Finite-Rate Feedback MIMO Broadcast Channels with a Large Number of Users

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***Abstract*— We analyze the sum-rate performance of a multi-antenna downlink system carrying more users than transmit antennas, with partial channel knowledge at the transmitter due to finite rate feedback. In order to exploit multiuser diversity, we show that the transmitter must have, in addition to directional information, information regarding the quality of each channel. Such information should reflect both the channel magnitude and the quantization error. Expressions for the SINR distribution and the sum-rate are derived, and tradeoffs between the number of feedback bits, the number of users, and the SNR are observed. In particular, for a target performance, having more users reduces feedback load.**

I. Introduction

Recent advances in multiuser downlink communication channels show that in multiple input multiple output (MIMO) systems with M transmit antennas and $K \geq M$ single antenna users, the full multiplexing gain M can be achieved by using space-division multiple access schemes such as dirty-paper coding (DPC) or transmit beamforming [1]–[3]. Moreover, in a large user regime $K \gg M$, the sum-capacity grows like $M \log \log K$ due to multiuser diversity [3]–[5]. Low-complexity schemes based on zero-forcing beamforming (ZFBF) or zero-forcing dirty-paper coding (ZF-DPC) have been proposed that achieve this optimal growth rate [6]–[8]. However, all these results are based on the assumption of perfect channel state information at the transmitter (CSIT), which may not be a practical assumption.

One of the popular models to address the lack of perfect CSIT is to provide the transmitter with imperfect CSI via a rate constrained feedback channel from each mobile, where each mobile quantizes its vector channel to one of $N = 2^B$ quantization vectors and feeds back the corresponding index. This feedback is used to capture information regarding only the spatial direction of the channel (referred to as channel direction information, or CDI), and not the channel magnitude. MIMO systems under limited feedback have been studied for single user systems [9]–[12] and recently applied to downlink systems for $K \leq M$ [13], [14]. For single user systems, it has been shown that only a few feedback bits (roughly on the order of M, the number of transmit antennas) are needed to achieve near perfect-CSIT performance. For downlink channels, however, the feedback load per mobile must be scaled with both the number of transmit antennas as well as the system SNR in order to achieve near-perfect CSIT performance and the full multiplexing gain [13].

When there are more users than antennas ($K \geq M$), CDI can be used to achieve the full multiplexing gain of the downlink channel, but cannot simultaneously benefit from multiuser diversity, i.e. obtain the double logarithmic growth with K. As we later show, the sum rate with only CDI at the transmitter is bounded as the number of users is taken to be large while all other parameters (feedback load, number of antennas, and SNR) are held constant. In order to scale the sum rate at the optimal $\log \log K$ rate, the transmitter must also have channel quality information (CQI; be it a channel magnitude or SINR information), to exploit selection diversity among users as well as control the effect of quantization error in the CDI. Indeed, the random beamforming (RBF) scheme proposed in [2] uses SINR feedback and a few ($\log_2 M$) additional feedback bits to perform user selection and achieves the asymptotic sum-capacity as $K \to \infty$. However, its performance is generally poor for practical values of K [6].

In this paper, we consider a limited feedback model where each user feeds back B-bit quantized CDI as well as (unquantized) CQI. We propose a low-complexity scheme with a user selection based on a semi-orthogonal user selection (SUS) principle [6], [7] and a ZFBF precoder. When $B = \log_2 M$, our model reduces to the RBF. We characterize the sum-rate performance of our limited feedback model and show how it scales with K. Our analysis reveals tradeoffs between B, K, and SNR, and provides useful design guidelines.

II. System Model

Consider a single cell MIMO broadcast channel with M transmit antennas at the base and $K \geq M$ single antenna users. We assume users are homogeneous and experience flat Rayleigh fading. The signal received by a user k can be represented as

$$y_k = \mathbf{h}_k \mathbf{x} + z_k, \quad k = 1, \cdots, K \tag{1}$$

where $\mathbf{h}_k \in \mathcal{C}^{1 \times M}$ is the channel gain vector with zero-mean unit variance i.i.d complex Gaussian entries, $\mathbf{x}$ is the transmitted symbol vector containing information symbols to a selected set of users $\mathcal{S} = \{\pi(1), \cdots, \pi(|\mathcal{S}|)\}$ with an average power constraint $E\{\|\mathbf{x}\|^2\} = P$, z_k is the additive noise with

This work was supported in part by the National Science Foundation (NSF) under CCR-0325639-001 and by LG electronics.

a unit variance, and y_k is the symbol received by user k. The transmit symbol vector $\mathbf{x}$ is related to information symbols $s_i,\ i \in \mathcal{S}$, via linear beamforming $\mathbf{x} = \sum_{i\in\mathcal{S}} \mathbf{w}_i s_i$. The user set $\mathcal{S}$ is chosen to maximize the sum-rate.

1) Finite rate feedback model for CDI quantization: We assume that each user has perfect knowledge (CSIR) of $\mathbf{h}_k$ and quantizes the direction of its channel $\mathbf{h}_k/\|\mathbf{h}_k\|$ to a unit norm vector $\hat{\mathbf{h}}_k$. The quantization is chosen from a codebook $\mathcal{C} = \{\mathbf{f}_1, \cdots, \mathbf{f}_N\}$, $N = 2^B$, of unit norm vectors according to the minimum distance criterion $\hat{\mathbf{h}}_k = \mathbf{f}_n$ with $n = \arg\max_{1\le n\le N} \left|\frac{\mathbf{h}_k}{\|\mathbf{h}_k\|}\mathbf{f}_n^*\right|$, and each mobile feeds back the index n to the transmitter [9]–[11]. After a user set $\mathcal{S}$ is selected (which is discussed later), the users in $\mathcal{S}$ are supported via ZFBF [1], in which the unit-norm beamforming vectors $\mathbf{w}_i \in \mathcal{C}^{M\times 1}$, $i \in \mathcal{S}$, are chosen to satisfy $\hat{\mathbf{h}}_j\mathbf{w}_i = 0$, $\forall j \neq i,\ j \in \mathcal{S}$. Such vectors can be readily determined from the pseudo-inverse $([\hat{\mathbf{h}}^{\mathrm{T}}_{\pi(1)}, \cdots, \hat{\mathbf{h}}^{\mathrm{T}}_{\pi(|\mathcal{S}|)}]^{\mathrm{T}})^{\dagger}$. Note that CDI feedback is sufficient for determining ZFBF.

2) CQI feedback model: In addition to the CDI, each user feeds back its CQI $g(\mathbf{h}_k)$. We consider two definitions of CQI: one using the channel norm $g(\mathbf{h}_k) = \|\mathbf{h}_k\|^2$ in Section III, and the other using the SINR as the CQI in Section IV, among which we show the latter achieves multiuser diversity. We assume the CQI is directly fed back *without quantization*, in order to concentrate on the effect of quantization of CDI. We expect the number of bits for quantizing CQI can be kept relatively small.

3) User selection: Based on $\{g(\mathbf{h}_k)\hat{\mathbf{h}}_k, k = 1, \cdots, K\}$, the transmitter performs user selection and ZFBF to support up to M out of K users at a time. Since finding the optimal user set that maximizes the sum-rate requires an exhaustive search which is not computationally feasible for moderate to large K, we use a heuristic user selection algorithm based on the semi-orthogonal user selection (SUS) procedure [6], [7]. Specifically, the transmitter selects the first user from the initial user set $\mathcal{A}_0 = \{1, \cdots, K\}$ as $\pi(1) = \arg\max_{k\in\mathcal{A}_0} g(\mathbf{h}_k)$. After selecting i users, the $(i+1)$th user is selected among the user set $\mathcal{A}_i = \{1 \le k \le K : |\hat{\mathbf{h}}_k\hat{\mathbf{h}}^*_{\pi(j)}| \le \epsilon,\ 1 \le j \le i\}$ as $\pi(i+1) = \arg\max_{k\in\mathcal{A}_i} g(\mathbf{h}_k)$, where ϵ is a design parameter that dictates the maximum spatial correlation allowed between quantized channels. In this way, the transmitter can choose users that have high channel qualities and are mutually semi-orthogonal in terms of their quantized directions $\hat{\mathbf{h}}_k$. Under perfect CSIT, this user selection method achieves the optimal sum-capacity growth rate $M \log\log K$ at large K and performs quite well for moderate K as well [6], [7].

III. CDI and Magnitude Feedback

In this section we analyze the performance of a naive CQI feedback scheme where each user feeds back its quantized CDI $\hat{\mathbf{h}}_k$ as well as its channel magnitude $g(\mathbf{h}_k) = \|\mathbf{h}_k\|^2$.

The effective channel after the ZFBF is expressed as

$$y_k = (\mathbf{h}_k\mathbf{w}_k)s_k + \sum_{j\in\mathcal{S}, j\neq k} (\mathbf{h}_k\mathbf{w}_j)s_j + z_k, \quad k \in \mathcal{S}. \tag{2}$$

If the CDI was perfect (i.e., $\hat{\mathbf{h}}_k = \mathbf{h}_k/\|\mathbf{h}_k\|$), the second term (multiuser interference) would be evaluated to zero. Under quantized CDI, however, the interference is not completely eliminated because the beamforming vectors are chosen orthogonal to the quantized channels and not the actual channels. Assuming $|\mathcal{S}| = M$ and allocating equal powers $\rho = P/M$ to the M users, the SINR of the selected users are given as

$$\mathrm{SINR}_k = \frac{\rho|\mathbf{h}_k\mathbf{w}_k|^2}{1 + \rho\sum_{j\neq k}|\mathbf{h}_k\mathbf{w}_j|^2},\ k \in \mathcal{S}. \tag{3}$$

Denote as θ_k the angle between $\mathbf{h}_k$ and $\hat{\mathbf{h}}_k$, i.e. $\cos\theta_k = \frac{|\mathbf{h}_k\hat{\mathbf{h}}_k^*|}{\|\mathbf{h}_k\|}$. Then, the expected SINR at user k is given by

$$\begin{aligned} E(\mathrm{SINR}_k) &= E\left(\frac{\rho|\mathbf{h}_k\mathbf{w}_k|^2}{1 + \rho\sum_{j\neq k}|\mathbf{h}_k\mathbf{w}_j|^2}\right) \\ &\ge \frac{\rho|\mathbf{h}_k\mathbf{w}_k|^2}{1 + \rho\|\mathbf{h}_k\|^2\sin^2\theta_k} \ge \frac{\rho\|\mathbf{h}_k\|^2\cos^2(\theta_k+\phi)}{1 + \rho\|\mathbf{h}_k\|^2\sin^2\theta_k} \triangleq \gamma_k(\phi), \end{aligned} \tag{4}$$

where the expectation is taken over beamforming vectors $\{\mathbf{w}_j : j \in \mathcal{S}, j \neq k\}$ which the user k knows lie in the subspace orthogonal to $\hat{\mathbf{h}}_k$, and ϕ is a constant given by $\phi = \cos^{-1}\sqrt{\frac{1-(M-1)\epsilon}{1-(M-2)\epsilon}(1+\epsilon)}$ [7]. We see that in order to maximize SINR, the transmitter should try to select mutually semi-orthogonal users (small ϵ). The inequality (4) is tight when ϵ is small, and when $\epsilon = 0$, the SINR itself becomes

$$\mathrm{SINR}_k = \frac{\rho\|\mathbf{h}_k\|^2\cos^2\theta_k}{1 + \rho\|\mathbf{h}_k\|^2\sin^2\theta_k} = \gamma_k(0) \triangleq \gamma_k. \tag{5}$$

Henceforth, for ease of analysis, we use γ_k which approximates SINR_k when ϵ is small.

To obtain an upper bound on the sum-rate, we upper-bound γ_k as

$$\gamma_k = \frac{\rho\|\mathbf{h}_k\|^2\cos^2\theta_k}{1 + \rho\|\mathbf{h}_k\|^2\sin^2\theta_k} \le \frac{\cos^2\theta_k}{\sin^2\theta_k} \triangleq \tilde{\gamma}_k, \tag{6}$$

whose distribution depends on the quantization codebook design. Since the optimal codebook design is in general unknown, we resort to a quantization cell approximation used in [11], [12]. The approximation is based on the ideal assumption that each quantization cell is a Voronoi region of a spherical cap with the surface area 2^{-B} of the total surface area of the unit sphere. Specifically, for a given codebook $\mathcal{C}$, we approximate the actual quantization cell $\mathcal{R}_i = \{\mathbf{h} : |\mathbf{h}\mathbf{f}_i^*|^2 \ge |\mathbf{h}\mathbf{f}_j^*|^2, \forall j \neq i\}$ as $\mathcal{R}_i \approx \{\mathbf{h} : |\mathbf{h}\mathbf{f}_i^*|^2 \ge 1-\delta\}$, where $\delta = 2^{-\frac{B}{M-1}}$ to give $P\{\mathcal{R}_i\} = 2^{-B}$. From this, the cumulative distribution function (CDF) of $\sin^2\theta_k$ is obtained as

$$F_{\sin^2\theta}(x) = \begin{cases} 2^B x^{M-1}, & 0 \le x \le \delta \\ 1, & x \ge \delta. \end{cases} \tag{7}$$

It is shown in [12] that for *any* quantization codebook $\tilde{\mathcal{C}}$ and its corresponding CDF $F_{\sin^2\tilde{\theta}}$, we have $F_{\sin^2\theta}(x) \ge F_{\sin^2\tilde{\theta}}(x)$. Therefore, the quantization cell approximation yields a performance upper bound, e.g. higher rate, lower outage probability, etc.[1] From (7) the probability density function (PDF) of $\tilde{\gamma}_k$

[1]Numerical results (not included in this paper due to space limitations) show that this bound is very tight.

can be derived as

$$f_{\tilde{\gamma}_k}(x) = \begin{cases} \frac{2^B(M-1)}{(x+1)^M}, & x \geq \frac{1}{\delta} - 1 \\ 0, & 0 \leq x < \frac{1}{\delta} - 1. \end{cases} \quad (8)$$

Finally, the sum-rate is given by

$$\begin{aligned} E\{R\} &\approx E\left\{\sum_{i\in\mathcal{S}} \log_2(1+\gamma_i)\right\} \leq E\left\{\sum_{i\in\mathcal{S}} \log_2(1+\tilde{\gamma}_i)\right\} \\ &= E_{\mathcal{S}}\left\{\sum_{i\in\mathcal{S}} \int_0^\infty \log_2(1+x)\ f_{\tilde{\gamma}}(x)dx \ \Big|\ \mathcal{S}\right\} \\ &= E_{\mathcal{S}}\left\{\sum_{i\in\mathcal{S}} \frac{B+\log_2 e}{M-1} \ \Big|\ \mathcal{S}\right\} = \frac{M}{M-1}(B+\log_2 e), \end{aligned} \quad (9)$$

which is independent of P and K. This means that the system not only becomes interference limited as either P or K increases, but also does not benefit from multiuser diversity even with $\|\mathbf{h}_k\|^2$ feedback.[2] This is because the SINR is essentially limited by the CDI quantization error θ_k, of which the transmitter has no knowledge. Note that the asymptotic sum-rate as $P \to \infty$ *without* any CQI feedback is also given by (9), since $\lim_{P\to\infty} \gamma_i = \tilde{\gamma}_i$. Therefore, we conclude that a good CQI should be based on both the channel magnitude ($\|\mathbf{h}_k\|^2$) and the CDI quantization error (θ_k), which motivates the use of $\gamma_k(\phi)$ as the CQI in the next section.

IV. CDI and SINR Feedback

In this section we analyze the sum-rate of the finite-rate feedback scheme with SINR-based CQI. Note that the exact SINR in (3) is unknown at either the transmitter or receiver. Therefore, we propose the use of $g(\mathbf{h}_k) = \gamma_k(\phi)$ in (4) which the user k can calculate based on $\mathbf{h}_k$, θ_k, and a given parameter ϕ. To simplify analysis, we assume the feedback takes the form $g(\mathbf{h}_k) = \gamma_k(0) = \gamma_k$. In the following theorem we derive the distribution of γ_k:

Theorem 1: Consider two independent random variables $X \sim \chi_2^2$ and $Y \sim \chi^2_{2(M-1)}$, and define

$$\gamma = \frac{\rho\,(X + (1-\delta)Y)}{1+\rho\delta Y}. \quad (10)$$

Then, under the distribution (7), γ_k and γ have identical distribution with a CDF for $x \geq \frac{1}{\delta} - 1$ given by[3]

$$F_\gamma(x) = 1 - \frac{2^B e^{-\frac{x}{\rho}}}{(x+1)^{M-1}}, \qquad x \geq \frac{1}{\delta} - 1 = 2^{\frac{B}{M-1}} - 1. \quad (11)$$

Proof: Omitted. ∎

Therefore, the interference term $\rho\|\mathbf{h}_k\|^2 \sin^2\theta_k$ has a $\chi^2_{2(M-1)}$ distribution scaled by $\rho\delta$, and the received signal power $\rho\|\mathbf{h}_k\|^2\cos^2\theta_k$ is described as the sum of two independent scaled chi-squares $\rho\chi_2^2 + \rho(1-\delta)\chi^2_{2(M-1)}$. Note that the signal and interference powers are correlated through Y.

By the user selection process, the ith user has the maximum SINR among $|\mathcal{A}_{i-1}|$ i.i.d. users. Therefore, it is necessary for us to characterize the behavior of the maximum of a number of i.i.d. random variables, for which extreme value theory [15]–[17], [2, Appendix A], [8, Appendix II] is useful.

Theorem 2: The ith largest order statistic among $\gamma_1, \cdots, \gamma_K$, denoted as $\gamma_{i:K}$, satisfies

$$P\left\{|\gamma_{i:K} - b_K| \leq \rho \log\log\sqrt{K}\right\} \geq 1 - O\left(\frac{1}{\log K}\right), \quad (12)$$

where

$$b_K = \rho\log\frac{2^B K}{\rho^{M-1}} - \rho(M-1)\log\log\frac{2^B K}{\rho^{M-1}}. \quad (13)$$

Proof: The choice of b_K in (13) may be inferred from solving $1 - F_\gamma(x) = \frac{1}{K}$. Using [16], [8, Theorem 6], with $a_K = \rho$ and b_K as in (13), we can show that $F_\gamma(x)$ belongs to the *domain of attraction of Gumbel type* [8], [15]. Then, the theorem is proved by utilizing [17], [8, Theorem 7], in a similar manner used in [8, Lemma 6]. ∎

Theorem 2 implies that for large K,

$$\gamma_{i:K} = \rho\log\frac{2^B K}{\rho^{M-1}} + O(\log\log K). \quad (14)$$

By the law of large numbers, the ratio $|\mathcal{A}_i|/K$ converges to some constant α_i, where $\alpha_0 = 1$ since $|\mathcal{A}_0| = K$, and $\alpha_i \geq I_{\epsilon^2}(i, M-i)$, $1 \leq i \leq M$ with $I_z(a,b)$ the regularized incomplete beta function [7]. Thus, the sum-rate is given by

$$\begin{aligned} E\{R\} &\approx E\left\{\sum_{i=1}^M \log_2\left(1 + \gamma_{i:|\mathcal{A}_{i-1}|}\right)\right\} \\ &\approx \sum_{i=1}^M \log_2\left(1 + \rho\log\frac{2^B K\alpha_{i-1}}{\rho^{M-1}}\right). \end{aligned} \quad (15)$$

Factoring $\gamma_{i:|\mathcal{A}_{i-1}|}$ into the SNR part ρ and the logarithmic term $\Delta \triangleq \log\frac{2^B K\alpha_{i-1}}{\rho^{M-1}}$, we can interpret the latter as the SNR improvement (degradation) factor which includes both the effect of quantization error and multiuser diversity. From the expression we can observe the following:

1) Multiuser diversity of an SNR improvement by a factor of $\log K$ [2], [4], [5] is still preserved under quantized CDI feedback.
2) The quantities 2^B and K are interchangeable. Thus, for a target sum-rate, every doubling of the number of users saves one feedback bit per user.
3) For a target SNR improvement (degradation), B and K should scale with P such that $B + \log_2 K = (M-1)\log_2 P + c$ for some constant c. That is, for a fixed K, every doubling (3dB increase) of power requires $M-1$ additional feedback bits. This result has also been observed in [13] for the case of $K = M$. Alternatively, for a fixed B, K could scale with P as $K \propto P^{M-1}$ for a target Δ, or both B and K could be adjusted simultaneously to meet $B + \log_2 K = (M-1)\log_2 P + c$.

[2]Note that SINR_k in (5) and the sum-rate bound in (9) are valid only for $|\mathcal{S}| = M$. Clearly, when $|\mathcal{S}| = 1$ (i.e. when one gives up the multiplexing gain), the system is limited only by noise and fully benefits from multiuser diversity.

[3]$F_\gamma(x)$ for $x < \frac{1}{\delta} - 1$ can also be found, but its expression is more involved.

A. High SNR or high resolution regimes

The formulas (12)-(15) are valid only when both K and $\frac{2^B K}{\rho^{M-1}}$ are large. Moreover, the CDF (11) is valid only for $\gamma_{i:|\mathcal{A}_{i-1}|} \geq 2^{\frac{B}{M-1}} - 1$. Some of these conditions may fail when either B or P is large to the degree that a given K is not large enough to satisfy the conditions. In this subsection we characterize these regimes by investigating two limiting cases, $P \to \infty$ and $B \to \infty$, which correspond to, respectively, interference limited and noise limited regimes.

1) High SNR regime: In this regime the SINR becomes $\lim_{P\to\infty} \gamma_k = \frac{\cos^2\theta_k}{\sin^2\theta_k} = \tilde{\gamma}_k$, whose extremal value is given by the following theorem:

Theorem 3: The ith largest order statistic among $\tilde{\gamma}_1, \cdots, \tilde{\gamma}_K$, denoted as $\tilde{\gamma}_{i:K}$, satisfies

$$P\Big\{ \log(2^B K) - \log\log\sqrt{K} \leq (M-1)\log(1+\tilde{\gamma}_{j:K}) \leq \log(2^B K) + \log\log\sqrt{K} \Big\} \geq 1 - O\left(\frac{1}{\log K}\right). \quad (16)$$

Proof: Choosing $a_K = (2^B K)^{\frac{1}{M-1}}$ and $b_K = -1$ we can show that $F_{\tilde{\gamma}}(x)$ has a *Frechet (M-1) type* limit. Then, (16) is proved by using [17], [8, Theorem 7]. ∎

Thus, for large K, $\log(1+\tilde{\gamma}_{i:K}) = \frac{1}{M-1}\log(2^B K) + O(\log\log K)$. The sum-rate then becomes

$$E\{R\} \approx E\left\{ \sum_{i=1}^{M} \log_2\left(1+\tilde{\gamma}_{i:K\alpha_{j-1}}\right) \right\} \approx \frac{M}{M-1}(B+\log_2 K) + \frac{\sum_{i=1}^{M}\log_2 \alpha_{i-1}}{M-1}. \quad (17)$$

We again observe the interchangeability between 2^B and K. Under finite B and K, however, we see that the sum-rate eventually converges to a constant value (17) as $P \to \infty$. This is because the system is interference limited at high SNR due to the unavoidable effect of quantization error. The limiting sum-rate (17), however, grows linearly (ignoring the additive term) with $B + \log_2 K$. In particular, the multiuser diversity amounts to a logarithmic increase to the sum-rate. This is in contrast to previous findings that the sum-rate increase by the multiuser diversity is only by a factor of $\log\log K$ [3]. Therefore, multiuser diversity is even more beneficial in this regime.

2) High resolution regime: As $B \to \infty$, $\theta_k \to 0$, and (5) reduces to $\gamma_k = \rho\|\mathbf{h}_k\|^2$. For K i.i.d. χ^2_{2M} random variables, it has been shown that their ith order statistic behaves like $\log K$ for a large K [2], [8]. Thus, $\gamma_{i:K} = \rho\log K + O(\log\log K)$. We see that the exchangeability between 2^B and K is no longer observed, i.e. in the high resolution regime, doubling the number of users is worth more than one additional feedback bit.

B. Relation to random beamforming (RBF)

In the random beamforming (RBF) scheme proposed in [2], M orthogonal random beams are generated at the transmitter. Then, each user calculates its SINR for each of the M beams and feeds back the maximum SINR value (without quantization) along with a corresponding beam index, after which the transmitter chooses the best user for each beam. Note that $\lceil \log_2 M \rceil$ bits are required for feeding back a user's beam index. Now, consider our limited feedback system that employs a randomly generated optimal codebook of size $N = M$ for CDI quantization. Since the optimal codebook design for $N \leq M$ is a set of orthonormal vectors [11], [18], this codebook is equivalent to the random beamformer. Also, note that both systems assume perfect SINR feedback. Therefore, the RBF scheme is essentially equivalent to our limited feedback scheme with $N = M$ and $\phi = 0$. The similarity can also be observed by comparing $F_\gamma(x)$ in (11) with $F_s(x)$ in [2, eq (15)]. Thus, our scheme can be understood as a generalization of the RBF to the case of $N > M$ and to the beamformers which are not necessarily orthonormal.

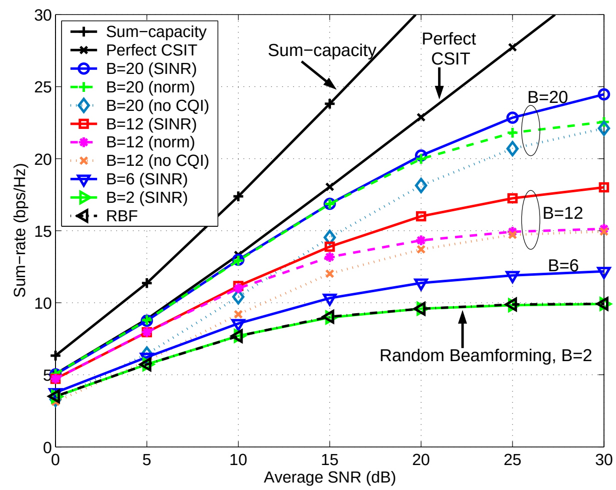

Fig. 1. Sum-rate R versus average SNR P under $M = 4$, $K = 100$, and various B and feedback types.

V. Numerical Results

In this section we present numerical results. In Figure 1 we present the sum-rate R vs. average SNR P for the system with $M = 4$ base-station antennas, $K = 100$ users, $\epsilon = 0.25$, and various quantization levels $B = 2$, 6, 12, and 20 bits. For CQI feedback we use three different feedback schemes: (A) SINR ($\gamma_k(\phi)$) feedback, (B) channel norm ($\|\mathbf{h}_k\|^2$) feedback, and (C) no CQI feedback. For the CDI quantization codebook for $B > \log_2 M = 2$, we use the quantization approximation in (7) that gives a performance upper bound. For $B = 2$, we use orthonormal codewords, which is optimal [11], [18]. From the figure it is seen that the sum-rate approaches the perfect CSIT sum-rate as B increases. Among the three CQI feedback schemes the SINR feedback performs the best. The channel norm feedback performs close to the SINR feedback at low P (where the system is noise-limited), while at high P (where the system is interference limited) it is only slightly better than having no CQI feedback. The rate increase of SINR feedback over no CQI feedback seems to be rather constant over a wide SNR range, whereas the benefit of increasing B becomes more pronounced at high SNR. This implies that a limited feedback resource should be spent more on CQI quantization at low SNR and on CDI quantization at high SNR. As $P \to \infty$ all the

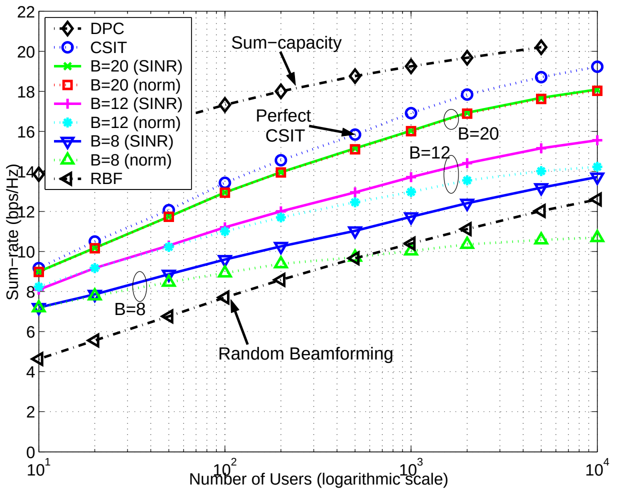

Fig. 2. Sum-rate R versus the number of users K under $M = 4$, $P = 10$, and various B and CQI types.

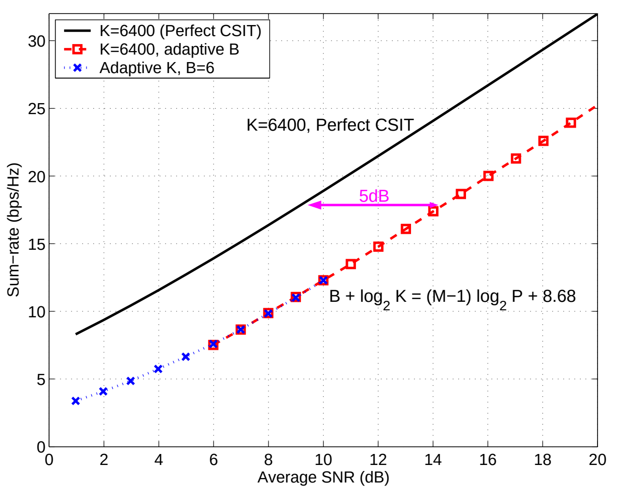

Fig. 3. Sum-rate R versus average SNR P under $M = 4$ and adaptive B and K such that $B + \log_2 K = (M - 1)\log_2 P + 8.68$.

sum-rate curves with quantized feedback eventually converge to a finite ceiling. Observe that the sum-rate with $B = 2$ is the same as that of the RBF.

In Figure 2 we plot the sum-rate vs. K at $P = 10$ dB. We see that with the SINR feedback the sum-rate benefits from multiuser diversity. With the norm feedback, however, the sum-rate increase is slowed down as K increases and is eventually upper bounded by (9), although the sum-rate increase is maintained longer for a larger B. For $B = 20$ both CQI feedback schemes perform reasonably close to the perfect CSIT case up to 10^4 users.

In Figure 3 we adapt B and K as $B + \log_2 K = (M - 1)\log_2 P + c$ so that a constant gap from the perfect CSIT sum-rate is maintained. By either fixing $K = 6400$ and adapting B, or fixing $B = 6$ and adapting K, we achieve an SNR gap of about 5dB from the perfect CSIT curve, confirming the third observation in Section IV. We note that this SNR gap is close to the $10\log_{10}\left(1 + P \cdot 2^{-\frac{B}{M-1}}\right)$ dB gap shown in [13].

VI. Conclusion

We have investigated a multiuser multi-antenna downlink system under partial channel knowledge at the transmitter, when there are more users than transmit antennas. The SINR distributions and the sum-rates under quantized channel direction information (CDI) and various channel quality information (CQI) are derived. We have shown that CDI alone does not achieve the full multiplexing and multiuser diversity gain simultaneously. To achieve both gains we have shown that CQI feedback is necessary, and that CQI should be the SINR rather than just the channel magnitude, since SINR captures both the channel magnitude and the quantization error. This implies that any quantization should be applied to SINR rather than directly to the channel magnitude. We have derived tradeoffs between the number of feedback bits, the number of users, and SNR. In particular, for a target performance, having more users reduces feedback load.

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