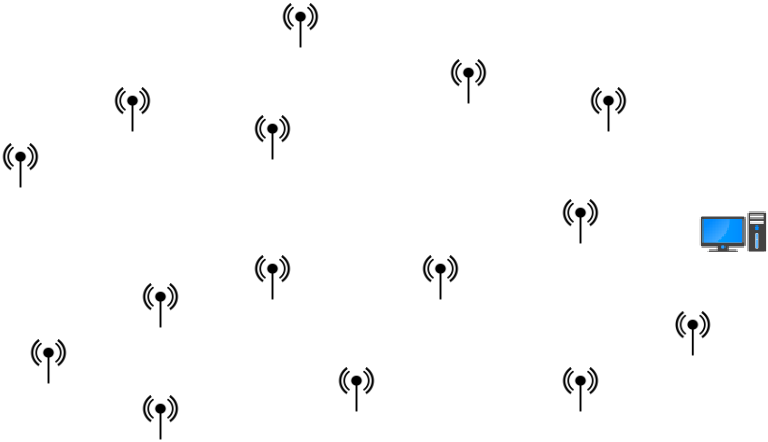
Radio Aggregation Scheduling

ALGOSENSORS 2015

Rajiv Gandhi, Magnús M. Halldórsson, Christian Konrad, Guy Kortsarz, Hoon Oh

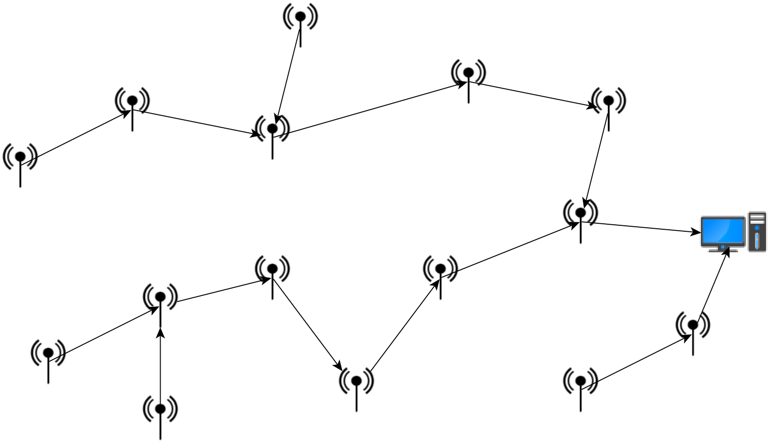
18.09.2015

Aggregation Scheduling in Radio Networks



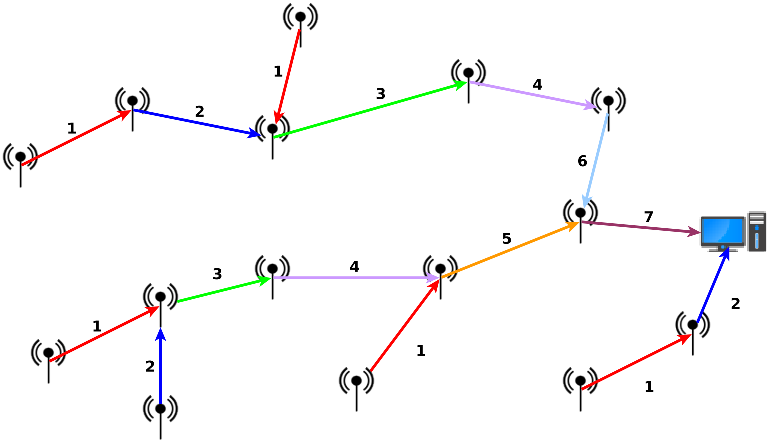
Goal: Convergecast, all nodes send data item to sink

Aggregation Scheduling in Radio Networks



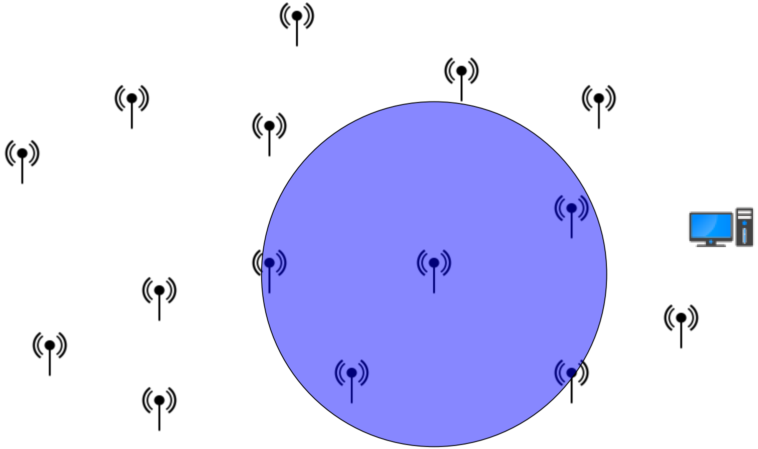
Goal: Spanning Tree

Aggregation Scheduling in Radio Networks



Goal: Conflict-free schedule of edge

Aggregation Scheduling in Radio Networks



Difficulty: Limited Transmission range

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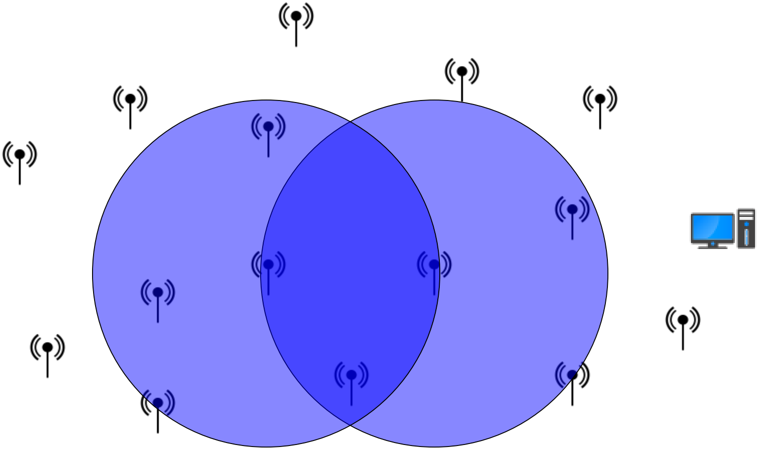
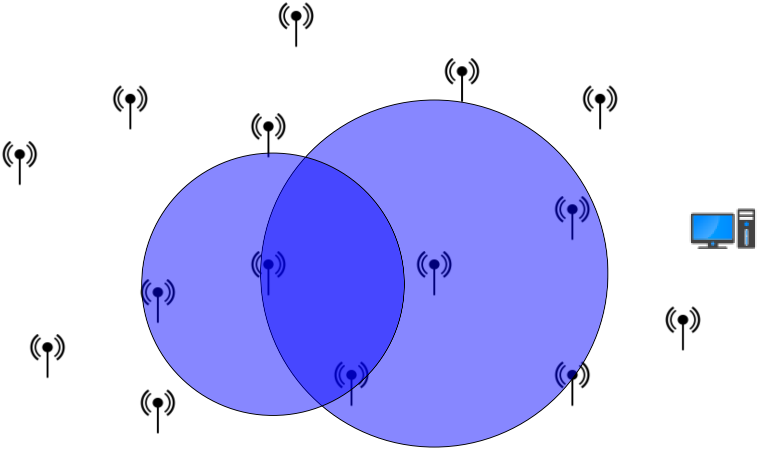


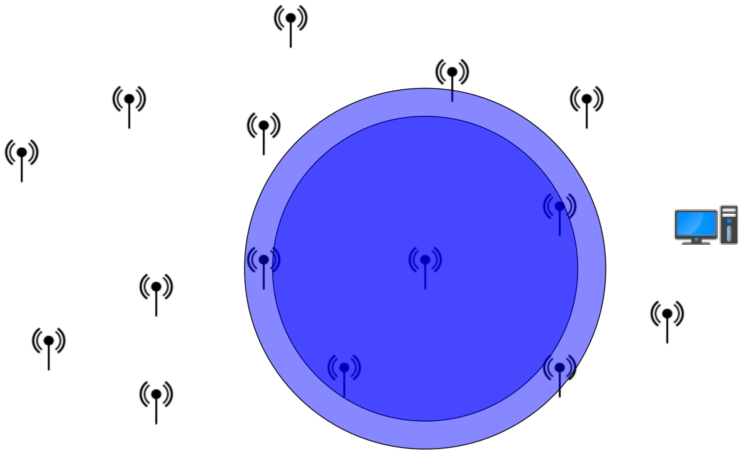
Difficulty: Interference

Aggregation Scheduling in Radio Networks



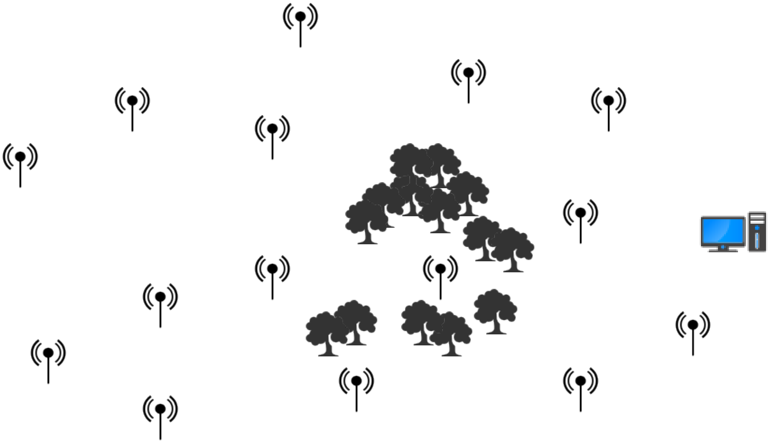
Difficulty: Transmission radii may vary

Aggregation Scheduling in Radio Networks



Difficulty: Transmission radii may be different from interference radii

Aggregation Scheduling in Radio Networks



Difficulty: Obstacles

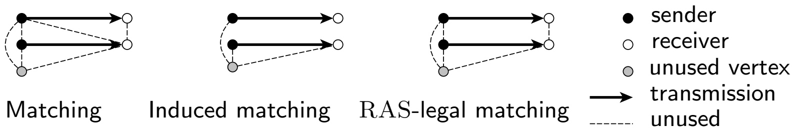
Radio Aggregation Scheduling

Problem Definition: Radio Aggregation Scheduling (RAS)

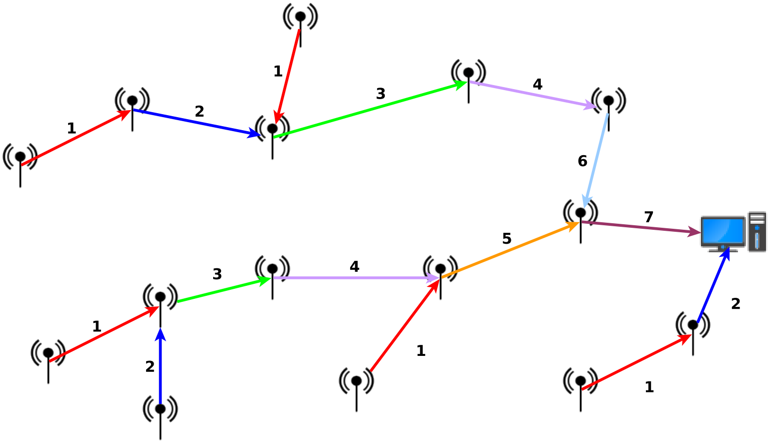
Given: Graph $G = (V, E)$ and sink node $s \in V$

Find: Directed matchings $M_1, M_2, \ldots, M_t \subseteq E$ so that:

1. $\cup_i M_i$ induce an in-arborescence directed towards s,
2. The M_i are conflict-free (RAS-legal matching),
3. t minimal.

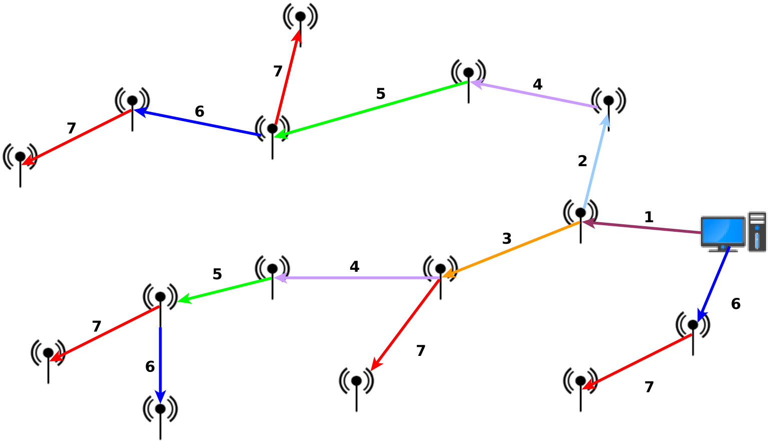


Broadcast: Reversing the Slots



Convergecast

Broadcast: Reversing the Slots



Broadcast, by reversing the slots

Broadcast in Radio-unicast Model

Brodcast in the Radio-unicast Model

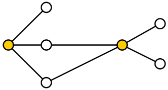
Given: Graph $G = (V, E)$ and source node $s \in V$

Each round, RAS-legal matching between informed & uninformed nodes

1. One-to-one communication (one sender to one receiver)
2. Interference constraint: Successful reception at receiver if exactly one neighbor transmits

Relation to other Models

Telephone model: One-to-one comm., no interference constraint



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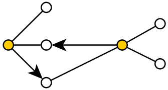

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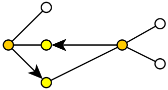

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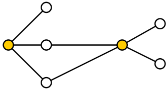
Radio model: One-to-many comm., interference constraint holds



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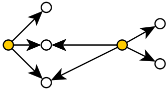

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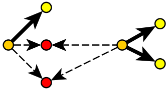

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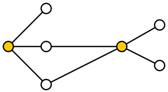

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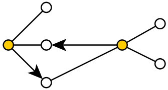

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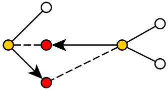

Known Results

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- Converge-cast schedule in $\Theta(diam + \omega(G))$ on unit-disc graph
- If interference radius larger than transmission radius in unit disc graph: $\mathrm{O}(1)$-approximation
- 2-approximation on unit interval graphs

[Wan et al., MobiHoc 2009], **[Xu et al., FOWANC 2009]**, **[Chen et al., MSN 2005]**
[An et al., I. J. Comput. Appl. 2011], **[Guo et al., J. of Combin. Opt. 2014]**

Our Objectives

- Systematic study of RAS, starting with general graphs
- Approximation algorithms for geometrically defined graph classes

Our Results

General Graphs

1. It is NP-hard to approximate RAS within factors $n^{1-\epsilon}$ or $\sqrt{\overline{d}n}$, where $\overline{d}$ is the average degree
2. Polynomial-time $\mathrm{O}(\sqrt{\overline{d}n})$-approximation algorithm

Interval Graphs

3. Polynomial-time $\mathrm{O}(\log n)$-approximation algorithm

Algorithm for General Graphs

Algorithm for General Graphs

Simulating the Radio Model

A round in the radio model can be simulated in Δ (max degree) rounds in the radio-unicast model

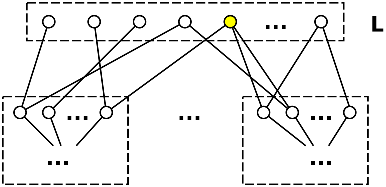
Theorem [Kowalski, Pelc, Dist. Comp. 2007]
Broadcast in the radio model can be done in $\mathrm{O}(\mathit{diam} + \log^2(n))$ rounds.

Corollary
Broadcast in the radio-unicast model can be done in $\mathrm{O}(\Delta(\mathit{diam} + \log^2(n)))$ rounds.

Lower Bound on OPT
diam is a trivial LB. Hence: $\tilde{\mathrm{O}}(\Delta)$-approximation

Algorithm for General Graphs

$G = (V, E)$, $\overline{d}$: average degree, informed node s, OPT known



1. $L \subseteq V$: nodes of degree at least $\sqrt{\overline{d}n}$ ($|L| \leq \sqrt{\overline{d}n}$)
2. Inform L sequentially along shortest paths from s in $\mathrm{O}(diam\sqrt{\overline{d}n})$ rounds
3. Inform $\sqrt{\overline{d}n} \cdot OPT$ centers adjacent nodes to L in $\mathrm{O}(\sqrt{\overline{d}n} \cdot OPT)$ rounds
4. Inform remaining nodes by simulating radio broadcast algorithm in $\mathrm{O}(\sqrt{\overline{d}n}(diam + \log^2(n))$ rounds

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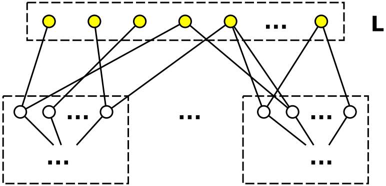

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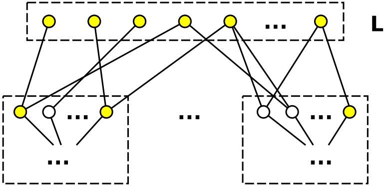

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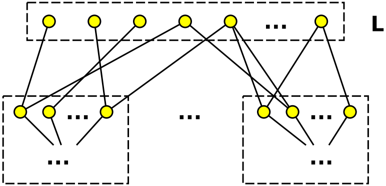

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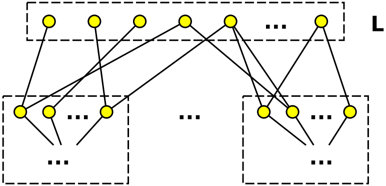

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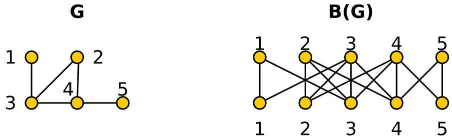
Theorem $\tilde{\mathrm{O}}(\sqrt{\overline{d}n})$-approximation for radio-unicast broadcast

Approximation Hardness for General Graphs

Connection IS/Coloring and RAS

Hardness of IS/Coloring: **[Feige, Kilian, J. Comput. Syst. Sci. 1998]**
Deciding whether a graph has chromatic number $\chi(G) \leq n^{\epsilon}$ or $\chi(G) \geq n^{1-\epsilon}$ is NP-hard.

Connection IS/Coloring and RAS



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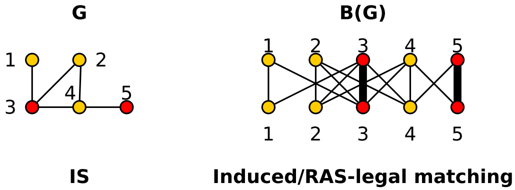

Large IS in G implies large RAS-legal matching in $B(G)$

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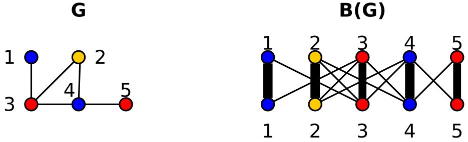

Coloring **Covering by RAS-legal matchings**

c-coloring in G implies RAS-legal matching cover of size c in $B(G)$

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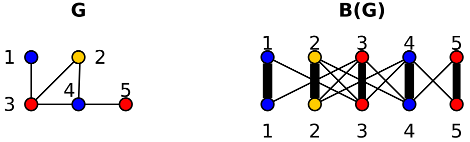
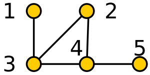


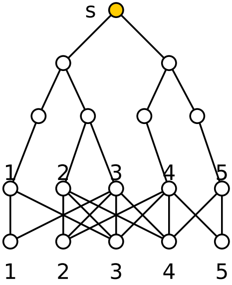
Coloring **Covering by RAS-legal matchings**

Converse is also true:
RAS-legal matching cover of size c in $B(G)$ implies c-coloring in G

Lower Bound Construction



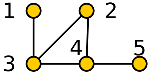
G



B(G)

Binary $+B(G)$

Lower Bound Construction



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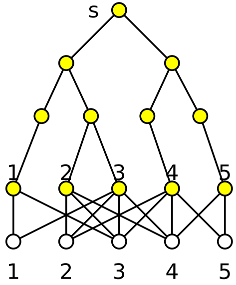
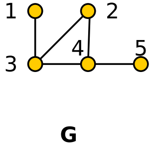
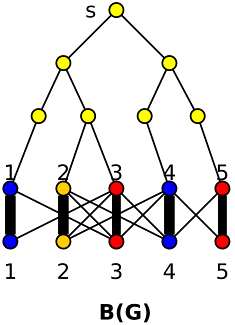


B(G)

One bipartition of $B(G)$ can be informed in $\mathrm{O}(\log n)$ rounds

Lower Bound Construction



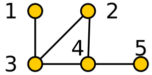


$OPT = \mathrm{O}(\log n)$ + size of *RAS*-legal matching cover

OPT small $\rightarrow$ induced *RAS*-legal matching cover small in $B(G)$ $\rightarrow$ coloring with few colors in G

LB Construction

Lower Bound Construction



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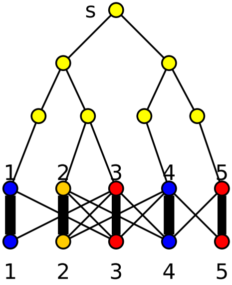


B(G)

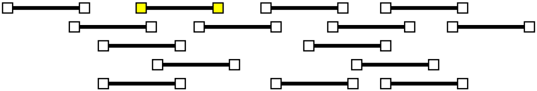
Theorem It is NP-hard to approximate *RAS* within factor $n^{1-\epsilon}$, for any $\epsilon > 0$.

Interval Graphs

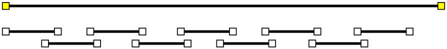
Interval Graphs

Unit Interval Graphs [Guo et al., J. of Combin. Opt. 2014]

- Inform a diameter path (dominating set)
- Each color class of a coloring can be informed in $\mathrm{O}(1)$ rounds
- Runtime: $\mathrm{O}(\mathit{diam} + \chi(G))$, *diam* and $\chi(G)$ are LBs $\Rightarrow$ $\mathrm{O}(1)$-approx.



Interval Graphs Difficulty: claws



- Splitting into $\mathrm{O}(\log n)$ length classes
- Informed length class informs other length class in $\mathrm{O}(OPT)$ rounds

Theorem There is a polynomial-time algorithm for RAS on interval graphs with approximation ratio $\mathrm{O}(\log n)$.

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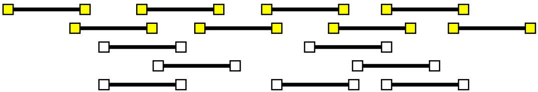

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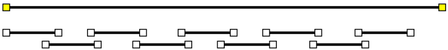

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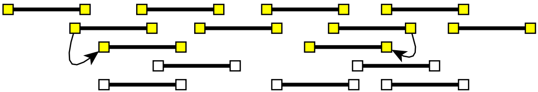
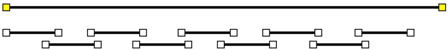


Interval Graphs Difficulty: claws



- Splitting into $\mathrm{O}(\log n)$ length classes
- Informed length class informs other length class in $\mathrm{O}(OPT)$ rounds

Theorem There is a polynomial-time algorithm for RAS on interval graphs with approximation ratio $\mathrm{O}(\log n)$.

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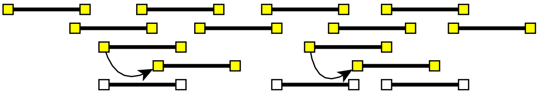

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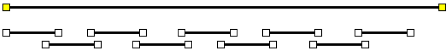

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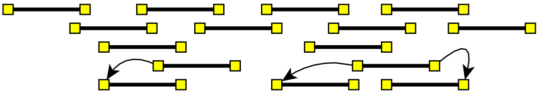

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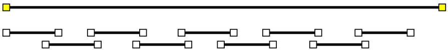

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Conclusion

Summary

- $\tilde{O}(\sqrt{d}n)$-approximation algorithm for *RAS* on general graphs
- $n^{1-\epsilon}$-approximation hardness on general graphs
- $O(\log n)$-approximation algorithm for *RAS* on interval graphs

Open Questions

- $O(1)$-approximation on interval graphs?
- Is there a const/poly-log approximation on unit disc graphs?
- Disc Graphs?

Thank you.