Proceedings of Machine Learning Research vol 120:1–10, 2020

Exploiting Model Sparsity in Adaptive MPC: A Compressed Sensing Viewpoint

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Abstract

This paper proposes an Adaptive Stochastic Model Predictive Control (MPC) strategy for stable linear time-invariant systems in the presence of bounded disturbances. We consider multi-input, multi-output systems that can be expressed by a Finite Impulse Response (FIR) model. The parameters of the FIR model corresponding to each output are unknown but assumed *sparse*. We estimate these parameters using the Recursive Least Squares algorithm. The estimates are then improved using set-based bounds obtained by solving the Basis Pursuit Denoising Chen et al. (2001) problem. Our approach is able to handle hard input constraints and probabilistic output constraints. Using tools from distributionally robust optimization, we reformulate the probabilistic output constraints as tractable convex second-order cone constraints, which enables us to pose our MPC design task as a convex optimization problem. The efficacy of the developed algorithm is highlighted with a thorough numerical example, where we demonstrate performance gain over the counterpart algorithm of Bujarbaruah et al. (2018), which does not utilize the sparsity information of the system impulse response parameters during control design.

Keywords: Adaptive MPC, Finite Impulse Response, Kalman Filtering, Convex Optimization, Compressed Sensing.

1. Introduction

The uncertainty in modeling of dynamical systems can be primarily attributed to two factors: (i) model uncertainty (e.g., modeling mismatch and inaccuracies), and (ii) exogenous disturbances (e.g., sensor noise). In recent times, utilizing tools from classical Adaptive Control, Adaptive Model Predictive Control (MPC) Tanaskovic et al. (2014); Lorenzen et al. (2019); Bujarbaruah et al. (2018); Köhler et al. (2019b) has established itself as a promising approach for control of uncertain systems subject to input and output constraints. For linear systems specifically, the literature on Adaptive MPC has extensively focused on either robust or probabilistic satisfaction of such imposed constraints on the system, using either state-space or input-output modeling.

In Soloperto et al. (2018); Bujarbaruah et al. (2019) additive model uncertainties are considered with known system matrices, and imposed state and input constraints are robustly satisfied for all such realizable uncertainties. In Lorenzen et al. (2019); Vicente and Trodden (2019); Köhler et al. (2019a) robust state-input constraint satisfaction is extended to include both unknown system dynamics matrices and additive disturbances. The approach introduced in Bujarbaruah et al. (2019) is also suited for satisfaction of probabilistic chance constraints on system states. Furthermore, the

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work in Hewing and Zeilinger (2017); Koller et al. (2018) uses Gaussian Process (GP) regression for real-time learning of an uncertain model and satisfies probabilistic state constraints with a traditional stochastic MPC Farina et al. (2016) controller. Although such state-space modeling based Adaptive MPC controllers have proven to be effective, they involve construction of positive invariant sets Blanchini (1999), which can become computationally cumbersome. As a consequence, input-output modeling of systems has been opted in literature for proposing computationally efficient Adaptive MPC algorithms for certain applications (e.g., for stable, slow systems).

Adaptive MPC algorithms using input-output modeling of the system are presented in Tanaskovic et al. (2014, 2019); Bujarbaruah et al. (2018); Parsi et al. (2019), both for robust and probabilistic satisfaction of imposed input-output constraints. The works of Tanaskovic et al. (2014, 2019); Parsi et al. (2019) deal with modeling errors in the Finite Impulse Response (FIR) domain, in the presence of a bounded additive disturbance, and prove recursive feasibility and stability (Borrelli et al., 2017, Chapter 12) of the proposed robust Adaptive MPC approaches. These ideas are extended in Bujarbaruah et al. (2018), where a recursively feasible adaptive stochastic MPC algorithm is presented, demonstrating satisfaction of probabilistic output constraints and hard input constraints. The proposed approach in Bujarbaruah et al. (2018) obtains a better performance compared to Tanaskovic et al. (2014) measured in terms of closed-loop cost, owing to the allowance of output constraint violations with a certain (low) probability.

In this paper, we build on the work of Bujarbaruah et al. (2018), and propose an *Adaptive Stochastic MPC* algorithm that similarly considers probabilistic output constraints and hard input constraints for a Multi Input Multi Output (MIMO) system. Similar to Bujarbaruah et al. (2018), we consider an uncertain FIR model of the system that is subject to bounded disturbances with known mean and variance. The support for the set of all possible models, which we call the Feasible Parameter Set (FPS), is adapted at each timestep using a set membership based approach. In contrast to Bujarbaruah et al. (2018), we additionally consider that the impulse response parameters corresponding to each output are sparse. Such sparse impulse response modeling can be motivated by Benesty et al. (2006); Etter (1985) for MIMO systems. Our goal is to utilize this additional sparsity information to demonstrate performance improvement over the algorithm in Bujarbaruah et al. (2018). Our main contributions can be summarized as follows:

- Offline before the control process, we compute a set containing all possible values of the unknown sparse FIR vectors corresponding to each output, with a very *high* probability. This set, which we call the Feasible Sparse Parameter Set (FSPS) is computed using the Basis Pursuit Denoising Chen et al. (2001) problem.

- Online during the control process, we obtain a point estimate of the unknown system inside the intersection of the FPS and the FSPS, using a Recursive Least Squares (RLS) estimator. Using this estimated system, we propagate our nominal predicted outputs used in the MPC controller objective function to improve performance. Simultaneously, we ensure satisfaction of the output chance constraints for the unknown true system.

- Through numerical simulations, we demonstrate that our algorithm exhibits better performance than the algorithm presented in Bujarbaruah et al. (2018).

2. Problem Description

2.1. System Modeling and Control Objective

We consider stable linear time-invariant systems described by a Finite Impulse Response (FIR) model of the form

$$y(t) = H_a\Phi(t) + w(t), \tag{1}$$

where the number of inputs and outputs considered is n_u and n_y, respectively. The FIR regressor vector of length m is denoted by $\Phi(t) \in \mathbb{R}^{n_u m} = [u_1(t-1), \dots, u_1(t-m), \dots, u_{n_u}(t-1), \dots, u_{n_u}(t-m)]^\top$, where $u_i(t)$ denotes the i^{th} input at time t. The matrix $H_a \in \mathbb{R}^{n_y \times n_u m}$ is a matrix comprising of the impulse response coefficients that relate inputs to the outputs of the system. The disturbance vector $w(t) \in \mathbb{R}^{n_y}$ is assumed to be a zero-mean random variable with a known variance, component-wise bounded as

$$|w_j(t)| \leq \bar{w}_j, \forall j = 1, 2, \dots n_y, \tag{2}$$

where the $\bar{w}_j$ are assumed known. Finally, $y(t) \in \mathbb{R}^{n_y}$ is the measured output of the system. Our goal is to control the output $y(t)$ while satisfying input and output constraints of the form

$$Cu(t) \leq g, \quad t = 0, 1, \dots, \tag{3a}$$

$$\mathbb{P}\{Ey(t) \leq p\} \geq 1 - \epsilon, \quad t = 0, 1, \dots, \tag{3b}$$

where $\epsilon \in (0, 1)$ is the maximum allowed probability of output constraint violation. Following Bujarbaruah et al. (2018), we consider a single linear output chance constraint, meaning E is a row vector and $p \in \mathbb{R}$.

Assumption 1 *We assume that each row of impulse response matrix H_a is sparse. Without loss of generality, we further assume that the sparsity index for each row, i.e., the number of nonzero entries, is at most $\bar{k}$.*

2.2. Method Outline

We assume in this paper that the system matrix H_a in (1) is unknown. This paper proposes a method for identifying this unknown system matrix H_a, and using the estimate in a robust control formulation to safely regulate the constrained uncertain system. Our proposed method uses the following steps:

1. <u>Offline before the control process begins</u>: Use q number of collected input sequence regressors $[\Phi_1, \Phi_2, \dots, \Phi_q]$ to compute a set $\mathcal{B}(H_a)$, called the Feasible Sparse Parameter Set (FSPS). We compute this set via the Basis Pursuit Denoising problem Chen et al. (2001). The FSPS contains the true unknown model H_a, with a *high* probability.

2. <u>Online during the control process</u>: At each timestep t,

   (a) Obtain the current output measurement $y(t)$ and, using the known disturbance bounds (2), update the time-varying set $\mathcal{F}(t)$, which we call the Feasible Parameter Set (FPS). The FPS is a set guaranteed to contain the true model H_a.

(b) Use the previous applied control inputs and measured outputs to construct an estimate $\mu_a(t)$ of H_a, lying in the intersection of $\mathcal{F}(t)$ and the offline-computed FSPS. The estimate is constructed using the Recursive Least Squares method.

(c) Compute the input sequence that minimizes the objective function obtained with $\mu_a(t)$ while satisfying the input and output constraints (3) for all models in the FPS $\mathcal{F}(t)$. Apply the first computed control input and continue to step 2a.

3. Model Estimation and Adaptation

We approximate system (1) with the form

$$y(t) = H(t)\Phi(t) + w(t), \tag{4}$$

where our model $H(t) \in \mathbb{R}^{n_y \times n_u m}$ is a random variable whose support we estimate online during the control process from the output measurements. The support for the set of all possible models $H(t)$ consistent with the recorded system data, which we call the Feasible Parameter Set (FPS), is guaranteed to contain the true model H_a. Based on the knowledge that system (1) has sparse impulse response properties, we also construct a Feasible Sparse Parameter Set (FSPS) offline by solving the Basis Pursuit Denoising Problem. During control run-time, a point estimate of H_a is then computed to lie in the intersection of the offline-computed FSPS and the online-updated FPS. This estimate is then used in the control design. We decouple the offline and online phases of this design process and delineate the steps in detail in the following sections:

3.1. Offline: Construct the Feasible Sparse Parameter Set

The Feasible Sparse Parameter Set (FSPS), denoted by $\mathcal{B}(H_a)$, is a function of the (unknown) true system response H_a, and is synthesized *offline* utilizing the sparsity aspect of system responses from Assumption 1. Proposition 2 clarifies how this set is synthesized.

Definition 1 (Restricted Isometry Property (RIP) Candes and Tao (2005)) *A matrix $\mathcal{A}$ satisfies the Restricted Isometry Property (RIP) of order $\bar{k}$, with constant $\delta \in [0, 1)$, if*

$$(1-\delta)\|x\|_2^2 \leq \|\mathcal{A}x\|_2^2 \leq (1+\delta)\|x\|_2^2, \ \forall x \ \bar{k} \text{ sparse}.$$

The order-$\bar{k}$ restricted isometry constant $\delta_{\bar{k}}(\mathcal{A})$ is the smallest number δ such that the above inequality holds.

Proposition 2 *Suppose we collect q output measurements offline. Suppose $\mathbf{y}_i = \mathcal{A}H_{a_i}^\top + \mathbf{w}_i$ for $i = 1, 2, \ldots, n_y$, where each $\mathbf{y}_i \in \mathbb{R}^{q\times 1}$ and $\mathcal{A} = [\Phi_1, \Phi_2, \ldots, \Phi_q]^\top \in \mathbb{R}^{q\times n_u m}$, $\mathbf{w}_i \in \mathbb{R}^{q\times 1}$ with $\|\mathbf{w}_i\|_2 \leq \sqrt{q}\bar{w}_i$, and $H_{a_i} \in \mathbb{R}^{1\times n_u m}$ denotes the i^{th} row of H_a. If $\delta_{2\bar{k}}(\mathcal{A}) < \sqrt{2} - 1$, then any solution $\hat{\mathbf{x}}$ to the Basis Pursuit Denoising optimization problem*

$$\begin{array}{ll} \min. & \|\mathbf{x}\|_1 \\ \textit{s.t.} & \|\mathcal{A}\mathbf{x} - \mathbf{y}_i\|_2 \leq \sqrt{q}\bar{w}_i, \ (\textit{denoted as } \mathcal{B}(H_{a_i})) \end{array} \tag{5}$$

satisfies $\|\hat{\mathbf{x}} - H_{a_i}\|_2 \leq \bar{C}\sqrt{q}\bar{w}_i$ for some constant $\bar{C} \in \mathbb{R}$, and for all $i = 1, 2, \ldots, n_y$.

Proof See (Wright and Ma, expected 2020, Theorem 3.5.1). ■

The set $\mathcal{B}(H_a)$ is obtained as $\mathcal{B}(H_a) = [\mathcal{B}(H_{a_1}), \dots, \mathcal{B}(H_{a_{n_y}})]^\top$. Note that this set $\mathcal{B}(H_a)$ is synthesized offline, as the regressor vectors $[\Phi_1, \Phi_2, \dots, \Phi_q]$ are required to come from a Gaussian distribution in order to ensure the RIP property of each matrix $\mathcal{A}_i$ for $i = 1, 2, \dots, n_y$. Such Gaussian inputs are not always allowable during control with system constraints (3). Details on our choice of these offline regressors $[\Phi_1, \Phi_2, \dots, \Phi_q]$ to ensure $\delta_{2\bar{k}}(\mathcal{A}) < \sqrt{2} - 1$ with *high* probability are described in Bujarbaruah and Vallon (2019).

Remark 3 *As pointed out in the proof of (Wright and Ma, expected 2020, Theorem 3.5.1), a possible choice of the numerical constant $\bar{C}$ in Proposition 2 is $\bar{C} = \frac{2}{\sqrt{\lambda}}$, with $\sqrt{\lambda} = \frac{1 - \delta_{2\bar{k}}(1+\sqrt{2})}{\sqrt{2(1+\delta_{2\bar{k}})}}$. However, since $\delta_{2\bar{k}}$ is not* exactly *known, computing $\bar{C}$ and hence $\mathcal{B}(H_a)$ accurately is not possible. We see that $\bar{C} \to 2\sqrt{2}$ as $\delta_{2\bar{k}} \to 0$. Therefore offline regressor vectors $[\Phi_1, \Phi_2, \dots, \Phi_q]$ should be chosen to ensure $\delta_{2\bar{k}} < \bar{\delta}$, with $\bar{\delta} \ll \sqrt{2} - 1$. Under such choice of the offline regressors as shown in the Appendix, we pick the constant $\bar{C} \approx 2\sqrt{2}$.*

3.2. Online: Update the Feasible Parameter Set

Following Tanaskovic et al. (2014); Bujarbaruah et al. (2018), a set-membership identification method is used for updating the time-varying FPS, denoted by $\mathcal{F}(t)$. The initialization of $\mathcal{F}(0)$ is done considering the fact that the true system (1) is stable. We update the FPS as given by

$$\mathcal{F}(t) = \mathcal{F}(t-1) \cap \{H(t) : H(t)\Phi(t) \leq y(t) + \bar{w}\} \cap \{H(t) : -H(t)\Phi(t) \leq -y(t) + \bar{w}\}, \quad (6)$$

where $\bar{w} = [\bar{w}_1, \dots, \bar{w}_{n_y}]^\top$ is the bound of the additive disturbance given by (2). In order to bound the computational complexity of (6) over time, an alternative algorithm to compute (6) is presented in Tanaskovic et al. (2014).

3.3. Online: Obtain Point Estimate $\mu_a(t)$

We rewrite (4) as $y(t) = \mathbf{\Phi}(t)\mathbf{H}(t) + w(t)$, where matrices $\mathbf{\Phi}(t) \in \mathbb{R}^{n_y \times n_y n_u m}$ and $\mathbf{H}(t) \in \mathbb{R}^{n_y n_u m \times 1}$ are shown in Bujarbaruah and Vallon (2019). Furthermore, let σ_w^2 be the variance of the disturbance $w(t)$. Let the initial prior mean and variance estimates for true system be $\mu_{\mathbf{a}}(0)$ and $\sigma_{\mathbf{a}}^2(0)$, respectively. Now, the conditional mean and variance estimates, given measurements up to $y(t)$, can be obtained using the Recursive Least Squares method (Anderson and Moore, 1979, Sec. (3.1)). We ensure that the mean point estimate $\mu_{\mathbf{a}}(t)$ at any time instant t is chosen as a point contained in a set $\mathcal{F}_p(t)$, that is, $\mu_a(t) \in \mathcal{F}_p(t)$, with

$$\mathcal{F}_p(t) = \mathcal{F}(t) \cap \mathcal{B}(H_a). \quad (7)$$

As shown in Bujarbaruah and Vallon (2019), after finding a $\mu_{\mathbf{a}}(t)$, this is achieved by solving

$$X^\star = \arg \min_{X \in \mathcal{F}_p(t)} (X - \mu_{\mathbf{a}}(t))^\top M (X - \mu_{\mathbf{a}}(t)), \text{ and then assigning } \mu_{\mathbf{a}}(t) = X^\star, \quad (8)$$

where $M = (\sigma_{\mathbf{a}}^2(t))^{-1} \succ 0$. The mean in matrix form, that is, $\mu_a(t) \in \mathbb{R}^{n_y \times n_u m}$ is obtained by reorganizing $\mu_{\mathbf{a}}(t) \in \mathbb{R}^{n_y n_u m \times 1}$ into $n_u m$ columns. This provides the (linear) minimum mean squared error estimate of the true system H_a. Note that (8) is a convex optimization problem.

Remark 4 *In Bujarbaruah et al. (2018) the set $\mathcal{F}_p(t)$ in (7) is set as the FPS $\mathcal{F}(t)$ for all timesteps.*

4. Control Synthesis

4.1. Prediction Model

Let $N > m$ be the prediction horizon for a predictive controller for system (1). We denote the predicted system outputs at time t by $y(k|t) = H(t)\Phi(k|t) + w(k)$, for some $H(t) \in \mathcal{F}(t)$. Similarly, $\Phi(k|t)$ denotes the *predicted regressor vector*, for $k \in \{t+1, \ldots, t+N\}$, and is computed as

$$\Phi(k|t) = W\Phi(k-1|t) + Zu(k-1|t), \tag{9}$$

where, the matrices W and Z are as reported in the Appendix. With these predicted regressor vectors, the estimated system $\mu_a(t)$ obtained in Section 3.3 is used to propagate the nominal predicted outputs as $\hat{y}(k+1|t) = \mu_a(t)\Phi(k+1|t)$, for all $k \in \{t, \ldots, t+N-1\}$. This is shown in the optimization problem presented in Section 4.3.

4.2. Reformulation of Chance Constraints

Within each prediction horizon we enforce $\mathbb{P}\{Ey(k|t) \leq p\} \geq 1-\epsilon$, where $y(k|t)$ is a function of some $H(t) \in \mathcal{F}(t)$. Therefore, to ensure satisfaction of (3b) despite uncertainty in the true system, we must satisfy the constraint for all $H(t) \in \mathcal{F}(t)$. Using the theory of distributionally robust optimization Calafiore and El Ghaoui (2006); Zymler et al. (2013), we can conservatively approximate the output chance constraints (3b) as

$$\kappa_\epsilon \sqrt{\bar{\Phi}^\top(k|t)\Gamma\bar{\Phi}(k|t)} + \Phi^\top(k|t)\bar{E}\mathbf{H}(t) - p \leq 0, \ \ \forall H(t) \in \mathcal{F}(t), \tag{10}$$

where we have $k \in \{t+1, \ldots, t+N\}$, $\kappa_\epsilon = \sqrt{\frac{1-\epsilon}{\epsilon}}$ and $\bar{\Phi}(k|t) = [\Phi^\top(k|t) \quad 1 \quad 1]^\top$. Here, $\Gamma \succeq 0$ is an appended covariance matrix shown in the Appendix. As $\mathcal{F}(t)$ is a polytope, (10) can be succinctly written as

$$\kappa_\epsilon \sqrt{\bar{\Phi}^\top(k|t)\Gamma\bar{\Phi}(k|t)} + \Phi^\top(k|t)\bar{E}f^j(t) - p \leq 0, \tag{11}$$

where $f^j(t)$ denote the vertices of the polytope $\mathcal{F}(t)$.

4.3. MPC Problem

We solve the following optimization problem for given weights $Q \in \mathbb{R}^{n_y \times n_y}, S \in \mathbb{R}^{n_u \times n_u} \succ 0$:

$$\begin{aligned}
\min_{U(t)}. \quad & \sum_{k=t}^{t+N-1} [\hat{y}^\top(k|t)Q\hat{y}(k|t) + u^\top(k|t)Su(k|t)] + \hat{y}^\top(t+N|t)Q\hat{y}(t+N|t) \\
\text{s.t.} \quad & \hat{y}(k+1|t) = \mu_a(t)\Phi(k+1|t), \\
& \hat{y}(t|t) = y(t), \\
& Cu(k|t) \leq g, \\
& \Phi(t+N|t) = W\Phi(t+N|t) + Zu(t+N-1|t), \\
& \kappa_\epsilon \sqrt{\bar{\Phi}^\top(k+1|t)\Gamma\bar{\Phi}(k+1|t)} + \Phi^\top(k+1|t)\bar{E}f^j(t) \leq p, \\
& \forall k = t, \ldots, t+N-1, \\
& \forall f^j(t) \in \text{vertex}(\mathcal{F}(t)), \ \mu_a(t) \in \mathcal{F}(t) \cap \mathcal{B}(H_a),
\end{aligned} \tag{12}$$

where $U(t) = [u(t|t)^\top, u(t+1|t)^\top, \ldots, u(t+N-1|t)^\top]^\top$, and the regressor $\Phi(k|t)$ is as in (9). We have included the terminal constraint on the regressor vector as given in Tanaskovic et al. (2014)

$$\Phi(t+N|t) = W\Phi(t+N|t) + Zu(t+N-1|t). \tag{13}$$

After solving (12), we apply the first input

$$u(t) = u^\star(t|t) \tag{14}$$

to system (1) in closed-loop. We then re-solve (12) at timestep $t+1$ with new estimated data $\mu_a(t+1)$ and $\mathcal{F}(t+1)$. This yields a receding-horizon control scheme. See Algorithm 1.

Algorithm 1 Adaptive Stochastic MPC: Sparse-FIR MIMO Systems

Initialize: $\mathcal{F}(0), \mu_{\mathbf{a}}(0), \sigma_{\mathbf{a}}(0)$.
Inputs: $q, \bar{w}, \bar{k}, t_{\text{end}}$

begin Basis Pursuit Denoising (offline)

Construct offline regressors $[\Phi_1, \Phi_2, \ldots, \Phi_q]$ such that operator $\mathcal{A}$ in Proposition 2 satisfies $\delta_{2\bar{k}}(\mathcal{A}) < \bar{\delta} \ll \sqrt{2} - 1$;
Solve (5) for $i = 1, 2, \ldots, n_y$ to obtain the FSPS $\mathcal{B}(H_a)$;

end Basis Pursuit Denoising

begin MPC control process (online)

for timestep $1 \leq t \leq t_{\text{end}}$ **do**
  Obtain $y(t)$ and update the FPS $\mathcal{F}(t)$ using (6);
  Estimate mean and variance $\mu_{\mathbf{a}}(t)$ and $\sigma^2_{\mathbf{a}}(t)$ with RLS estimator. Project the mean with (8) to the set $\mathcal{F}_p(t)$;
  Solve (12) and apply $u(t) = u^\star(t|t)$ to system (1);
end for

Remark 5 *See Bujarbaruah and Vallon (2019) for a detailed proof of recursive feasibility.*

5. Numerical Simulations

We compare the performance of our Algorithm 1 with that from the adaptive stochastic MPC presented in Bujarbaruah et al. (2018). This performance is measured in terms of the expected closed-loop cost $\mathbb{E}[\mathcal{V}]$. The closed-loop cost of any trajectory which is a function of the disturbance realization $\bar{\mathbf{w}} = [w(0), w(2), \ldots, w(t_{\text{end}})]$, is given by $\mathcal{V}(\bar{\mathbf{w}}, \Phi(0), \mathcal{F}_p(0), \mu_{\mathbf{a}}(0), \sigma_{\mathbf{a}}(0)) = \sum_{t=0}^{t_{\text{end}}} y^\top(t)Qy(t) + (u^\star(t))^\top S u^\star(t)$, where $\mathcal{F}_p(\cdot)$ for Algorithm 1 is obtained as in (7), and for the algorithm in Bujarbaruah et al. (2018), $\mathcal{F}_p(\cdot) = \mathcal{F}(\cdot)$, as Remark 4 points out. For simulating both the algorithms, we use the parameters given in Table 1, with the true system response given as $H_a = [-1, 0, 0, 0, 0, 0, 0, 0, 0, -2, 0]$, which is $\bar{k} = 2$ sparse. We run 100 Monte Carlo simulations with both algorithms for 100 randomly chosen disturbance sequences $\bar{\mathbf{w}}$. We approximate the average closed-loop cost $\mathbb{E}[\mathcal{V}]$ with the empirical average

$$\hat{\mathcal{V}}(\Phi(0), \mathcal{F}_p(0), \mu_{\mathbf{a}}(0), \sigma_{\mathbf{a}}(0)) = \frac{1}{100} \sum_{\tilde{m}=1}^{100} \mathcal{V}((\bar{\mathbf{w}})^{\star\tilde{m}}, \Phi(0), \mathcal{F}_p(0), \mu_{\mathbf{a}}(0), \sigma_{\mathbf{a}}(0)), \tag{15}$$

where $(\cdot)^{\star\tilde{m}}$ represents the $\tilde{m}^{\text{th}}$ Monte Carlo sample.

Table 1: Simulation Parameters

Parameter	Value	Parameter	Value
m	10	N	12
t_{end}	20	w	$U \sim [-0.1, 0.1]$
n_u	1	n_y	1
ϵ	0.1	κ_ϵ	3
E	1	p	5
C	$\text{diag}(1, -1)$	g	$[1, 1]^\top$
Q	$\text{diag}(20, 20)$	S	$\text{diag}(2, 2)$
$\mu_{\mathbf{a}}(0)$	$\mathbf{1}_{10\times1}$	$\sigma^2_{\mathbf{a}}(0)$	$0.1 \times \mathbb{I}_{10}$
$\Phi(0)$	$0.1 \times \mathbf{1}_{10\times1}$	$\mathcal{F}(0)$	$-\mathbf{3}_{10\times1} \leq H^\top \leq \mathbf{3}_{10\times1}$

5.1. Cost Comparison

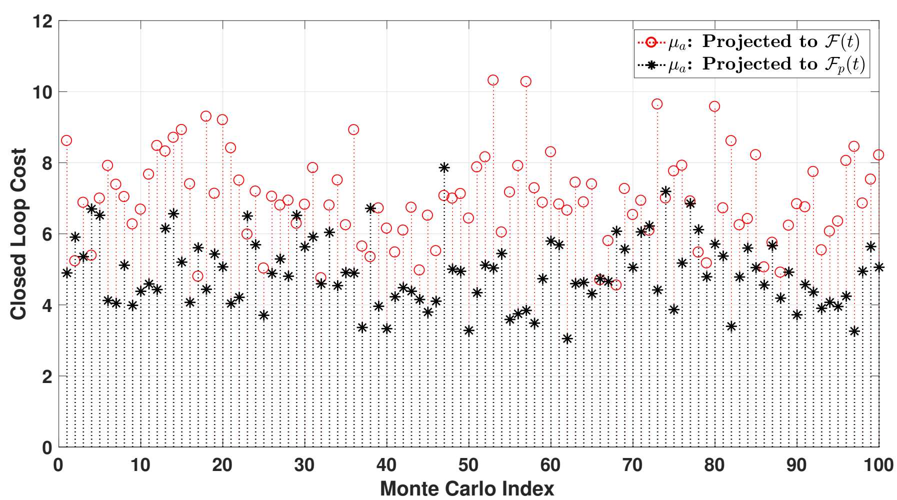

Figure 1: Closed-Loop Cost $\sum_{t=0}^{20} y^\top(t)Qy(t) + (u^\star(t))^\top Su^\star(t)$ Along 100 Monte Carlo Simulation of Trajectories.

Fig. 1 shows the comparison of closed-loop cost expressed as $\mathcal{V}(\bar{\mathbf{w}}, \Phi(0), \mathcal{F}_p(0), \mu_{\mathbf{a}}(0), \sigma_{\mathbf{a}}(0)) = \sum_{t=0}^{20} y^\top(t)Qy(t) + (u^\star(t))^\top Su^\star(t)$ for 100 different Monte Carlo draws of trajectories, obtained with Algorithm 1 and the algorithm in Bujarbaruah et al. (2018). We see that the empirical average closed-loop cost obtained as (15) for Algorithm 1 is around 30% lower than the corresponding value obtained with Bujarbaruah et al. (2018). This demonstrates performance gain by Algorithm 1 as a consequence of leveraging sparsity information of H_a via the FSPS set $\mathcal{B}(H_a)$. This improvement is cost can be explained in detail from system trajectories. See Bujarbaruah and Vallon (2019).

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