## Supplementary Materials for "Kernel Learning for Data-Driven Spectral Analysis of Koopman Operators"

## Appendix A. Additional Figures for The Results in Section 5.3

Figures A.1 and A.2 correspond to the second and the third rows of Table 1 in the main manuscript, respectively. Figures A.3 and A.4 correspond to the second and the thid rows of Table 2 in the main manuscript, respectively.



Figure A.1: Histograms of estimation error of leading Koopman eigenfrequency on the short TORUS datasets with q = 3, without or with the kernel learning. (*left*)  $\beta = 0.05$ , (*center*)  $\beta = 0.1$ , and (*right*)  $\beta = 0.2$ . Best viewed in color.



Figure A.2: Histograms of estimation error of leading Koopman eigenfrequency on the short TORUS datasets with q = 5, without or with the kernel learning. (*left*)  $\beta = 0.05$ , (*center*)  $\beta = 0.1$ , and (*right*)  $\beta = 0.2$ . Best viewed in color.



Figure A.3: Histograms of estimation error of leading Koopman eigenfrequency on the short LORENZ datasets with q = 20, without or with the kernel learning. (*left*)  $\beta = 0.05$ , (*center*)  $\beta = 0.1$ , and (*right*)  $\beta = 0.2$ . Best viewed in color.



Figure A.4: Histograms of estimation error of leading Koopman eigenfrequency on the short LORENZ datasets with q = 30, without or with the kernel learning. (*left*)  $\beta = 0.05$ , (*center*)  $\beta = 0.1$ , and (*right*)  $\beta = 0.2$ . Best viewed in color.





Figure B.1: Leading eigenfrequencies estimated on long TORUS datasets of varying length with q = 100.

Figure B.2: Leading eigenfrequencies estimated on long LORENZ datasets of varying length with q = 1,000.

## Appendix B. Estimation Results on Long Datasets

In the experiments on the TORUS and the LORENZ datasets, we used the estimations of the leading Koopman eigenfrequencies on the long datasets (length 5,000) as the pseudo ground truths. Figures B.1 and B.2 show that how the estimations converge with regard to the dataset lengths. In both plots, the results converge quite well after length of 5,000.