An Iterative and Truthful Multi-Unit Auction Scheme for Coordinated Sharing of Spectrum White Spaces

Saravana Manickam and Mahesh K. Marina
The University of Edinburgh

Sofia Pediaditaki
Intel

Maziar Nekovee
Samsung R & D

ABSTRACT

We consider the use of dynamic auctions for coordinating spectrum sharing among secondary users, and propose an online multi-unit, iterative auction mechanism called VERUM that is truthful and efficient (the item is always won by the bidder who values it the most). VERUM is an adaptation of the well known Ausubel's clinching auction [1] to suit the dynamic spectrum sharing context. As a use case for VERUM, we consider TV white space (TVWS) spectrum sharing among home networks, and compare VERUM with two existing efficient and truthful multi-unit spectrum auction schemes, VERITAS and SATYA. Our evaluations, using real distributions of homes in a dense urban neighborhood in London and realistic TVWS spectrum availability maps for the UK, show that VERUM outperforms the other two schemes in terms of revenue, spectrum utilization and percentage of winning bidders.

1. INTRODUCTION

Auctions have been extensively used over the years for dynamic spectrum management with several different types of auction mechanisms developed to suit different scenarios [2]. We consider dynamic auctions to coordinate the sharing of spectrum white spaces among secondary users. In such a dynamic spectrum auction problem, bidders (secondary users) may request multiple items (channels) based on their private and independent valuations. As we seek an efficient and truthful auction, it would suggest the use of multi-unit Vickrey auction proposed in the seminal paper [3]. In the classical multi-unit Vickrey auction, the bidder winning M items will pay the amount of the M^{th} highest losing bid for the first item, $(M-1)^{th}$ highest losing bid for the second item and so on. When we apply the above multi-unit Vickrey auction mechanism in the spectrum allocation context where spatial reuse is allowed for efficient spectrum utilization and conflict (interference) relationships need to be accounted, computing Vickrey pricing described above becomes complex as each bidder could be allocated a channel and there may as such be no losing bid. We could simplify the pricing scheme but that can come at the expense of truthfulness as discussed in [4].

To ensure truthfulness, existing multi-unit spectrum auction schemes [4, 5] employ different means to realize Vickrey pricing and end up being polynomial only under certain constraints or have an exponential run time. All these schemes are variants of the sealed bid auction scheme where the bidders submit sealed bids and the auctioneer computes the winners based on all the bids. VERITAS [4] and SATYA [5] are two such auction schemes. In VERITAS [4], the auctioneer collects sealed bids from all the bidders and uses a greedy algorithm for allocation. It then determines a critical neighbor for each of the bidders based on which the winning price is computed. However, VERITAS does not support channel sharing among neighbors. SATYA [5] uses bucketing and ironing of bids to maintain monotonicity for truthfulness. While it is the first scheme to support channel sharing it has a few drawbacks. Firstly, it has an exponential run time and is only polynomial under some restrictions. Secondly, the performance of the scheme is highly dependant on the bucketing function which is not part of the SATYA mechanism and is abstracted out. Finally, it does not support marginal valuations, which refer to the values a bidder associates with the first channel in its demand and every additional channel.

In this paper, we take an iterative approach that is fundamentally different from the above mentioned schemes. Iterative auctions have multiple advantages over sealed bid auctions. The foremost advantage of an iterative auction is that it provides a simpler means to achieve truthfulness. Another compelling advantage is that iterative auctions are transparent in the way they determine the outcome of an auction. In other words, bidders can verify and validate the auction outcome. To see why this might be important, notice that when bidders do not pay the bidding price, they can doubt the correctness of the auction scheme. In fact, a frequently mentioned problem with sealed bid auction schemes is that the auctioneer could create a fake second-highest bid after receiving all the sealed bids from the bidders in order to increase its revenue. Doing the equivalent with iterative auctions, however, might result in revenue loss for the auctioneer. Yet another advantage of iterative auctions is that they are better at protecting the privacy of bidders' valuations. It is preferable for bidders not to be required to disclose their private values as they could be based on sensitive information. When compared to sealed bid auctions, iterative auctions incur lower information revelation as they do not share their value with the auctioneer. While several solutions are available to protect the privacy of bidders in a sealed bid auction, they either require a third party to compute the outcome of the auction or require cryptographic techniques.

Our proposed auction scheme called VERUM is based on the ascending-bid multi-unit clinching auction proposed in [1]. Ausubel's clinching auction [1] while being simple has also been shown to be efficient (the bidder with the highest valuation for the channel always wins it) and also replicate the outcome of Vickrey auction. But it was not intended for dynamic spectrum sharing. As such it does not account for any of the unique characteristics associated with the dynamic spectrum allocation in general. Spatial reuse, which allows multiple users to be allocated the same channel provided they do not interfere with each other, is one such characteristic. Our contribution in terms of auction design lies in adapting the clinching auction in [1] to factor in all such characteristics unique to dynamic spectrum sharing while preserving its desirable properties like simplicity, truthfulness and efficiency. To the best of our knowledge, this is the first time Ausubel's clinching auction has been applied in the dynamic spectrum management context even though it has been around for some time.

As a use case for VERUM, we consider TV white space (TVWS) spectrum sharing. TV white spaces are portions of the UHF TV band that are unused at any given location by primary users of the TV spectrum (e.g., nearby TV transmitters). TVWS spectrum is (being) made available for unlicensed secondary use provided it is accessed via a geolocation database to ensure interference protection for primary users. However uncoordinated use of TVWS spectrum can lead to inefficient utilization of this new source of spectrum, especially with a heterogeneous set of secondary users. With this in mind, we propose the use of short-term auctions as a means to enable coordinated TVWS spectrum use. Specifically, we consider the case where home networks are TVWS users and define an auctioning based TVWS spectrum coordination framework using VERUM as shown in Fig. 1; in the figure, bidders are the participating Home White Space Networks (HWSNs) with non-zero demand for TVWS channels available in their location, and the auctioneer is the spectrum manager (could be the geolocation database provider).

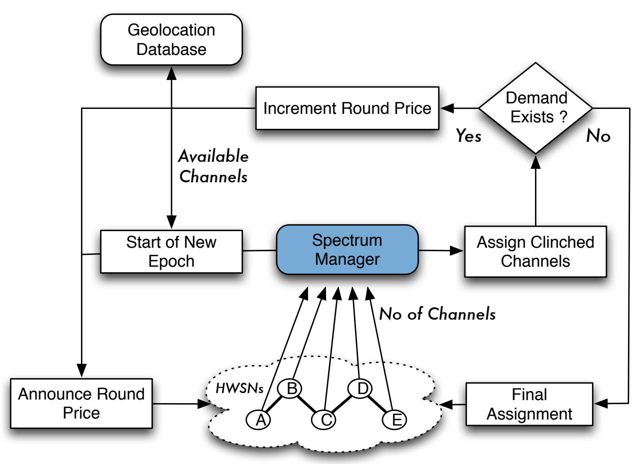


Figure 1: Auction based framework for coordinating TVWS spectrum using VERUM.

2. VERUM AUCTION MECHANISM

We now describe VERUM. The system model consists of users bidding for access to channels available in their location. Each bidder is assumed to have a private value function to generate marginal valuations for the channels. Marginal valuations of a bidder refer to values that the bidder associates with the first channel in its demand and every additional channel. We assume that marginal valuations of each bidder are weakly decreasing. Fig. 2(a) shows example marginal valuations (with V: above each bidder). Considering bidder A in Fig. 2(a) as a specific example, A's marginal valuations show that it values the first channel it can get at 13, the second channel at 8 and a third channel at 6.

The auctioneer initially announces a round price p_1 for the channels; bidders then respond with the number of channels they are willing to buy at price p_1. The round price controls the demand $D_i(p_1)$ from each bidder in the sense that the number of channels it can bid is determined by the number of channels within its private marginal valuations that have higher valuations than the current round price. In the example shown in Fig. 2(a), if the current round price is 13, then demand from node B is 1 ($D_B(13) = 1$) as it has only one channel that has a higher valuation than 13.

At each round t with price p_t, the auctioneer determines if for any bidder i the aggregate demand of bidder i's neighbors in the conflict graph $D_{-i}(p_t) = \sum_{j \in N_i} D_j(p_t)$ is less than x_i, the number of channels available at i. If so, the difference is deemed clinched and the new channels clinched in this round are considered won by the bidder i at that round price p_t. Note that to allow for *spatial reuse*, we view only the neighbors of a node in the conflict graph as its competing bidders. For example in the conflict graph shown in Fig. 2(a): A competes with B and C in the auction; C competes with A, B and D; and E competes only with D. This is unlike the classical multi-unit Vickrey auction or Ausubel's clinching auction where all bidders compete with each other.

In reality, bidders may have access to different subsets of the channels. For example, consider the conflict graph shown in Fig. 2 (a). Suppose that a license primary user is active only in the vicinity of bidder E, then the channels used by that primary user would be unavailable only for bidder E while the spectrum availability for other bidders may remain unaffected. We refer to this as *heterogeneous spectrum availability* and to handle this, we introduce the notion of *exclusive channels*. A channel k is considered exclusive to HSWN i if it is available for use only by i and not by any of its neighbors. Some channels maybe exclusive to a bidder right at the first round of the auction. Alternatively, channels may become exclusive to a bidder in subsequent rounds (associated with higher round prices) when the demand of any of its neighbors reduces to zero. In each round t, we identify the set of exclusive channels and consider them clinched by the respective bidders at the round price p_t.

The above process repeats with increasing round prices until there is no demand from the bidders. Fig. 2(b) illustrates the working of the VERUM auction mechanism for the example in Fig. 2(a). Note that in Fig. 2(a), the set of channels available at each bidder are shown with "A:". As a specific example, bidder A has two channels available (1 and 2). The number of channels a bidder is assigned is limited by the number of channels it has available. In the example, A can be assigned at most 2 channels even if its demand is more.

In the first round of the auction with price $p = 1$, the five bidders A, B, C, D, and E bid for 3 channels each (as they have higher valuations than the current round price). Since there is excess demand, the auction proceeds to subsequent rounds with the price being incremented at each round. At price $p = 6$, the cumulative demand of bidder E's neighbors is 1, where as 2 channels are available for use by E hence it is assured of winning at least one channel. Considering this, a channel is deemed clinched by bidder E at price $p = 6$. Similarly at price $p = 8$, when bidder D's demand drops to 0, E clinches another channel. At this point bidder E has clinched two channels as $x_E - D_{-E} = 2$; in other words, bidder E is assured of at least two channels. At price $p = 12$, the cumulative demand of bidder C's neighbors (A, B and D) is 2 whereas there are 3 channels available for use by C. Hence bidder C clinches a channel at price $p = 12$. Finally, at price $p = 13$, bidder A's demand drops to 0 and bidders B and C win a channel each. Since no more channels can be assigned beyond this point, the auction comes to an end with bidders B, C and E winning 1, 2, and 2 channels, respectively.

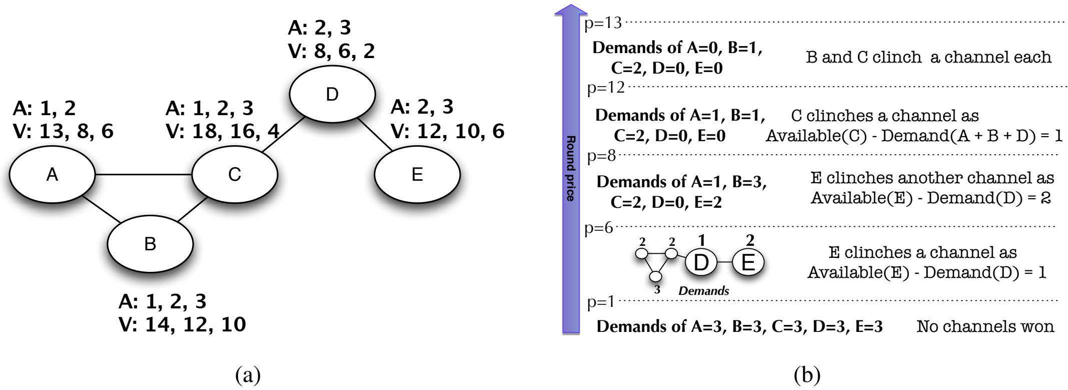


Figure 2: (a) Example conflict graph with 5 bidders A, B, C, D and E showing available channels (A) and marginal valuations (V). (b) Example illustrating the working of VERUM auction mechanism

It can be clearly seen from the above example that the result of the auction is efficient: the auction has allocated the channels to bidders who value them the most. The formal proof is provided in [6]. It can also be seen that the resultant pricing for channels won is equivalent to that of multi-unit Vickery auction. For example, bidder C wins its first channel at the second highest losing bid ($p = 12$) among its neighbors in the conflict graph and the second channel at the highest losing bid ($p = 13$). Similarly, bidder B wins a channel at the highest losing bid amongst its neighbors.

Although the mechanism computes winner determination and payments effectively, it does not allocate channels. This is because we consider all items (channels) as substitutes in the auction mechanism. We determine the actual channel allocation using a greedy heuristic. We select the channel that is available in the least number of the winning node's neighbors. This directly reduces the number of bidders for whom a channel becomes unavailable.

THEOREM 2.1. *VERUM is truthful.*

PROOF. It is well known that for an auction mechanism to be truthful, the price paid by the winning bidder should not depend on its own bid *and* the allocation strategy should be optimal. If a heuristic is used for allocation, then the resulting auction scheme may not be strategy proof. However, as shown in [7], for single parameter settings, use of a heuristic auction allocation scheme that satisfies monotonicity is sufficient for the scheme to be truthful. Given this and the fact that ours is a single parameter setting, to prove that VERUM is truthful, we need to show: (i) the pricing function does not depend on the bid of the winning bidder; and (ii) it is monotonic, i.e., if bidder i wins a channel at bid p then it will win the channel at any bid $p^* > p$.

It is indeed the case that pricing function in VERUM does not depend on the bid of the winning bidder. In any given round, the number of channels won by a bidder i is not dependant on i's demand but instead on the cumulative demand of i's conflicting neighbors. Even more crucially, the price that i needs to pay for the channels it clinches in a round t is the round price p_t, which does not have any relation with i's bid.

Now to the monotonicity. Assume bidder i won a channel at bid p, then at bid $p^* > p$, the cumulative demand of i's neighbours $D_{-i}(p) >= D_{-i}(p^*)$. The only way i could not win the channel at higher price p^* is if the aggregate demand of i's neighbors increases with the bid p^*. This is not possible since we have assumed that the marginal valuations are weakly decreasing, which results in monotonically non-increasing demands with each new round. Thus i will always win the channel at any bid $p^* > p$. $\square$

2.1 Channel Sharing Case

Here we show how to extend VERUM to support channel sharing among interfering bidders. Towards this end, we introduce the notion of *usable channel opportunities*. The usable channel opportunities represent the total number of opportunities potentially available to each bidder to use the available channels. In the exclusive use case, each bidder has no more than one opportunity to use an available channel among a set of conflicting bidders. However with channel sharing enabled, conflicting bidders may have more than one opportunity between them to use a channel, since the same channel may be used by conflicting bidders.

The usable channel opportunities of a bidder i, C_i^{opp}, is defined as:

$$C_i^{opp} = \sum_{k=1}^{C} \sum_{j \in N_i} F_j(k) \times X_i(k) \tag{1}$$

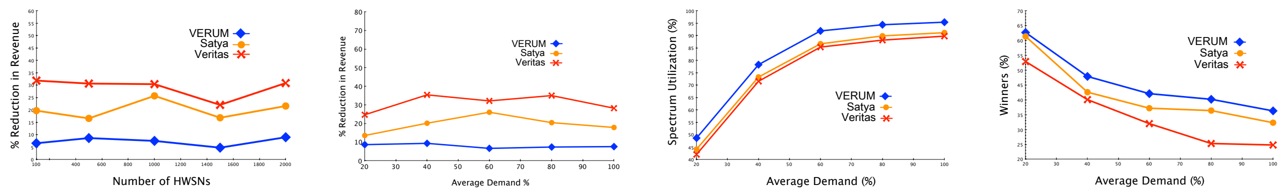
where $F_j(k) \in \{0, 1\}$ is the channel usability factor that indicates if channel k can be used by bidder j and $X_i(k)$ is the channel availability vector.

The mechanism to determine if a bidder can use a channel can be as simple as fixing the number of bidders per channel, or based on a more realistic function like we do as elaborated in [6].

With usable channel opportunities C_i^{opp} defined as above, extending VERUM to allow channel sharing reduces to changing the criteria for clinching channels. Specifically, a bidder i clinches m channels if its number of usable channel opportunities C_i^{opp} exceeds the aggregate demand of its neighbors by m. The price paid by the winning bidder for a channel k would now be a fraction of the exclusive price: $Price_i^{shared}(k) = Price_i(k) * b_i(k)$, where $Price_i(k)$ is the price paid if k was an exclusive channel and $b_i(k)$ is the fraction of channel k that bidder i utilizes.

3. EVALUATION

We evaluate VERUM in comparison with VERITAS [4] and SATYA [5], the two most relevant auction schemes from the literature, in the context of the framework for TVWS spectrum sharing among home



(a) (b) (c) (d)

Figure 3: Revenue, spectrum utilization and percentage of winners with VERUM in comparison with VERITAS and SATYA.

networks shown in Fig. 1. Recall that VERITAS does not support channel sharing, whereas SATYA supports channel sharing as well as heterogeneous channel availability. For SATYA, we use a bid price based bucketing function as was done by its authors in [5]. To benchmark the above three mechanisms with respect to the optimum, we formulate the problem as an ILP with Vickrey pricing as described in [6] and solve it using the GUROBI solver[1]. Our evaluations use real distributions of houses from a dense-urban neighborhood in London with 5456 houses/buildings in a 1 square kilometer area, and a realistic TVWS channel availability map for the UK.

We use three metrics in our evaluations: (1) *Revenue*: This is the sum of the market clearing prices paid by all k winning bidders in the auction $R = \sum_{i=1}^{k} S_i$ where S_i is the winning price paid by HWSN i; (2) *Spectrum Utilization*: The percentage of available channels at each HWSN that are allocated (to any HWSN), averaged across all HWSNs; (3) *Percentage of Winners*: The percentage of bidders who are allocated at least one channel at the end of the auction.

Figs. 3 (a) and (b) show the percentage reduction in revenue with respect to the optimal case (ILP solution) obtained with different auction schemes with varying number of active subscribed HWSNs and demand in a dense urban scenario. The non-monotonic trend seen for individual curves in those figures can be attributed to the fact that auction mechanisms and the optimal case have different objectives — while the optimal seeks a revenue maximizing solution, the three auction mechanisms assign channels to the bidders with the highest valuations. This is also a key reason behind the reduced amount of revenue generated by `VERUM`, SATYA and VERITAS to different degrees compared to the optimal (non-zero percentage reduction in revenue).

Comparing the three mechanisms in Figs. 3 (a) and (b), we see that VERITAS causes the most drop in revenue with respect to the optimal by as much as 35%. This is because it does not support channel sharing. SATYA relatively fares better primarily due to its support for channel sharing. Even SATYA too results in close to 30% reduction in revenue in some cases because the bucketing and ironing techniques it employs limit channel sharing opportunities and hence the revenue. Specifically, bidders in SATYA are not allowed to share the channel with a neighbor placed in a higher bucket. As `VERUM` does not impose such constraints to ensure truthfulness, it offers the best relative performance in all cases, mostly within around 10% of the optimal and around 20% revenue drop in the worst case.

We now look at the relative performance of `VERUM`, SATYA and VERITAS in terms of the other two metrics: spectrum utilization and percentage of winners. We can observe from Fig. 3 (c) and (d) that `VERUM` does comparatively better both in terms of spectrum utilization and percentage of winners. VERITAS performs worse in all cases as it lacks support for channel sharing. SATYA gains in comparison with VERITAS as it allows channel sharing. But the bucketing approach underlying SATYA makes it lose out on some channel sharing opportunities, explaining the performance gap between SATYA and `VERUM`. The impact of parameters such as reserve price, step size, interference ranges on `VERUM` along with the evaluation results for urban scenario are presented and discussed in [6].

[1] `http://www.gurobi.com/`

4. CONCLUSIONS

In this paper, we have considered the use of short-term auctions for coordinating dynamic sharing of spectrum white spaces among secondary users and presented an iterative multi-unit auction scheme called `VERUM`, which not only leads to simpler means to achieve truthfulness but also offers desirable properties like transparency and privacy. `VERUM` also supports channel sharing, heterogeneous spectrum availability and marginal valuations. We showed the benefits of `VERUM` with respect to the state-of-the-art auction schemes, VERITAS and SATYA, in the context of TVWS spectrum sharing among home networks. Our evaluations, using real distributions of homes in a dense urban neighborhood in London and realistic TVWS spectrum availability maps for the UK, show that `VERUM` outperforms the other two schemes in terms of revenue, spectrum utilization and percentage of winning bidders. These results demonstrate that `VERUM` offers an effective alternative for dynamic spectrum sharing with incentives for both subscribed users of the auction as well as for the auctioneer.

5. REFERENCES

[1] L. Ausubel. An Efficient Ascending-Bid Auction for Multiple Objects. *American Economic Review*, 94(5), Dec 2004.

[2] G. Iosifidis and I. Koutsopoulos. Challenges in Auction Theory Driven Spectrum Management. *IEEE Communications*, 49(8), Aug 2011.

[3] W. Vickrey. Counterspeculation, Auctions, and Competitive Sealed Tenders. *The Journal of Finance*, 16(1):8–37, 1961.

[4] X. Zhou, S. Gandhi, S. Suri, and H. Zheng. eBay in the Sky: Strategy-Proof Wireless Spectrum Auctions. In *Proc. ACM MobiCom*, 2008.

[5] I. Kash, R. Murty, and D. Parkes. Enabling Spectrum Sharing in Secondary Market Auctions. *IEEE Transactions on Mobile Computing*, 2013.

[6] S. Manickam, M. K. Marina, S. Pediaditaki, and M. Nekovee. Auctioning based Coordinated TV White Space Spectrum Sharing for Home Networks. http://arxiv.org/abs/1307.0962, Dec 2013.

[7] A. Archer and E. Tardos. Truthful Mechanisms for One-Parameter Agents. In *Proc. IEEE FOCS*, 2001.