Strategic Information Diffusion: Spread vs. Exploit *

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ABSTRACT

Frequent drops of prices to zero is a common phenomenon in price trends of many smartphone applications. The only means by which many of these *apps* spread is the *word of mouth* of their users. Motivated by these observations, we study the problem of optimal dynamic pricing in a social network where agents can only get informed about the product via word of mouth from a friend who has already bought the product. We show that for a durable product such as many apps, the optimal policy should drop the price to zero infinitely often, giving away the immediate profit in full to expand the informed network in order to exploit it in future. We further show that, beside the word of mouth nature of the information diffusion, this behavior crucially depends on the type of the product being offered. For a nondurable product, although the firm may initially make some free offers to expand its network, after a finite period, it will fix the price at a level that extracts the maximum profit from the already informed population.

Keywords

Information diffusion, word of mouth, dynamic pricing, social networks

1. INTRODUCTION

> "Word of mouth communication involves the passing of information between a non-commercial communicator (i.e. someone who is not rewarded) and a receiver concerning a brand, a product, or a service", Dichter [5].

Information diffusion via word of mouth (WOM hereafter) marketing in social networks has become ubiquitous, recently attracting a lot of attention.[1] Managers are interested in better understanding WOM as some evidence suggests that traditional forms of communication may be losing effectiveness.[2] WOM communication strategies are appealing because they combine the prospect of overcoming consumer resistance with significantly lower costs and fast delivery – especially through the Internet and more recently through smartphone applications. A recent example is the smartphone application *WhatsApp* which was sold to *facebook* in early 2014 for $19 billion. WOM was the key to *WhatsApp* popularity. As noted by Bloomberg [10], "They [*WhatsApp* management] eschewed marketing and did not employ a public relations person, relying on the word of mouth recommendations of its users instead".

During the last decade, the technology of smartphone applications has seen a huge growth. Smartphone applications (apps) are typically cheap, and the only low budget mean by which many of these apps spread is the word of mouth communication of their users.[3] The price a firm offers for its product is also a big driver for spreading the information. As such, posting volatile prices is a common marketing tool for expanding information about the existence of a new app among the users. Figure 1 depicts the (dynamic) price history for *tadaa 3D*, an iPhone photo and video application since its release.[4] An interesting observation from this chart is the *frequent* drops of the price to zero. The same patten can be seen in the price trends of many other apps.

Motivated by these observations, this paper studies the problem of optimal dynamic pricing of a firm selling a product in a large social network where agents can only get informed about the product via word of mouth from a friend who has already bought the product. A key feature of our work is modeling the effect of the price on the information diffusion via WOM. The (dynamic) price is a control variable by which the firm directly affects the information diffusion of its new product through the underlying social network. Firm's problem is then to decide, at each time step, between optimally exploiting the informed network it already has or charging a lower price in favor of a higher spread of information. In particular, the role of (potential) volatile prices in the information diffusion is in play for selling smartphone applications in social networks.

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[1] For example [9] noted that 50% of service provider replacements were found via WOM.

[2] For example, one survey showed consumer attitudes toward advertising was declining from September 2002 to June 2004. More specifically, [6] reported survey showing that 59% fewer respondents say they buy products because of their ads, 40% fewer agree that ads are a good way to learn about new products, and 59% fewer say they buy products because of their ads.

[3] Many apps ask users for permission to send notifications about the product to contacts in their address books or to post a message on their social pages, when they start using the app.

[4] The figure is plotted using data gathered from an application called *appsfire*.

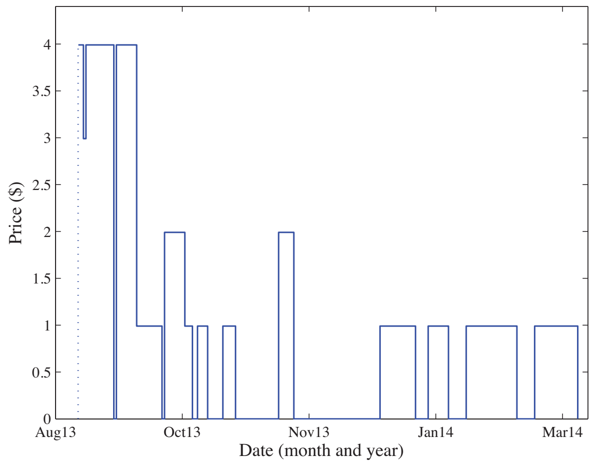

Figure 1: Price history for the iphone application tadaa 3D since it was debuted on Aug. 16, 2013.

The main contribution of this work is to describe and analyze a model that explains this phenomenon consistent with the real world evidence from smartphone applications. We show that for a durable product[5] the optimal policy should drop the price to zero *infinitely* often, essentially giving away the immediate profit in full to expand the informed network in order to exploit it in future. We further analyze the case for a nondurable product, for which we show that although the firm may initially make some free offers to expand its network, after a finite time it will fix the price at a level that extracts the maximum profit from the already informed population. In addition to the type of the product, the WOM nature of the information diffusion is another effective factor in play. Indeed, in the absence of WOM, in which the agents are assumed to be (ex ante) informed about the product, the optimal pricing policy will become monotonically decreasing. As for the final size of the informed population, it turns out that the pricing policy maximizing the profit for a durable product also maximizes the spread of the information in the network, while for a nondurable product the maximum profit is obtained by exploiting a smaller set of informed agents.

This work is related to the growing literature in economics studying optimal policies for a decision maker who would like to optimize an objective function controlled by agents' actions embedded in a social network. Examples include ([1], [4], [8], [7], [2], [3]). The closest work to ours is [3] which studies dynamic pricing for a nondurable product, where the spread of information about the product is carried out via word of mouth of buyers. However, the author models the word of mouth as a branching process, where he also emphasizes that a branching model is only valid at the early stages of introducing a product to the network. This model thus cannot be used to study behaviors related to the asymptotic properties of the model such as the frequent price drops to zero discussed in this paper.

[5]We deal with smartphone applications as durable products. This is because when a user buys an app, she usually does not need to buy the same product any more.

2. MODEL

2.1 General Description

The economy consists of a unit measure continuum of agents indexed by $i \in I = [0, 1]$. Agents reside in a social network, the structure of which is captured by an undirected random graph G with Poisson degree distribution with mean λ. More precisely, each agent i has a total of $d_i \sim \text{Poiss}(\lambda)$ friends uniformly distributed in I. We denote the set of the friends of i in G with N_i. For every $i \in I$, the set of her friends N_i forms a random Poisson process in I.

At each time step $t = 0, 1, 2, \cdots$, a firm is selling a product in this social network at price $u(t) \in \mathcal{U}$, where $\mathcal{U}$ is a finite set of admissible prices. The set of admissible prices $\mathcal{U}$ can represent any set of quantized price levels in $[0, 1]$. In particular, we assume $0 \in \mathcal{U}$ to allow for the *free offer* of the product. We can write the set of admissible prices as $\mathcal{U} = \{p_0 = 0 < p_1 < \ldots < p_m \leq 1\}$, where $m \geq 1$ is the number of nonzero price levels.

Each agent has a private valuation $\theta \sim \text{Unif}[0, 1]$ of the product. The valuations of the agents are time-invariant and are independent of their degrees and the valuations of their friends. Moreover, agents' valuations and their positions in the network are their private information, and hence, not known to the firm.

In order for an agent to buy the product, she should first be informed about its existence. At $t = 0$ to initiate the spread of information, a uniformly randomly chosen subset of the population becomes informed about the product directly by the firm. Later on, at any time $t \geq 1$, other agents can only get informed via word of mouth from a friend who has already bought the product. Note that at any time t, an informed agent buys the product if the offered price does not exceed her valuation, i.e. $u(t) \leq \theta$. Upon buying the product, she will inform her friends about the product.

In this framework, firm's objective is to devise an optimal dynamic pricing policy maximizing its accumulated discounted profit over an infinite time horizon. We first study this problem for the case of a *durable* product, such as many smartphone applications, in order to justify the behavior pointed out in the previous section. We then investigate the role of the type of the product in the price drops to zero by studying the problem for a nondurable product. It is to be noted that an informed agent may buy a nondurable product at each time step, given that its price is lower than her valuation. However, if she buys a durable product at some time, she will not buy it thereafter.

2.2 WOM Diffusion Dynamics

In this subsection, we first present a few notations, definitions, and observations that will be used later to derive the dynamics of the information diffusion in the network. We denote the set of informed agents at time t by $X(t)$ and its size by $x(t)$. $X(0)$ is therefore the set of those agents who are directly informed by the firm, with $x(0) = x_0$ denoting the size of this set. Considering that we are dealing with a unit measure continuum of agents, an informal use of the strong law of large numbers will let us write $x(t) = \text{Prob}(i \in X(t))$. As we will see in the sequel, this will prove very convenient in deriving the dynamics of the information diffusion.[6] The set of informed agents $X(t)$ is increasing, that is

[6]Following the same logic, we may exchangeably use the

$X(t-1) \subseteq X(t)$, where $Y(t) = X(t) - X(t-1)$ represents the set of freshly informed agents at time t, whose size is denoted by $y(t)$. The set of agents that contribute to diffusion is comprised of two subsets. One set is the freshly informed agents in $Y(t)$ that buy the product, which we denote by $B_Y(t)$. Noting that θ has the same uniform distribution in $Y(t)$ as it has in I, the fraction of agents from $Y(t)$ that buy the product when offered the price $u(t)$ is $(1-u(t))y(t)$. These new buyers spread the information about the product and their friends will constitute a part of $Y(t+1)$.

Another contribution to diffusion comes from the set of the agents which were previously informed about the product but have not yet bought it. For such agents, the price has not fallen below their valuation since the time they were informed about the product. We denote the set of such agents at time t by $Z(t)$. An agent in this set may buy the product at time t and thus inform her friends, if the offered price at time t is below her valuation. Unlike $Y(t)$, the distribution of θ is not uniform for the agents in $Z(t)$ and depends on the price history. However, we can use a stack of m variables (recall that m is the number of nonzero price levels) to fully describe the distribution of θ in $Z(t)$. We can partition $Z(t)$ as $\bigcup_{j=1}^{m} Z_j(t)$, where $Z_j(t)$ is the set of those agents in $Z(t)$ whose valuations lie between price levels p_{j-1} and p_j, that is, $Z_j(t) = \{i \in Z(t) | p_{j-1} \leq \theta_i < p_j\}$. Then, the distribution of θ in $Z(t)$ is fully determined by the sizes of these sets, which are denoted by $z(t) = [z_1(t) \dots z_m(t)]^T$. If the firm chooses $u(t) = p_r$ as the price at time t, then all the agents in $B_Z(t) = \bigcup_{j=r+1}^{m} Z_j(t)$ will buy the product and will subsequently inform their friends, while the rest of the agents in $Z(t)$ will be carried over to $Z(t+1)$. However, $Z(t+1)$ also has another component coming from those freshly informed agents in $Y(t)$ whose valuations are below p_r, having a size of $p_r y(t)$. θ has a uniform distribution of Unif$[0, p_r]$ in this set. Summarizing this, we arrive at the following update rule for z_j's:

$$z_j(t+1) = \begin{cases} z_j(t) + (p_j - p_{j-1})y(t), & 1 \leq j \leq r \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

assuming that the offered price at time t is p_r. As can be seen from the above discussion, in a WOM model of diffusion, the fresh buyers $B(t) = B_Y(t) \cup B_Z(t)$ are responsible for expanding the network of informed agents. The size of this set is given by

$$b(t) = (1 - p_r)y(t) + \sum_{j=r+1}^{m} z_j(t), \quad (2)$$

where p_r is the price level chosen by firm at time t ($u(t) = p_r$). An agent $i \in I$ will be uninformed at time $t+1$ if and only if she neither is informed nor has a friend among fresh buyers at time t. The WOM dynamics can be written as

$$\begin{aligned} \text{Prob}(i \notin X(t+1)) =& \text{Prob}(i \notin X(t) \wedge N_i \cap B(t) = \emptyset) \\ =& \text{Prob}(i \notin X(t)) \times \\ & \text{Prob}(N_i \cap B(t) = \emptyset | i \notin X(t)). \quad (3) \end{aligned}$$

Using the *stationary increments property* of Poisson random processes, the number of friends that an agent $i \notin X(t)$ has in $B(t)$ is a Poisson random variable with mean $\lambda b(t)$. Therefore, the probability of having no friend in $B(t)$ for an agent $i \notin X(t)$ is given by $e^{-\lambda b(t)}$. It is worth mentioning that $i \notin X(t)$ implies that i does not have any friend among the previous buyers $\cup_{\tau=0}^{t-1} B(\tau)$. However, this does not affect the distribution of her friends in $B(t)$ due to the *independent increments property* of Poisson random processes. Using (3), we can write the dynamics of the informed population $x(t)$ as

$$1 - x(t+1) = (1 - x(t))e^{-\lambda b(t)}, \quad (4)$$

where $b(t)$ is given by (2), $z(t)$ is updated using (1), and $y(t+1) = x(t+1) - x(t)$. Moreover, $y(0) = x(0) = x_0$ and $z_j(0) = 0$ for $1 \leq j \leq m$.

2.3 Durable and Nondurable Products

The profit of the firm for the case of a durable product is given by

$$\Pi^D(u(\cdot)) = \sum_{t=0}^{\infty} \beta^t u(t) b(t), \quad (5)$$

where $0 < \beta < 1$ is the discount factor, and the marginal cost of the product is assumed to be zero. Firm's objective is to find a pricing policy that maximizes the above profit, which we denote with $u^D(\cdot)$.

For a nondurable product, every agent $i \in X(t)$ can buy the product if the offered price is below her valuation. Recalling that θ has a uniform distribution in $X(t)$, the size of the buyers at time t is $(1 - u(t))x(t)$, and therefore the accumulated discounted profit of the firm over an infinite time horizon is given by

$$\Pi^{ND}(u(\cdot)) = \sum_{t=0}^{\infty} \beta^t u(t)(1 - u(t))x(t). \quad (6)$$

Firm's objective is to find the optimal pricing policy $u^{ND}(\cdot)$ that maximizes the above profit.

It is to be noted that the dynamics of the diffusion is the same for both cases, as in both an agent informs her friends about the product as soon as she buys it.

3. FIRM'S PROBLEM: TO SPREAD OR EXPLOIT?

Given the dynamics of the information diffusion for the WOM model developed in previous section and the profit of the firm given by (5) and (6), the firm's problem is to decide at each time step, between optimally exploiting the network it already has by offering a price that results in the maximum immediate profit, or offering a lower price in favor of a larger spread.

A related problem is to find the maximum achievable size of the informed network via WOM. For any price function $u(\cdot)$, $x(t)$ is bounded and increasing thus having a limit as $t \to \infty$. Define $q(x_0; u(\cdot)) = \lim_{t\to\infty} x(t)$. q is the asymptotic size of the population that can be informed about the product via WOM, starting from a uniformly randomly chosen informed population of size x_0 and following a given pricing policy $u(\cdot)$. It is easy to see that for $x_0 < 1$ this asymptotic size is always less than 1, implying that the product cannot take over the entire population I via only WOM. A quick reason for this is that, there are $e^{-\lambda}$ isolated agents (with no friend) in I, out of which $(1 - x_0)e^{-\lambda}$ of them are not in $X(0)$ and therefore will never hear about the product via WOM.

The case in which the product is offered for free, i.e. $u \equiv 0$, gives an upperbound on the asymptotic size of the informed

words, size, fraction, and probability in the paper.

population, that is $q(x_0;u(\cdot)) \leq q(x_0;0)$. In this case, every agent that gets informed about the product will in turn inform her friends about it. The information will then spread throughout the network and all the agents that are reachable from an agent $i \in X(0)$ will eventually get informed about the product. In this case, $Z(t) = \emptyset$ and $B(t) = Y(t)$, thus the dynamics of diffusion governed by (1), (2), and (4) simplifies to

$$1 - x(t+1) = (1 - x(t))e^{-\lambda y(t)}, \tag{7}$$

$$y(t+1) = x(t+1) - x(t), \tag{8}$$

where $y(0) = x(0) = x_0$. Using this recursively for $t, t-1, \ldots, 0$, we obtain

$$1 - x(t+1) = (1 - x_0)e^{-\lambda x(t)}. \tag{9}$$

The asymptotic size of the informed network for a free product can be obtained noting that $q(x_0;0)$ should satisfy the above relation:

$$1 - q(x_0;0) = (1 - x_0)e^{-\lambda q(x_0;0)}. \tag{10}$$

Based on this equation, we present several properties for $q(x_0;0)$ in the following proposition.

Proposition 1. *For every $0 < x_0 \leq 1$, the asymptotic size of the informed population for a free product is given by the unique solution of $1 - q(x_0;0) = (1 - x_0)e^{-\lambda q(x_0;0)}$ in $[0,1]$. The solution is concave and monotonically increasing in x_0. Moreover, $q(x_0;0) > 1 - \frac{1}{\lambda}$.*

One interesting aspect of Proposition 1 is the discontinuity in $q(x_0;0)$ at $x_0 = 0$ for $\lambda > 1$. Although $q(0;0) = 0$, for any nonzero x_0 and $\lambda > 1$, $q(x_0;0)$ is lowerbounded by a positive constant independent of x_0. This implies that no matter how small the size of the initially informed population is, a free product can take over a large portion of the network via WOM given the typically large average number of friends in the networks.

As the main objective of this paper, we next consider the case of a durable product and we show that under the optimal policy price should drop to zero infinitely often. This is in line with the real world evidence from smartphone applications discussed in Section 1, where price histories witness frequent drops of the price to zero for many apps. Moreover, using these frequent price drops to zero we can show that the optimal pricing policy maximizing the profit for a durable product also maximizes the spread of the information in the network. These results are summarized in the next theorem.

Theorem 1. *Under the optimal pricing policy $u^D(\cdot)$ for a durable product, the price drops to zero infinitely often. Moreover, $q(x_0;u^D(\cdot)) = q(x_0;0)$, that is, the network of informed agents achieves its maximum asymptotic size under this optimal policy.*

The WOM nature of the information diffusion is apparently a key deriver for dropping the price to zero. If agents (users) are not involved in spreading the information about the product, the firm will not have any incentive to drop the price to zero. In fact, it is easy to show that for the case of full information, in which everybody is directly informed by the firm, the optimal pricing policy is monotone (decreasing) exploiting those who are willing to pay more first and then gradually lowering the price. Beside the WOM nature of the information diffusion, these drops are also rooted in the type of the product being offered. It can be shown that, for a nondurable product, when the size of the informed population gets large enough, the optimal policy is to set the price at a level that extracts the maximum profit out of the already informed population. This is stated in the next theorem.

Theorem 2. *Given the optimal pricing policy $u^{ND}(\cdot)$ for a nondurable product, there exists a finite time T after which the price is set to the fixed level u^* maximizing the immediate profit $u(t)(1-u(t))x(t)$, that is $u^{ND}(t) = u^*$ for $t \geq T$. The price level u^* is the closest price level in $\mathcal{U}$ to $\frac{1}{2}$.*[7]

Although the above theorem assures that there will be no free offer of the product after some finite time in the nondurable case, it is possible that the firm drops the price to zero during the early stages in order to expand its network, as shown in the next theorem. We also show how effective these free offers can be in expanding the network of informed agents.

Theorem 3. *Consider the optimal pricing policy $u^{ND}(\cdot)$ for a nondurable product and assume that $\beta > \frac{1-u^*}{p_1(\lambda^*-1)}$, where $\lambda^* = (1-u^*)\lambda$. Then, there exists $x^c > 0$ such that for $x^{ND}(t) < x^c$:*
i) $u^{ND}(t)u^{ND}(t+1) = 0$,
ii) $x^{ND}(t+1) > \lambda^* x^{ND}(t)$.

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[7] If there are two such price levels in $\mathcal{U}$, denote the smaller one with u^*.