Effects of Topographic Variability and Lidar Sampling Density on Several DEM Interpolation Methods

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Abstract

This study aims to quantify the effects of topographic variability (measured by coefficient variation of elevation, CV) and lidar (Light Detection and Ranging) sampling density on the DEM (Digital Elevation Model) accuracy derived from several interpolation methods at different spatial resolutions. Interpolation methods include natural neighbor (NN), inverse distance weighted (IDW), triangulated irregular network (TIN), spline, ordinary kriging (OK), and universal kriging (UK). This study is unique in that a comprehensive evaluation of the combined effects of three influencing factors (CV, sampling density, and spatial resolution) on lidar-derived DEM accuracy is carried out using different interpolation methods. Results indicate that simple interpolation methods, such as IDW, NN, and TIN, are more efficient at generating DEMs from lidar data, but kriging-based methods, such as OK and UK, are more reliable if accuracy is the most important consideration. Moreover, spatial resolution also plays an important role when generating DEMs from lidar data. Our results could be used to guide the choice of appropriate lidar interpolation methods for DEM generation given the resolution, sampling density, and topographic variability.

Introduction

As defined by the U.S. Geological Survey, a grid Digital Elevation Model (DEM) is the digital cartographic representation of the elevation of the land at regularly spaced intervals in the *x* and *y* directions, using *z*-values referenced to a common vertical datum (Aguilar *et al.*, 2005). DEMs are essential to various applications, such as terrain modeling, soil-landscape modeling, and hydrological modeling (Anderson *et al.*, 2006; Walker and Willgoose, 1999). Consequently, the quality of the DEM and its derived terrain attributes becomes important in a range of spatial modeling techniques (Thompson *et al.*, 2001).

Recently, lidar (Light Detection and Ranging) has emerged as an important technology for the acquisition of high quality DEMs due to its ability to generate 3D terrain point data with high density and accuracy (Lohr, 1998; Wehr and Lohr, 1999; Lefsky *et al.*, 2002). Lidar is an optical remote sensing technology that measures properties of scattered light to find the range and/or other information of a distant object. The range to an object is calculated by measuring the time delay between transmission of a laser pulse and detection of the reflected signal (Wehr and Lohr, 1999). With high-density lidar data, very detailed high-resolution DEMs can be generated with great accuracy using appropriate interpolation methods (Liu *et al.*, 2007a). Compared to the traditional DEM derived from photogrammetric techniques, such as the U.S. Geological Survey 30 m DEM data, the lidar-derived DEM is more reliable and accurate with a higher resolution. The principles of lidar and its application to produce high-quality DEMs have been well documented (Lohr, 1998; Wehr and Lohr, 1999; Lloyd and Atkinson, 2002; Liu *et al.*, 2007b). The use of airborne lidar sensors for topographic mapping is rapidly becoming a standard practice in a range of applications, such as storm water assessment, flood control, visualization, etc. (Hodgson and Bresnahan, 2004).

As the DEM plays an important role in spatial modeling, it is necessary to consider the accuracy of the DEM and its derived terrain attributes (Thompson *et al.*, 2001). Several studies have indicated that morphology-derived variables, such as average terrain slope, are positively correlated with the increase in the global error of the modeled surface (Toutin, 2002). Despite the ability of lidar to gather point samples at very small separation distances, these points are obtained irregularly, and thus interpolation is necessary to generate continuous surfaces. As a result, the interpolation from points to a grid introduces uncertainties into the DEM (Lloyd and Atkinson, 2002; Smith *et al.*, 2004).

Previous studies have demonstrated that the accuracy of derived DEMs is significantly influenced by various factors, such as topographic variability, sampling density, interpolation methods, spatial resolution, etc. (Quattrochi and Goodchild, 1997; Caruso and Quarta, 1998; Gong *et al.*, 2000; Thompson *et al.*, 2001; Kienzle, 2004; Smith *et al.*, 2004; Aguilar *et al.*, 2005; Anderson *et al.*, 2006; Liu *et al.*, 2007a). For example, MacEachren and Davidson (1987) studied the relationship between observation point density and the accuracy of the derived DEM, and they demonstrated that as the density of observation points increases, the accuracy of the resulting DEM increases. Anderson *et al.* (2006) investigated the effects of data density reduction on DEMs of various horizontal resolutions, and their research showed that lidar datasets could withstand substantial data reductions without decreasing the DEM quality, but the level of reduction that

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could be withstood is significantly influenced by the DEM horizontal resolution. Liu *et al.* (2007a) further demonstrated that lidar data set density reduction can increase the efficiency of DEM generation in terms of file size and processing time, and the extent to which a data set can be reduced depends on the original data density, terrain characteristics, the interpolation method for DEM generation, and DEM resolution. Further comparisons with different interpolation methods and DEM resolutions are required for a comprehensive guide on the use of different interpolation methods and resolutions for lidar-derived DEMs (Liu *et al.*, 2007a). Aguilar *et al.* (2005) studied the effects of terrain morphology, sampling density, and interpolation methods for scattered sample data on the accuracy of the DEM, using a factorial scheme and an analysis of variance. Their research found that morphology has the greatest influence on DEM quality, followed by the lidar sampling density and interpolation method. Behan (2000) quantified the error within models produced from different interpolation algorithms by examining the global or average error differences between two interpolation methods. The study found that the most accurate surfaces were created using grids that had a spacing similar to the original points (Smith *et al.*, 2004). Hodgson and Bresnahan (2004) also indicated that the accuracy of the DEM was significantly different between land-cover categories, but the DEMs derived from lidar were less sensitive to terrain slope than those derived from digital photogrammetry.

Although a considerable amount of literature has studied the relationship between lidar-derived DEM accuracy and topographic variability, lidar sampling density, spatial resolution, and interpolation methods, few studies have comprehensively studied the effects of all the aforementioned factors together. Previous studies primarily focus on one or two aspects of these influencing factors (Lloyd and Atkinson, 2002; Smith *et al.*, 2004; Anderson *et al.*, 2006; Liu *et al.* 2007a). Therefore, the objective of this study is to quantify the effects of topographic variability and lidar sampling density on DEM accuracy with respect to different interpolation methods and spatial resolutions, and thereby aims to provide a relatively comprehensive understanding of DEM generation from lidar data.

Data and Methods

Study Area and Lidar Data

Our study area is located northeast of Oakhurst, California, and encompasses approximately 118 km^2 (Figure 1). The average elevation of the study area is 1,631 m with a standard deviation of 427 m. The minimum and maximum elevations are 758 m and 2,652 m, respectively. We contracted the National Center of Airborne Laser Mapping (NCALM) at the University of Florida to fly over and map our study area. The survey used an Optech GEMINI Airborne Laser Terrain Mapper (ALTM) mounted in a twin-engine Cessna Skymaster (Tail Number N337P). The entire study area was covered in five survey flights: two each on 13 and 14 September (days-of-year 256 and 257) and one final flight on 15 September 2007 (day-of-year 258). TerraSolid's TerraScan (**http://terrasolid.fi**) software was used to classify the lidar points and generate the "bare-earth" data points.

The raw lidar data consist of 103 flight strip files that range in size from 50 to 610 MB in the LAS binary format with a total file size of 36.8 GB. As processing all lidar data is highly computationally intensive and time-consuming, we selected 20 tiles to represent different topographic characteristics of the study area (Table 1). Figure 2 shows selected tiles with different slopes. The size of each tile is 500 m^2, and the average "bare-earth" data density is about 1.32 points per m^2. Most of the elevations are over 1,000 m, but their standard deviations are quite different. The slope varies from 6 to 26 degrees. We calculated the CV (coefficient of variation) of elevation as the ratio of the standard deviation of elevation to the elevation mean, which is a normalized measurement of dispersion as defined in Equation 1:

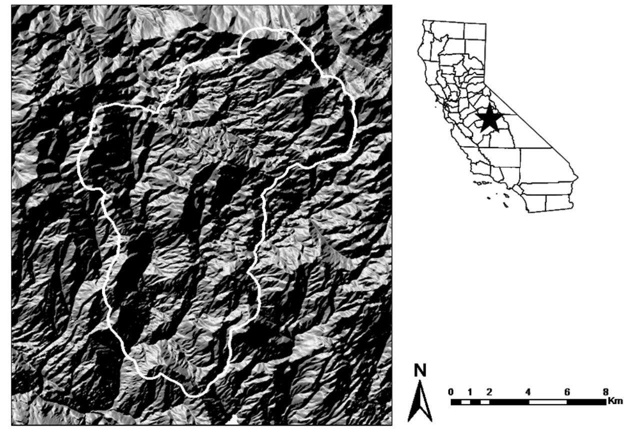


Figure 1. Study area overlaid with hill shading of USGS 30 m DEM, Fish Camp, California.

$$CV = \frac{\sqrt{\sum (z_i - \overline{z})^2 \Big/ n}}{\overline{z}} \tag{1}$$

where z_i refers to the elevation at point i, $\overline{z}$ is the mean of elevation, and n is the total number of points in a tile.

Interpolation Methods

There are many methods to derive a DEM from point data, and each has its own advantages and disadvantages depending on the characteristics of the data sets (Caruso and Quarta, 1998). In this study, we compared several commonly used interpolation methods: natural neighbor (NN), inverse distance weighted (IDW), triangulated irregular network (TIN), spline, ordinary kriging (OK), and universal kriging (UK). Detailed descriptions of each method follow.

Natural neighbor (NN) is a simple interpolation method that finds the closest subset of input samples to an unknown point and applies weights to them based on proportionate areas in order to interpolate a value (Sibson, 1981). A Voronoi diagram is first constructed of all the given points, and the natural neighbors of any point are those associated with its neighboring Voronoi polygons. A new Voronoi polygon is then created around the interpolation point, and the proportion of overlap between the new polygon and the initial polygons are then used as the weights. This method is simple, requiring no parameterization from the user, and it works equally well with regularly and irregularly distributed data (Watson, 1992; Sambridge *et al.*, 1995).

Inverse distance weighted (IDW) is also a simple interpolation method that estimates the value of a point by averaging the values of sample data points within its neighborhood (Bartier and Keller, 1996). Based on the fundamental geographic principle that objects that are closer together tend to be more alike than objects that are farther apart (Tobler, 1970), the idea of this method is to give more weight to nearby points than to distant points (Caruso and

Table 1. Summary of the Descriptive Statistics of the Lidar Tiles used in this Study

Tile number	Elevation (m) Min	Max	Mean	SD	CV	Slope (degree)
1	1612.42	1669.07	1630.62	7.27	0.00	6.66
2	1615.36	1659.03	1632.87	8.23	0.01	7.79
3	2501.75	2578.48	2530.32	13.60	0.01	12.12
4	1917.75	2006.92	1968.83	16.74	0.01	13.25
5	898.01	940.10	914.88	8.39	0.01	10.16
6	1397.25	1454.59	1417.61	13.66	0.01	11.49
7	1416.30	1499.67	1453.36	18.00	0.01	15.37
8	897.98	976.37	934.65	14.40	0.02	10.79
9	2318.70	2484.13	2418.55	39.25	0.02	18.24
10	1904.38	2034.28	1981.24	33.30	0.02	17.16
11	1799.08	1959.56	1903.73	33.67	0.02	20.39
12	1163.31	1283.08	1237.19	22.72	0.02	16.17
13	1117.25	1245.46	1178.79	29.87	0.03	19.33
14	1601.21	1818.81	1712.97	43.67	0.03	24.42
15	836.35	946.47	895.24	24.76	0.03	14.15
16	1489.80	1709.53	1591.91	46.06	0.03	22.34
17	1450.18	1668.41	1542.66	50.13	0.03	23.31
18	979.90	1167.89	1087.77	40.93	0.04	21.42
19	1063.39	1279.52	1164.16	46.16	0.04	26.44
20	1129.25	1353.33	1233.10	53.61	0.04	25.38

Quarta, 1998). The influence of known points on the interpolated values based on their distance from the output point can be controlled by defining the power. A higher power places more emphasis on the nearest points and results in a less smooth surface with more detail, while a lower power gives more influence to the points that are farther away, and results in a smoother surface with less detail. The characteristics of the interpolated surface can be controlled by applying a fixed or variable search radius, which limits the number of input points that can be used for calculating each interpolated cell. In our study, we used a power of two and a variable search radius with 12 minimum points for interpolation, which is most commonly used with IDW for interpolation comparison (Chaplot *et al.*, 2006).

Triangulated irregular network (TIN) is an alternative terrain representation approach that partitions a surface into a set of contiguous, non-overlapping triangles (Polis and McKeown, 1992). Elevation is recorded for each triangle node, while elevations between nodes can be interpolated, thus allowing the generation of a continuous surface. We adopted an interpolation approach similar to Hu *et al.* (2009): the interpolated elevation is the weighted sum of elevations of its surrounding triangle vertices, and the weights are defined as the areal proportions of the sub-triangles to the original triangle.

The spline method estimates values using a mathematical function that minimizes overall surface curvature, resulting in a smooth surface that passes exactly through the sample points (Bojanov *et al.*, 1993). In this study, we chose the regularized spline (Mitáová and Hofierka, 1993; Mitáová and Mitá, 1993), which needs two parameters to be defined: weight and number of points. The weight parameter defines the weight of the third derivative of the surface in the curvature minimization expression. The value of this parameter must be equal to or greater than zero. Generally, a higher weight would generate a smoother surface; in this study, we found the weight of 0.1 and 12 points produced reasonably good results.

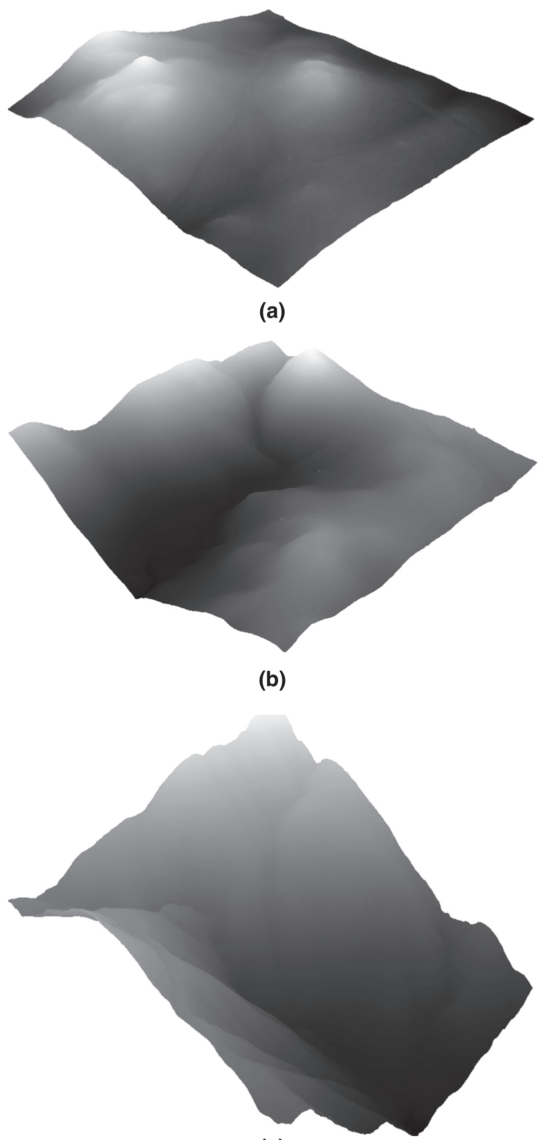
Kriging is an advanced geostatistical procedure that generates an estimated surface from a scattered set of points with *z*-values (Cressie, 1990; Caruso and Quarta, 1998). It is based on the regionalized variable theory that assumes that the spatial variation in the phenomenon represented by the *z*-values is statistically homogeneous throughout the surface. To quantify the spatial variation, the semivariogram is estimated by the sample semivariogram, which is computed from the input point data set. We evaluated two commonly used kriging approaches in this study: ordinary kriging (OK) and universal kriging (UK). Ordinary kriging assumes that the variation in *z*-values is free of any structural component (drift) (Cressie, 1988). Universal kriging assumes that the spatial variation in *z*-values is the sum of three components: (a) a structural component (drift) representing a constant trend over the surface, (b) a random but spatially correlated component, and (c) random noise representing the residual error (Armstrong, 1984; Zimmerman *et al.*, 1999). We applied ordinary kriging with a spherical model and universal kriging with a linear drift model in this study. All parameters were determined by weighted least squares methods, which are commonly used to fit semivariogram models (Zhang *et al.*, 1995).

Accuracy Assessment

To evaluate the interpolation accuracy, a ten-fold cross-validation (Kohavi, 1995; Picard and Cook, 1997) was applied to our data sets: the lidar point data were first randomly divided into 10 sub-samples. We retained one of the ten sub-samples as the validation data for testing the model performance, and used the remaining nine sub-samples as training data for DEM interpolation. We repeated the process ten times so that all sample points were used for both training and validation. Root Mean Squared Errors (RMSE), a widely used global accuracy measure for evaluating the performance of DEMs (Aguilar *et al.*, 2005), were calculated to assess the accuracy of derived DEMs:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(z_i^{predicted} - z_i^{real}\right)^2}{n}} \tag{2}$$

where $Z^{predicted}$ is the predicted elevation, Z^{real} is the real elevation from lidar ground points, and n is the total number of points. Note that the objective of this study focuses on the interpolation errors only. Implications of uncertainty from lidar measurement errors will be addressed in the discussion section.



(a)

(b)

(c)

Figure 2. Selected tiles with different average slopes: (a) 7.79 degrees, (b) 15.37 degrees, and (c) 25.38 degrees.

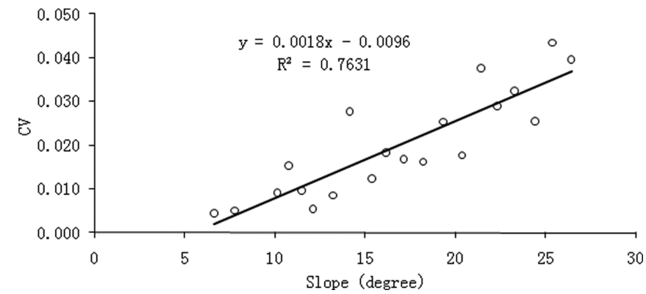


Figure 3. Correlation between elevation CV and slope.

Factors Influencing DEM Accuracy

Topographic variability is considered to be a main source of uncertainties for DEM production. In order to investigate its influence on DEM accuracy, we used 20 data sets of different topographic characteristics to generate DEMs and estimated their interpolation errors (RMSE). Different interpolation methods with different spatial resolutions were tested in each data set. We used elevation coefficient of variation (CV) to measure the topographic variability. When data sets have very different means, it is recommended that the coefficient of variation (CV) is employed for comparison instead of the standard deviation (Zar, 1999). In this study, we used CV to represent the topographic variability and investigate its impact on interpolation methods. It should be noted that, in our study, we found that the CV was strongly correlated with the slope ($R^2 = 0.76$) due to the fact that areas with high topographic variability tend to occur in high slope regions (Figure 3). To avoid the collinearity impact on the regression analysis, we only studied the impact of CV on the DEM accuracy. Therefore, CV used in this study represents not just the impact of topographic variation, but to some extent, it also represents the mixed impact of the terrain morphology because of the collinearity problem of terrain attributes. We will address the implications of this issue in the discussion section.

In addition to CV, lidar sampling density, commonly termed "nominal posting density" (Tullis *et al.*, 2009), also plays an import role in DEM accuracy (Raber *et al.*, 2007). Under-sampling will decrease the DEM accuracy, while over-sampling will result in redundant data and extra computer time (Balce, 1987). Lidar is able to gather point samples at very small separation distances and provides highly dense point data from which to generate a high-quality DEM. However, lidar points are obtained irregularly, some of which might represent over-sampled terrain and provide redundant information (Liu *et al.*, 2007a). Processing all lidar datasets would be computationally intensive and time-consuming and may not provide a significant improvement on accuracy. In this case, data reduction might be useful and efficient to produce a more manageable and operationally sized terrain data set for DEM generation (Anderson *et al.*, 2005; Anderson *et al.*, 2006).

Statistical Test

We compared the performance of each interpolation method identified in the Interpolation Methods Sub-section for each tile of lidar data. The resolution of the DEM was specified as 0.5, 1, 5, and 10 m. In order to investigate the influence of sampling density on the accuracy of lidar-derived DEM, all tiles of data sets were randomly reduced to 90 percent, 80 percent, 70 percent, 60 percent, 50 percent, 40 percent, 30 percent, 20 percent, and 10 percent of their original point density, and the above processes of DEM generation and accuracy assessment were repeated for each density (Figure 4). All of the data processing procedures are automated in our program, which was developed in Visual Basic 6.0 and ESRI ArcObjects® 9.3.

Finally, we used multivariate regression analysis to quantify the effects of CV and sampling density on the interpolation errors (RMSE) for each interpolation method at different spatial resolutions. We chose CV and density as the independent variables or predictors and RMSE as the dependent variable. Before applying the regression analysis, we first examined the bivariate relationship between RMSE versus CV and density, respectively, in order to test the regression assumption that there exists a linear relationship between the independent and dependent variables. We found that RMSE had a linear relationship with CV but

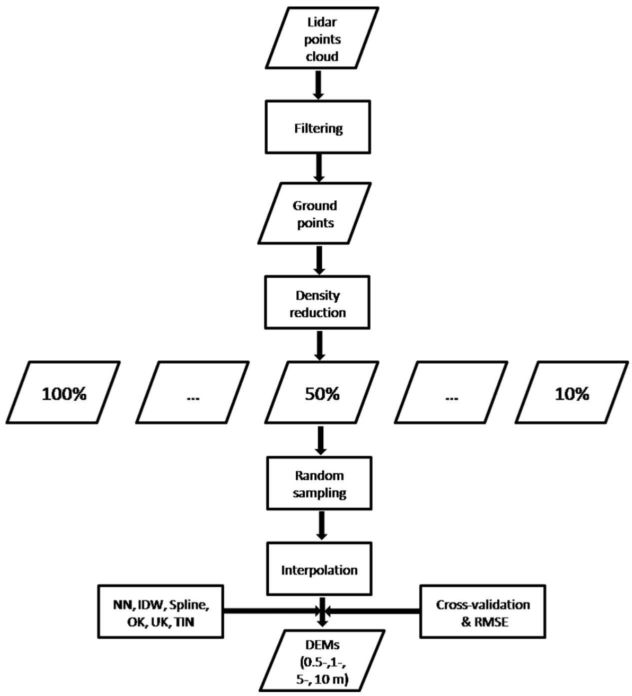


Figure 4. The scheme of evaluating the effect of topographic variability, sampling density on different interpolation methods.

exhibited a non-linear relationship with density. However, we discovered that the relationship between RMSE and the logarithms of density with natural base "*e*", labeled $\log_e$ *density*, was nearly linear. Therefore, the variable "density" was transformed using the logarithms function with the natural base "*e*." We then compared different regression models, and we found that a multiple linear regression model, as described in Equation 3, was sufficient to explain the variability of RMSE.

$$RMSE = a \times CV + b \times \log_e density + C \quad (3)$$

where a and b refer to the coefficients of CV and $\log_e$ *density*, respectively, and C is the constant. Because CV and density have different units and scales, their coefficients were standardized for evaluating their relative influences on RMSE (Zar, 1999).

Furthermore, to compare the performance of interpolation methods, we applied Tukey's tests (Zar, 1999) to conduct the multiple comparisons. We implemented both the multivariate regression analysis and Tukey's tests in SPSS (Statistical Package for the Social Sciences). Figure 4 summarizes the scheme implemented in this study.

Results

Topographic Variability

Plate 1 illustrates the relationship between the DEM RMSE and CV for different interpolation methods at different resolutions. The results show that CV has positive effects on the error in DEMs. A higher CV will result in higher uncertainties and errors no matter what interpolation method and resolution are used, and the relationship between them is approximately linear. Although the accuracy is affected by CV, OK and UK always provide better predictions compared to other methods. At higher resolutions (0.5 and 1 m), IDW, NN, and TIN seem to be more sensitive to CV, and produce relatively higher RMSE, particularly when CV is high. However, at lower resolutions (5 and 10 m), their prediction performances became almost as good as OK and UK, while spline produced the worst prediction.

To study the impact of lidar sampling density on DEM accuracy, we reduced the point density of lidar datasets to 90 percent, 80 percent, 70 percent, 60 percent, 50 percent, 40 percent, 30 percent, 20 percent, and 10 percent using a random sampling technique, and performed a similar analysis for each density. The relationships between RMSE and CV were quite similar as mentioned above, but RMSE was affected by sampling density as well, which is described in the next section.

Lidar Sampling Density

In this section, we analyze the impact of data density on DEM accuracy with different interpolation methods and across different CVs. The average density of the lidar data set representing bare ground is around 1.32 points per m^2. Table 2 shows their corresponding densities in terms of points per grid. It should be noted that the density values in Table 2 are equal to the average number of points that fall into a grid.

We applied different interpolation methods to generate DEMs at multiple resolutions, and Plate 2 illustrates the relationships between RMSE and density with respect to interpolation methods and resolutions. The RMSE in Plate 2 is an average across the range of CV, which allows us to examine the relationship between RMSE and lidar sampling density. According to Plate 2, in general, the RMSE decreases as the density increases, but the effects of density on DEM accuracy are quite different depending on spatial resolution and interpolation methods. At higher resolutions (0.5 and 1 m), RMSE seems to decrease exponentially as density increases, and the changes of RMSE of IDW and spline are slightly more sensitive to changes of density than other methods, particularly when the density is low. However, when the density is over 70 percent, the impact of density becomes relatively small for each method. As we can see in Table 2, the number of points falling into each grid increases dramatically when the grid size increases. Consequently, at lower resolutions (5 and 10 m), the RMSEs are nearly the same at each density, and no significant impact resulting from density was observed. However, the case of spline is more complicated as its RMSE changes in different directions. Overall, we still found that OK and UK provided the highest accuracy of all the methods.

Multivariate Regression Analysis

The regression analysis was conducted to investigate those effects quantitatively, and the corresponding R^2 and coefficients with significance level are shown in Table 3. The R^2 values of IDW, spline, OK, and UK range from 0.7 to 0.9, and decrease as the resolution becomes lower. For example, the R^2 value for spline is 0.88 at 0.5 m resolution, but decreases to 0.44 at 10 m resolution. The cases for NN and TIN are quite different and complicated. Their R^2 values are relatively smaller than those of other interpolation methods, and they increase as the resolution decreases. Overall, the regression models have relatively high R^2 values, which indicate the good fit of the models and a strong ability to explain most of the variability in RMSE by differences in CV and density. We tested the normality assumption by examining the distribution of residuals (Cook and Weisberg, 1982), and found that the residuals followed the normal distributions. Meanwhile, we also checked the multi-collinearity issue among the independent variables (i.e., CV and $\log_e$

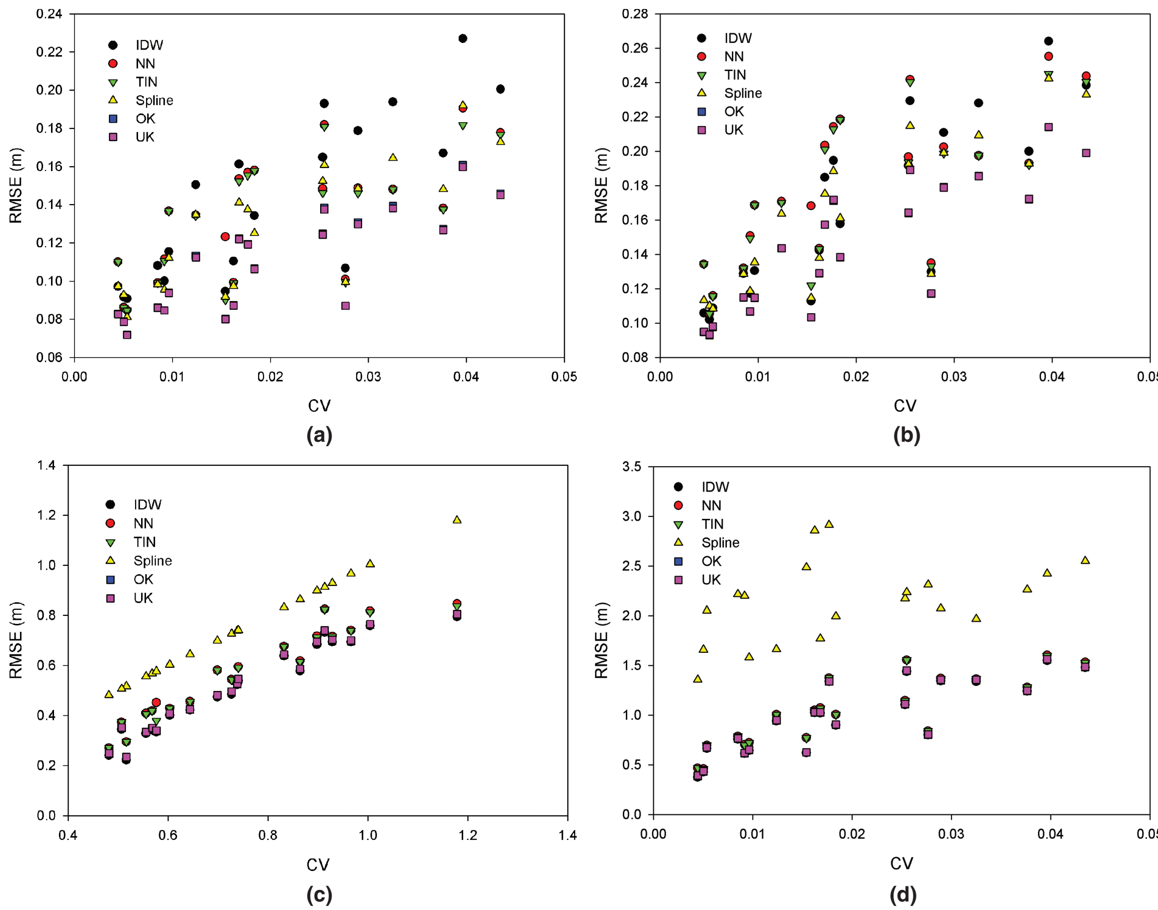


Plate 1. Relationships between the RMSE and elevation CV at multiple resolutions: (a) 0.5 m, (b) 1 m, (c) 5 m, and (d) 10 m.

TABLE 2. SAMPLING DENSITY UNDER DIFFERENT DATA REDUCTIONS AND RESOLUTIONS (POINTS/GRID)

Data reduction (%)	Resolution (m)			
	0.5	1	5	10
100	0.34	1.34	33.54	134.15
90	0.30	1.21	30.18	120.73
80	0.27	1.07	26.83	107.32
70	0.23	0.94	23.48	93.90
60	0.20	0.80	20.12	80.49
50	0.17	0.67	16.77	67.07
40	0.13	0.54	13.41	53.66
30	0.10	0.40	10.06	40.24
20	0.07	0.27	6.71	26.83
10	0.03	0.13	3.35	13.41

density) for the multivariate regression analysis, and found that the variance inflation factor was only 1.02, which is much less than the recommended cut-off value of 10 (Kutner, *et al.* 2004).

According to the coefficients and their significance levels, there is a strong linear correlation between RMSE and CV at different resolutions. The linear correlation between RMSE and $\log_e$ *density* is significant at fine resolutions (0.5 and 1 m), but becomes insignificant at coarse resolutions (5 and 10 m), which indicates that the effects of density on RMSE are quite small at such coarse spatial scales.

The relative contributions of CV and density to RMSE were evaluated by comparing the standardized coefficients, which take into account the different units and scales of the variables. As shown in Table 3 (values in parentheses), the standardized coefficients of $\log_e$ *density* are relatively high and compatible with those of CV under the resolutions of 0.5 and 1 m, but they decrease to nearly zero under the resolutions of 5 and 10 m. By contrast, coefficients of CV increase as the resolution becomes coarser. The spline method is more sensitive to density than other methods as its coefficient of $\log_e$ *density* is higher than the others.

Comparison of Interpolation Methods

In addition to CV and data density, the interpolation method is another important factor that influences the accuracy of a DEM. We compared and evaluated the performances of different interpolation methods in terms of the mean and standard deviation of RMSE. As shown in Plate 3, the performances of OK and UK are quite similar, and they always provide the smallest RMSE and standard deviation. IDW produces the largest RMSE and standard deviation under the resolutions of 0.5 and 1 m. However, its performance is nearly as good as OK and UK under the resolutions of 5 and 10 m. By contrast, spline provides a relatively small RMSE and standard deviation at high resolutions, but its RMSE and standard deviation are the highest at low resolutions.

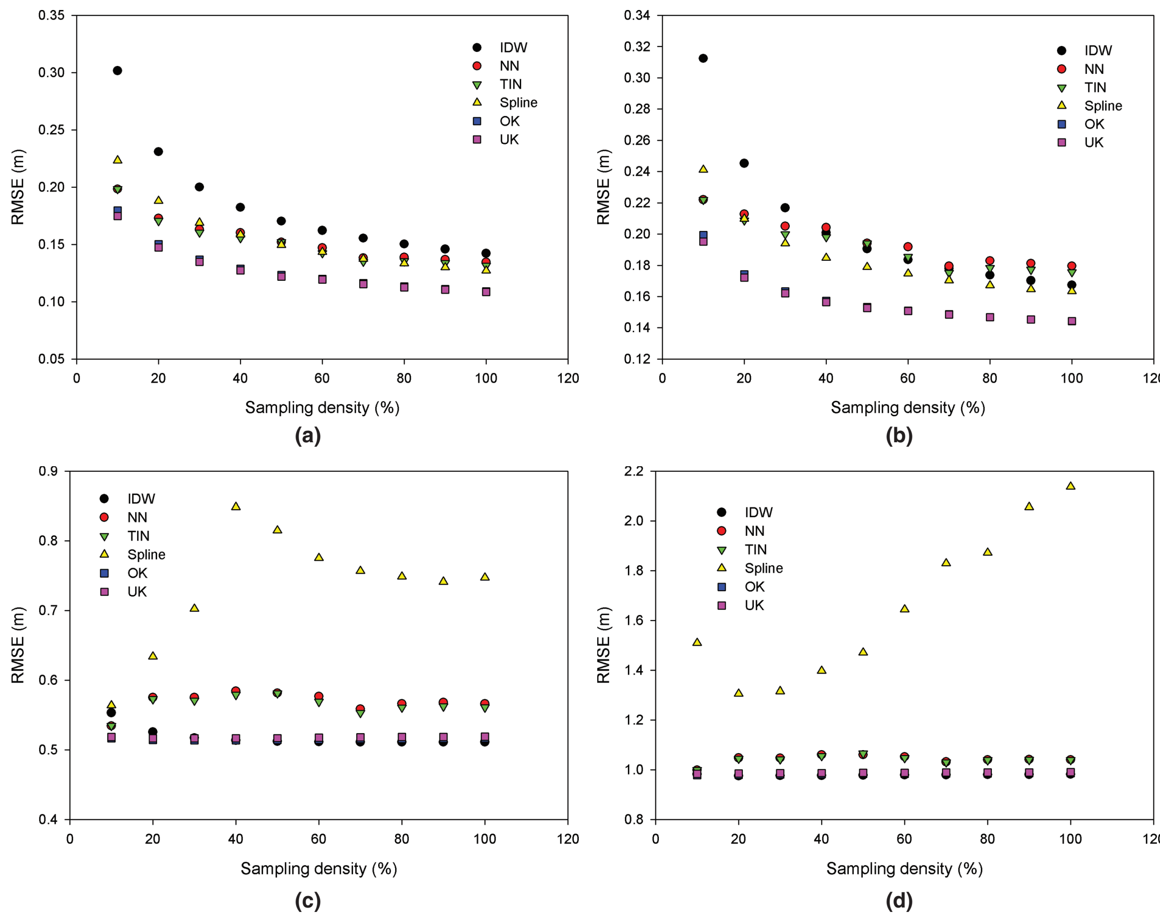


Plate 2. Relationships between the RMSE and sampling density at multiple resolutions: (a) 0.5 m, (b) 1 m, (c) 5 m, and (d) 10 m.

TABLE 3. COEFFICIENTS OF REGRESSION MODELS

		R^2 and Coefficients					
Resolution	Predictors	IDW	NN	TIN	Spline	OK	UK
0.5m	Constant	0.08**	0.10**	0.09**	0.09**	0.07**	0.08**
	CV	3.53**	2.09**	2.02**	2.08**	1.85**	1.80**
		(0.53)	(0.52)	(0.51)	(0.51)	(0.57)	(0.58)
	$\log_e$density	−0.07**	−0.03**	−0.03**	−0.05**	−0.03**	−0.03**
		(−0.67)	(−0.49)	(−0.52)	(−0.72)	(−0.65)	(−0.64)
	R^2	0.84	0.58	0.60	0.88	0.85	0.84
1m	Constant	0.09**	0.12**	0.12**	0.11**	0.09**	0.09**
	CV	4.09**	2.96**	2.88**	2.88**	2.72**	2.69**
		(0.60)	(0.58)	(0.58)	(0.65)	(0.71)	(0.72)
	$\log_e$density	−0.06**	−0.02**	−0.03**	−0.04**	−0.03**	−0.03**
		(−0.60)	(−0.30)	(−0.34)	(−0.55)	(−0.45)	(−0.44)
	R^2	0.82	0.47	0.50	0.84	0.81	0.80
5m	Constant	0.25**	0.31**	0.31**	0.48**	0.26**	0.26**
	CV	12.71**	12.90**	12.79**	14.13**	12.51**	12.53**
		(0.83)	(0.82)	(0.82)	(0.74)	(0.84)	(0.84)
	$\log_e$density	−0.03**	−0.01	−0.01	0.05**	−0.01	−0.01
		(−0.12)	(−0.03)	(−0.04)	(0.16)	(−0.06)	(−0.06)
	R^2	0.74	0.68	0.68	0.54	0.73	0.73
10m	Constant	0.46**	0.53**	0.53**	1.36**	0.47**	0.47**
	CV	25.37**	25.54**	25.39**	22.79**	25.30**	25.35**
		(0.84)	(0.84)	(0.84)	(0.55)	(0.84)	(0.84)
	$\log_e$density	−0.02	−0.01	−0.01	0.30**	−0.02	−0.02
		(−0.05)	(−0.03)	(−0.03)	(0.47)	(−0.05)	(−0.05)
	R^2	0.72	0.71	0.71	0.44	0.72	0.72

** Significant at 0.01 level; * Significant at 0.05 level; Coefficients without "**" or "*" were insignificant at 0.1 level; Values in parentheses are standardized coefficients.

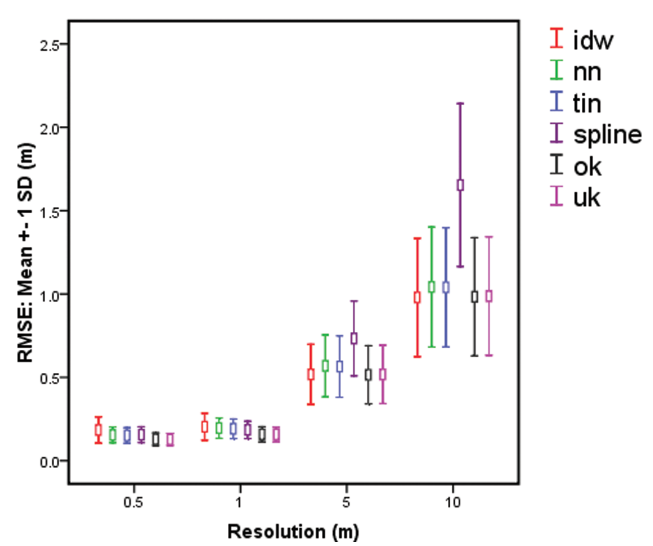


Plate 3. Error bars of different methods at multiple resolutions.

Similarly, the RMSE and standard deviation of NN and TIN are relatively small at high resolutions but become larger at low resolutions. We also performed Tukey's test to examine whether the differences in errors produced by different methods were significant at the 0.05 significance level, and the results are displayed in Table 4. The differences in performances of interpolators change at different resolutions. At the fine resolutions (0.5 and 1 m), OK and UK provide significantly better results than the other methods. At the coarse resolutions (5 and 10 m), the differences between interpolators are insignificant except for spline, which yields the poorest results of all methods.

As lidar gathers point samples at very small separation distances and provides huge amounts of 3D point data, processing entire lidar datasets would be highly computationally intensive and time-consuming. Therefore, computation time should be taken into account when choosing the appropriate interpolation methods. Although the absolute computation time may change under different computation conditions, such as the computer's CPU, available memory, and software used, comparison of the relative computation time under the same conditions could provide useful information for choosing a more efficient interpolation method. We recorded the average processing time for each method to generate one tile of lidar-derived DEM at 100 percent density, and the results are shown in Table 5. The computations ran under a Windows™ server with Intel® Quad-Core 2.93 GHZ processors and 8 GB memory using

Table 4. Comparing RMSEs Averaged over 20 Tiles based on Different Interpolation Methods by Tukey's Tests

Resolution	Mean of RMSE*
0.5m	IDW>Spline=NN=TIN>OK=UK
1m	IDW= NN=TIN=Spline >OK=UK
5m	Spline>NN=TIN=IDW =UK=OK
10m	Spline>NN =TIN=UK=OK=IDW

* Significance level: 0.05

Table 5. Average Computation Time of Different Methods for Each Tile (Unit: Second)

Resolution	IDW	NN	TIN	Spline	OK	UK
0.5m	377.29	154.00	248.57	1224.71	1428.00	1538.71
1m	81.57	110.57	197.14	337.29	796.29	841.14
5m	112.43	159.43	171.42	297.71	1215.00	705.29
10m	126.00	118.43	171.42	398.57	2216.71	2218.43

ESRI ArcGIS® 9.3. OK and UK generate the most accurate DEM, but their processing time is also the greatest. IDW, NN, and TIN prove to be simple and fast methods, while spline is moderate in computation time compared with the other methods.

Discussion

In our study, we used the elevation CV to represent the topographic variability (Chaplot *et al.*, 2006). One advantage of using the CV is that its calculation can be based on the raw lidar data. This is an important feature in our study, as generating DEM grids also depends on interpolation methods, which need to be separated from the topographic factor. There are many ways to represent topographic variability, such as fractal dimension, semivariogram, slope, and elevation variation (Frederiksen *et al.*, 1986; Chaplot *et al.*, 2006). It is difficult to apply all these factors together in a single analysis due to the strong collinearity among them. For example, the semivariogram is strongly correlated with the fractal dimension (Palmer, 1988). In this study, we also found that the elevation coefficient of variation was linearly correlated with slope ($R^2 = 0.763$). Our results show that the topographic variability contributes significantly to the DEM RMSE. However, future research is needed to explore what are the best variables to represent topographic variability that will have the greatest effect on the interpolation accuracy.

According to our study results, it is obvious that topographic variability and sampling density have significant influences on the accuracy of lidar-derived DEMs. As the complexity of the terrain increases, the uncertainties in the derived DEM increase. This result is similar to other findings. For example, Hodgson and Bresnahan (2004) indicated that the observed elevation error in steeper slopes (e.g., 25°) was estimated to be twice as large as those in flatter slopes (e.g., 1.5°). The pattern of highest magnitude error was also observed to occur in the areas of greater surface roughness (Smith *et al.*, 2004). In addition, the effects of topographic variability also changed when we used different interpolation methods and resolutions. For instance, IDW, NN, and TIN seem to be more sensitive to CV than other interpolation methods, particularly when CV and spatial resolution are relatively high. As demonstrated by other research (Aguilar *et al.*, 2005; Liu *et al.*, 2007a; Hu *et al.*, 2009), lidar sampling density was also an important factor affecting the accuracy of derived DEM. Increasing sampling density could reduce the interpolation error. Unlike the effects of CV, the relationship between sampling density and interpolation error is non-linear, which has also been reported in the literature (Aguilar *et al.*, 2006; Hu *et al.*, 2009). At high resolutions (0.5 and 1 m), increasing the density would significantly reduce the errors, but this impact becomes quite small after the density reached 70 percent of the original lidar data density. Similarly, several other studies also demonstrated that a lidar data set can be reduced by up to 50 percent of its original data density without degradation of the quality of the DEM. This means that lidar data can be reduced to a certain level without significantly decreasing the accuracy of the DEM, while reducing the processing time for DEM generation significantly (Anderson *et al.*, 2006; Liu *et al.*, 2007a). However, it is recommended that critical elements be kept while removing less important elements when conducting data reduction (Chou *et al.*, 1999; Liu *et al.*, 2007a), and the level of data reduction depends on terrain complexity and spatial resolution (Anderson *et al.*, 2006; Liu *et al.*, 2007a). For instance, higher data density is required to reduce the uncertainty of the derived DEM if CV and spatial resolution are high.

The effects of CV and sampling density on interpolation errors were also quantified through the regression models. The relatively high R^2 indicates that our regression models are able to explain most of the variability in RMSE using CV and density, and the coefficients with a significant level provide useful information on the levels of impacts of corresponding factors. Through an analysis of variance, Aguilar *et al.* (2005) concluded that DEM accuracy (RMSE) is more affected by morphology than sampling density. However, we found that this might be different depending on different spatial resolutions. At high resolutions (0.5 and 1 m), both CV and density contributed similarly to the RMSE, while CV had a higher level of impact than density at low resolutions (5 and 10 m). Provided that CV and sampling density are known, these models could then be used to estimate the approximate errors of generated DEMs produced by different interpolation methods with different resolutions and provide some guidance on choosing the appropriate method. Aguilar *et al.* (2006) also proposed an empirical model with the non-linear form to quantify the relationship between DEM accuracy, slope, and sampling density. Although the power factor for the slope was 0.89145, it was close to 1 and may indicate a quasi-linear relationship between DEM accuracy and slope. This is similar to our result that indicated a linear relationship between DEM accuracy and CV (Plate 1). Note that the CV and slope are linearly correlated (Figure 3). In this study, we did not consider the interactions of factors in our regression models. Further research is required to quantify the effects of factors' interactions on DEM accuracy.

In addition to the characteristics of the initial data points, DEM quality is also controlled by the interpolators. Since the final error of a DEM grid depends on both the measurement error as well as the interpolation error, the total error could be derived from the rule of error propagation if we know both errors. For example, assuming that the measurement error and the interpolation error are independent, we can apply the following equation to estimate the total error of a DEM grid (Thapa and Bossler, 1992):

$$\text{total error} = \sqrt{e_1^2 + e_2^2} \tag{4}$$

where e_1 is the measurement error, and e_2 the interpolation error. Our lidar data were collected by the Optech GEMINI ALTM, which typically had an elevation accuracy of 0.05 to 0.1 m. Absolute calibration was conducted by NCALM based on a calibration site consisting of 682 checkpoints surveyed with vehicle-mounted GPS, and the RMSE was 0.1 m. Note that the land-cover and vegetation structure play important roles in lidar accuracy (Hodgson *et al.*, 2003). Hodgson *et al.* (2005) reported higher RMSE in areas with tall vegetation, as the checkpoints are mostly constrained to the paved areas, and this may result in an underestimate of instrument errors. In this study, the RMSE varies with different methods and resolutions. For example, at the 0.5 m resolution, OK and UK (the two best interpolators) produce an RMSE ranging from 0.07 to 0.14 m, while IDW (the worst interpolator) results in an RMSE ranging from 0.09 to 0.2 m. This indicates that the errors produced by interpolators are as significant as the measurement errors and should be taken into account when generating high quality DEMs from lidar data. In the above discussion, we made the assumption that the instrument errors and interpolation errors were independent. However, it is possible that two errors are correlated, such as both errors are correlated to terrain complexity. In such a situation, the general rule of error propagation that takes into consideration the higher order error terms due to the correlation (Arbia *et al.*, 2003; Oksanen and Sarjakoski, 2005), or alternative error measurements (Wieczorek *et al.*, 2004; Hu *et al.*, 2009) are suggested. When the assumption that DEM errors are random and independent is difficult to satisfy because of spatial autocorrelation, Hu *et al.* (2009) demonstrated that the maximum error rooted in approximation theory is preferable to RMSE rooted in propagation theory.

Choosing the appropriate interpolator can become difficult since each method has its own advantages and disadvantages. Generally, kriging-based methods like OK and UK are able to produce more accurate DEMs, but they are computationally intensive and time-consuming. IDW, NN, and TIN are simple and fast methods of generating DEMs, and they are able to generate relatively accurate DEMs, but their performances decrease and become sensitive to topographic variability as the spatial resolution of the DEM increases. Spline seems to provide a trade-off between computation time and accuracy at high resolutions, but it becomes less reliable at the low resolutions, generating the highest RMSE and standard deviation (Plate 3 and Table 4). Some research also showed that when applied to large volumes of data, spline may suffer from the numerical instability that depends on the data density and the smoothness of the radial basis functions that have been used (Mitáová and Mitá, 1993; Lazzaro and Montefusco, 2002; Aguilar *et al.*, 2005). Our results indicate that there is no universal interpolation method superior in all aspects of performance; thus choosing an appropriate method should depend on the initial data characteristics and the research objectives. In addition, the spatial resolution should also be taken into account to evaluate the optimal method for generating lidar DEMs. In general, simple methods such as IDW, NN, and TIN would be more efficient for generating lidar-derived DEMs where there is a high sampling density (Lloyd and Atkinson, 2002; Anderson *et al.*, 2006; Chaplot *et al.*, 2006; Liu *et al.*, 2007a). However, if accuracy is of the greatest concern, then OK and UK would be good choices for generating a high-quality DEM.

It should be noted that although kriging-based methods produce the best accuracy in this study across different resolutions and lidar sampling densities, it does not mean that kriging methods are always the optimal interpolators for all terrain conditions. One important assumption for kriging methods is that there exists a spatial autocorrelation of the elevation. These assumptions could be met in most terrain conditions, particularly for the lidar data that produce high-resolution DEMs and discover the fine-scale terrain variability. However, there are cases where kriging methods do not perform well (Desmet, 1997), such as terrain with faults or spatial outliers with extremely high or low values. Terrain with faults or other types of abrupt changes will violate the intrinsic stationarity assumption of kriging methods (Isaaks and Srivastava, 1990). For the spatial outliers with extreme values, it will not only cause the incorrect estimate of the unknown points surrounding the outliers, but it will also cause the incorrect estimate of the semivariograms that could introduce errors to all the points estimated in the entire study area. On the other hand, methods such as IDW, NN, and TIN, which mainly use local known points to estimate the unknown points and require fewer assumptions on the data distribution, may perform better in those conditions. Further studies are needed to evaluate the influence of those terrain conditions on lidar DEM accuracy. In addition, Li (1992) found that there was a relationship between the spatial configuration of the sampling points and the DEM accuracy. In this study, our lidar data exhibited more or less regular sampling configurations, which is very common for airborne lidar data due to the flight line constraint and the laser scanning technique. However, when dealing with irregular sampling configurations, we may not find the same result as in this study.

Spatial resolution is also very important when generating DEMs from lidar data. The effects of CV and density are quite different at different resolutions. Changes in spatial resolution also have significant effects on the magnitude of error produced by different interpolation methods (Smith *et al.*, 2004). Generally, a higher resolution will provide more detailed spatial information, while a lower resolution will result in more uncertainties. Gao (1997) found that the DEM RMSES were linearly correlated with spatial resolutions from 10 to 60 m. Plate 3 also shows that, from 1 to 10 m resolutions, different interpolation methods exhibit similar linear trends except the spline method, which shows a relatively drastic change of RMSEs with respect to the resolution. However, from 0.5 to 1 m resolution, the linear trends are less obvious. In addition, the RMSE of DEMs derived from OK with 10 m resolution is about 10 times that of the 0.5 m resolution. However, choosing the appropriate resolution of a DEM is constrained by the source data density (Florinsky, 1998) and depends on the study purpose as well. Generating a high-resolution DEM from very sparse terrain data will be problematic, while generating a low-resolution DEM from high density terrain data will devalue the accuracy of the original data (Florinsky, 1998; Florinsky, 2002; Liu *et al.*, 2007a). Several studies have indicated that the optimal spatial resolution of a DEM should be as close as possible to, or slightly less than, the original point spacing, so that the number of grid cells would be roughly equivalent to the number of terrain data points in the covered area (Behan, 2000; Smith *et al.*, 2004; Liu *et al.*, 2007a). Therefore, the resolution of 0.5 or 1 m is appropriate to match the density of the original lidar point data in this study, which is about 1.32 points per m^2. In fact, many lidar data sets are able to generate 3D terrain point data with high density and accuracy. Consequently, high-quality DEMs can be derived with an equivalently high resolution. However, previous studies on the accuracy of lidar-derived DEM primarily focus on relatively low resolution, such as 5, 10, and 30 m (Gao 1997, Anderson *et al.*, 2006; Liu *et al.*, 2007a). As we demonstrate in this study, the relative contribution of CV and sampling density to the DEM RMSE are different between the high spatial resolutions (0.5 and 1 m) and the low spatial resolutions (5 and 10 m). However, future studies are required to evaluate the reliability and accuracy of lidar-derived DEM with a very high-resolution, such as less than 0.5 m, and test if they will behave differently from 0.5 and 1 m spatial resolutions.

Conclusion

This study aims to quantify the effects of topographic variability (measured by CV) and lidar sampling density on interpolation accuracy with respect to different interpolation methods and spatial resolutions. Unique to this study are the high-resolution DEMs (0.5 and 1 m) generated from lidar, as well as the comprehensive evaluation of the four factors influencing DEM accuracy, whereas previous studies primarily focus on coarse resolution DEMs (>1 m) and mainly examine one or two of the influencing factors. We found that topographic variability, lidar sampling density, interpolation methods, and spatial resolution have significant effects on the accuracy of lidar-derived DEMs. The errors of DEMs (RMSE) were observed to have a linear correlation with topographic variability (CV) and a non-linear correlation with lidar sampling density. The effects of CV and density on DEM accuracy vary with different interpolation methods and spatial resolutions. At the fine spatial resolutions (0.5 and 1 m), both CV and sampling density influence RMSE, and OK and UK provide significantly better accuracy than other methods. However, at the coarse spatial resolutions (5 and 10 m), RMSEs are mainly influenced by CV, and the accuracy differences between interpolators are insignificant except for spline, which is significantly less accurate than the other methods. Although we cannot change the terrain characteristics of the data, its effects on the accuracy of DEMs can be reduced through controlling the sampling density, interpolation methods, and spatial resolution. As lidar provides 3D terrain point data with very high density and accuracy, proper data reduction can significantly reduce the processing time of DEM generation without decreasing the accuracy. Simple interpolation methods, such as IDW, NN, and TIN, are more efficient in generating DEMs from lidar data, but kriging-based methods, such as OK and UK, are more reliable if accuracy is the most important consideration. In addition, spatial resolution is very important when generating DEMs from lidar data. With the increasing use of lidar data in environmental and urban applications, DEM generation is the essential step for lidar data processing. Our results could be used to guide the choice of appropriate interpolation methods for generating DEMs from lidar data given the resolution, sampling density, and topographic variability.

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