirect autocatalysis mechanism:

- When $\Gamma_2 \gg \Gamma_1 \frac{K_B}{K_A}$, the flux tends to $\varphi = \frac{\alpha}{\beta} V_A - \delta V_B$: the system is non-autocatalytic.
- When $\Gamma_2 \approx \Gamma_1 \frac{K_B}{K_A}$, the flux tends to $\varphi = \frac{\alpha}{\gamma} V_A V_B - \delta V_B$: the system is simply autocatalytic.
- When $\Gamma_2 \ll \Gamma_1 \frac{K_B}{K_A}$, the flux tends to $\varphi = \frac{\alpha}{\Gamma_1} V_B - \delta V_B$: the system presents an undamped autocatalysis.

Following the kinetic analysis, the behavior is similar to the time evolution of autocatalytic systems (See Fig. 4(c)). The behavioral equivalence of these two systems (kinetically equivalent but mechanistically very different) will be investigated in more details in the next section.

The mechanism of Iwamura et al. (2004) is an autoinductive autocatalysis, with a slightly more complex mechanism (see Fig. 3(e)). The core principle is a reaction $A+X \rightarrow AX$,

catalyzed by P, the product AX catalyzing the first catalytic step $P+A \rightarrow PA$. This chemical system can be decomposed into two different fluxes $A + X \rightarrow AX$, one coupled to a catalytic cycle $[P \rightarrow PA \rightarrow PAX \rightarrow P|AX \rightarrow P]$, and one coupled to a catalytic cycle $[PA \rightarrow PAX \rightarrow P|AX \rightarrow PA]$. The first one contains the slow reaction of A on P, and corresponds to a slow flux ε. The second one only contains fast reactions, and corresponds to a fast flux φ. These two fluxes can be shown to be related by:

$$\frac{\varphi}{\varepsilon} = \alpha V_A + \beta V_{AX} \quad (28)$$

α and β being constants depending on the kinetic parameters of the system. This implies an increase of the effective rate production φ as a function of the concentration in product.

Network vs Autoinductive Autocatalysis: Autoinductive autocatalysis is mechanistically different from network or template autocatalysis. The balance equation is rather of the form $A + \alpha B \rightarrow (1 + \alpha)B$, with $\alpha \ll 1$. The linear transformation $A \rightarrow B$ is only weakly coupled to the cycle of B back to itself, this latter one being subject to a much lower flux than the linear flux. However, autoinduction is kinetically and dynamically equivalent to network autocatalysis, leading to the same kind of differential equation, and thus of behavior. It can be noted that the undamped exponential profile due to a flux only proportional to the products and not to the reactant is not characteristic of autoinductive processes (Iwamura et al., 2004) but can also be explained by network autocatalytic mechanisms, when the consumption of the reactant is not limiting the kinetic of the network.

Embedded Autocatalyses

Autocatalysis is not so important *per se* but as a way of giving birth to rich non-linear behaviors like bifurcation, multistability or chemical oscillations. It becomes capital to study the interaction of autocatalytic mechanisms and their ability to generate such behaviors when embedded in a larger chemical network.

Dynamical Distinctions

Different behaviors depending on the order n of the autocatalysis can be observed in biochemical competitive systems. They are classically studied in population evolution (Szathmáry, 1991; Nowak, 2006) and described as "survival of the all" in the case of $0 < n < 1$ (characterized by the coexistence of all compounds), as "survival of the fittest" in the case of $n = 1$ (when the only stable solution retains the fittest compound or the most "reproductible") and as "survival of the first" in the case of $n > 1$ (when the final solution just retains the product initially present in the highest concentration).

The case $0 < n < 1$ is the least interesting, as it hardly leads to a clear selectionnist process. However, real mechanism that seems to possess a first order autocatalysis may actually present a lower autocatalytic order. This is typically the case for direct template autocatalysis, in which the order falls to $1/2$ on account of the high stability of the dimeric intermediate—which is actually a necessary condition for the selectivity of template replication (von Kiedrowski, 1986, 1993; Wills et al., 1998). This turns out to be a fundamental problem for understanding the emergence of the first replicative molecules (Szathmáry and Gladkih, 1989; Lifson and Lifson, 1999; Scheuring and Szathmáry, 2001).

More complex mechanisms may lead to higher orders, typically by the formation of dimeric autocatalysts (Wagner and Ashkenasy, 2009). This is the case of the Soai reaction whose high sensitivity to initial conditions may potentially be explained by the formation of trimeric (Gridnev et al., 2003) or even hexameric complexes (Schiaffino and Ercolani, 2008).

Comparative Efficiency of Direct and Autoinductive Autocatalyses

Bifurcations appear when installing two autocatalytic processes in competition, placing them in a non-equilibrium open-flow system, both being fed by the same incoming compound and with cross-inhibition between them:

$$\rightarrow A \quad \text{(incoming flux)} \quad (29)$$

$$A \rightleftharpoons B_1 \quad \text{(Direct AC)} \quad (30)$$

$$A \rightleftharpoons B_2 \quad \text{(Autoinduced AC)} \quad (31)$$

$$B_1 + B_2 \rightarrow (P) \quad \text{(cross inhibition)} \quad (32)$$

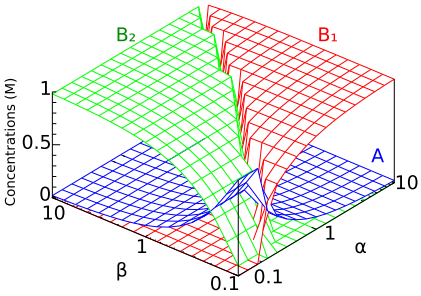
$$B_1 \rightarrow \quad \text{(outgoing flux)} \quad (33)$$

$$B_2 \rightarrow \quad \text{(outgoing flux)} \quad (34)$$

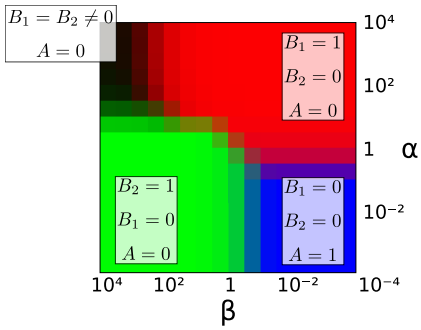
In the case of total symmetry between B_1 and B_2, with the same direct autocatalystic mechanism, this system would correspond to the classical Frank model for the emergence of homochirality (Frank, 1953), leading to a the same probability to end up with either B_1 or B_2.

The kinetic equivalence between template autocatalysis and autoinductive autocatalysis can be shown by making these two mechanisms to compete, replacing Eq. (30) and (31) by the corresponding mechanism. Kinetic parameters have first been normalized so that both reaction leads to the same kinetic behavior (sigmoidal evolution, half-reaction at 10^5 s), and then multiplied by respectively α and β parameters in order to tune the respective velocity of each mechanism. The result is actually quite symmetrical between the two processes and only the fastest product is maintained in the system: B_1 when $\alpha > \beta$, and B_2 when $\alpha < \beta$ (see Fig. 5(a)).

This selection is independent of the relative stability of B_1 and B_2, but is only possible for kinetics that are well adapted to the global influx of matter. For slow kinetics, there is a flush of the system, and no B_1 nor B_2 compound can be maintained. For fast kinetics, the system is close to equilibrium,



(a) Sharp bifurcation depending on the relative values of α and β for moderate reactivities.



(b) Different zones of behaviors: majority of A for $\alpha, \beta \ll 1$, majority of B_1 for $\alpha > \beta$, majority of B_2 for $\alpha < \beta$, and coexistence of B_1 and B_2 for $\alpha, \beta \gg 1$.

Figure 5: Competition between template and autoinductive autocatalysis, generating respectively B_1 and B_2 compounds from the same A compound. Incoming flux of A, and outgoing fluxes of B_1 and B_2, 10^{-5} M.s^{-1}. $K_A = 1$, $K_{B_1} = K_{B_2} = 100$. Direct autocatalysis: $\Gamma_{AC} = 10^{-2} \cdot \alpha$, $\Gamma_{NC} = 10^{-6} \cdot \alpha$. Autoinduction, according to Fig. 3(d): $\Gamma_1 = \beta$, $\Gamma_2 = \Gamma_3 = 100 \cdot \beta$, $K_C = K_E = 1$; $K_{E^*} = 10$.

the compounds B_1 and B_2 being both present in proportion to their respective stability (see Fig. 5(b)). Such result is well known for open flow Frank systems (Cruz et al., 2008).

From Autocatalytic Processes towards Autocatalytic Sets

These competitive systems are able to dynamically maintain a set of components, to the detriment of others. The notion of autocatalytic set (requiring the system to be materially closed and self-maintained by a crossing energetical flux) is rather popular in the artificial life literature and relies much more on the cooperation between autocatalytic mechanisms than on the competition that has just been detailed here. It implies a notion of closure of the system and of self maintenance of the whole network (Kauffman, 1986; Hordijk and Steel, 2004; Benkö et al., 2009). Confusion among these different phenomena can be pinpointed in the literature (Blackmond, 2009), when the failure of autoinductive sets to be maintained do not originate from a difference of behavior between autocatalytic and autoinductive mechanisms, but from a defect in the closure of the system.

Conclusion

Important distinctions need to be done between mechanistic and dynamic aspects of autocatalysis. The same mechanisms can produce different dynamics, while identical dynamics can originate from different mechanisms. But all these different autocatalytic processes are able to generate autocatalytic kinetics, that may constitute a pathways towards the onset of "self-sustaining autocatalytic sets", as a chemical attractor in non-equilibrium networks. However, the problem of the evolvability of such systems must be kept in mind (Vasas et al., 2010). If a system evolves towards a stable attractor, no evolution turns out to be possible. There is the necessity of "open-ended" evolution (Ruiz-Mirazo, 2007) i.e. the possibility of a dynamic set not only to maintain itself (i.e. a strictly autocatalytic system) but act as a "general autocatalytic set", redounding upon the concept originally introduced by Muller (1922) for the autocatalytic power linked to mutability of genes. Insights can be gained by a deeper and renewed study of the evolution of prions as a simple mechanism of mutable autocatalytic systems (Li et al., 2010).

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