Hindawi
Complexity
Volume 2017, Article ID 6875874, 13 pages
https://doi.org/10.1155/2017/6875874

Research Article

Existence and Globally Asymptotic Stability of Equilibrium Solution for Fractional-Order Hybrid BAM Neural Networks with Distributed Delays and Impulses

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Received 7 May 2017; Revised 10 July 2017; Accepted 20 July 2017; Published 13 September 2017

Academic Editor: Amr Elsonbaty

This paper investigates the existence and globally asymptotic stability of equilibrium solution for Riemann-Liouville fractional-order hybrid BAM neural networks with distributed delays and impulses. The factors of such network systems including the distributed delays, impulsive effects, and two different fractional-order derivatives between the U-layer and V-layer are taken into account synchronously. Based on the contraction mapping principle, the sufficient conditions are derived to ensure the existence and uniqueness of the equilibrium solution for such network systems. By constructing a novel Lyapunov functional composed of fractional integral and definite integral terms, the globally asymptotic stability criteria of the equilibrium solution are obtained, which are dependent on the order of fractional derivative and network parameters. The advantage of our constructed method is that one may directly calculate integer-order derivative of the Lyapunov functional. A numerical example is also presented to show the validity and feasibility of the theoretical results.

1. Introduction

Since fractional derivatives are nonlocal and have weakly singular kernels, the subject of fractional calculus has been attracting attention and interest in various fields of diffusion [1], physics [2], market dynamics [3], engineering [4], control system [5], biological system [6], financial system [7], epidemic model [8], and so on. At the same time, fractional-order differential equations have been proved to be an excellent tool in the modelling of many phenomena [9–11]. Recently, some important advances on dynamical behaviors such as chaos phenomena, Hopf bifurcation, synchronization control, and stabilization problems for fractional-order systems or fractional-order practical models have been reported in [12–16]. These proposed results show the superiority and importance of fractional calculus and effectively motivate the development of new applied fields.

Note that various classes of neural networks such as Hopfield neural networks [17, 18], recurrent neural networks [19, 20], cellular neural networks [21], Cohen-Grossberg neural networks [22], and bidirectional associative memory (BAM) neural networks [23–25] have been widely used in solving some signal processing, optimization, and image processing problems. In the last few years, some researchers have introduced fractional operators to neural networks to form fractional-order neural models [26–30], which could better describe the dynamical behaviors of the neurons. As an important dynamic behavior, stability is one of the most concerned problems for any dynamic system. For example, Song and Cao [26] have established some sufficient conditions to ensure the existence and uniqueness of the nontrivial solution by using the contraction mapping principle, Krasnoselskii fixed point theorem, and the inequality technique, in which uniform stability conditions of fractional-order neural networks are also derived in fixed time-intervals. Note that time-delay (see [23–25, 31–37]) is a common phenomenon and is inevitable in practice, which often exists in almost every neural network and has an important effect on the stability and performance of system.

There are also several recent results discussing the topics including stability analysis for fractional-order dynamical systems in [38, 39]. For instance, the stability problems of main concern for control theory in finite-dimensional linear fractional-order systems have been considered [38], in which both internal and external stabilities for fractional-order differential systems in state-space form have been studied. For fractional-order differential systems in polynomial representation, the external stability has been thoroughly discussed. In [39], Matouk has investigated the stability conditions of a class of fractional-order hyperchaotic systems; then the stability conditions have been applied to a novel fractional-order hyperchaotic system. Based on the Routh-Hurwitz theorem, the conditions for controlling hyperchaos via feedback control approach have also been derived. At the same time, the various kinds of stability of delayed fractional-order neural networks have been extensively investigated. For example, Mittag-Leffler stability of fractional-order delayed neural networks has been investigated by applying fractional Lyapunov direct method [28, 30, 32]. The finite-time stability of Caputo fractional-order delayed neural networks has been studied by applying Gronwall's inequality approach and inequality scaling techniques [33, 34]. The delay-independent stability criteria of Riemann-Liouville fractional-order neutral-type delayed neural networks have been proposed based on classical Lyapunov functional method [35]. The uniform stability and global stability of fractional neural networks with delay are considered based on the fractional calculus theory and analytical techniques [36]. Global $o(t^{-\alpha})$ stability and global asymptotical periodicity for a class of fractional-order complex-valued neural networks with time-varying delays are discussed by using the fractional Lyapunov method and a Leibniz rule for fractional differentiation [37].

Although most dynamical systems are analyzed in either the continuous-time or discrete-time domain, many real systems in physics, engineering, chemistry, biology, and information science may experience abrupt changes as certain instants during the continuous dynamical processes. This kind of impulsive behaviors can be modelled by impulsive systems [23, 25, 29, 32, 40–42]. On the other hand, bidirectional associative memory (BAM) neural networks attract many studies due to its extensive applications in many fields [22–25, 43–46]. In [43], Kosko first introduced hybrid BAM neural network models. The remarkable feature of the proposed BAM neural networks lies in the close relation of the neurons between the U-layer and V-layer. That is, the neurons in one layer are fully interconnected to the one in the other layer, but there are not any interconnections among neurons in the same layer. It is worth mentioning that many contributions have been made concerning the dynamics of fractional-order BAM delayed neural networks (see [44–46]) including finite-time stability [44] and Mittag-Leffler synchronization [45]. In [46], globally asymptotic stability problem of impulsive fractional-order neural networks with discrete delays has been studied, yet the existence of the equilibrium solution for fractional-order BAM neural networks has not been taken into account. On the other hand, it should be pointed out that the finite-time stability and asymptotic stability in the sense of Lyapunov are different concepts, because finite-time stability does not contain Lyapunov asymptotic stability and vice versa [34, 47]. Although the signal transmission is sometimes instantaneous modelling with discrete delays, it may be sometimes a distribution propagation delay over a period of time so that distributed delays (see [20, 23, 25]) should not be ignored in the model. Compared to the advances of integer-order neural networks with or without time delays, the research on the stability of fractional-order BAM delayed neural networks is still at the stage of exploiting and developing [44–46]. To the best of our knowledge, there are few papers on investigating the global stability of the fractional-order hybrid BAM neural networks with both impulse and distributed delay in the current literature.

Motivated by the above discussions, this paper investigates the existence and globally asymptotic stability of equilibrium solution for impulsive Riemann-Liouville fractional-order hybrid BAM neural networks with distributed delays. The factors of such network systems including the distributed delays, impulses, and two different fractional-order derivatives between the U-layer and V- layer are taken into account synchronously. Based on the contraction mapping principle, the sufficient conditions are presented for the existence and uniqueness of the equilibrium solution for such network systems. By constructing a suitable Lyapunov functional associated with fractional integral terms, the globally asymptotic stability criteria of the equilibrium point are derived. The advantage of constructing the Lyapunov functional is that one can directly calculate its first-order derivative to check global stability. A numerical example is also given to show the validity and feasibility of the theoretical results.

This paper is organized as follows. In Section 2, we recall some definitions concerning fractional calculus and describe impulsive Riemann-Liouville fractional-order BAM neural networks with distributed delays. In Section 3, the existence and uniqueness of the equilibrium solution for such network systems are discussed based on the contraction mapping principle. In Section 4, the globally asymptotic stability criteria of the equilibrium solution are derived. An illustrative example is given to show the effectiveness and applicability of the proposed results in Section 5. Finally, some concluding remarks are drawn in Section 6.

2. Preliminaries and Model Description

In this section, we recall the definitions of fractional calculus and several basic lemmas. Moreover, we describe a class of impulsive fractional-order hybrid BAM neural network models with distributed delays.

Definition 1 (see [10]). The Riemann-Liouville fractional integral of order q for a function f is defined as

$$ {}_{t_0}D_t^{-q} f(t) = \frac{1}{\Gamma(q)} \int_{t_0}^{t} (t-s)^{q-1} f(s)\, ds, \quad (1) $$

where $q > 0,\ t \geqslant t_0$. The Gamma function $\Gamma(q)$ is defined by the integral

$$ \Gamma(z) = \int_0^{+\infty} s^{z-1} e^{-s} ds, \quad (\mathfrak{Re}(z) > 0). \quad (2) $$

Currently, there exist several definitions about the fractional derivative of order $q > 0$ including Grünwald-Letnikov (GL) definition, Riemann-Liouville (RL) definition, and Caputo definition [9–11]. In this paper, our consideration is the fractional-order neural networks with Riemann-Liouville derivative, whose definition and properties are given below.

Definition 2 (see [10]). The Riemann-Liouville fractional derivative of order q for a function f is defined as

$$ \begin{aligned} {}_{t_0}^{\mathrm{RL}}D_t^{q} f(t) &= \frac{d^m}{dt^m}\left[{}_{t_0}D_t^{-(m-q)} f(t)\right] \\ &= \frac{1}{\Gamma(m-q)} \frac{d^m}{dt^m} \int_{t_0}^{t} (t-s)^{m-q-1} f(s)\, ds, \end{aligned} \quad (3) $$

where $0 \leqslant m-1 < q < m,\ m \in \mathbb{Z}^+$.

In particular, for $\alpha \in (0,1)$ case, the Riemann-Liouville fractional derivative of order α for a constant x^* is

$$ {}_{0}^{\mathrm{RL}}D_t^{\alpha} x^* = \frac{t^{-\alpha}}{\Gamma(1-\alpha)} x^*. \quad (4) $$

Lemma 3 (see [10]). *If $f(t),\ g(t) \in \mathbf{C}^m[t_0, b]$, and $m-1 \leqslant p < m \in \mathbb{Z}^+$, then*

(1) ${}_{t_0}^{\mathrm{RL}}D_t^{q}(L_1 f(t) + L_2 g(t)) = L_1\, {}_{t_0}^{\mathrm{RL}}D_t^{q} f(t) + L_2\, {}_{t_0}^{\mathrm{RL}}D_t^{q} g(t)$, $L_1, L_2 \in \mathbb{R},\ q > 0$;

(2) ${}_{t_0}D_t^{-p}({}_{t_0}D_t^{-q} f(t)) = {}_{t_0}D_t^{-(p+q)} f(t),\ p, q > 0$;

(3) ${}_{t_0}^{\mathrm{RL}}D_t^{p}({}_{t_0}D_t^{-q} f(t)) = {}_{t_0}^{\mathrm{RL}}D_t^{p-q} f(t),\ p > q > 0$;

(4) ${}_{t_0}^{\mathrm{RL}}D_t^{p}({}_{t_0}D_t^{-q} f(t)) = {}_{t_0}D_t^{-(q-p)} f(t),\ q > p > 0$.

The following lemmas will be used in the proof of our main results.

Lemma 4 (contraction mapping principle [48]). *Suppose that (X, ρ) is a complete metric space, $\Phi : X \to X$, and there is some real number $0 < k < 1$ such that*

$$ \rho(\Phi(x), \Phi(y)) \leqslant k\rho(x, y), \quad \forall x, y \in X; \quad (5) $$

then there is a unique point $x_0 \in X$ such that $\Phi(x_0) = x_0$.

Lemma 5 (fractional Barbalat lemma [42]). *If $\int_{t_0}^{t} w(s)ds$ has a finite limit as $t \to +\infty$, and ${}_{t_0}^{\mathrm{RL}}D_t^{\alpha} w(t)$ is bounded, then $w(t) \to 0$ as $t \to +\infty$, where $0 < \alpha < 1$.*

In this paper, we consider the Riemann-Liouville fractional-order hybrid BAM neural network models with distributed delay and impulsive effects described by the following states equations:

$$ \begin{aligned} {}_{0}^{\mathrm{RL}}D_t^{\alpha} x_i(t) &= -a_i x_i(t) + \sum_{j=1}^{m} b_{ij} f_j(y_j(t)) \\ &\quad + \sum_{j=1}^{m} \int_0^{\tau} r_{ij}(s) f_j(y_j(t-s))\, ds + I_i, \\ &\qquad t > 0,\ t \neq t_k, \\ \Delta x_i(t_k) &= \gamma_k^{(1)}(x_i(t_k)), \\ &\qquad i = 1, 2, \ldots, n;\ k = 1, 2, \ldots, \\ {}_{0}^{\mathrm{RL}}D_t^{\beta} y_j(t) &= -c_j y_j(t) + \sum_{i=1}^{n} d_{ji} g_i(x_i(t)) \\ &\quad + \sum_{i=1}^{n} \int_0^{\tau} p_{ji}(s) g_i(x_i(t-s))\, ds + J_j, \\ &\qquad t > 0,\ t \neq t_k, \\ \Delta y_j(t_k) &= \gamma_k^{(2)}(y_j(t_k)), \\ &\qquad j = 1, 2, \ldots, m;\ k = 1, 2, \ldots, \end{aligned} \quad (6) $$

where $U = \{x_1, x_2, \ldots, x_n\}$ and $V = \{y_1, y_2, \ldots, y_m\}$ are two layers in the BAM model (6); $x_i(t)$ and $y_j(t)$ are state variables of ith neuron in the U-layer and jth neuron in the V-layer, respectively; ${}_{0}^{\mathrm{RL}}D_t^{\alpha} x_i(\cdot)$ and ${}_{0}^{\mathrm{RL}}D_t^{\beta} y_j(\cdot)$ denote an α and a β order Riemann-Liouville fractional-order derivative of $x_i(\cdot)$ and $y_j(\cdot)$, respectively; the constants α and β satisfy $0 < \alpha < 1,\ 0 < \beta < 1$. $a_i > 0$ and $c_j > 0$ denote decay coefficients of signals from neurons x_i to y_j, respectively; f_i and g_j are the neuron activation functions; $b_{ij}, d_{ji}, r_{ij}(t)$ and $p_{ji}(t)$ represent the weight coefficients of the neurons; I_i and J_j denote the external inputs of U-layer and V-layer, respectively; $\tau > 0$ denotes the maximum possible transmission delay from neuron to another. Moreover, impulsive moments $\{t_k \mid k = 1, 2, \ldots\}$ satisfy $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots,\ t_k \to +\infty$ as $k \to +\infty$, and

$$ \begin{aligned} \Delta x_i(t_k) &= x_i(t_k^+) - x_i(t_k^-), \\ x_i(t_k^+) &= \lim_{\varepsilon \to 0^+} x_i(t_k + \varepsilon),\ x_i(t_k^-) = x_i(t_k), \\ \Delta y_j(t_k) &= y_j(t_k^+) - y_j(t_k^-), \\ y_j(t_k^+) &= \lim_{\varepsilon \to 0^+} y_j(t_k + \varepsilon),\ y_j(t_k^-) = y_j(t_k), \end{aligned} \quad (7) $$

where $x_i(t_k^+)$ and $x_i(t_k^-)$ represent the right and left limits of $x_i(t)$ at $t = t_k$, respectively; $x_i(t_k^-) = x_i(t_k)$ and $y_j(t_k^-) = y_j(t_k)$ imply that $x_i(t)$ and $y_j(t)$ are both left continuous at $t = t_k$. The initial conditions associated with Riemann-Liouville fractional-order network system (6) can be expressed as (see [9–11])

$$
\begin{aligned}
&{}_0D_t^{-(1-\alpha)}x_i(t)=\varphi_i(t),\\
&{}_0D_t^{-(1-\alpha)}y_j(t)=\psi_j(t),\\
&\qquad i=1,2,\ldots,n;\ j=1,2,\ldots,m,\ t\in[-\tau,0].
\end{aligned}\tag{8}
$$

Throughout this paper, we assume that the neuron activation functions f_j, g_i and impulsive operators $\gamma_k^{(1)}(x_i(t_k))$, $\gamma_k^{(2)}(y_j(t_k))$ satisfy the following conditions:

(**H1**) For $i=1,2,\ldots,n$; $j=1,2,\ldots,m$, the functions $r_{ij}(\cdot)$ and $p_{ji}(\cdot)$ are continuous on $[0,\tau]$. Thus, there exist positive constants $R_{ij}, P_{ji}\in\mathbb{R}^+$ such that

$$
\begin{aligned}
&\left|r_{ij}(s)\right|\leqslant R_{ij},\\
&\left|p_{ji}(s)\right|\leqslant P_{ji},\\
&\qquad \forall s\in[0,\tau].
\end{aligned}\tag{9}
$$

(**H2**) The neuron activation functions $f_j(\cdot), g_i(\cdot)$ ($i=1,2,\ldots,n$; $j=1,2,\ldots,m$) are Lipschitz continuous. That is, there exist positive constants $F_j, G_j\in\mathbb{R}^+$ such that

$$
\begin{aligned}
&\left|f_j(x)-f_j(y)\right|\leqslant F_j\left|x-y\right|,\\
&\left|g_i(x)-g_i(y)\right|\leqslant G_i\left|x-y\right|,\\
&\qquad \forall x,y\in\mathbb{R}.
\end{aligned}\tag{10}
$$

(**H3**) The impulsive operators $\gamma_k^{(1)}(x_i(t_k))$ and $\gamma_k^{(2)}(y_j(t_k))$ satisfy

$$
\begin{aligned}
&\gamma_k^{(1)}(x_i(t_k))=-\lambda_{ik}^{(1)}(x_i(t_k)-x_i^*),\\
&\qquad i=1,2,\ldots,n;\ k=1,2,\ldots,\\
&\gamma_k^{(2)}\left(y_j(t_k)\right)=-\lambda_{jk}^{(2)}\left(y_j(t_k)-y_j^*\right),\\
&\qquad j=1,2,\ldots,m;\ k=1,2,\ldots,
\end{aligned}\tag{11}
$$

where $\lambda_{ik}^{(1)}\in(0,2)$ ($i=1,2,\ldots,n$; $k=1,2,\ldots$), and $\lambda_{jk}^{(2)}\in(0,2)$ ($j=1,2,\ldots,m$; $k=1,2,\ldots$).

Remark 6. The purpose of this paper is to investigate the existence and globally asymptotic stability conditions of the equilibrium solution for fractional-order BAM network model (6). In discussing the stability of neural networks, the neuron activation functions are usually assumed to be bounded, monotonic [23], and differential [36, 37]. In system (6), the neuron activation functions are not necessarily bounded, monotonic, and differential. Therefore, the globally asymptotic stability criteria are more general and less conservative in this paper.

3. Existence of Equilibrium Solution

In this section, the sufficient conditions for the existence and uniqueness of the equilibrium solution of system (6) are derived based on the contraction mapping principle [48].

Similar to integer-order differential systems, we first define the equilibrium solution of fractional-order network systems. It should be pointed out that Riemann-Liouville fractional-order derivative of a nonzero constant is not equal to zero, which leads to the remarkable difference of the equilibrium solution between integer-order systems and Riemann-Liouville fractional-order systems.

Definition 7. A constant vector $(x^{*T}, y^{*T})^T=(x_1^*,x_2^*,\ldots,x_n^*,y_1^*,y_2^*,\ldots,y_m^*)^T\in\mathbb{R}^{n+m}$ is an equilibrium solution of system (6) if and only if $x^*=(x_1^*,x_2^*,\ldots,x_n^*)^T$ and $y^*=(y_1^*,y_2^*,\ldots,y_m^*)^T$ satisfy the following equations:

$$
\begin{aligned}
&{}_0^{\mathrm{RL}}D_t^{\alpha}\{x_i^*\}=-a_ix_i^*+\sum_{j=1}^{m}b_{ij}f_j\left(y_j^*\right)\\
&\qquad+\sum_{j=1}^{m}\int_0^{\tau}r_{ij}(s)\,f_j\left(y_j^*\right)ds+I_i,\\
&\qquad\qquad i=1,2,\ldots,n,\\
&{}_0^{\mathrm{RL}}D_t^{\beta}\left\{y_j^*\right\}=-c_jy_j^*+\sum_{i=1}^{n}d_{ji}g_i\left(x_i^*\right)\\
&\qquad+\sum_{i=1}^{n}\int_0^{\tau}p_{ji}(s)\,g_i\left(x_i^*\right)ds+J_j,\\
&\qquad\qquad j=1,2,\ldots,m,
\end{aligned}\tag{12}
$$

and the impulsive jumps $\gamma_k^{(1)}(x_i(t_k))$ and $\gamma_k^{(2)}(y_j(t_k))$ are assumed to satisfy

$$
\begin{aligned}
&\gamma_k^{(1)}\left(x_i^*\right)=0,\\
&\gamma_k^{(2)}\left(y_j^*\right)=0,\\
&\qquad i=1,2,\ldots,n;\ j=1,2,\ldots,m;\ k=1,2,\ldots.
\end{aligned}\tag{13}
$$

In what follows, we use the following vector norm of $\mathbb{R}^{n+m}$:

$$
\|u\|=\sum_{i=1}^{n+m}\left|u_i\right|,\quad u=\left(u_1,u_2,\ldots,u_{n+m}\right)^T\in\mathbb{R}^{n+m}.\tag{14}
$$

Theorem 8. *Suppose that conditions (**H1**)–(**H3**) hold; then there exists a unique equilibrium solution for system (6), if the following inequalities simultaneously hold for a small enough constant $\varepsilon>0$*

$$
\begin{aligned}
\omega_1=\max_{1\leqslant i\leqslant n}&\left\{\frac{\varepsilon}{\Gamma(1-\alpha)}\cdot\frac{1}{a_i}+\frac{G_i}{a_i}\sum_{j=1}^{m}\left[\left|d_{ji}\right|+\tau P_{ji}\right]\right\}\\
&<1,
\end{aligned}
$$

$$\omega_2 = \max_{1\leqslant j\leqslant m}\left\{\frac{\varepsilon}{\Gamma(1-\beta)}\cdot\frac{1}{c_j}+\frac{F_j}{c_j}\sum_{i=1}^{n}\left[\left|b_{ij}\right|+\tau R_{ij}\right]\right\} < 1. \tag{15}$$

Proof. According to Definition 2, for $\alpha, \beta \in (0,1)$, the Riemann-Liouville fractional-order derivatives of the constants u_i^* and v_j^* can be written as the following forms:

$$\begin{aligned}
{}^{\mathrm{RL}}_{0}D_t^{\alpha}u_i^* &= \frac{t^{-\alpha}}{\Gamma(1-\alpha)}u_i^*,\\
{}^{\mathrm{RL}}_{0}D_t^{\beta}v_j^* &= \frac{t^{-\beta}}{\Gamma(1-\beta)}v_j^*,\\
&i=1,2,\ldots,n;\ j=1,2,\ldots,m.
\end{aligned} \tag{16}$$

Define a mapping $\Phi:\mathbb{R}^{n+m}\to\mathbb{R}^{n+m}$, where $\mathbf{u}=(u_1,\ldots,u_n,v_1,\ldots,v_m)^T\in\mathbb{R}^{n+m}$ and

$$\Phi(\mathbf{u}) = \begin{bmatrix}
\sum_{j=1}^{m}b_{1j}f_j\left(\frac{v_j}{c_j}\right)+\sum_{j=1}^{m}\int_0^{\tau}r_{1j}(s)f_j\left(\frac{v_j}{c_j}\right)ds+I_1-\frac{t^{-\alpha}}{\Gamma(1-\alpha)}\frac{u_1}{a_1}\\
\vdots\\
\sum_{j=1}^{m}b_{nj}f_j\left(\frac{v_j}{c_j}\right)+\sum_{j=1}^{m}\int_0^{\tau}r_{nj}(s)f_j\left(\frac{v_j}{c_j}\right)ds+I_n-\frac{t^{-\alpha}}{\Gamma(1-\alpha)}\frac{u_n}{a_n}\\
\sum_{i=1}^{n}d_{1i}g_i\left(\frac{u_i}{a_i}\right)+\sum_{i=1}^{n}\int_0^{\tau}p_{1i}(s)g_i\left(\frac{u_i}{a_i}\right)ds+J_1-\frac{t^{-\beta}}{\Gamma(1-\beta)}\frac{v_1}{c_1}\\
\vdots\\
\sum_{i=1}^{n}d_{mi}g_i\left(\frac{u_i}{a_i}\right)+\sum_{i=1}^{n}\int_0^{\tau}p_{mi}(s)g_i\left(\frac{u_i}{a_i}\right)ds+J_m-\frac{t^{-\beta}}{\Gamma(1-\beta)}\frac{v_{\mathrm{m}}}{c_m}
\end{bmatrix}. \tag{17}$$

Consider $\forall\overline{\mathbf{u}}=(\overline{u}_1,\ldots,\overline{u}_n,\overline{v}_1,\ldots,\overline{v}_m)^T\in\mathbb{R}^{n+m}$; then it follows from (14) that

$$\begin{aligned}
\|\Phi(\mathbf{u})-\Phi(\overline{\mathbf{u}})\| \leqslant{}& \sum_{i=1}^{n}\left|\sum_{j=1}^{m}\left\{b_{ij}\left[f_j\left(\frac{v_j}{c_j}\right)-f_j\left(\frac{\overline{v}_j}{c_j}\right)\right]+\int_0^{\tau}r_{ij}(s)\left[f_j\left(\frac{v_j}{c_j}\right)-f_j\left(\frac{\overline{v}_j}{c_j}\right)\right]ds\right\}\right|\\
&+\sum_{j=1}^{m}\left|\sum_{i=1}^{n}\left\{d_{ji}\left[g_i\left(\frac{u_i}{a_i}\right)-g_i\left(\frac{\overline{u}_i}{a_i}\right)\right]+\int_0^{\tau}p_{ji}(s)\left[g_i\left(\frac{u_i}{a_i}\right)-g_i\left(\frac{\overline{u}_i}{a_i}\right)\right]ds\right\}\right|\\
&+\sum_{i=1}^{n}\left|\frac{t^{-\alpha}}{\Gamma(1-\alpha)}\left[\frac{u_i}{a_i}-\frac{\overline{u}_i}{a_i}\right]\right|+\sum_{j=1}^{m}\left|\frac{t^{-\beta}}{\Gamma(1-\beta)}\left[\frac{v_j}{c_j}-\frac{\overline{v}_j}{c_j}\right]\right|.
\end{aligned} \tag{18}$$

According to (**H1**)-(**H2**), one has

$$\begin{aligned}
\|\Phi(\mathbf{u})-\Phi(\overline{\mathbf{u}})\| \leqslant{}& \sum_{i=1}^{n}\sum_{j=1}^{m}\left\{\left|b_{ij}\right|F_j\left|\frac{v_j-\overline{v}_j}{c_j}\right|\right.\\
&\left.+\int_0^{\tau}\left|r_{ij}(s)\right|F_j\left|\frac{v_j-\overline{v}_j}{c_j}\right|ds\right\}\\
&+\sum_{j=1}^{m}\sum_{i=1}^{n}\left\{\left|d_{ji}\right|G_i\left|\frac{u_i-\overline{u}_i}{a_i}\right|\right.\\
&\left.+\int_0^{\tau}\left|p_{ji}(s)\right|G_i\left|\frac{u_i-\overline{u}_i}{a_i}\right|ds\right\}+\frac{t^{-\alpha}}{\Gamma(1-\alpha)}\\
&\cdot\max_{1\leqslant i\leqslant n}\left\{\frac{1}{a_i}\right\}\cdot\sum_{i=1}^{n}\left|u_i-\overline{u}_i\right|+\frac{t^{-\beta}}{\Gamma(1-\beta)}\cdot\max_{1\leqslant j\leqslant m}\left\{\frac{1}{c_j}\right\}\\
&\cdot\sum_{j=1}^{m}\left|v_j-\overline{v}_j\right|,
\end{aligned} \tag{19}$$

For $\alpha,\beta\in(0,1)$, we have $\lim_{t\to+\infty}t^{-\alpha}=0$, $\lim_{t\to+\infty}t^{-\beta}=0$. Therefore, there exists a small enough constant $\varepsilon>0$ such that $t^{-\alpha}<\varepsilon$, $t^{-\beta}<\varepsilon$. Thus, it follows from (19) that

$$\begin{aligned}
\|\Phi(\mathbf{u})-\Phi(\overline{\mathbf{u}})\| \leqslant{}& \sum_{i=1}^{n}\sum_{j=1}^{m}\left[\frac{\left|b_{ij}\right|+\tau R_{ij}}{c_j}F_j\left|v_j-\overline{v}_j\right|\right]\\
&+\sum_{j=1}^{m}\sum_{i=1}^{n}\left[\frac{\left|d_{ji}\right|+\tau P_{ji}}{a_i}G_i\left|u_i-\overline{u}_i\right|\right]+\frac{\varepsilon}{\Gamma(1-\alpha)}
\end{aligned}$$

$$
\begin{aligned}
&\cdot \max_{1\leqslant i\leqslant n}\left\{\frac{1}{a_i}\right\}\cdot\sum_{i=1}^{n}\left|u_i-\overline{u}_i\right| + \frac{\varepsilon}{\Gamma(1-\beta)}\cdot\max_{1\leqslant j\leqslant m}\left\{\frac{1}{c_j}\right\} \\
&\cdot\sum_{j=1}^{m}\left|v_j-\overline{v}_j\right| \\
&\leqslant \max_{1\leqslant j\leqslant m}\left\{\frac{\varepsilon}{\Gamma(1-\beta)}\cdot\frac{1}{c_j}+\frac{F_j}{c_j}\sum_{i=1}^{n}\left[\left|b_{ij}\right|+\tau R_{ij}\right]\right\} \\
&\cdot\sum_{j=1}^{m}\left|v_j-\overline{v}_j\right| \\
&+\max_{1\leqslant i\leqslant n}\left\{\frac{\varepsilon}{\Gamma(1-\alpha)}\cdot\frac{1}{a_i}+\frac{G_i}{a_i}\sum_{j=1}^{m}\left[\left|d_{ji}\right|+\tau P_{ji}\right]\right\} \\
&\cdot\sum_{i=1}^{n}\left|u_i-\overline{u}_i\right|.
\end{aligned}
\tag{20}
$$

Let $k = \max\{\omega_1,\omega_2\}$, where ω_1 and ω_2 are defined in (15). Hence, we have

$$
\begin{aligned}
\left\|\Phi(\mathbf{u})-\Phi(\overline{\mathbf{u}})\right\| &\leqslant k\left[\sum_{i=1}^{n}\left|u_i-\overline{u}_i\right|+\sum_{j=1}^{m}\left|v_j-\overline{v}_j\right|\right] \\
&= k\left\|\mathbf{u}-\overline{\mathbf{u}}\right\|.
\end{aligned}
\tag{21}
$$

Thus, it follows from (15) that $0 < k < 1$, which implies that $\Phi : \mathbb{R}^{n+m}\to\mathbb{R}^{n+m}$ is a contraction mapping. Therefore, from Lemma 4, there exists a unique fixed point of the map $\Phi : \mathbb{R}^{n+m}\to\mathbb{R}^{n+m}$, such that $\Phi(\mathbf{u}^*)=\mathbf{u}^*$. Thus, from (17), we get

$$
\begin{aligned}
&\sum_{j=1}^{m}b_{ij}f_j\left(\frac{v_j^*}{c_j}\right)+\sum_{j=1}^{m}\int_0^{\tau}r_{ij}(s)\,f_j\left(\frac{v_j^*}{c_j}\right)ds+I_i \\
&\quad-\frac{t^{-\alpha}}{\Gamma(1-\alpha)}\frac{u_i^*}{a_i}=u_i^*,\quad i=1,2,\ldots,n, \\
&\sum_{i=1}^{n}d_{ji}g_i\left(\frac{u_i^*}{a_i}\right)+\sum_{i=1}^{n}\int_0^{\tau}p_{ji}(s)\,g_i\left(\frac{u_i^*}{a_i}\right)ds+J_j \\
&\quad-\frac{t^{-\beta}}{\Gamma(1-\beta)}\frac{v_j^*}{c_j}=v_j^*,\quad j=1,2,\ldots,m.
\end{aligned}
\tag{22}
$$

Let $x_i^*=u_i^*/a_i$, $y_j^*=v_j^*/c_j$; then it follows from (22) that

$$
\begin{aligned}
&\sum_{j=1}^{m}b_{ij}f_j\left(y_j^*\right)+\sum_{j=1}^{m}\int_0^{\tau}r_{ij}(s)\,f_j\left(y_j^*\right)ds+I_i \\
&\quad-\frac{t^{-\alpha}}{\Gamma(1-\alpha)}x_i^*=a_ix_i^*,\quad i=1,2,\ldots,n, \\
&\sum_{i=1}^{n}d_{ji}g_i\left(x_i^*\right)+\sum_{i=1}^{n}\int_0^{\tau}p_{ji}(s)\,g_i\left(x_i^*\right)ds+J_j \\
&\quad-\frac{t^{-\beta}}{\Gamma(1-\beta)}y_j^*=c_jy_j^*,\quad j=1,2,\ldots,m;
\end{aligned}
\tag{23}
$$

that is

$$
\begin{aligned}
&\sum_{j=1}^{m}b_{ij}f_j\left(y_j^*\right)+\sum_{j=1}^{m}\int_0^{\tau}r_{ij}(s)\,f_j\left(y_j^*\right)ds+I_i-a_ix_i^* \\
&\quad={}^{\mathrm{RL}}_{\ 0}D_t^{\alpha}\left\{x_i^*\right\},\quad i=1,2,\ldots,n, \\
&\sum_{i=1}^{n}d_{ji}g_i\left(x_i^*\right)+\sum_{i=1}^{n}\int_0^{\tau}p_{ji}(s)\,g_i\left(x_i^*\right)ds+J_j-c_jy_j^* \\
&\quad={}^{\mathrm{RL}}_{\ 0}D_t^{\beta}\left\{y_j^*\right\},\quad j=1,2,\ldots,m.
\end{aligned}
\tag{24}
$$

According to (**H3**), we know that

$$
\begin{aligned}
&\gamma_k^{(1)}\left(x_i^*\right)=0, \\
&\gamma_k^{(2)}\left(y_j^*\right)=0, \\
&\qquad i=1,2,\ldots,n;\ j=1,2,\ldots,m;\ k=1,2,\ldots.
\end{aligned}
\tag{25}
$$

Thus, it follows from Definition 7 that $(x_1^*,x_2^*,\ldots,x_n^*,y_1^*,y_2^*,\ldots,y_m^*)^T\in\mathbb{R}^{n+m}$ is a unique equilibrium solution for system (6). The proof is complete. □

The following corollary is the direct result of Theorem 8.

Corollary 9. *Suppose that conditions (**H1**)–(**H3**) hold; then there exists a unique equilibrium solution for system (6), if the following inequalities simultaneously hold for a small enough constant* $\varepsilon>0$

$$
\begin{aligned}
&\min_{1\leqslant i\leqslant n}\left\{a_i-\frac{\varepsilon}{\Gamma(1-\alpha)}-G_i\sum_{j=1}^{m}\left[\left|d_{ji}\right|+\tau P_{ji}\right]\right\}>0, \\
&\min_{1\leqslant j\leqslant m}\left\{c_j-\frac{\varepsilon}{\Gamma(1-\beta)}-F_j\sum_{i=1}^{n}\left[\left|b_{ij}\right|+\tau R_{ij}\right]\right\}>0.
\end{aligned}
\tag{26}
$$

Remark 10. Theorem 8 and Corollary 9 reveal that the conditions of existence and uniqueness of the equilibrium solution for system (6) are based on the contraction mapping principle, which can be expressed in terms of the algebraic inequalities. The conditions of existence and uniqueness of the equilibrium point for system (6) reflect the close relation between the coefficients, neuron activation functions, and time-delay of network parameters, which are also dependent on the orders α and β of Riemann-Liouville derivatives. On the other hand, if we only assume that (**H1**)–(**H3**) hold, then there exists at least an equilibrium solution for system (6) by applying Schauder fixed point theorem, whose proof is omitted here.

4. Globally Asymptotic Stability Criteria

In this section, by constructing a novel Lyapunov functional, we obtain the sufficient conditions to ensure the globally asymptotic stability of the equilibrium solution for system (6) based on fractional Barbalat theorem and classical Lyapunov stability theory.

Theorem 11. *Suppose that conditions (**H1**)–(**H3**) hold; then a unique equilibrium solution for system (6) is globally asymptotically stable, if the following inequalities simultaneously hold for a small enough constant $\varepsilon > 0$*

$$\eta_1 = \min_{1\leqslant i\leqslant n}\left\{a_i - G_i\sum_{j=1}^{m}\left[\left|d_{ji}\right| + \tau P_{ji}\right]\right\} > \frac{\varepsilon}{\Gamma(1-\alpha)},$$
$$\eta_2 = \min_{1\leqslant j\leqslant m}\left\{c_j - F_j\sum_{i=1}^{n}\left[\left|b_{ij}\right| + \tau R_{ij}\right]\right\} > \frac{\varepsilon}{\Gamma(1-\beta)}. \quad (27)$$

Proof. From Corollary 9, there exists a unique equilibrium solution $(x^{*T}, y^{*T})^T$ for system (6). By using the variable transformation method, we can shift the equilibrium point to the origin. Let $u_i(t) = x_i(t) - x_i^*$, $v_j(t) = y_j(t) - y_j^*$; then system (6) is transformed into

$$\begin{aligned}
&{}^{\mathrm{RL}}_{0}D_t^{\alpha} u_i(t) \\
&\quad = -a_i u_i(t) + \sum_{j=1}^{m} b_{ij}\left[f_j\left(y_j(t)\right) - f_j\left(y_j^*\right)\right] \\
&\qquad + \sum_{j=1}^{m}\int_0^{\tau} r_{ij}(s)\left[f_j\left(y_j(t-s)\right) - f_j\left(y_j^*\right)\right] ds, \\
&\qquad\qquad t \neq t_k, \\
&u_i(t_k^+) = \left(1 - \lambda_{ik}^{(1)}\right) u_i(t_k^-), \\
&\qquad\qquad i = 1,2,\ldots,n;\ k = 1,2,\ldots, \\
&{}^{\mathrm{RL}}_{0}D_t^{\beta} v_j(t) \\
&\quad = -c_j v_j(t) + \sum_{i=1}^{n} d_{ji}\left[g_i(x_i(t)) - g_i(x_i^*)\right] \\
&\qquad + \sum_{i=1}^{n}\int_0^{\tau} p_{ji}(s)\left[g_i x_i(t-s) - g_i(x_i^*)\right] ds, \\
&\qquad\qquad t \neq t_k, \\
&v_j(t_k^+) = \left(1 - \lambda_{jk}^{(2)}\right) v_j(t_k^-), \\
&\qquad\qquad j = 1,2,\ldots,m;\ k = 1,2,\ldots.
\end{aligned} \quad (28)$$

Construct a novel Lyapunov functional composed of fractional-order integral and definite integral terms:

$$\begin{aligned}
V(t) &= {}_0D_t^{-(1-\alpha)}\left[\sum_{i=1}^{n}\left|u_i(t)\right|\right] + {}_0D_t^{-(1-\beta)}\left[\sum_{j=1}^{m}\left|v_j(t)\right|\right] \\
&\quad + \sum_{i=1}^{n}\sum_{j=1}^{m} F_j\left|b_{ij}\right|\int_{t-\tau}^{t}\left|v_j(s)\right| ds \\
&\quad + \sum_{j=1}^{m}\sum_{i=1}^{n} G_i\left|d_{ji}\right|\int_{t-\tau}^{t}\left|u_i(s)\right| ds \\
&\quad + \sum_{i=1}^{n}\sum_{j=1}^{m} F_j R_{ij}\int_0^{\tau}\int_{t-s}^{t}\left|v_j(\eta)\right| d\eta\, ds \\
&\quad + \sum_{j=1}^{m}\sum_{i=1}^{n} G_i P_{ji}\int_0^{\tau}\int_{t-s}^{t}\left|u_i(\eta)\right| d\eta\, ds.
\end{aligned} \quad (29)$$

The time derivative of $V(t)$ along the trajectories of system (6) can be calculated, which are carried out for the following cases.

Case 1. For $t \neq t_k$, from Lemma 3, we obtain

$$\begin{aligned}
\frac{d^+V(t)}{dt} &= {}^{\mathrm{RL}}_{0}D_t^{\alpha}\left[\sum_{i=1}^{n}\left|u_i(t)\right|\right] + {}^{\mathrm{RL}}_{0}D_t^{\beta}\left[\sum_{j=1}^{m}\left|v_j(t)\right|\right] \\
&\quad + \sum_{i=1}^{n}\sum_{j=1}^{m} F_j\left|b_{ij}\right|\left[\left|v_j(t)\right| - \left|v_j(t-\tau)\right|\right] \\
&\quad + \sum_{j=1}^{m}\sum_{i=1}^{n} G_i\left|d_{ji}\right|\left[\left|u_i(t)\right| - \left|u_i(t-\tau)\right|\right] \\
&\quad + \sum_{i=1}^{n}\sum_{j=1}^{m} F_j R_{ij}\int_0^{\tau}\left[\left|v_j(t)\right| - \left|v_j(t-s)\right|\right] ds \\
&\quad + \sum_{j=1}^{m}\sum_{i=1}^{n} G_i P_{ji}\int_0^{\tau}\left[\left|u_i(t)\right| - \left|u_i(t-s)\right|\right] ds.
\end{aligned} \quad (30)$$

An application of Definition 2 yields

$$\begin{aligned}
{}^{\mathrm{RL}}_{0}D_t^{\alpha}\left|u_i(t)\right| &= \mathrm{sgn}\left(u_i(t)\right)\cdot\left({}^{\mathrm{RL}}_{0}D_t^{\alpha}u_i(t)\right), \\
{}^{\mathrm{RL}}_{0}D_t^{\beta}\left|v_j(t)\right| &= \mathrm{sgn}\left(v_j(t)\right)\cdot\left({}^{\mathrm{RL}}_{0}D_t^{\beta}v_j(t)\right),
\end{aligned} \quad (31)$$

where $\mathrm{sgn}(\cdot)$ denotes the standard signum function. Thus, (30) can be rewritten as

$$\begin{aligned}
\frac{d^+V(t)}{dt} &= \sum_{i=1}^{n}\mathrm{sgn}\left(u_i(t)\right)\left[{}^{\mathrm{RL}}_{0}D_t^{\alpha}\left(u_i(t)\right)\right] \\
&\quad + \sum_{j=1}^{m}\mathrm{sgn}\left(v_j(t)\right)\left[{}^{\mathrm{RL}}_{0}D_t^{\beta}\left(v_j(t)\right)\right] \\
&\quad + \sum_{i=1}^{n}\sum_{j=1}^{m} F_j\left|b_{ij}\right|\left[\left|v_j(t)\right| - \left|v_j(t-\tau)\right|\right]
\end{aligned}$$

$$+\sum_{j=1}^{m}\sum_{i=1}^{n}G_i\left|d_{ji}\right|\left[\left|u_i(t)\right|-\left|u_i(t-\tau)\right|\right]$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{m}F_jR_{ij}\int_0^{\tau}\left[\left|v_j(t)\right|-\left|v_j(t-s)\right|\right]ds$$
$$+\sum_{j=1}^{m}\sum_{i=1}^{n}G_iP_{ji}\int_0^{\tau}\left[\left|u_i(t)\right|-\left|u_i(t-s)\right|\right]ds. \tag{32}$$

Combining (28) and (32) yields

$$\begin{aligned}\frac{d^+V(t)}{dt} &= \sum_{i=1}^{n}\operatorname{sgn}(u_i(t))\left\{-a_iu_i(t)\right.\\
&+\sum_{j=1}^{m}b_{ij}\left[f_j\left(y_j(t)\right)-f_j\left(y_j^*\right)\right]\\
&\left.+\sum_{j=1}^{m}\int_0^{\tau}r_{ij}(s)\left[f_j\left(y_j(t-s)\right)-f_j\left(y_j^*\right)\right]ds\right\}\\
&+\sum_{j=1}^{m}\operatorname{sgn}\left(v_j(t)\right)\left\{-c_jv_j(t)\right.\\
&+\sum_{i=1}^{n}d_{ji}\left[g_i\left(x_i(t)\right)-g_i\left(x_i^*\right)\right]\\
&\left.+\sum_{i=1}^{n}\int_0^{\tau}p_{ji}(s)\left[g_ix_i(t-s)-g_i\left(x_i^*\right)\right]ds\right\}\\
&+\sum_{i=1}^{n}\sum_{j=1}^{m}F_j\left|b_{ij}\right|\left[\left|v_j(t)\right|-\left|v_j(t-\tau)\right|\right]\\
&+\sum_{j=1}^{m}\sum_{i=1}^{n}G_i\left|d_{ji}\right|\left[\left|u_i(t)\right|-\left|u_i(t-\tau)\right|\right]+\sum_{i=1}^{n}\sum_{j=1}^{m}F_jR_{ij}\\
&\cdot\int_0^{\tau}\left[\left|v_j(t)\right|-\left|v_j(t-s)\right|\right]ds+\sum_{j=1}^{m}\sum_{i=1}^{n}G_iP_{ji}\\
&\cdot\int_0^{\tau}\left[\left|u_i(t)\right|-\left|u_i(t-s)\right|\right]ds.\end{aligned} \tag{33}$$

By computations, we have

$$\begin{aligned}\frac{d^+V(t)}{dt} &\leqslant -\sum_{i=1}^{n}a_i\left|u_i(t)\right|-\sum_{j=m}^{n}c_j\left|v_j(t)\right|\\
&+\sum_{i=1}^{n}\sum_{j=1}^{m}F_j\left|b_{ij}\right|\left|v_j(t-\tau)\right|\\
&+\sum_{j=1}^{m}\sum_{i=1}^{n}G_i\left|d_{ji}\right|\left|u_i(t-\tau)\right|\\
&+\sum_{i=1}^{n}\sum_{j=1}^{m}F_jR_{ij}\int_0^{\tau}\left|v_j(t-s)\right|ds\\
&+\sum_{j=1}^{m}\sum_{i=1}^{n}G_iP_{ji}\int_0^{\tau}\left|u_i(t-s)\right|ds\\
&+\sum_{i=1}^{n}\sum_{j=1}^{m}F_j\left|b_{ij}\right|\left[\left|v_j(t)\right|-\left|v_j(t-\tau)\right|\right]\\
&+\sum_{j=1}^{m}\sum_{i=1}^{n}G_i\left|d_{ji}\right|\left[\left|u_i(t)\right|-\left|u_i(t-\tau)\right|\right]\\
&+\sum_{i=1}^{n}\sum_{j=1}^{m}F_jR_{ij}\int_0^{\tau}\left[\left|v_j(t)\right|-\left|v_j(t-s)\right|\right]ds\\
&+\sum_{j=1}^{m}\sum_{i=1}^{n}G_iP_{ji}\int_0^{\tau}\left[\left|u_i(t)\right|-\left|u_i(t-s)\right|\right]ds\\
&\leqslant -\sum_{i=1}^{n}a_i\left|u_i(t)\right|-\sum_{j=m}^{n}c_j\left|v_j(t)\right|\\
&+\sum_{i=1}^{n}\sum_{j=1}^{m}F_j\left|b_{ij}\right|\left|v_j(t)\right|\\
&+\sum_{j=1}^{m}\sum_{i=1}^{n}G_i\left|d_{ji}\right|\left|u_i(t)\right|\\
&+\sum_{i=1}^{n}\sum_{j=1}^{m}F_jR_{ij}\int_0^{\tau}\left|v_j(t)\right|ds\\
&+\sum_{j=1}^{m}\sum_{i=1}^{n}G_iP_{ji}\int_0^{\tau}\left|u_i(t)\right|ds\\
&\leqslant \sum_{i=1}^{n}\left\{-a_i+G_i\sum_{j=1}^{m}\left(\left|d_{ji}\right|+\tau P_{ji}\right)\right\}\left|u_i(t)\right|\\
&+\sum_{j=1}^{m}\left\{-c_j+F_j\sum_{i=1}^{n}\left(\left|b_{ij}\right|+\tau R_{ij}\right)\right\}\left|v_j(t)\right|\\
&\leqslant -\frac{\varepsilon}{\Gamma(1-\alpha)}\sum_{i=1}^{n}\left|u_i(t)\right|\\
&-\frac{\varepsilon}{\Gamma(1-\beta)}\sum_{j=1}^{m}\left|v_j(t)\right|,\quad t\neq t_k,\end{aligned} \tag{34}$$

which implies that $d^+V(t)/dt \leqslant 0$ as $t \neq t_k$. Hence, for any $t \in [t_{k-1}, t_k)$, we get

$$\begin{aligned}V(t)+\int_{t_{k-1}}^{t_k}&\left[\frac{\varepsilon}{\Gamma(1-\alpha)}\sum_{i=1}^{n}\left|u_i(s)\right|\right.\\
&\left.+\frac{\varepsilon}{\Gamma(1-\beta)}\sum_{j=1}^{m}\left|v_j(s)\right|\right]ds \leqslant V\left(t_{k-1}^+\right).\end{aligned} \tag{35}$$

Case 2. For $t = t_k$, from (29), one has

$$\begin{aligned} V\left(t_k^+\right) = {} & {}_0D_{t_k^+}^{-(1-\alpha)}\left[\sum_{i=1}^{n}\left|u_i\left(t_k^+\right)\right|\right] \\ & + {}_0D_{t_k^+}^{-(1-\beta)}\left[\sum_{j=1}^{m}\left|v_j\left(t_k^+\right)\right|\right] \\ & + \sum_{i=1}^{n}\sum_{j=1}^{m}F_j\left|b_{ij}\right|\int_{t_k^+-\tau}^{t_k^+}\left|v_j\left(s\right)\right|ds \\ & + \sum_{j=1}^{m}\sum_{i=1}^{n}G_i\left|d_{ji}\right|\int_{t_k^+-\tau}^{t_k^+}\left|u_i\left(s\right)\right|ds \\ & + \sum_{i=1}^{n}\sum_{j=1}^{m}F_jR_{ij}\int_0^{\tau}\int_{t_k^+-s}^{t_k^+}\left|v_j\left(\eta\right)\right|d\eta\, ds \\ & + \sum_{j=1}^{m}\sum_{i=1}^{n}G_iP_{ji}\int_0^{\tau}\int_{t_k^+-s}^{t_k^+}\left|u_i\left(\eta\right)\right|d\eta\, ds. \end{aligned} \tag{36}$$

From (**H3**), we get

$$\begin{aligned} V\left(t_k^+\right) = {} & {}_0D_{t_k^+}^{-(1-\alpha)}\left[\sum_{i=1}^{n}\left|1-\lambda_{ik}^{(1)}\right|\left|u_i\left(t_k^-\right)\right|\right] \\ & + {}_0D_{t_k^+}^{-(1-\beta)}\left[\sum_{j=1}^{m}\left|1-\lambda_{jk}^{(2)}\right|\left|v_j\left(t_k^-\right)\right|\right] \\ & + \sum_{i=1}^{n}\sum_{j=1}^{m}F_j\left|b_{ij}\right|\int_{t_k^+-\tau}^{t_k^+}\left|v_j\left(s\right)\right|ds \\ & + \sum_{j=1}^{m}\sum_{i=1}^{n}G_i\left|d_{ji}\right|\int_{t_k^+-\tau}^{t_k^+}\left|u_i\left(s\right)\right|ds \\ & + \sum_{i=1}^{n}\sum_{j=1}^{m}F_jR_{ij}\int_0^{\tau}\int_{t_k^+-s}^{t_k^+}\left|v_j\left(\eta\right)\right|d\eta\, ds \\ & + \sum_{j=1}^{m}\sum_{i=1}^{n}G_iP_{ji}\int_0^{\tau}\int_{t_k^+-s}^{t_k^+}\left|u_i\left(\eta\right)\right|d\eta\, ds. \end{aligned} \tag{37}$$

Note that the inequalities $|1-\lambda_{ik}^{(1)}| < 1$ and $|1-\lambda_{jk}^{(2)}| < 1$ hold; then

$$\begin{aligned} V\left(t_k^+\right) \leqslant {} & {}_0D_{t_k^-}^{-(1-\alpha)}\left[\sum_{i=1}^{n}\left|u_i\left(t_k^-\right)\right|\right] \\ & + {}_0D_{t_k^-}^{-(1-\beta)}\left[\sum_{j=1}^{m}\left|v_j\left(t_k^-\right)\right|\right] \\ & + \sum_{i=1}^{n}\sum_{j=1}^{m}F_j\left|b_{ij}\right|\int_{t_k^+-\tau}^{t_k^+}\left|v_j\left(s\right)\right|ds \\ & + \sum_{j=1}^{m}\sum_{i=1}^{n}G_i\left|d_{ji}\right|\int_{t_k^+-\tau}^{t_k^+}\left|u_i\left(s\right)\right|ds \\ & + \sum_{i=1}^{n}\sum_{j=1}^{m}F_jR_{ij}\int_0^{\tau}\int_{t_k^+-s}^{t_k^+}\left|v_j\left(\eta\right)\right|d\eta\, ds \\ & + \sum_{j=1}^{m}\sum_{i=1}^{n}G_iP_{ji}\int_0^{\tau}\int_{t_k^+-s}^{t_k^+}\left|u_i\left(\eta\right)\right|d\eta\, ds = V\left(t_k^-\right) \\ = {} & V\left(t_k\right). \end{aligned} \tag{38}$$

Let $U(t) = \sum_{i=1}^{n}|u_i(t)| + \sum_{j=1}^{m}|v_j(t)|$, for any $t \in [t_{k-1}, t_k)$; then we have the following estimations:

$$\begin{aligned} V\left(t\right) & \leqslant -\int_{t_{k-1}}^{t}U\left(s\right)ds + V\left(t_{k-1}^+\right) \\ & \leqslant -\int_{t_{k-1}}^{t}U\left(s\right)ds + V\left(t_{k-1}^-\right) \\ & \leqslant -\int_{t_{k-2}}^{t}U\left(s\right)ds + V\left(t_{k-2}^-\right) \leqslant \cdots \\ & \leqslant -\int_{0}^{t}U\left(s\right)ds + V\left(0\right); \end{aligned} \tag{39}$$

Thus, we can get the following inequality:

$$V\left(t\right) + \int_0^t U\left(s\right)ds \leqslant V\left(0\right), \tag{40}$$

which implies that $\lim_{t\to+\infty}U(t)$ is bounded. From (28), $|{}_0^{\mathrm{RL}}D_t^{\alpha}u_i(t)|$ and $|{}_0^{\mathrm{RL}}D_t^{\beta}v_j(t)|$ are also bounded. From Lemma 5, we have $\lim_{t\to+\infty}\sum_{i=1}^{n}|u_i(t)| = 0$ and $\lim_{t\to+\infty}\sum_{j=1}^{m}|v_j(t)| = 0$. Therefore, according to Lyapunov stability theory, a unique equilibrium solution $(x^{*T}, y^{*T})^T$ for system (6) is globally asymptotically stable. This completes the proof. □

The following corollary is the direct result of Theorem 11.

Corollary 12. *Suppose that (**H1**)–(**H3**) hold; then a unique equilibrium solution for system (6) is globally asymptotically stable, if the following inequalities simultaneously hold for a small enough constant* $\varepsilon > 0$

$$\begin{aligned} \omega_1 & = \max_{1\leqslant i\leqslant n}\left\{\frac{\varepsilon}{\Gamma\left(1-\alpha\right)}\cdot\frac{1}{a_i} + \frac{G_i}{a_i}\sum_{j=1}^{m}\left[\left|d_{ji}\right| + \tau P_{ji}\right]\right\} \\ & < 1, \\ \omega_2 & = \max_{1\leqslant j\leqslant m}\left\{\frac{\varepsilon}{\Gamma\left(1-\beta\right)}\cdot\frac{1}{c_j} + \frac{F_j}{c_j}\sum_{i=1}^{n}\left[\left|b_{ij}\right| + \tau R_{ij}\right]\right\} \\ & < 1. \end{aligned} \tag{41}$$

Remark 13. Different from fractional Lyapunov functional method in [30, 32, 37], an appropriate Lyapunov functional composed of fractional integral and definite integral terms in the proof of Theorem 11 is presented, and we only need to calculate its first-order derivative to derive stability conditions. As discussed in [35], in general speaking, it is very difficult to calculate the fractional-order derivatives of a Lyapunov functional. The main advantage of our constructed method is that we can avoid computing the fractional-order derivatives of the Lyapunov functional.

Remark 14. The globally asymptotic stability criteria of a unique equilibrium solution for system (6) are described by the algebraic inequalities, which are dependent on the orders α and β of fractional derivatives and reflect the close relation between the coefficients, neuron activation functions, and time-delay of network parameters. Moreover, the globally asymptotic stability criteria are more easily checked and contribute to reducing the computational burden.

Remark 15. When $\alpha = \beta = 1$, system (6) is reduced to integer-order BAM neural networks with distributed delays and impulses [23]. Note that the Riemann-Liouville derivative is a continuous operator of the order (see [9–11]); then we can obtain globally asymptotic stability criteria for impulsive integer-order hybrid BAM neural networks from the proof of Theorem 11.

Remark 16. In [33, 34, 44], the authors have focused on studying the finite-time stability of fractional-order delayed neural networks. However, it should be pointed out that the finite-time stability and asymptotic stability in the sense of Lyapunov are different concepts, because finite-time stability does not contain Lyapunov asymptotic stability and vice versa [34, 47]. This is also the motivation of this paper.

5. An Illustrative Example

In this section, an example for impulsive fractional-order hybrid BAM neural networks with distributed delays is given to illustrate the effectiveness and feasibility of the theoretical results.

Example 17. Consider the four-state Riemann-Liouville fractional-order hybrid BAM neural network model with distributed delays and impulsive effects described by

$$\begin{aligned}
{}^{\mathrm{RL}}_{0}D_t^{0.2} x_1(t) &= -0.7x_1(t) - 0.2f_1(y_1(t)) \\
&\quad + 0.1f_2(y_2(t)) \\
&\quad + 2\int_0^{0.2} sf_1(y_1(t-s))\,ds \\
&\quad + \int_0^{0.2} sf_2(y_2(t-s))\,ds, \\
{}^{\mathrm{RL}}_{0}D_t^{0.2} x_2(t) &= -0.6x_2(t) + 0.3f_1(y_1(t)) \\
&\quad + 0.2f_2(y_2(t)) \\
&\quad + \int_0^{0.2} sf_1(y_1(t-s))\,ds \\
&\quad - \int_0^{0.2} s^3 f_2(y_2(t-s))\,ds, \\
\Delta x_i(t_k) &= -0.3(x_i(t_k) - x_i^*), \\
&\qquad i = 1,2;\ k = 1,2,\ldots, \\
{}^{\mathrm{RL}}_{0}D_t^{0.6} y_1(t) &= -0.7y_1(t) + 0.4g_1(y_1(t)) \\
&\quad + 0.2g_2(y_2(t)) \\
&\quad - \int_0^{0.2} sg_1(y_1(t-s))\,ds \\
&\quad + \int_0^{0.2} s^2 g_2(y_2(t-s))\,ds, \\
{}^{\mathrm{RL}}_{0}D_t^{0.6} y_2(t) &= -0.6y_2(t) + 0.1g_1(y_1(t)) \\
&\quad - 0.3g_2(y_2(t)) \\
&\quad + \int_0^{0.2} s^2 g_1(y_1(t-s))\,ds \\
&\quad + \int_0^{0.2} sg_2(y_2(t-s))\,ds, \\
\Delta y_j(t_k) &= -0.4(y_j(t_k) - y_i^*), \\
&\qquad j = 1,2;\ k = 1,2,\ldots,
\end{aligned} \tag{42}$$

where $\alpha = 0.2$, $\beta = 0.6$, $\tau = 0.2$, $a_1 = c_1 = 0.7$, $a_2 = c_2 = 0.6$, $b_{11} = -0.2$, $b_{12} = 0.1$, $b_{21} = 0.3$, $b_{22} = 0.2$, $d_{11} = 0.4$, $d_{12} = 0.2$, $d_{21} = 0.1$, $d_{22} = -0.3$, $r_{11}(s) = 2s$, $r_{12}(s) = s$, $r_{21}(s) = s$, $r_{22}(s) = -s^3$, $p_{11}(s) = -s$, $p_{12}(s) = s^2$, $p_{21}(s) = s^2$, $p_{22}(s) = s$, and

$$\begin{aligned}
f_j(y_j) &= \frac{1}{2}(|y_j+1| - |y_j-1|), \quad j = 1,2, \\
g_i(x_i) &= \frac{1}{2}(|x_i+1| - |x_i-1|), \quad i = 1,2.
\end{aligned} \tag{43}$$

From (43), we know that $F_1 = F_2 = G_1 = G_2 = 1$. Since $f_1(0) = f_2(0) = 0$, $g_1(0) = g_2(0) = 0$, then $(x_1^*, x_2^*, y_1^*, y_1^*)^T = (0,0,0,0)^T$ is an equilibrium solution for system (42). Next, we apply Theorem 11 or Corollary 12 to check the uniqueness and global asymptotic stability of the equilibrium point for system (42).

In fact, by computations, one can get that $R_{11} = 0.4$, $R_{12} = R_{21} = 0.2$, $R_{22} = 0.008$, $P_{11} = P_{22} = 0.2$, and $P_{12} = P_{21} = 0.04$. Choosing a positive constant $\varepsilon = 0.04 > 0$, then we can obtain

$$\begin{aligned}
\eta_1 &= \min_{1\leqslant i\leqslant 2}\left\{a_i - G_i\sum_{j=1}^{2}\left[|d_{ji}| + \tau P_{ji}\right]\right\} = 0.052 \\
&> \frac{\varepsilon}{\Gamma(1-\alpha)} = 0.044, \\
\eta_2 &= \min_{1\leqslant j\leqslant 2}\left\{c_j - F_j\sum_{i=1}^{2}\left[|b_{ij}| + \tau R_{ij}\right]\right\} = 0.116 \\
&> \frac{\varepsilon}{\Gamma(1-\beta)} = 0.045,
\end{aligned}$$

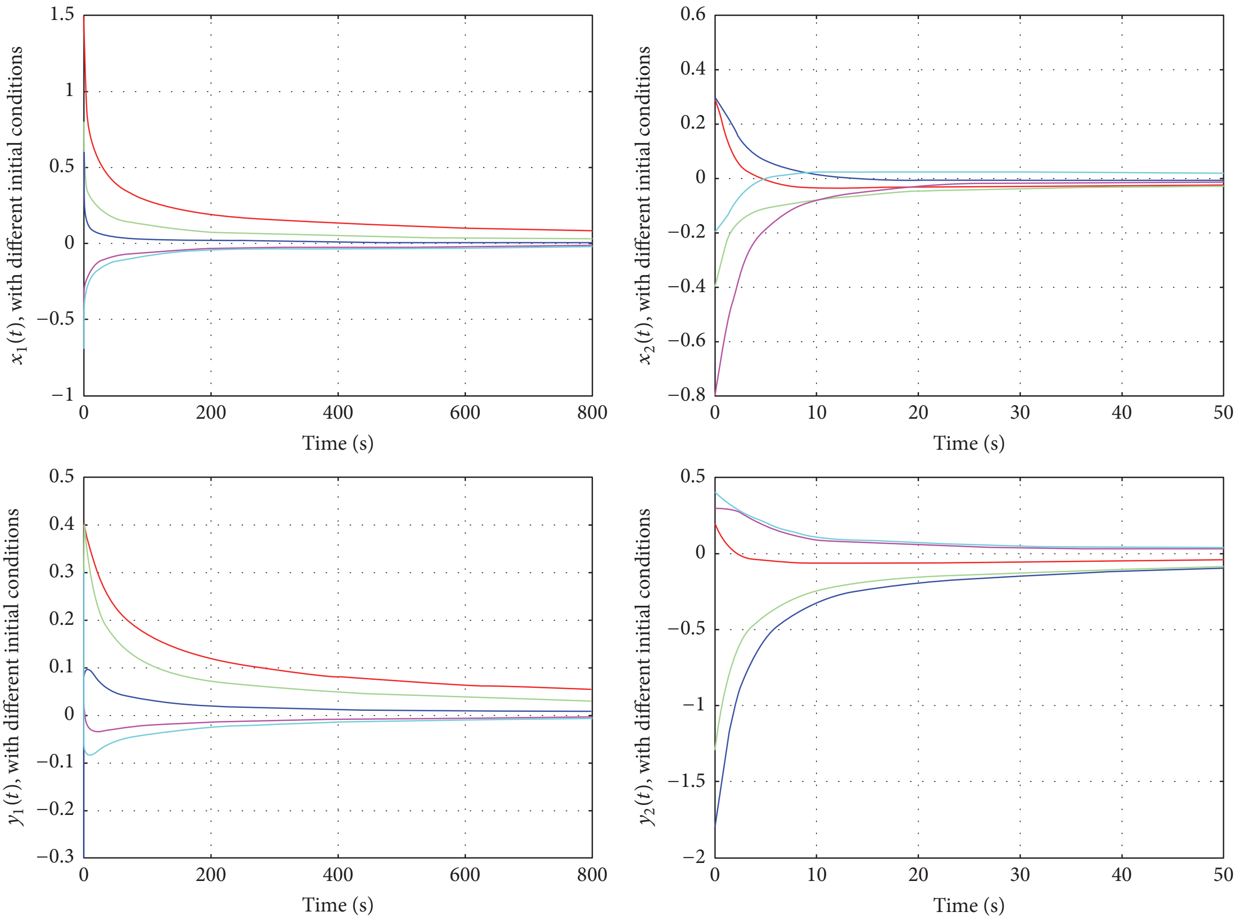

FIGURE 1: State trajectories of BAM neural network (42) with $\alpha = 0.2$; $\beta = 0.6$ under different initial conditions.

$$\omega_1 = \max_{1\leqslant i\leqslant 2}\left\{\frac{\varepsilon}{\Gamma(1-\alpha)}\cdot\frac{1}{a_i}+\frac{G_i}{a_i}\sum_{j=1}^{2}\left[\left|d_{ji}\right|+\tau P_{ji}\right]\right\}$$
$$= 0.856 < 1,$$
$$\omega_2 = \max_{1\leqslant j\leqslant 2}\left\{\frac{\varepsilon}{\Gamma(1-\beta)}\cdot\frac{1}{c_j}+\frac{F_j}{c_j}\sum_{i=1}^{2}\left[\left|b_{ij}\right|+\tau R_{ij}\right]\right\}$$
$$= 0.722 < 1. \quad (44)$$

Thus, the conditions of Theorem 11 or Corollary 12 are satisfied. For numerical simulations, Figure 1 depicts the state trajectories of system (42) under different initial conditions with $\alpha = 0.2$, $\beta = 0.6$. It can be directly observed that the unique equilibrium solution $(0,0,0,0)^T$ for system (42) is globally asymptotically stable with $\alpha = 0.2$, $\beta = 0.6$. Therefore, the numerical simulations further confirm the theoretical results of this paper.

6. Conclusions

In this paper, the sufficient conditions for the existence and uniqueness of the equilibrium solution are presented based on the contraction mapping principle. By constructing a suitable Lyapunov functional composed of fractional integral and definite integral terms, we calculate its first-order derivative to derive global asymptotic stability of the equilibrium point. The constructed method avoids calculating the fractional-order derivative of the Lyapunov functional. Furthermore, the presented results are described as the algebraic inequalities, which are convenient and feasible to verify the existence and asymptotic stability of the equilibrium solution. For further research, it is interesting and challenging to discuss the chaos phenomena, Hopf bifurcation, and synchronization control problem for fractional-order memristor-based hybrid BAM neural networks with leakage, time-varying, and distributed delays.

Conflicts of Interest

The authors declare that there are no conflicts of interest with regard to the publication of this paper.

Acknowledgments

This work is jointly supported by the National Natural Science Fund of China (11301308, 61573096, and 61272530), the 333 Engineering Fund of Jiangsu Province of China (BRA2015286), the Natural Science Fund of Anhui Province of China (1608085MA14), the Key Project of Natural Science Research of Anhui Higher Education Institutions of China (gxyqZD2016205 and KJ2015A152), and the Natural Science Youth Fund of Jiangsu Province of China (BK20160660).

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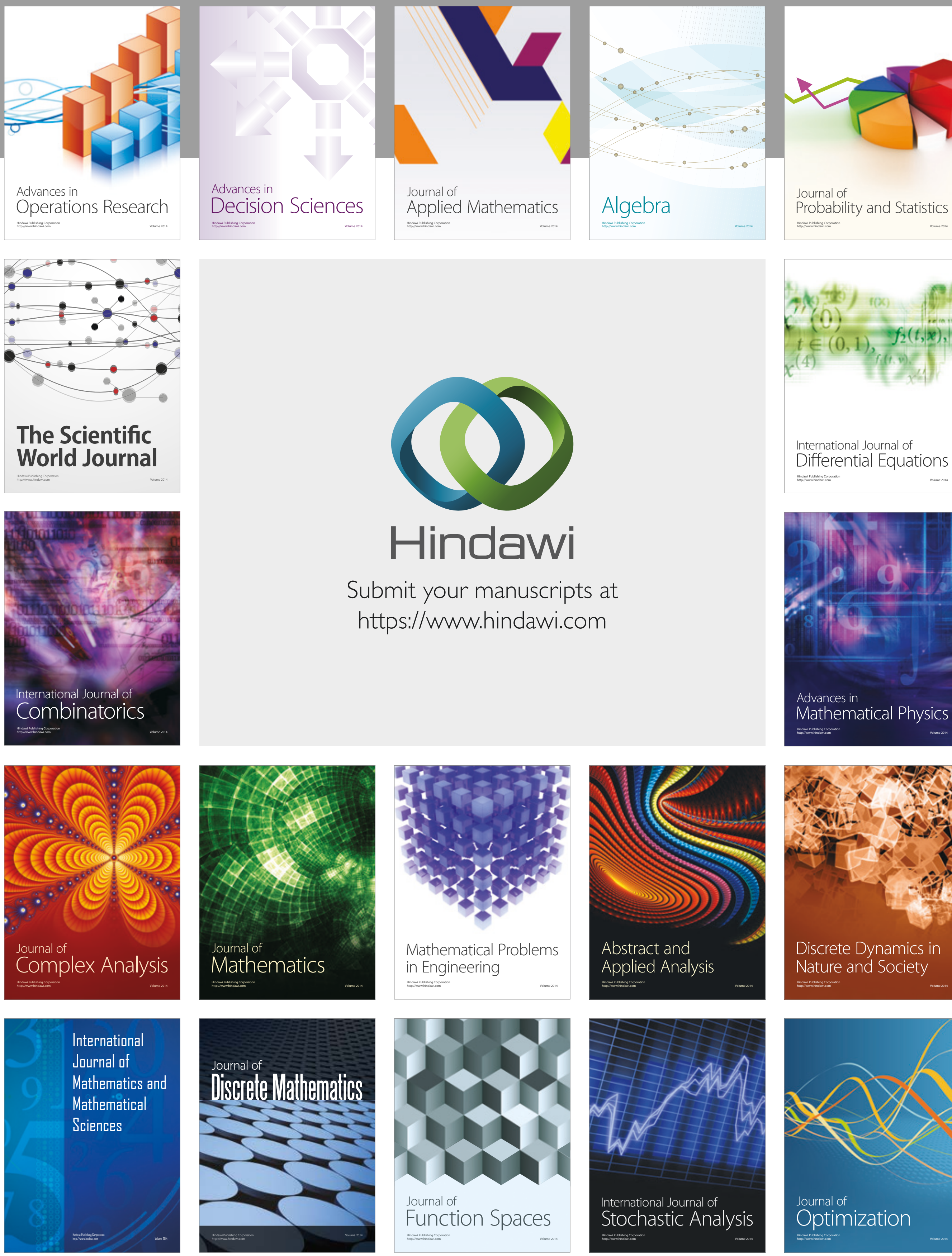
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