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Fractional Calculus in Modelling of Measuring Transducers

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Introduction

Measuring transducer with a seismic mass is a mechanical oscillating system with one degree of freedom, consisting of seismic mass conducted on the relevant elastic suspensions [1–7]. The whole is placed in the center damping. Such a system can be described by the general differential equation (1)

$$\ddot{w}(t) + 2\zeta\omega_0\dot{w}(t) + \omega_0{}^2 w(t) = -\ddot{x}(t)\,, \qquad (1)$$

where ω_0 – natural angular frequency, ζ – damping, w(t) – displacement mass m towards base.

In this case

$$F(t) = ma(t)\,, \qquad (2)$$

where $a(t)$ - acceleration of the transducer housing.

Seismic mass transducers, depending on their execution (selection of parameters characterising their dynamic properties), can be used to measure displacement, speed or acceleration. In practical vibration measurements, transducers are applied to acceleration measurements. The parameters of speed and displacement are determined by applying elements that integrate accelerometer signals.

In our work a laboratory system for examining the characteristics of dynamic measuring transducer with a seismic mass as depicted in Fig. 1 was used. The system consisting of an accelerometer, conditioner and a measuring card was modelled. This station was built in the Department of Automatics and Electrical Engineering at the Faculty of Transport and Electrical Engineering of the Technical University of Radom. The voltage signal from the end of the examined measurement chain is an identified signal, whereas the comparative signal is the one from the model accelerometer being a response to the sinusoidal input of a generator having the frequency of 200 Hz.

The recent dynamic development of research into the use of fractional calculus for the analysis of dynamic systems encouraged the authors to make an attempt at using it for modelling of measuring transducers [3, 5–7].

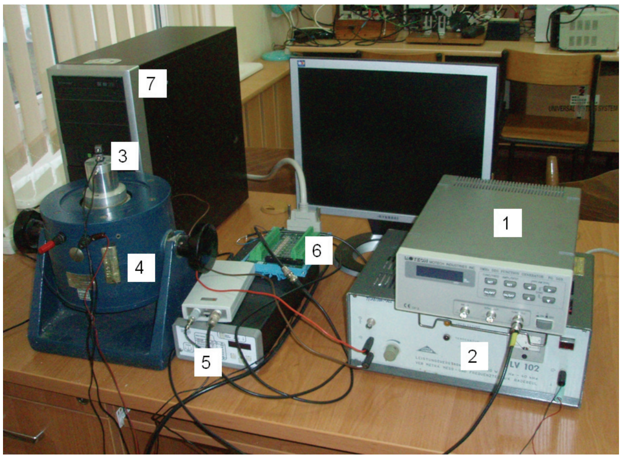

Fig. 1. View of the laboratory measurement system examining measuring transducers with a seismic mass: 1 – amplifier, 2 – signal generator, 3 – accelerometer, 4 – exciter, 5 – conditioner, 6 – measuring card, 7 computer / software (MATLAB/SIMULINK)

Identification algorithm of measuring transducer using fractional calculus

Many physical phenomena, such as liquid permeation through porous substances, load transfer through an actual insulator, or heat transfer through a heat barrier are described more accurately by means of derivative-integral equations. The dynamics of physical processes such as acceleration, displacement, liquid flow, electric current power or magnetic field flux are modelled by means of differential equations. The courses of these processes are actually continuous variables, $m+1$ - fold differentiable, where m is determined subject to the order of the examined fractional derivative. Mass cannot be relocated from one place to another in an infinitely short time. Neither is it possible to change temperature or pressure in an actual object infinitely fast.

A classical notation of the measuring transducer dynamics is based on differential equations (1), which constitute their mathematical model in the time domain.

In the case of fractional calculus, operators of the function differentiation and integration are combined into one operator (3) D^n.
For differentiation, the n order assumes positive values of $n = 1, 2, 3, ...$ and for integration – negative values $-n = -1, -2, -3, ...$. The neutral operator for the order of $n = 0$ is also defined [3, 7]:

$$D\,f(t) = \begin{cases} \frac{df(t)}{dt}, \frac{d\,f(t)}{dt}, \frac{d\,f(t)}{dt}, ..., \frac{d\,f(t)}{dt}, & \text{where } n > 0, \\ f(t), & \text{where } n = 0, \\ \int\left[...\int\left[\int f(\tau)\,d\tau \right]...d\tau \right]d\tau, & \text{where } n < 0. \end{cases} \quad (3)$$

In fractional calculus the arbitrary order derivative is treated as an interpolation of the sequence of discrete order operators (3) by continuous order operators. A notation introduced by H. D. Davis is applied here where the fractional order derivative of the $f(t)$ function can be presented in the following way

$$_{t_0}D_t^{\nu} f(t)\,, \quad (4)$$

where t_0 and t - terminals of fractional differentiation or integration; ν- order of the derivative of the integral.

It must be emphasized that in formula (4) the $[t_0, t]$ differentiation range is defined, identical with that which appears in classical definite integrals. Hence, a derivative and an integral of the fractional order are defined in the $[t_0, t]$ range, which in the case of the derivative narrows down to the $[t, t]$ point (range) for the integer order n. To distinguish between integer and fractional orders, fractional orders are labelled by Greek letters ν and μ. For integer orders the commonly applied letters m and n are reserved.

The order of a derivative or an integral satisfies the condition

$$\nu \in G_+, n \in Z_+\,, \quad (5)$$

where G_+ – set of real, fractional, positive numbers; Z_+ – set of non-negative integers.

In order to emphasize the difference between a derivative and an integral, in H.D. Davis's notation it is written down that $_{t_0}D^{-\nu} f(t)$, where the integration order fulfills condition (5), and the minus sign next to the D operator informs that it is the integration operation.

For measuring transducer equation (1) can be noted as a difference equation

$$a_2 w_k + a_1 w_{k-1} + a_0 w_{k-2} = b_2 x_k + b_1 x_{k-1} + a_0 x_{k-2} \quad (6)$$

or a matrix equation (7)

$$\begin{bmatrix} a_2 & a_1 & a_0 \end{bmatrix} \begin{bmatrix} w_k \\ w_{k-1} \\ w_{k-2} \end{bmatrix} = \begin{bmatrix} b_2 & b_1 & b_0 \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-1} \\ x_{k-2} \end{bmatrix}. \quad (7)$$

The expression of (6) becomes (8)

$$A_2 \Delta_k{}^{(2)} w_k + A_1 \Delta_{k-1}{}^{(1)} + A_0 w_{k-2} =$$
$$= B_2 \Delta_k{}^{(2)} w_k + B_1 \Delta_k{}^{(1)} x_{k-1} + B_0 w_{k-2}, \quad (8)$$

where $\Delta_k^{(n)}$ is the reverse difference of the discrete function [3, 7], defined as

$$\Delta_k^{(n)} f_{(k)} = \sum_{j=0}^{k} a_j^{(n)} f_{(k-j)}\,. \quad (9)$$

Once (9) is taken into consideration, (8) noted as a matrix equation becomes (10)

$$\begin{bmatrix} a_2 & -a_1 - 2a_0 & a_2 + a_1 + a_0 \end{bmatrix} \begin{bmatrix} \Delta_k^{(2)} w_k \\ \Delta_k^{(1)} w_k \\ \Delta_k^{(0)} w_k \end{bmatrix} =$$
$$= \begin{bmatrix} b_0 & -b_1 - 2b_0 & b_2 + b_1 + b_0 \end{bmatrix} \begin{bmatrix} \Delta_k^{(2)} x_k \\ \Delta_k^{(1)} x_k \\ \Delta_k^{(0)} x_k \end{bmatrix}. \quad (10)$$

In our examinations we compared responses of the measurement transducer to the sinusoidal input signal with noise. It is described by three models [5, 6]:

- A continuous model (transfer function model) described by the transfer function

$$G(s) = \frac{-s^2}{s^2 + 70s + 350}\,. \quad (11)$$

- A discrete model (discrete transfer function model) obtained from a continuous model (11) described by discrete transfer function

$$G(z) = \frac{-z^2 + 2z - 1}{z^2 - 1.966z + 0.9656}\,. \quad (12)$$

- A discrete model (fractional model) determined by the derivative-integral notation takes the following form

$$G(z) = \frac{-z^2 + 0.0345z - 8.599 \cdot 10^{-5}}{z^2 - 4.324 \cdot 10^{-5} z}\,. \quad (13)$$

Fig. 2 depicts a block diagram of the measurement system executed in the MATLAB programme. An irreplaceable work tool here was SIMULINK, an interactive package built on the basis of MATLAB. It offers a possibility of an analysis and synthesis of continuous and discrete dynamic systems. SIMULINK is a graphic environment where the dynamic system simulation is accomplished on the basis of a block diagram built with the use of library blocks.

Fig. 3 depicts responses of all models of the measurement transducer to the sinusoidal input signal with Gaussian noise.

Following the comparison of model responses to the input signal (Fig. 3) it can be concluded that: the fractional model of the measurement transducer (13) from the very beginning of the simulation reproduces the input signal

amplitude correctly. The models determined in the classical way - (11) and (12) – reproduce the input signal amplitude correctly after leaving the transient state.
Fig. 4 compares responses of the presented models to the impulse signal.

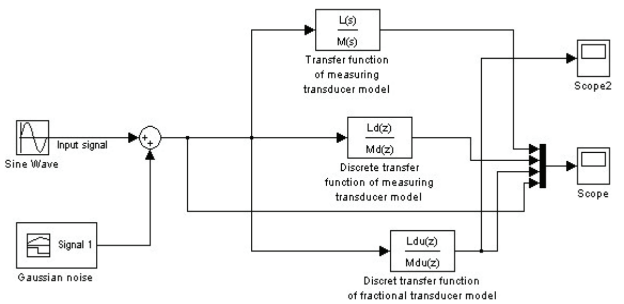

Fig. 2. Block diagram of the measurement system for the measurement transducer

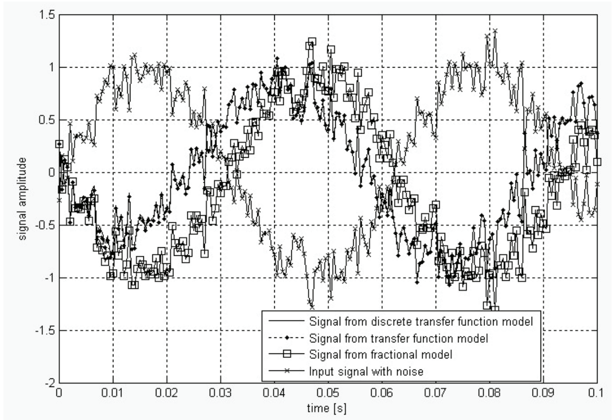

Fig. 3. Comparison of responses of the measuring transducer models (11), (12) and (13)

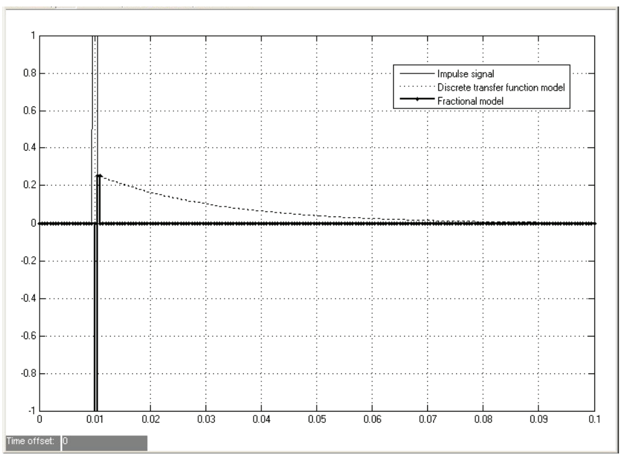

Fig. 4. Comparison of responses of the measurement transducer models (12) and (13) to the impulse signal [6]

The classical model response reaches the steady state after 0.08 s from the moment the signal occurs. In the case of the derivative-integration model this time is reduced to 0.001 s.

Fig. 5 depicts a comparison of the presented model responses to the step function. The classical model response passes into the steady state after 0.06 s from the moment the signal occurs. In the case of the derivative-integration model the steady state occurs after 0.005 s and assumes the value approximating that of the input function amplitude with an opposite sign.

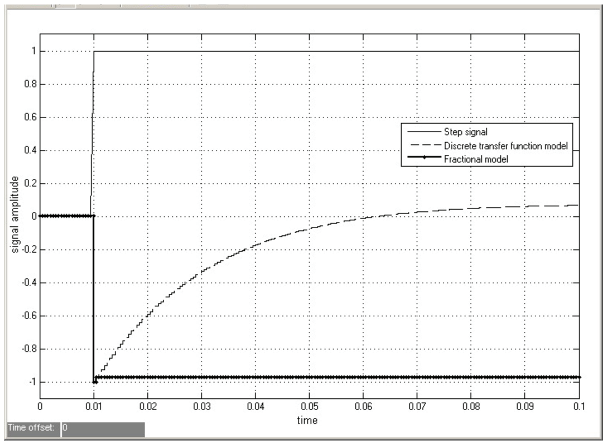

Fig. 5. Comparison of the measurement transducer model (12) and (13) responses to the step function [6]

Fig. 6 depicts a comparison of Bode frequency plots for the discrete model (12) and discrete model of the measurement transducer determined by the derivative-integral notation (13).

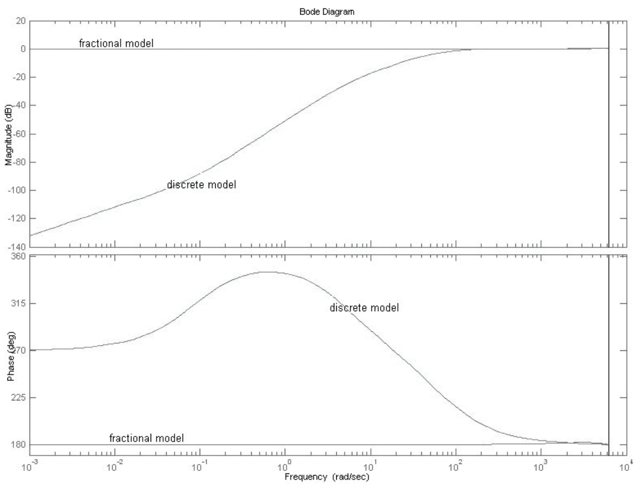

Fig. 6. Comparison of Bode plots for models (12) and (13) of the measuring transducer

The Bode plots (Fig. 6) indicate that for the measurement transducer model determined by the derivative-integral method when compared with the model determined in the classical way, the range of the input signal processing is extended by low frequencies. For the presented characteristics, the amplification of the derivative-integration model amplitude equal 0 dB is reached for the frequencies from 0.001 rad/s, and for the "classical" model - from 100 Hz, at a stable phase shift of 180°.

Conclusions

Fractional calculus open unimaginable possibilities in the field of dynamic system identification and creation of new, earlier inaccessible, algorithms of feedback system control. It must be emphasized here that the commonly known derivatives are special cases of the calculus presented in this paper.

While applying fractional model for the creation of the measuring transducer models one obtains models of ideal, in the case of amplitude reproduction, input signal processing. In the case of the measurement system model, the phase shift obtained was reduced and signal amplitude lower than the model signal.

It seems necessary to continue the examinations in order to check how the presented models determined by the fractional method reflect the actual models and whether they reflect the dynamics of the input signal processing more accurately than the models described by the “classical” differential equations.

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The paper is inspired by developments of fractional calculus in areas of control theory and electrical measurements. It outlines an example of modelling the measuring transducers with the use of this calculus. The algorithm determining these models is presented in the form of a fractional calculus notation and then the models are compared to the ones described by means of classical differential equations. Tests are executed in the Matlab/Simulink programme. Ill. 6, bibl. 7 (in English; abstracts in English and Lithuanian).

M. Luft, R. Cioc, D. Pietruszczak. Dalinių skaičiavimų modeliavimas matavimo keitikliuose // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2011. – Nr. 4(110). – P. 97–100.

Analizuojama dalinių skaičiavimų įtaka tokiose srityse kaip valdymo teorija, elektriniai matavimai. Apžvelgti matavimo keitikliuose tokių skaičiavimų modeliavimo rezultatai, kuriems nusakyti pasiūlytas algoritmas bei modeliai. Modeliai yra palyginti tarpusavyje ir apibrėžti diferencialinėmis lygtimis. Atliktas patikrinimas programų paketu Matlab/Simulink. Il. 6, bibl. 7 (anglų kalba; santraukos anglų ir lietuvių k.).