Appendix

A. Details of Environments

In this section, we explain the details of the environments used in our experiments. The properties of each environment are summarized in Table 2.

Environment	State dim	Action dim	Control	Episode Length
CartPole-v0	4	2	Discrete	200
Pendulum-v0	3	1	Continuous	200
MountainCar-v0	2	3	Discrete	200
MountainCarContinuous-v0	2	1	Continuous	999
Acrobot-v1	6	3	Discrete	500
Ant-v2	111	8	Continuous	1000
HalfCheetah-v2	17	6	Continuous	1000
Hopper-v2	11	3	Continuous	1000
Walker2d-v2	17	6	Continuous	1000
Humanoid-v2	376	17	Continuous	1000
cheetah run	17	6	Continuous	1000
reacher easy	6	2	Continuous	1000
ball_in_cup catch	8	2	Continuous	1000
Reacher	11	2	Continuous	50
Pointmaze	4	2	Continuous	150

Table 2. The details of Open AI Gym and DeepMind Control Suite Environments used in this paper.

CartPole The states of CartPole environment are composed of 4-dimension real values, cart position, cart velocity, pole angle, and pole angular velocity. The initial states are uniformly randomized. The action space is discretized into 2-dimension. The cumulative reward corresponds to the time steps in which the pole isn't fallen down.

Pendulum The states of Pendulum environment are composed of 3-dimension real values, cosine of pendulum angle $\cos\varphi$, sine of pendulum angle $\sin\varphi$, and pendulum angular velocity $\dot{\varphi}$. The initial states are uniformly randomized. The action space is continuous and 1-dimension. The reward is calculated by the following equation;

$$r_t = -(\varphi_t^2 + 0.1 * \dot{\varphi_t}^2 + 0.001 * \|\boldsymbol{a}_t\|_2^2).$$

MountainCar The states of MountainCar environment are composed of 2-dimension real values, car position, and its velocity. The initial states are uniformly randomized. The action space is discretized into 3-dimension. The cumulative reward corresponds to a negative value of the time steps in which the car doesn't reach the goal region.

MountainCarContinuous The 1-dimensional continuous action space version of MountainCar environment. The reward is calculated by the following equation;

$$r_t = -0.1 * \|\boldsymbol{a}_t\|_2^2 + 100 * \mathbb{1}[x_t \geq x_g \text{ and } \dot{x}_t \geq \dot{x}_g].$$

Acrobot The states of Acrobot environment consist of 6-dimension real values, sine, and cosine of the two rotational joint angles and the joint angular velocities. The initial states are uniformly randomized. The action space is continuous and 3-dimension. The cumulative reward corresponds to a negative value of the time steps in which the end-effector isn't swung up at a height at least the length of one link above the base.

Ant The state space is 111-dimension, position and velocity of each joint, and contact forces. The initial states are uniformly randomized. The action is an 8-dimensional continuous space. The reward is calculated by the following equation;

$$r_t = \dot{x}_t - 0.5 * \|\boldsymbol{a}_t\|_2^2 - 0.0005 * \|\boldsymbol{s}_t^{\text{contact}}\|_2^2 + 1.$$

HalfCheetah The state space is 17-dimension, position and velocity of each joint. The initial states are uniformly randomized. The action is a 6-dimensional continuous space. The reward is calculated by the following equation;

$$r_t = \dot{x}_t - 0.1 * \|\boldsymbol{a}_t\|_2^2.$$

Hopper The state space is 11-dimension, position and velocity of each joint. The initial states are uniformly randomized. The action is a 3-dimensional continuous space. This environment is terminated when the agent falls down. The reward is calculated by the following equation;

$$r_t = \dot{x}_t - 0.01 * \|\boldsymbol{a}_t\|_2^2 + 1.$$

Walker2d The state space is 17-dimension, position and velocity of each joint. The initial states are uniformly randomized. The action is a 6-dimensional continuous space. This environment is terminated when the agent falls down. The reward is calculated by the following equation;

$$r_t = \dot{x}_t - 0.01 * \|\boldsymbol{a}_t\|_2^2 + 1.$$

Humanoid The state space is 376-dimension, position and velocity of each joint, contact forces, and friction of actuator. The initial states are uniformly randomized. The action is a 17-dimensional continuous space. This environment is terminated when the agent falls down. The reward is calculated by the following equation;

$$r_t = 1.25 * \dot{x}_t - 0.1 * \|\boldsymbol{a}_t\|_2^2 - \min(5e^{-7} * \|\boldsymbol{s}_t^{\text{contact}}\|_2^2,\ 10) + 5.$$

cheetah run The state space is 17-dimension, position and velocity of each joint. The initial states are uniformly randomized. The action is a 6-dimensional continuous space. The reward is calculated by the following equation;

$$r_t = \begin{cases} 0.1 * \dot{x}_t & (0 \le \dot{x}_t \le 10) \\ 1 & (\dot{x}_t > 10). \end{cases}$$

reacher easy The state space is 6-dimension, position and velocity of each joint. The initial states are uniformly randomized. The action is a 2-dimensional continuous space. The reward is calculated by the following equation;

$$r_t = \mathbb{1}[x_t - x_g \le \epsilon].$$

ball_in_cup catch The state space is 8-dimension, position and velocity of each joint. The initial states are uniformly randomized. The action is a 2-dimensional continuous space. The reward is calculated by the following equation;

$$r_t = \mathbb{1}[x_t - x_g \le \epsilon].$$

Reacher The state space is 11-dimension, position and velocity of each joint. The initial states are uniformly randomized. The action is a 2-dimensional continuous space. We modify the reward function for the experiments (see Appendix G).

Pointmaze We use the implementation provided by Fu et al. (2020) (Figure 12). The state space is 4-dimension, position and velocity. The initial states are uniformly randomized. The action is a 2-dimensional continuous space. We modify the reward function for the experiments (see Appendix G).

B. Additional Details of Synthetic Experiments

We provide the rest of the results, presented in Section 5, and then show additional comparisons between PIC or POIC and marginal entropies in this setting.

B.1. Answer to (2) in Section 5

First, Figure 5 shows the full results of Figure 2, which holds the same tendency overall.

Similar to the results of POIC, presented in Section 5, we also investigate what happens to PIC throughout a realistic learning process, optimizing μ in $p(\theta)$ to solve the MDP by evolution strategy and observing this metric and performance of the agent during training. Following the experimental settings of POIC, we assume the Gaussian prior, $\mathcal{N}(\mu, I)$, and vary μ initializations using $[-10, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 10]$. We set $N = M = 1000$, and horizon is $T = 3$. Figure 6 reveals that the initial prior with high PIC (e.g. $\mu = 0$) actually solves the environment faster than those with low PIC (e.g. $\mu = -3.0, -4.0, -5.0$), which seems the same as the case of POIC. Interestingly, Figure 6 also shows that high PIC corresponds to regions of fastest learning.

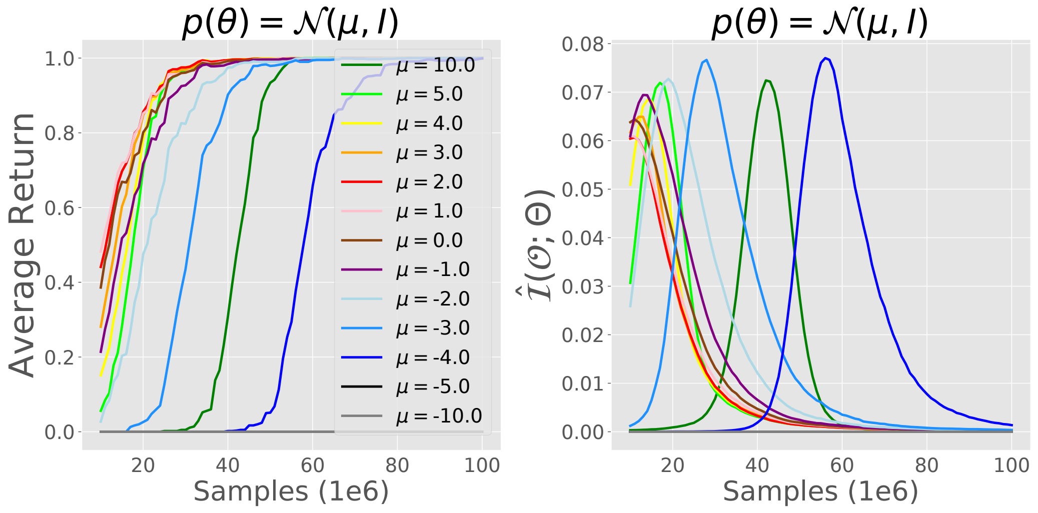
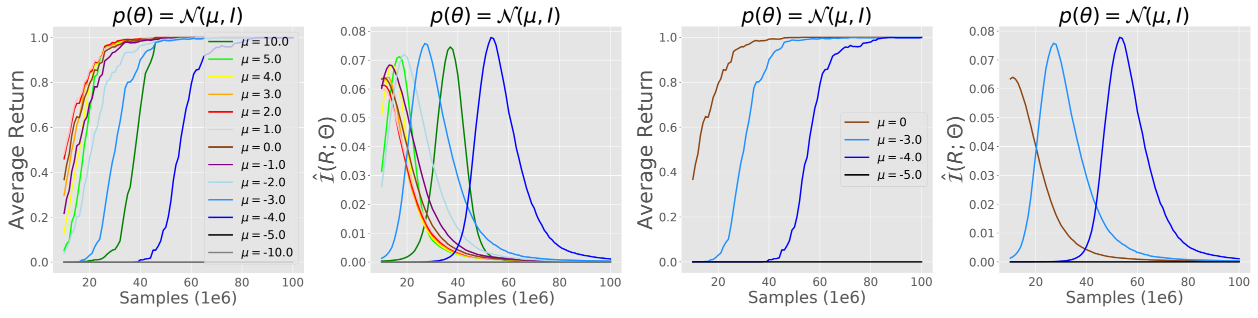


Figure 5. Average return (left) and POIC (right) during the training of evolution strategies. We change the parameter of initial prior distribution μ and optimize it.



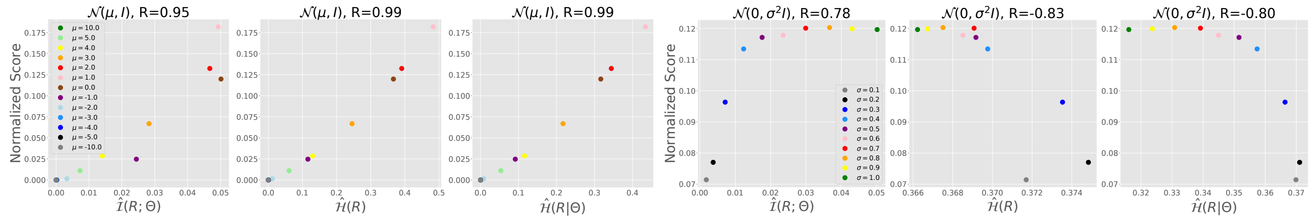
(a) All results (b) $\mu = 0.0, -3.0, -4.0, -5.0$

Figure 6. Average return and PIC during the training of evolution strategies. We vary the parameter of initial prior distribution μ and optimize it. (a) shows the all results, and in (b), we extract a few of them ($\mu = 0.0, -3.0, -4.0, -5.0$; we separate these for the visibility). High PIC surely corresponds to regions of faster learning, which is similar trends of POIC.

B.2. Are PIC and POIC more suitable for evaluating task solvability than other alternatives?

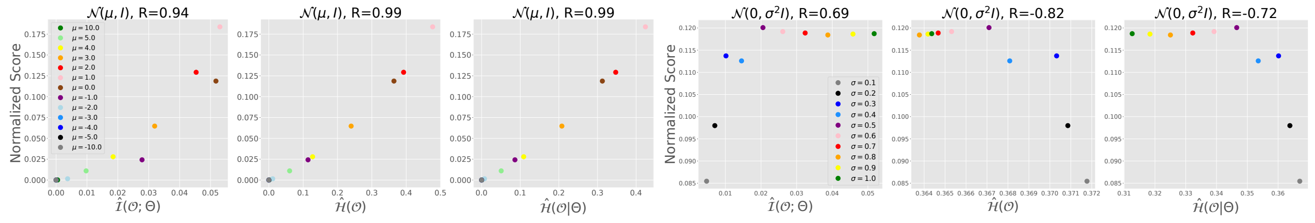
We consider the relations between the normalized score and each reward-based metric ($\mathcal{I}(R;\Theta)$, $\mathcal{H}(R)$, $\mathcal{H}(R|\Theta)$), or optimality-based metric ($\mathcal{I}(\mathcal{O};\Theta)$, $\mathcal{H}(\mathcal{O})$, $\mathcal{H}(\mathcal{O}|\Theta)$) on the MDP $\mathcal{M}$, for a variety of the prior distributions. We test two variants of Gaussian prior, (a) $\mathcal{N}(\mu, I)$ (changing $\mu \in \{-10, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 10\}$), and (b) $\mathcal{N}(0, \sigma^2 I)$ (changing $\sigma \in \{0.1, 0.2, ..., 0.9, 1.0\}$). We set $N = M = 1000$, and the horizon is $T = 3$ (note that as far as we observed, $N = M = 100$ could not estimate POIC metric properly). Figure 7 indicates the relation between each metric of reward and normalized score. In (a), we change μ, and in (b) we change σ. We see that PIC shows positive correlation to normalized score both in (a) ($R = 0.95$) and (b) ($R = 0.78$). The marginal and conditional entropies show, however, positive correlation in (a) (both $R = 0.99$), but negative in (b) ($R = -0.83$ and $R = -0.80$). This suggests that there are cases where the marginal or conditional entropy alone might not reflect task solvability appropriately. Similar to PIC, we see in Figure 8 that POIC shows positive correlation to normalized score both in (a) ($R = 0.94$) and (b) ($R = 0.69$). The marginal and conditional entropies show, however, positive correlation in (a) (both $R = 0.99$), but negative in (b) ($R = -0.82$ and $R = -0.72$).

These observations suggest that there are cases where the marginal or conditional entropy alone might not reflect task solvability appropriately.



(a) $p(\theta) = \mathcal{N}(\mu, I)$ (b) $p(\theta) = \mathcal{N}(0, \sigma^2 I)$

Figure 7. Relation between each metric (x-axis; PIC (left), marginal entropy (middle), conditional entropy (right)) and normalized score (y-axis). We compute the Pearson correlation coefficient (above the plots). In (a), we change μ, and in (b) we change σ. PIC is the only metric that shows a consistently positive correlation. The marginal and conditional entropy doesn't have such consistency.



(a) $p(\theta) = \mathcal{N}(\mu, I)$ (b) $p(\theta) = \mathcal{N}(0, \sigma^2 I)$

Figure 8. Relation between each metric (x-axis; POIC (left), marginal entropy (middle), conditional entropy (right)) and normalized score (y-axis). We compute the Pearson correlation coefficient (above the plots). In (a), we change μ, and in (b) we change σ. Same as the case of PIC (Figure 7), POIC is the only metric that shows a consistently positive correlation. The marginal and conditional entropy doesn't have such consistency.

C. Normalized Score as Task Complexity Measure

In this section, we explain the details of normalized score. One intuitive and brute-force approach to measure the ground-truth task complexity in complex environments is to compare the performances of just an *average* agent among the environments. When we try to realize this impractical method, we face two difficulties: (1) how can we obtain an *average* agent? (2) how do we manage the different reward scale among the different environments?

To deal with (1), we have two options: substituting average score among diverse RL algorithms or random policy sampling score, for the performance of average agent. We can also solve (2) by normalizing some average quantity divided by max-min value. In the following section, we explain the details of each approach.

C.1. Algorithm-based Normalized Score

To compute the normalized score, we prepare the set of algorithms for evaluation and then execute them all. Since algorithms with different hyper-parameter behave differently, we treat them as different "algorithms". After the evaluation, we compute the average return over the algorithms $r^{\text{algo}}_{\text{ave}}$. This value, however, completely differs from all environments, due to the range of reward. To normalize average return for the comparison over the environments, we compute the maximum return $r_{\max}$ and minimum return $r_{\min}$. In practice, we take the maximum between this algorithm-based and random-sampling-based maximum scores, and use the minimum return obtained by random policy sampling:

$$\text{Normalized Score} = \frac{r^{\text{algo}}_{\text{ave}} - r^{\text{rand}}_{\min}}{\max(r^{\text{rand}}_{\max},\ r^{\text{algo}}_{\max}) - r^{\text{rand}}_{\min}}.$$

We use three types of environments, classic control, MuJoCo, and DeepMind Control Suite (DM Control). For MuJoCo and DM Control, we test SAC, MPO and AWR. To simulate the diverse set of algorithms, we employ the leaderboard scores reported in previous SoTA works (Fujimoto et al., 2018; Peng et al., 2019; Laskin et al., 2020).

C.1.1. Classic Control

For classic control, we run 23 algorithms, based on PPO, DQN and Evolution Strategy for discrete action space environments, and PPO, DDPG, SAC, and Evolution Strategy for continuous action space environments with different hyper-parameters, such as network architecture or discount factor (Table 3). We average each performance with 5 random seeds, and also average over algorithms.

C.1.2. MuJoCo

We test SAC, MPO and AWR, following hyper parameters in original implementations. We average each performance with 10 random seeds and train each agent 1M steps for Hopper, 3M for Ant, HalfCheetah, Walker2d, and 10M for Humanoid (Table 5). To simulate the diverse set of algorithms, we employ the leaderboard scores reported in previous works (Fujimoto et al., 2018; Peng et al., 2019) (Table 6 and Table 7). Totally, we use 17 algorithms for Ant, HalfCheetah, Hopper and Walker2d, and 10 algorithms for Humanoid to compute $r^{\text{algo}}_{\text{ave}}$ and $r^{\text{algo}}_{\max}$ (Table 4).

C.1.3. DeepMind Control Suite

We also test SAC, MPO and AWR, following hyper parameters in original implementations (Table 9). We average each performance with 10 random seeds and train each agent 500k steps. To simulate the diverse set of algorithms, we employ the leaderboard scores reported in previous work (Laskin et al., 2020) (Table 10). Totally, we use 11 algorithms for cheetah run and ball_in_cup catch, and 10 algorithms for reacher easy to compute $r^{\text{algo}}_{\text{ave}}$ and $r^{\text{algo}}_{\max}$ (Table 8).

C.2. Random-Sampling-based Normalized Score

In Oller et al. (2020), they compute some representatives (e.g. 99.9 percentile score) obtained via random weight guessing and compare them qualitatively among the variety of environments. We extend this idea to our settings – quantitative comparison of the task difficulty.

Through the random policy sampling, we can compute the average cumulative reward $r^{\text{rand}}_{\text{ave}}$ and then normalize it using maximum return $r_{\max}$ and minimum return $r_{\min}$. In practice, we take the maximum between this algorithm-based and random-search-based maximum scores, and use the minimum return obtained by random policy search:

$$\text{Normalized Score} := \frac{r^{\text{rand}}_{\text{ave}} - r^{\text{rand}}_{\text{min}}}{\max(r^{\text{rand}}_{\text{max}},\ r^{\text{algo}}_{\text{max}}) - r^{\text{rand}}_{\text{min}}}.$$

This method seems easy to use since we do not need extensive evaluations by a variety of RL algorithms. However, this random-sampling-based normalized score is highly biased towards the early stage of learning. It might not reflect the overall nature of environments properly.

C.3. Correlation to Obvious Properties of MDP

In this section, we verify that these brute-force task solvability metrics do not just depend on obvious properties of MDP or policy networks, such as state and action dimensionalities, horizon, and the other type of normalized score.

Figure 9 summarizes the correlation among those metrics. While some properties such as action dimensions or episode length have negative correlations with algorithm-based normalized score, compared to Figure 3, our proposed POIC seems much better than those metrics (see also Table 11 for the details).

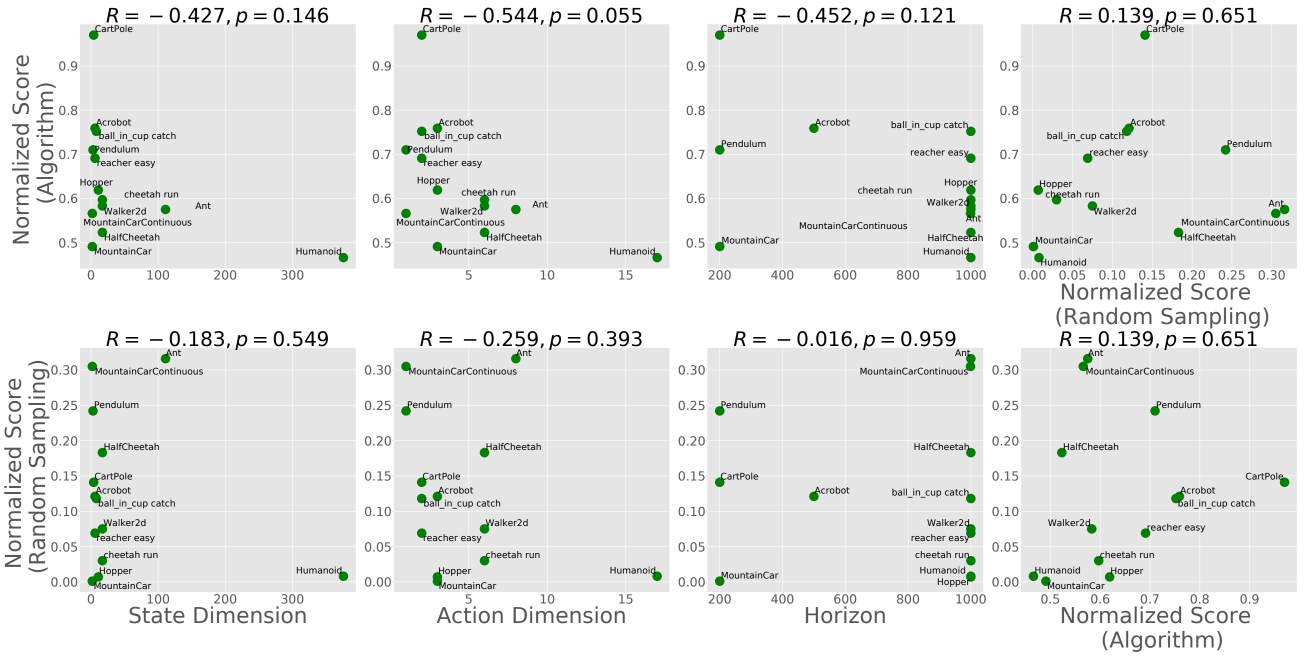


Figure 9. 2D-Scatter plots between each metric (State dimension, action dimension, episode horizon, and normalized scores; x-axis) and each normalized score (algorithm-based (top) and random-sampling-based (bottom); y-axis).

Algorithm	Hyper-Parameters	CartPole	Pendulum	MountainCar	MountainCarContinuous	Acrobot
PPO	(64, 64), $\gamma = 0.995$	200.00	-1045.16	-97.48	65.05	-66.84
PPO	(64, 64), $\gamma = 0.99$	200.00	-582.53	-117.83	18.82	-67.15
PPO	(64, 64), $\gamma = 0.95$	200.00	-170.45	-158.77	9.53	-67.42
PPO	(128, 64), $\gamma = 0.995$	200.00	-1152.09	-118.02	95.89	-67.42
PPO	(128, 64), $\gamma = 0.99$	200.00	-467.42	-104.73	0.00	-72.83
PPO	(128, 64), $\gamma = 0.95$	199.75	-151.43	-116.44	0.00	-68.09
PPO	(128, 128), $\gamma = 0.995$	200.00	-1143.59	-97.22	56.81	-64.83
PPO	(128, 128), $\gamma = 0.99$	200.00	-707.25	-97.21	30.75	-64.90
PPO	(128, 128), $\gamma = 0.95$	199.79	-161.28	-98.82	0.00	-67.25
ES	(16, 16), $\sigma = 0.1$	200.00	-1062.54	-128.82	0.00	-80.30
ES	(16, 16), $\sigma = 0.1$, rand	200.00	-1059.30	-127.67	0.00	-79.57
ES	(16, 16), $\sigma = 0.3$, rand	143.19	-1175.70	-200.00	0.00	-182.21
ES	(64, 64), $\sigma = 0.1$	200.00	-999.26	-136.48	0.00	-80.97
ES	(64, 64), $\sigma = 0.1$, rand	200.00	-1012.32	-131.85	0.00	-80.83
ES	(64, 64), $\sigma = 0.3$, rand	158.43	-1180.50	-200.00	0.00	-257.48
DQN	(100, 100), $\gamma = 0.95$	188.80	–	-200.00	–	-360.57
DQN	(100, 100), $\gamma = 0.99$	200.00	–	-164.76	–	-250.53
DQN	(200, 200), $\gamma = 0.95$	176.69	–	-192.70	–	-296.48
DQN	(200, 200), $\gamma = 0.99$	200.00	–	-150.86	–	-381.99
DQN	(50, 50), $\gamma = 0.95$	200.00	–	-154.91	–	-318.57
DQN	(50, 50), $\gamma = 0.99$	200.00	–	-167.02	–	-329.15
DQN	(50, 50, 50), $\gamma = 0.95$	200.00	–	-167.91	–	-274.28
DQN	(50, 50, 50), $\gamma = 0.99$	200.00	–	-167.22	–	-169.92
SAC	(128, 128), $\gamma = 0.99$	–	-129.30	–	0.00	–
SAC	(128, 128), $\gamma = 0.95$	–	-138.26	–	0.00	–
SAC	(256, 256), $\gamma = 0.99$	–	-128.63	–	0.00	–
SAC	(256, 256), $\gamma = 0.95$	–	-138.27	–	19.21	–
DDPG	(150, 50), $\gamma = 0.95$	–	-138.76	–	0.00	–
DDPG	(150, 50), $\gamma = 0.99$	–	-130.35	–	0.00	–
DDPG	(400, 300), $\gamma = 0.95$	–	-138.61	–	0.00	–
DDPG	(400, 300), $\gamma = 0.99$	–	-131.21	–	0.00	–
$r^{\text{algo}}_{\text{ave}}$	–	194.20	-571.49	-143.12	12.87	-162.92
$r^{\text{algo}}_{\max}$	–	200.00	-128.63	-97.21	95.89	-64.83

Table 3. Performance of a variety of algorithms in classic control. The results are averaged over 5 random seeds. We change 2 hyper-parameters, architecture of neural networks and discount factor γ.

Environments	Average	Maximum
Ant	2450.8	6584.2
HalfCheetah	6047.2	15266.5
Hopper	2206.7	3564.1
Walker2d	3190.8	5813.0
Humanoid	3880.8	8264.0

Table 4. Average and maximum scores in MuJoCo environments, calculated from 17 algorithms (10 for Humanoid) (Table 5, Table 6 and Table 7).

Environments	SAC	MPO	AWR
Ant	5526.4	6584.2	1126.7
HalfCheetah	15266.5	11769.6	5742.4
Hopper	2948.9	2135.5	3084.7
Walker2d	5771.8	3971.5	4716.6
Humanoid	8264.0	5708.7	5572.6

Table 5. SAC, MPO and AWR results in MuJoCo environments. These results are averaged across 10 seeds.

Environments	TRPO	PPO	DDPG	TD3	SAC	RWR	AWR
Ant	2901	1161	72	4285	5909	181	5067
HalfCheetah	3302	4920	10563	4309	9297	1400	9136
Hopper	1880	1391	855	935	2769	605	3405
Walker2d	2765	2617	401	4212	5805	406	5813
Humanoid	552	695	4382	81	8048	509	4996

Table 6. Performance of a variety of algorithms in MuJoCo environments, reported by Peng et al. (2019). These results are averaged across 5 seeds.

Environments	TD3	DDPG(1)	DDPG(2)	PPO	TRPO	ACKTR	SAC
Ant	4372	1005	889	1083	-76	1822	655
HalfCheetah	9637	3306	8577	1795	-16	1450	2347
Hopper	3564	2020	1860	2165	2471	2428	2997
Walker2d	4683	1844	3098	3318	2321	1217	1284

Table 7. Performance of a variety of algorithms in MuJoCo environments, reported by Fujimoto et al. (2018) after 1M steps. These results are averaged across 10 seeds.

Environments	Average	Maximum
cheetah run	474.4	795.0
reacher easy	691.5	961.2
ball_in_cup catch	751.7	978.2

Table 8. Average and maximum scores in DM Control environments, calculated from 11 algorithms (10 for reacher easy) (Table 9 and Table 10).

Environments	SAC	MPO	AWR
cheetah run	536.0	253.9	125.2
reacher easy	961.2	841.5	530.2
ball_in_cup catch	971.9	957.3	135.2

Table 9. SAC, MPO and AWR results in DM Control environments after 500k steps. These results are averaged across 10 seeds.

Environments	RAD	CURL	PlaNet	Dreamer	SAC+AE	SLACv1	Pixel SAC	State SAC
cheetah run	728	518	305	570	550	640	197	795
reacher easy	955	929	210	793	627	–	145	923
ball_in_cup catch	974	959	460	879	794	852	312	974

Table 10. Performance of a variety of algorithms in DM Control environments, reported by Laskin et al. (2020), after 500k training steps. These results are averaged across 10 seeds.

D. Policy and Policy-Optimal Information Capacity during ES Training

We evaluate how PIC and POIC behave during RL training (reward maximization) on complex benchmarking environments, in contrast to synthetic ones in Section 5. We train linear, (4, 4) and (16, 16) neural networks with evolution strategy (ES) (Salimans et al., 2017). 100 parameters are sampled from the trainable prior distribution $p_\mu(\theta) = \mathcal{N}(\mu, \sigma = 0.1)$ (ES optimizes μ) in each epoch, and the agent runs 100 episodes per parameters to calculate both PIC and POIC. Section 10 shows the learning curves (top row), corresponding POIC $\mathcal{I}(\mathcal{O}; \Theta)$ (middle) and PIC $\mathcal{I}(R; \Theta)$ (bottom). As we observed in Section 5, both information capacity metrics increase during training, and after each performance converges, they gradually decrease. It might be related to higher correlation of POIC to algorithm-based normalized score shown in Section 6 that POIC seems to follow these trends better than PIC (e.g. Pendulum and HalfCheetah).

Another interesting observation of POIC can be seen in HalfCheetah, where (4, 4) network (green; top) converges to sub-optimal solution and (16, 16) network (blue; top) gets away from there. The agents that can have multi-modal solutions keeps high POIC (green; middle) after sub-optimal convergence, while POIC decreases as improving performance (blue; middle). This might suggests that a sub-optimal prior distribution $p_\mu(\theta) = \mathcal{N}(\mu, \sigma = 0.1)$ still can be easy to minimize the rewards (green), though the further improvements (blue) make it lean towards maximization (i.e. less controllable). Measuring PIC and POIC with more familiar on-policy algorithms such as PPO or TRPO remains as future work.

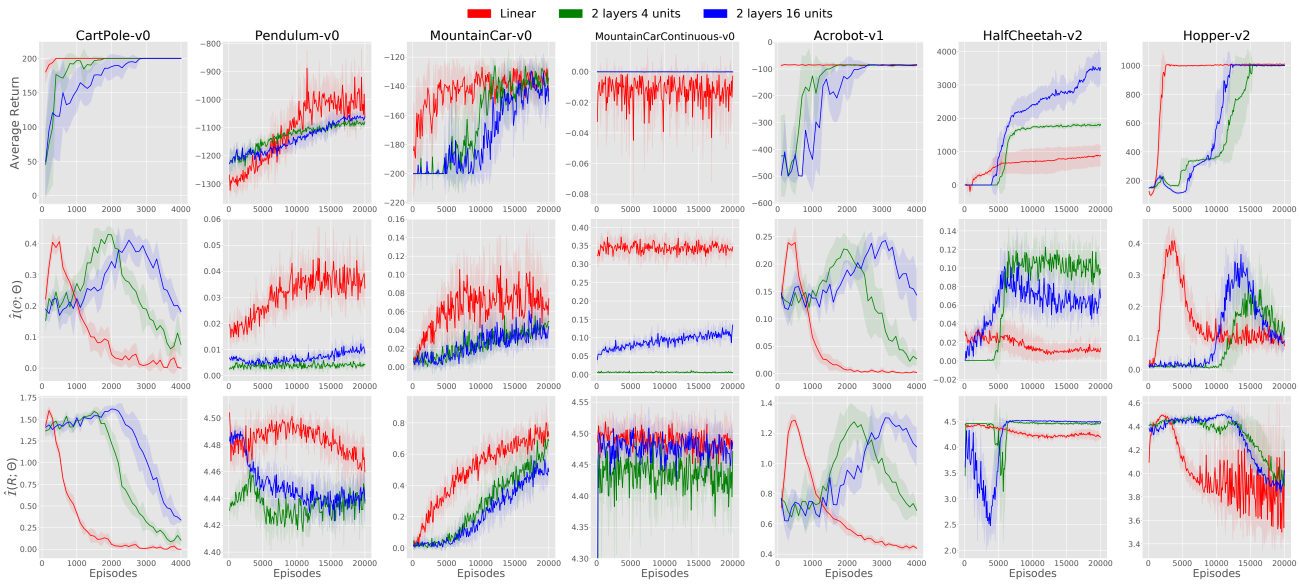


Figure 10. The average returns (top), POIC (middle) and PIC (bottom) during ES training. We use several classic control and MuJoCo tasks. Both information capacity metrics increase during training, and after each performance converges, they gradually decrease. It might be related to higher correlation of POIC to algorithm-based normalized score shown in Section 6 that POIC seems to follow these trends better than PIC (e.g. Pendulum and HalfCheetah).

E. A Proof of Proposition 1

Assume that θ_1 is better than θ_2 without loss of generality.

The proof relies on the following Chernoff's bound tailored for a normal distribution (Boucheron et al., 2013): for N independent samples $(X_n)_{n=1,\dots,N}$ from a normal distribution $\mathcal{N}(\mu, \sigma^2)$, and μ's N-sample-estimate $\hat{\mu} := \sum_{n=1}^{N} X_n/N$,

$$\Pr\{\hat{\mu} \le \mu - \varepsilon\} \le \exp\left(-\frac{N\varepsilon^2}{2\sigma^2}\right)$$

holds. Applying this bound to our case with $\hat{\mu} = \hat{\mu}_1 - \hat{\mu}_2$, $\mu = \mu_{\theta_1} - \mu_{\theta_2}$, and $\varepsilon = \mu$, we obtain

$$\Pr\{\hat{\mu}_1 - \hat{\mu}_2 \le 0 | \theta_1, \theta_2\} \le \exp\left(-\frac{N}{2}\left(\frac{\mu_{\theta_1} - \mu_{\theta_2}}{\sigma_{\theta_1} + \sigma_{\theta_2}}\right)^2\right).$$

Since the entropy of $\mathcal{N}(\mu_\theta, \sigma_\theta^2)$ is $\log(\sigma_\theta\sqrt{2\pi e})$, the upper bound can be rewritten as

$$\Pr\{\hat{\mu}_1 - \hat{\mu}_2 \le 0 | \theta_1, \theta_2\} \le \exp\left(-\pi e N\left(\frac{\mu_{\theta_1} - \mu_{\theta_2}}{\exp(\mathcal{H}_1) + \exp(\mathcal{H}_2)}\right)^2\right).$$

Taking the expectation of both sides with respect to θ_1 and θ_2, the proof is concluded.

F. Full Results on Deep RL Experiment

In this section, we provide the the full results on the experiment in Section 6.2.

We used max 40 CPUs for the experiments in Section 6.2 and it took about at most 2 hours per each random sampling (e.g. HalfCheetah-v2). For estimating brute-force normalized score (Section 6.1 and Appendix C), we mainly used 4 GPUs (NVIDIA V100; 16GB) and it took about 4 hours per seed.

Correlation of Policy-Optimal Information Capacity As seen in Figure 3, the relation between POIC and the algorithm-based normalized score might seem a bit concentrated or skewed with some outliers[9]. To check the validity of correlation, we remove these outliers and recompute the correlation. Figure 11 exhibits the strong positive correlation still holds ($R = 0.780$, statistically significant with $p < 0.01$.) after we remove top-3 outliers of POIC (CartPole, Acrobot, and MountainCarContinuous).

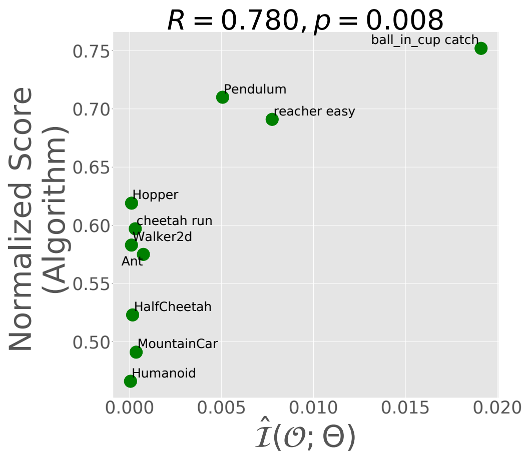


Figure 11. The correlation between POIC and the algorithm-based normalized score. The strong positive correlation still holds ($R = 0.780$, statistically significant with $p < 0.01$.) after we remove top-3 outliers of POIC (CartPole, Acrobot, and MountainCarContinuous).

[9]We tried an early stopping when finding the $\arg\max$ temperature with a black-box optimizer, but it didn't ease these concentrations.

Environment	Score(A)	Score(R)	$\hat{\mathcal{I}}(\mathcal{O};\Theta)$	$\hat{\mathcal{H}}(\mathcal{O})$	$\hat{\mathcal{H}}(\mathcal{O}\|\Theta)$	$\hat{\mathcal{I}}(R;\Theta)$	$\hat{\mathcal{H}}(R)$	$\hat{\mathcal{H}}(R\|\Theta)$	Variance	State dim	Action dim	Horizon
CartPole	0.970	0.141	0.153418	0.210	0.056	1.227	3.262	2.035	12.610	4	2	200
Pendulum	0.710	0.242	0.005060	0.374	0.369	3.708	10.520	6.812	23.223	3	1	200
MountainCar	0.491	0.001	0.000357	0.073	0.073	0.019	0.027	0.008	0.054	2	3	200
MountainCarContinuous	0.566	0.305	0.030092	0.424	0.394	5.953	10.095	4.142	8.022	2	1	999
Acrobot	0.759	0.121	0.106989	0.285	0.178	0.551	1.355	0.803	35.891	6	3	500
Ant	0.575	0.316	0.000751	0.501	0.500	1.767	8.010	6.243	37.830	111	8	1000
HalfCheetah	0.523	0.183	0.000165	0.503	0.503	2.488	8.764	6.276	23.468	17	6	1000
Hopper	0.619	0.007	0.000107	0.453	0.453	3.572	7.088	3.516	3.006	11	3	1000
Walker2d	0.583	0.075	0.000102	0.227	0.227	2.211	6.350	4.139	0.403	17	6	1000
Humanoid	0.466	0.008	5e-05	0.275	0.275	2.792	7.359	4.567	1.917	376	17	1000
cheetah run	0.597	0.030	0.000308	0.487	0.487	2.635	8.743	6.108	1.972	17	6	1000
reacher easy	0.691	0.069	0.007751	0.036	0.029	0.594	3.863	3.268	20.669	6	2	1000
ball_in_cup catch	0.752	0.118	0.019111	0.303	0.284	0.367	1.391	1.024	91.454	8	2	1000
Correlation Coefficient: Score(A)	–	0.139	0.807	-0.212	-0.418	-0.295	-0.349	-0.327	0.372	-0.427	-0.544	-0.452
p-Value: Score(A)	–	0.651	0.001	0.487	0.156	0.328	0.242	0.275	0.211	0.146	0.055	0.121
Correlation Coefficient: Score(R)	0.139	–	0.121	0.457	0.378	0.401	0.455	0.414	0.314	-0.183	-0.259	-0.016
p-Value: Score(R)	0.651	–	0.693	0.116	0.203	0.175	0.118	0.160	0.297	0.549	0.393	0.959

Table 11. The raw results of POIC $\hat{\mathcal{I}}(\mathcal{O};\Theta)$, optimality marginal entropy $\hat{\mathcal{H}}(\mathcal{O})$, optimality conditional entropy $\hat{\mathcal{H}}(\mathcal{O}|\Theta)$, PIC $\hat{\mathcal{I}}(R;\Theta)$, reward marginal entropy $\hat{\mathcal{H}}(R)$, reward conditional entropy $\hat{\mathcal{H}}(R|\Theta)$, normalized variance of return in standard RL benchmark environments, Pearson correlation coefficient to the algorithm-based normalized score (Score(A)), and the random-sampling-based normalized score (Score(R)). We prepare bags of policy architectures, totally 56 variants of architectures; ([0] layers $+$ [1, 2] layers $\times$ [4, 32, 64] units) $\times$ [Gaussian prior $\mathcal{N}(0, I)$, Uniform prior *Unif*$(-1, 1)$, Xavier Normal, Xavier Uniform] $\times$ [w/ bias, w/o bias]. The results suggest that POIC seems to positively correlate well with algorithm-based normalized score better than any other alternatives, such as marginal reward entropy or variance of returns (Oller et al., 2020).

G. Details of Reward Shaping Experiments

Here, we present hyper-parameters of the reward functions and raw experimental results (Table 12 for Reacher and Table 13 for Pointmaze (Figure 12)). In the experiment, we employ the following four families of goal-oriented reward function:

1. L1 norm: $r(s, s_g) = -\alpha\|s - s_g\|_1$,
2. L2 norm: $r(s, s_g) = -\alpha\|s - s_g\|_2$,
3. Fraction: $r(s, s_g) = \frac{\beta}{\gamma + \|s - s_g\|_2}$,
4. Sparse: $r(s, s_g) = -\mathbb{1}[\|s - s_g\|_2 \geq \epsilon]$.

This notations of hyper-parameter correspond to Table 12 and Table 13. To estimate information capacity metrics, we use the small neural networks (2 layers, 4 hidden units, Gaussian initialization $\mathcal{N}(0, I)$, and without bias term) for the simplicity. For the following RL training, we employ the same policy architectures, while keeping original value networks sizes (2 layers, 64 hidden units). We normalize the scores of PPO, trained 500 steps and averaged among 5 seeds for fair comparisons among different reward-scale environments.

Reward	Hyper-parameter	$r^{\text{rand}}_{\text{max}}$	PPO (500k)	Normalized Score	$\hat{\mathcal{I}}(R;\Theta)$	$\hat{\mathcal{I}}(\mathcal{O};\Theta)$
L1-norm	$\alpha = 1.0$	-0.557	-8.776	0.507	0.796	5.801e-3
L1-norm	$\alpha = 0.5$	-0.316	-4.245	0.524	0.798	5.785e-3
L1-norm	$\alpha = 2.0$	-1.335	-17.009	0.512	0.791	5.943e-3
L1-norm	$\alpha = 5.0$	-2.940	-42.702	0.524	0.794	5.965e-3
L2-norm	$\alpha = 1.0$	-0.417	-6.604	0.530	0.837	5.743e-3
L2-norm	$\alpha = 0.5$	-0.241	-3.238	0.558	0.841	5.841e-3
L2-norm	$\alpha = 2.0$	-0.969	-13.339	0.522	0.837	5.815e-3
L2-norm	$\alpha = 5.0$	-2.524	-34.760	0.502	0.837	5.778e-3
Fraction	$(\beta, \gamma) = (0.01, 0.01)$	31.182	5.874	0.133	0.522	9.279e-5
Fraction	$(\beta, \gamma) = (0.1, 0.1)$	45.758	23.787	0.306	0.772	1.270e-3
Fraction	$(\beta, \gamma) = (0.01, 0.1)$	4.546	2.326	0.281	0.779	1.318e-3
Fraction	$(\beta, \gamma) = (0.05, 0.1)$	22.839	11.673	0.278	0.772	1.284e-3
Sparse	$\epsilon = 0.05$	0.000	-44.220	0.111	0.297	1.348e-3
Sparse	$\epsilon = 0.01$	0.000	-49.660	0.009	0.024	9.171e-6
Sparse	$\epsilon = 0.1$	-10.000	-30.520	0.376	0.541	0.0103
Sparse	$\epsilon = 0.15$	0.000	-18.460	0.586	0.561	0.0329

Table 12. Hyper-parameter and results of reward shaping experiments in Reacher, appeared in Section 6.3.

Reward	Hyper-parameter	$r^{\text{rand}}_{\text{max}}$	PPO (500k)	Normalized Score	$\hat{\mathcal{I}}(R;\Theta)$	$\hat{\mathcal{I}}(\mathcal{O};\Theta)$
L1-norm	$\alpha = 1.0$	-4.095	-177.385	0.706	1.740	0.0232
L1-norm	$\alpha = 0.5$	-1.556	-100.371	0.664	1.806	0.0226
L1-norm	$\alpha = 2.0$	-7.418	-388.604	0.677	1.800	0.0226
L1-norm	$\alpha = 5.0$	-13.431	-1009.456	0.661	1.802	0.0227
L2-norm	$\alpha = 1.0$	-3.314	-154.742	0.641	1.845	0.0225
L2-norm	$\alpha = 0.5$	-1.983	-81.826	0.621	1.842	0.0225
L2-norm	$\alpha = 2.0$	-5.971	-293.556	0.660	1.836	0.0223
L2-norm	$\alpha = 5.0$	-11.693	-811.083	0.622	1.839	0.0216
Fraction	$(\beta, \gamma) = (0.01, 0.01)$	59.037	6.253	0.095	1.051	1.962e-5
Fraction	$(\beta, \gamma) = (0.1, 0.1)$	123.128	37.607	0.270	1.443	2.888e-4
Fraction	$(\beta, \gamma) = (0.01, 0.1)$	13.017	3.546	0.238	1.443	2.570e-4
Fraction	$(\beta, \gamma) = (0.05, 0.1)$	67.266	18.154	0.236	1.424	2.357e-4
Sparse	$\epsilon = 0.5$	0.000	-110.000	0.258	0.258	0.0186
Sparse	$\epsilon = 0.1$	0.000	-146.740	0.019	0.019	4.339e-4
Sparse	$\epsilon = 0.2$	0.000	-137.800	0.076	0.076	1.574e-3
Sparse	$\epsilon = 1.0$	0.000	-40.240	0.726	0.726	0.0768

Table 13. Hyper-parameter and results of reward shaping experiments in Pointmaze, appeared in Section 6.3.

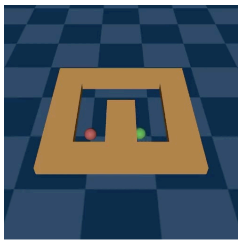


Figure 12. Pointmaze environment (Fu et al., 2020).

H. Evaluating the Goodness of Network Architecture and Initialization

Can we also use PIC and POIC to evaluate the goodness of network architecture or initialization? We investigate the correlation between PIC or POIC and the normalized score of the PPO policy with different network configurations. For the comparison, we prepare 7 policy network architectures without bias term (0 layers + [1, 2] layers × [4, 32, 64] hidden units; while keeping original value networks sizes, 2 layers 64 units), and 4 initializations (normal $\mathcal{N}(0, I)$, uniform *Unif*$(-1, 1)$, Xavier normal and Xavier uniform). Xavier Normal and Uniform are the typical initialization methods of neural networks (Glorot & Bengio, 2010). First, we measure both PIC and POIC for each policy, and then train it with PPO, during 500k steps, except for 50k steps in CartPole (see Appendix I for the detailed results).

The results are shown in Table 15, and Table 14. We can see valid positive correlations in CartPole, Pendulum, HalfCheetah, Hopper, and Walker2d with specific initialization, which are statistically significant with $p < 0.05$, while we also observe the weak positive, negative, or no trends in other environments. In several domains, PIC and POIC might be used for architectural tuning without extensive RL trainings. In addition, some negative results seem consistent with empirical observations in recent RL research; the performance of many deep RL algorithms require architectural tuning for best performances (Schulman et al., 2015; 2017; Engstrom et al., 2019; Andrychowicz et al., 2021), and can be sensitive to architecture and initialization (Rajeswaran et al., 2017).

PIC	CartPole	Pendulum	MountainCar	MountainCarContinuous	Acrobot	HalfCheetah	Hopper	Walker2d
Normal (p-Value)	**0.938** (0.002)	0.121 (0.796)	0.681 (0.092)	0.124 (0.791)	-0.455 (0.304)	**0.891** (0.007)	**0.839** (0.018)	**0.782** (0.038)
Uniform (p-Value)	0.658 (0.108)	-0.172 (0.712)	-0.783 (0.037)	-0.021 (0.965)	0.502 (0.251)	**0.881** (0.009)	0.649 (0.115)	0.617 (0.140)
Xavier (N) (p-Value)	0.483 (0.272)	0.438 (0.325)	-0.544 (0.207)	-0.079 (0.866)	-0.474 (0.283)	0.186 (0.689)	-0.660 (0.106)	-0.789 (0.035)
Xavier (U) (p-Value)	0.171 (0.713)	0.406 (0.366)	-0.244 (0.599)	0.667 (0.102)	-0.259 (0.574)	-0.003 (0.995)	-0.709 (0.075)	-0.965 (> 0.001)

Table 14. Pearson correlation coefficient between the normalized score of different policies (network architecture and initialization) and PIC. We can see valid positive correlations in CartPole, HalfCheetah, Hopper, and Walker2d with specific initialization, which are statistically significant with $p < 0.05$. The 2D-plots can be seen in Figure 13.

POIC	CartPole	Pendulum	MountainCar	MountainCarContinuous	Acrobot	HalfCheetah	Hopper	Walker2d
Normal (p-Value)	0.392 (0.385)	**0.769** (0.043)	-0.229 (0.621)	0.004 (0.994)	0.177 (0.704)	**0.855** (0.014)	0.476 (0.280)	0.624 (0.134)
Uniform (p-Value)	-0.087 (0.852)	0.451 (0.309))	-0.842 (0.018)	-0.059 (0.900)	-0.288 (0.532)	**0.864** (0.012)	0.188 (0.686)	0.523 (0.229)
Xavier (N) (p-Value)	0.127 (0.787)	0.579 (0.173)	-0.503 (0.250)	0.373 (0.409)	-0.720 (0.068)	0.344 (0.450)	**0.957** (> 0.001)	-0.946 (0.001)
Xavier (U) (p-Value)	**0.930** (0.002)	0.495 (0.258)	-0.406 (0.367)	-0.649 (0.114)	-0.381 (0.399)	0.080 (0.865)	0.006 (0.990)	-0.726 (0.065)

Table 15. Pearson correlation coefficient between the normalized score of different policies (network architecture and initialization) and POIC. We can see valid positive correlations in CartPole, Pendulum, HalfCheetah and Hopper with specific initialization, which are statistically significant with $p < 0.05$. The 2D-plots can be seen in Figure 14.

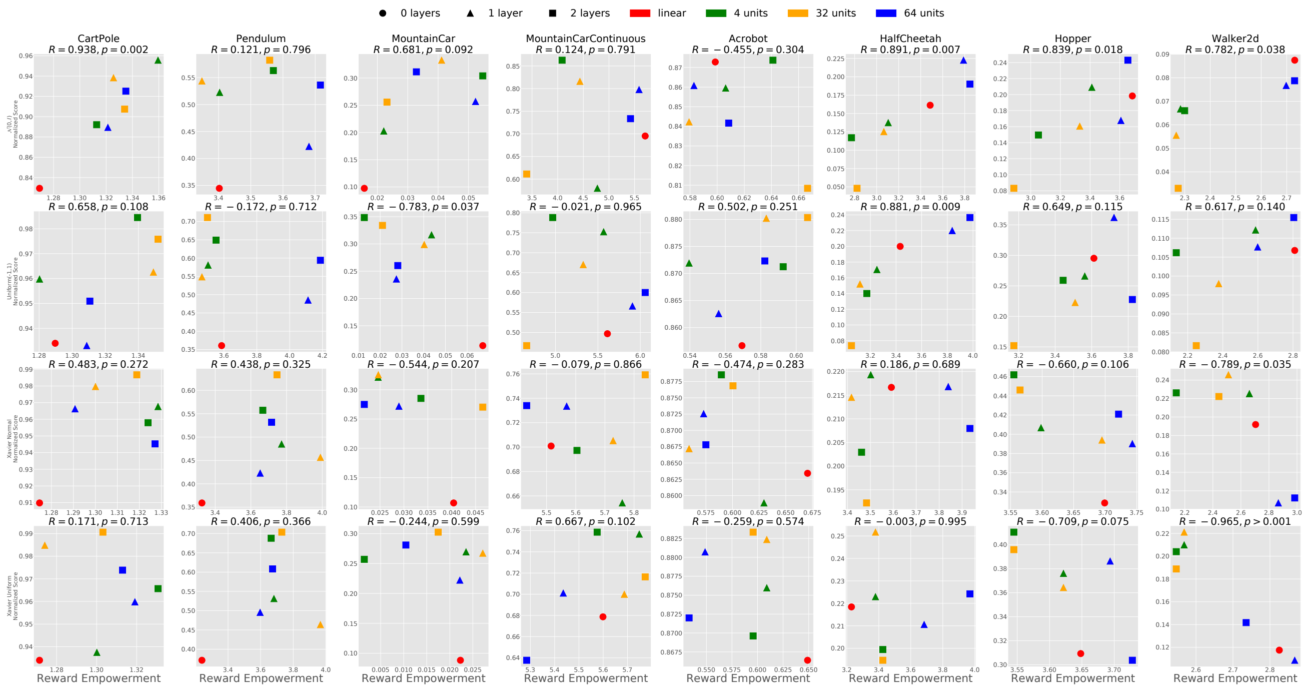


Figure 13. 2D-Scatter plots between PIC (x-axis) and the normalized score (y-axis), trained during 500k steps and averaged over 5 seeds. We test combinations of 7 network architectures (0 layers, 1 layer 4 units, 2 layers 4units) and 4 kinds of network initialization (normal, uniform, Xavier normal, Xavier uniform).

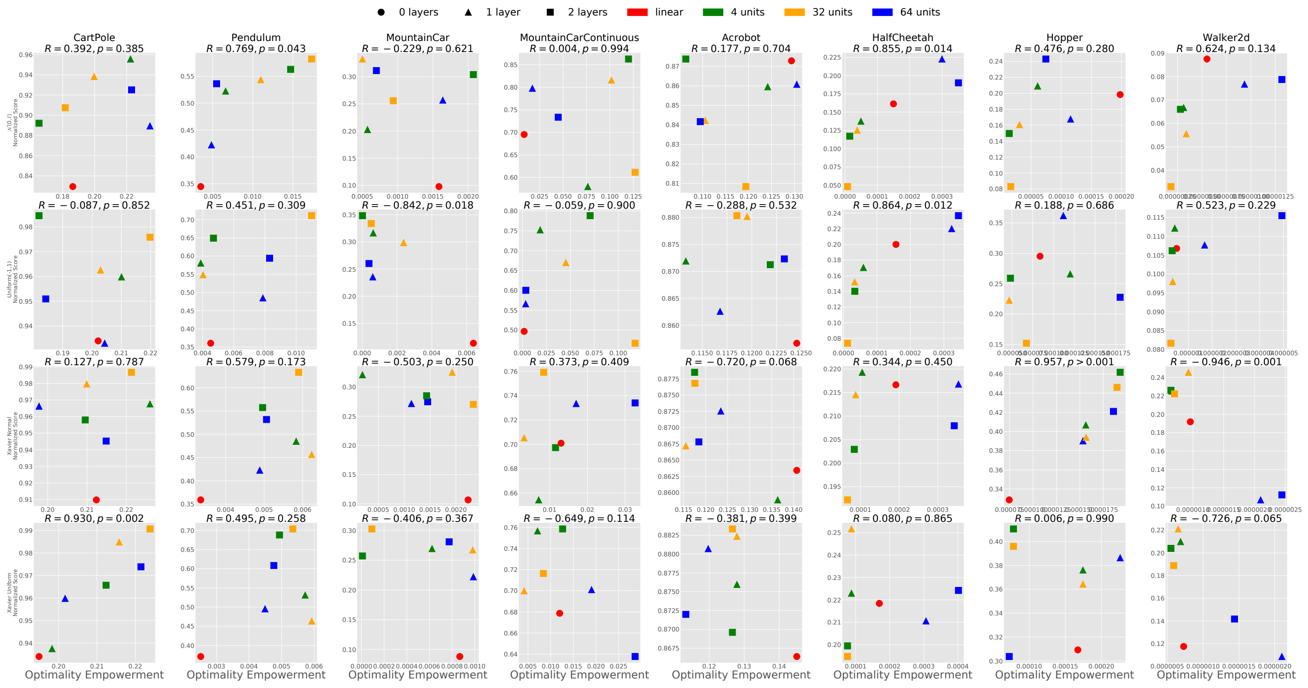


Figure 14. 2D-Scatter plots between POIC (x-axis) and the normalized score (y-axis), trained during 500k steps and averaged over 5 seeds. We test combinations of 7 network architectures (0 layers, 1 layer 4 units, 2 layers 4units) and 4 kinds of network initialization (normal, uniform, Xavier normal, Xavier uniform).

I. Details of Architecture and Initialization Experiments

In this section, we presents the raw scores observed through the experiments in Appendix H. We train PPO with (64, 64) neural networks and 3 different discount factor $\gamma \in \{0.95, 0.99, 0.995\}$. For normalized score, we refer the algorithm-max scores in Appendix C. All results are summarized in Table 16 (average cumulative rewards), Table 17 (normalized score), Table 18 (PIC), and Table 19 (POIC).

	CartPole	Pendulum	MountainCar	MountainCarContinuous	Acrobot	HalfCheetah	Hopper	Walker2d
0L Normal	167.38	-1268.00	-189.97	37.57	-120.18	154.04	488.97	705.56
1L 4U Normal	178.85	-1134.51	-173.60	56.37	-125.43	1221.59	427.17	596.35
1L 32U Normal	191.51	-962.21	-179.17	13.59	-125.97	-221.18	367.72	743.74
1L 64U Normal	188.20	-926.62	-165.81	59.85	-133.48	-344.62	303.13	570.78
2L 4U Normal	185.68	-931.49	-168.00	43.72	-133.74	618.83	435.76	864.46
2L 32U Normal	179.36	-883.60	-168.76	69.01	-119.79	-369.87	364.67	531.89
2L 64U Normal	182.30	-852.51	-173.71	19.79	-148.21	-404.71	173.00	294.66
0L Uniform	187.37	-1243.73	-188.33	31.21	-127.21	780.81	600.25	1051.04
1L 4U Uniform	187.19	-1017.40	-175.79	18.77	-124.63	1057.68	591.57	1289.52
1L 32U Uniform	192.31	-857.24	-167.46	47.44	-120.57	652.51	632.90	946.86
1L 64U Uniform	192.84	-908.35	-169.29	31.27	-116.97	316.07	548.21	792.32
2L 4U Uniform	190.61	-829.38	-173.24	24.82	-120.39	1361.30	637.16	810.50
2L 32U Uniform	197.04	-737.45	-164.19	54.40	-120.86	364.60	598.17	922.05
2L 64U Uniform	195.37	-629.24	-165.67	-8.47	-116.90	-186.69	456.83	540.52
0L Xavier (N)	182.75	-1249.18	-189.02	37.42	-124.27	1111.14	1096.75	1170.13
1L 4U Xavier (N)	193.55	-1132.60	-172.06	43.82	-120.29	1219.57	599.91	1389.66
1L 32U Xavier (N)	193.81	-1022.30	-166.99	31.09	-126.25	1644.45	1292.61	1448.79
1L 64U Xavier (N)	196.11	-1069.38	-166.55	43.21	-122.63	1593.45	1410.29	1403.11
2L 4U Xavier (N)	189.53	-937.73	-171.75	43.82	-122.36	1048.78	634.12	1499.48
2L 32U Xavier (N)	191.96	-893.65	-170.69	36.78	-117.69	1459.96	1297.16	1645.11
2L 64U Xavier (N)	197.45	-764.92	-172.21	49.70	-118.43	1507.47	1275.05	1588.82
0L Xavier (U)	187.37	-1223.45	-190.93	37.53	-122.97	1117.23	663.19	1101.14
1L 4U Xavier (U)	192.31	-1005.49	-177.17	37.54	-116.74	1218.68	611.44	1376.33
1L 32U Xavier (U)	188.04	-945.72	-172.31	49.55	-118.81	1467.81	1202.81	1339.44
1L 64U Xavier (U)	197.10	-1055.70	-172.53	42.33	-116.03	1977.95	1268.04	1297.33
2L 4U Xavier (U)	195.00	-806.39	-171.13	25.00	-120.53	1370.19	804.70	1081.45
2L 32U Xavier (U)	193.43	-670.05	-173.58	49.16	-121.57	1758.48	1168.65	1462.36
2L 64U Xavier (U)	198.21	-640.18	-168.91	42.80	-115.60	1677.97	1080.10	1410.35

Table 16. Cumulative rewards averaged over 3 different discount factor $\gamma \in \{0.95, 0.99, 0.995\}$ and 5 random seeds.

	CartPole	Pendulum	MountainCar	MountainCarContinuous	Acrobot	HalfCheetah	Hopper	Walker2d
0L Normal	0.829	0.345	0.098	0.695	0.873	0.161	0.087	0.198
1L 4U Normal	0.889	0.422	0.257	0.798	0.861	0.223	0.077	0.168
1L 32U Normal	0.956	0.522	0.203	0.580	0.859	0.137	0.067	0.209
1L 64U Normal	0.938	0.544	0.333	0.816	0.842	0.125	0.056	0.161
2L 4U Normal	0.925	0.536	0.311	0.734	0.842	0.190	0.079	0.243
2L 32U Normal	0.892	0.563	0.304	0.863	0.874	0.117	0.066	0.150
2L 64U Normal	0.907	0.582	0.256	0.611	0.808	0.048	0.033	0.083
0L Uniform	0.934	0.361	0.114	0.497	0.857	0.200	0.107	0.295
1L 4U Uniform	0.933	0.485	0.236	0.566	0.863	0.220	0.108	0.362
1L 32U Uniform	0.960	0.581	0.317	0.752	0.872	0.170	0.112	0.266
1L 64U Uniform	0.963	0.549	0.299	0.670	0.880	0.152	0.098	0.223
2L 4U Uniform	0.951	0.594	0.260	0.600	0.872	0.237	0.115	0.228
2L 32U Uniform	0.985	0.649	0.348	0.788	0.871	0.140	0.106	0.259
2L 64U Uniform	0.976	0.711	0.334	0.467	0.880	0.074	0.082	0.152
0L Xavier (N)	0.910	0.359	0.107	0.701	0.863	0.217	0.192	0.329
1L 4U Xavier (N)	0.966	0.423	0.272	0.733	0.873	0.217	0.107	0.390
1L 32U Xavier (N)	0.968	0.485	0.321	0.654	0.859	0.219	0.225	0.407
1L 64U Xavier (N)	0.980	0.456	0.325	0.705	0.867	0.215	0.246	0.394
2L 4U Xavier (N)	0.945	0.532	0.275	0.734	0.868	0.208	0.112	0.421
2L 32U Xavier (N)	0.958	0.557	0.285	0.697	0.879	0.203	0.226	0.462
2L 64U Xavier (N)	0.987	0.633	0.270	0.759	0.877	0.192	0.222	0.446
0L Xavier (U)	0.934	0.371	0.088	0.679	0.866	0.218	0.118	0.309
1L 4U Xavier (U)	0.960	0.495	0.222	0.701	0.881	0.211	0.109	0.386
1L 32U Xavier (U)	0.937	0.531	0.269	0.757	0.876	0.223	0.210	0.376
1L 64U Xavier (U)	0.985	0.464	0.267	0.700	0.882	0.252	0.221	0.364
2L 4U Xavier (U)	0.974	0.608	0.281	0.638	0.872	0.224	0.142	0.304
2L 32U Xavier (U)	0.966	0.688	0.257	0.759	0.870	0.199	0.204	0.411
2L 64U Xavier (U)	0.991	0.704	0.302	0.716	0.883	0.195	0.189	0.396

Table 17. Algorithm-based normalized scores. We use the minimum values during random-sampling and the maximum values reported in Table 3 and Table 4.

	CartPole	Pendulum	MountainCar	MountainCarContinuous	Acrobot	HalfCheetah	Hopper	Walker2d
0L Normal	1.269	3.402	0.016	5.707	0.599	3.486	2.732	3.685
1L 4U Normal	1.321	3.681	0.052	5.588	0.583	3.789	2.698	3.608
1L 32U Normal	1.359	3.403	0.022	4.774	0.606	3.105	2.283	3.412
1L 64U Normal	1.326	3.348	0.041	4.433	0.579	3.065	2.267	3.330
2L 4U Normal	1.335	3.716	0.033	5.421	0.608	3.847	2.731	3.655
2L 32U Normal	1.313	3.570	0.055	4.086	0.641	2.771	2.300	3.052
2L 64U Normal	1.334	3.559	0.023	3.395	0.667	2.822	2.274	2.886
0L Uniform	1.290	3.586	0.067	5.612	0.569	3.432	2.810	3.612
1L 4U Uniform	1.309	4.112	0.027	5.907	0.556	3.838	2.599	3.722
1L 32U Uniform	1.280	3.503	0.044	5.568	0.540	3.251	2.587	3.562
1L 64U Uniform	1.349	3.466	0.040	5.327	0.583	3.119	2.376	3.509
2L 4U Uniform	1.311	4.187	0.028	6.057	0.582	3.976	2.804	3.822
2L 32U Uniform	1.339	3.552	0.013	4.968	0.593	3.172	2.134	3.443
2L 64U Uniform	1.352	3.500	0.021	4.661	0.606	3.052	2.248	3.174
0L Xavier (N)	1.275	3.326	0.040	5.516	0.670	3.592	2.705	3.699
1L 4U Xavier (N)	1.291	3.652	0.029	5.569	0.573	3.840	2.867	3.743
1L 32U Xavier (N)	1.329	3.772	0.025	5.759	0.630	3.502	2.660	3.598
1L 64U Xavier (N)	1.300	3.990	0.025	5.728	0.559	3.418	2.511	3.695
2L 4U Xavier (N)	1.327	3.716	0.022	5.432	0.575	3.934	2.984	3.721
2L 32U Xavier (N)	1.324	3.667	0.034	5.604	0.590	3.462	2.139	3.555
2L 64U Xavier (N)	1.319	3.746	0.047	5.838	0.600	3.483	2.443	3.565
0L Xavier (U)	1.271	3.240	0.022	5.599	0.649	3.229	2.825	3.648
1L 4U Xavier (U)	1.319	3.597	0.022	5.436	0.549	3.683	2.867	3.694
1L 32U Xavier (U)	1.300	3.682	0.024	5.749	0.609	3.379	2.568	3.621
1L 64U Xavier (U)	1.274	3.967	0.027	5.687	0.609	3.379	2.568	3.621
2L 4U Xavier (U)	1.313	3.673	0.010	5.285	0.533	3.970	2.736	3.728
2L 32U Xavier (U)	1.331	3.665	0.001	5.575	0.596	3.425	2.547	3.545
2L 64U Xavier (U)	1.303	3.730	0.017	5.773	0.596	3.425	2.547	3.545

Table 18. Estimated Policy Information Capacity.

	CartPole	Pendulum	MountainCar	MountainCarContinuous	Acrobot	HalfCheetah	Hopper	Walker2d
0L Normal	0.186	0.003516	0.001586	0.008363	0.129	0.000150	0.000000551	0.000198
1L 4U Normal	0.235	0.004850	0.001641	0.017159	0.131	0.000300	0.000000935	0.000116
1L 32U Normal	0.222	0.006613	0.000573	0.076397	0.124	0.000049	0.000000312	0.000061
1L 64U Normal	0.200	0.010994	0.000502	0.101649	0.111	0.000037	0.000000339	0.000031
2L 4U Normal	0.223	0.005492	0.000699	0.044784	0.109	0.000351	0.000001318	0.000075
2L 32U Normal	0.165	0.014715	0.002074	0.119891	0.106	0.000014	0.000000280	0.000014
2L 64U Normal	0.182	0.017335	0.000939	0.126900	0.119	0.000008	0.000000182	0.000016
0L Uniform	0.202	0.004520	0.006387	0.001799	0.125	0.000157	0.000000507	0.000084
1L 4U Uniform	0.204	0.007866	0.000603	0.003397	0.117	0.000323	0.000001682	0.000112
1L 32U Uniform	0.210	0.003882	0.000628	0.018515	0.113	0.000059	0.000000427	0.000120
1L 64U Uniform	0.203	0.004031	0.002376	0.045212	0.120	0.000033	0.000000341	0.000047
2L 4U Uniform	0.184	0.008297	0.000383	0.003576	0.123	0.000343	0.000004932	0.000180
2L 32U Uniform	0.182	0.004696	0.000007	0.070520	0.122	0.000034	0.000000319	0.000048
2L 64U Uniform	0.220	0.010994	0.000516	0.117416	0.119	0.000012	0.000000266	0.000067
0L Xavier (N)	0.212	0.003354	0.002235	0.013028	0.141	0.000192	0.000000957	0.000075
1L4U Xavier (N)	0.198	0.004885	0.001138	0.017027	0.123	0.000353	0.000002099	0.000153
1L32U Xavier (N)	0.226	0.005830	0.000191	0.007107	0.136	0.000104	0.000000659	0.000156
1L 64U Xavier (N)	0.210	0.006233	0.001928	0.003256	0.115	0.000088	0.000000927	0.000157
2L 4U Xavier (N)	0.215	0.005060	0.001453	0.032625	0.118	0.000342	0.000002443	0.000186
2L 32U Xavier (N)	0.210	0.004961	0.001432	0.011564	0.117	0.000084	0.000000642	0.000193
2L 64U Xavier (N)	0.221	0.005896	0.002337	0.008424	0.118	0.000067	0.000000702	0.000190
0L Xavier (U)	0.195	0.002517	0.000868	0.011988	0.145	0.000169	0.000000728	0.000167
1L 4U Xavier (U)	0.202	0.004492	0.000988	0.019024	0.120	0.000306	0.000002099	0.000226
1L 32U Xavier (U)	0.198	0.005724	0.000623	0.007098	0.128	0.000088	0.000000685	0.000174
1L 64U Xavier (U)	0.216	0.005925	0.000983	0.004143	0.128	0.000088	0.000000652	0.000174
2L 4U Xavier (U)	0.221	0.004763	0.000774	0.028628	0.113	0.000402	0.000001439	0.000072
2L 32U Xavier (U)	0.212	0.004936	0.000009	0.012626	0.127	0.000076	0.000000551	0.000078
2L 64U Xavier (U)	0.224	0.005346	0.000092	0.008398	0.127	0.000076	0.000000589	0.000078

Table 19. Estimated Policy-Optimal Information Capacity.

J. Evaluating the Dynamics and Initialization Noises

In this section, we evaluate one of our information capacity metrics' properties: can we use PIC and POIC for tuning the noise levels in MDPs that might help to learn? To answer this question, we design the experiments to observe the correlations between each metric (POIC, optimality marginal entropy, optimality conditional entropy, PIC, reward marginal entropy, reward conditional entropy, and variance) and algorithm-based normalized scores.

For the experiments, we prepare 12 cartpole environments with 3 initialization and 4 dynamics noises. In these environments, we initialize 4-dimensional states with uniform distribution $\mathit{Unif}(-u_{\text{init}}, u_{\text{init}})$, where u_{init} is a tunable parameter $u_{\text{init}} \in \{0.05, 0.1.0.15\}$, and add 1-dimensional transition noise to angular velocity with uniform distribution $\mathit{Unif}(-u_{\text{dyn}}, u_{\text{dyn}})$, where u_{dyn} is also a tunable parameter $u_{\text{dyn}} \in \{0.0, 0.03, 0.05, 0.1\}$. As in Section 6.2, we prepare the bag-of-architectures to measure PIC and POIC: ([0] layers + [1, 2] layers × [4, 32, 64] hidden units) × [Gaussian prior $\mathcal{N}(0, I)$, Uniform prior $\mathit{Unif}(-1, 1)$, Xavier Normal, Xavier Uniform] × [w/ bias, w/o bias], and bag-of-algorithms (PPO, ES, DQN with different hyper-parameters) to compute the algorithm-based normalized scores (see Table 21 for each raw score).

The results are shown in Table 20. The POIC seems the best metric that positively correlates to the algorithm-based normalized scores ($R = 0.860$; statistically significant with $p < 0.001$), but PIC doesn't show such a trend. This suggests that POIC might be used for tuning initialization and dynamics noises that help to learn.

Environment	Initial Noise	Dynamics Noise	$\hat{\mathcal{I}}(\mathcal{O};\Theta)$	$\hat{\mathcal{H}}(\mathcal{O})$	$\hat{\mathcal{H}}(\mathcal{O}\vert\Theta)$	$\hat{\mathcal{I}}(R;\Theta)$	$\hat{\mathcal{H}}(R)$	$\hat{\mathcal{H}}(R\vert\Theta)$	Variance	Score(A)
CartPole	0.05	0.0	0.207	0.292	0.085	1.315	3.632	2.317	17.586	0.886
	0.05	0.03	0.172	0.252	0.080	1.268	3.696	2.428	15.540	0.848
	0.05	0.05	0.151	0.222	0.071	1.233	3.723	2.490	14.024	0.856
	0.05	0.1	0.111	0.172	0.061	1.151	3.746	2.595	11.302	0.827
	0.1	0.0	0.140	0.200	0.060	1.110	3.823	2.713	12.102	0.849
	0.1	0.03	0.130	0.185	0.055	1.095	3.843	2.748	11.430	0.849
	0.1	0.05	0.120	0.172	0.052	1.076	3.853	2.776	10.826	0.847
	0.1	0.1	0.093	0.143	0.050	1.019	3.866	2.847	9.343	0.820
	0.15	0.0	0.110	0.160	0.050	0.922	3.886	2.964	9.710	0.850
	0.15	0.03	0.105	0.150	0.046	0.913	3.895	2.981	9.326	0.848
	0.15	0.05	0.098	0.142	0.044	0.901	3.899	2.999	8.951	0.828
	0.15	0.1	0.077	0.120	0.043	0.859	3.902	3.042	7.891	0.824
Correlation Coefficient: Score(A)			0.860	0.824	0.698	0.613	-0.625	-0.625	0.783	–
p-Value: Score(A)			>0.001	>0.001	0.012	0.034	0.030	0.030	0.003	–

Table 20. PIC and POIC under noisy initialization and dynamics.

		(Initialization Noise, Dynamics Noise)											
Algorithm	Hyper-Parameters	(0.05, 0.0)	(0.05, 0.03)	(0.05, 0.05)	(0.05, 0.1)	(0.1, 0.0)	(0.1, 0.03)	(0.1, 0.05)	(0.1, 0.1)	(0.15, 0.0)	(0.15, 0.03)	(0.15, 0.05)	(0.15, 0.1)
PPO	(64, 64), $\gamma = 0.995$	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	199.6	200.0	197.8	199.4
PPO	(64, 64), $\gamma = 0.99$	200.0	199.7	200.0	200.0	200.0	199.2	196.5	200.0	200.0	200.0	196.2	197.2
PPO	(64, 64), $\gamma = 0.95$	194.0	200.0	197.2	192.5	193.7	196.9	182.4	193.2	189.8	186.5	200.0	193.5
PPO	(128, 64), $\gamma = 0.995$	200.0	200.0	200.0	197.2	200.0	200.0	200.0	200.0	199.6	199.9	200.0	199.9
PPO	(128, 64), $\gamma = 0.99$	200.0	200.0	200.0	200.0	200.0	200.0	193.1	198.5	200.0	196.8	198.3	193.8
PPO	(128, 64), $\gamma = 0.95$	198.1	196.1	199.3	192.2	196.1	191.0	195.3	196.1	197.4	182.5	189.3	184.9
PPO	(128, 128), $\gamma = 0.995$	200.0	199.6	200.0	200.0	197.8	200.0	200.0	199.7	191.1	196.5	200.0	195.9
PPO	(128, 128), $\gamma = 0.99$	200.0	200.0	200.0	200.0	200.0	200.0	197.8	200.0	198.6	200.0	199.3	199.4
PPO	(128, 128), $\gamma = 0.95$	197.1	195.2	196.4	197.5	192.9	193.4	196.5	191.7	195.6	193.4	191.2	185.8
ES	(16, 16), $\sigma = 0.1$	188.1	165.5	166.5	147.1	185.3	196.3	179.1	130.2	170.3	167.2	155.1	144.6
ES	(16, 16), $\sigma = 0.1$, rand	197.1	161.7	175.1	175.9	186.4	170.7	185.8	140.9	167.7	173.0	159.8	167.3
ES	(64, 64), $\sigma = 0.1$	168.6	159.6	187.2	170.0	160.9	150.2	171.9	115.7	173.3	150.0	170.4	141.5
ES	(64, 64), $\sigma = 0.1$, rand	168.0	177.0	170.8	112.7	161.4	174.3	169.4	131.1	173.4	166.6	138.9	128.2
DQN	(100, 100), $\gamma = 0.95$	133.2	127.1	131.4	131.5	123.4	102.5	87.3	128.8	116.0	134.8	130.7	117.8
DQN	(100, 100), $\gamma = 0.99$	185.2	169.6	114.7	145.0	163.1	174.3	177.2	193.1	169.0	157.8	163.6	171.4
DQN	(200, 200), $\gamma = 0.95$	132.2	97.2	73.9	118.6	96.2	135.6	113.0	128.8	106.0	107.3	117.0	122.6
DQN	(200, 200), $\gamma = 0.99$	195.5	196.0	179.7	194.9	195.9	149.4	172.2	182.4	184.8	183.2	167.3	189.4
DQN	(50, 50), $\gamma = 0.95$	161.3	123.5	166.4	91.5	98.6	101.3	156.2	113.3	132.4	127.5	121.0	90.2
DQN	(50, 50), $\gamma = 0.99$	176.3	153.7	169.4	172.8	170.0	179.8	163.6	137.0	168.8	155.2	150.8	167.9
DQN	(50, 50, 50), $\gamma = 0.95$	87.7	102.7	103.0	101.6	84.4	114.5	64.8	134.2	108.1	119.4	93.1	111.0
DQN	(50, 50, 50), $\gamma = 0.99$	159.8	164.1	192.6	164.7	185.6	163.3	180.2	160.5	153.9	188.6	165.2	185.3
$r^{\text{algo}}_{\text{ave}}$	–	178.2	170.9	172.6	166.9	171.0	171.1	170.6	165.5	171.2	170.8	166.9	166.0
$r^{\text{algo}}_{\max}$	–	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	199.9
$r^{\text{rand}}_{\min}$	–	8.742	8.752	8.759	8.789	8.137	8.139	8.161	8.184	7.479	7.482	7.482	7.476

Table 21. Performance of a variety of algorithms in CartPole with initialization and dynamics noises. The results are averaged over 5 random seeds. We change 2 hyper-parameters, architecture of neural networks and discount factor γ.

K. Our Information Capacity Metrics based on Channel Capacity

In previous works (Klyubin et al., 2005; Tishby & Polani, 2011), *empowerment* is originally defined with channel capacity between future state and n-step action sequence, instead of mutual information.

Following these definition, we extensively evaluate the variants of our PIC and POIC, based on *channel capacity*:

$$\begin{aligned}\text{Policy Information Capacity:} \quad & \max_{p(\theta)} \mathcal{I}(R;\Theta), \\ \text{Policy-Optimal Information Capacity:} \quad & \max_{p(\theta)} \mathcal{I}(\mathcal{O};\Theta).\end{aligned} \tag{5}$$

The experimental settings are similar to Section 6.2; we investigate the correlation between channel-capacity-based information capacity metrics, and algorithm- or random-sampling-based normalized score. To take the maximum over parameter distribution in Equation 5, we prepare a "bag-of-policy-architectures" in practice: ([0] layers + [1, 2] layers × [4, 32, 64] hidden units) × [Gaussian prior $\mathcal{N}(0, I)$, Uniform prior *Unif*$(-1, 1)$, Xavier Normal, Xavier Uniform] × [w/ bias, w/o bias], which amounts to 56 different parameter distributions $p(\theta)$.

The results are shown in Figure 16 (also shown in Figure 15 and Table 22). POIC seems to positively correlate to the algorithm-based normalized score ($R = 0.707$; statistically significant with $p < 0.01$), which can be regarded as the more realistic of the two task difficulty scores. On the other hand, PIC also shows a weak positive correlation with the random-sampling-based normalized score ($R = 0.421$). These results show the similar trends with our definition of PIC and POIC, based on mutual information.

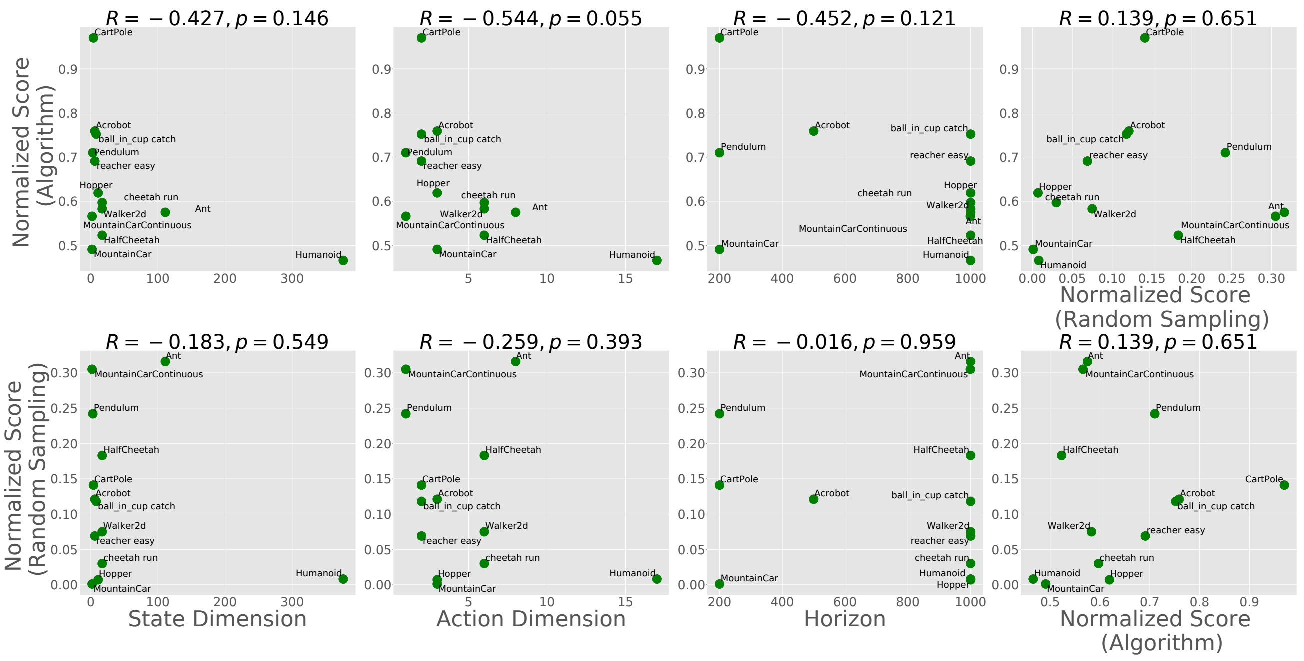


Figure 15. **[Channel Capacity version]**: 2D-Scatter plots between each metric (State dimension, action dimension, episode horizon, and normalized scores; x-axis) and each normalized score (algorithm-based (top) and random-sampling-based (bottom); y-axis).

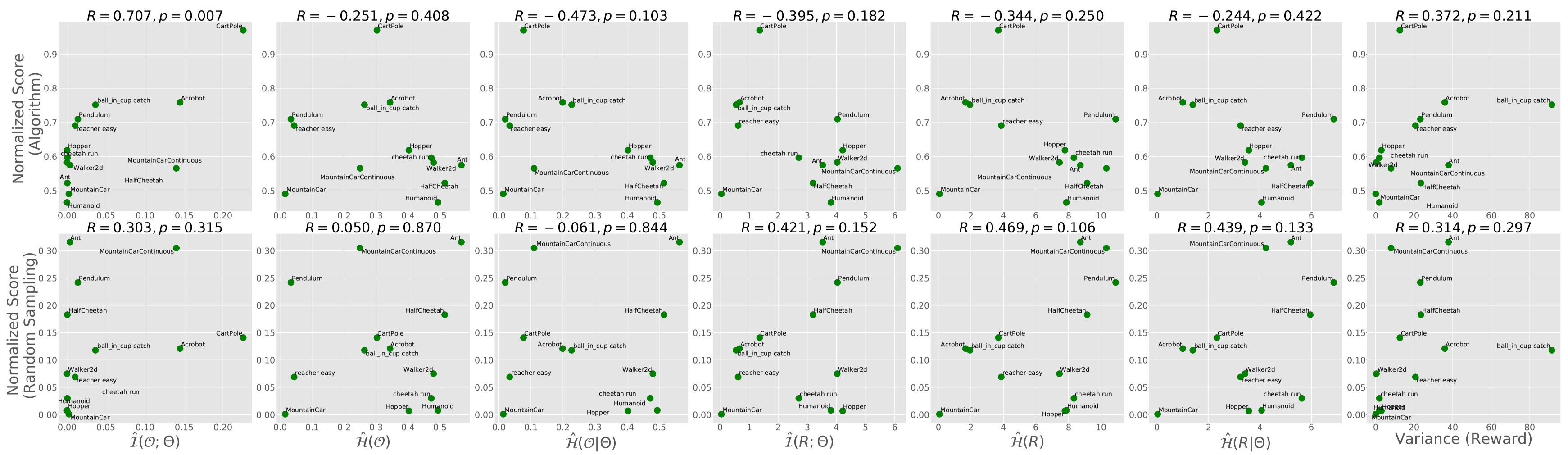


Figure 16. **[Channel Capacity version]**: 2D-Scatter plots between each metric (x-axis) and normalized scores (algorithm-based (top) and random-sampling-based (bottom) ; y-axis); see Table 22 for the details. Variance (last column) approximately corresponds to the metric proposed by Oller et al. (2020). POIC positively correlates with algorithm-based normalized score ($R = 0.707$; statistically significant with $p < 0.01$), the more realistic of the two task difficulty scores.

Environment	Score(A)	Score(R)	$\hat{\mathcal{I}}(\mathcal{O};\Theta)$	$\hat{\mathcal{H}}(\mathcal{O})$	$\hat{\mathcal{H}}(\mathcal{O}\|\Theta)$	$\hat{\mathcal{I}}(R;\Theta)$	$\hat{\mathcal{H}}(R)$	$\hat{\mathcal{H}}(R\|\Theta)$	Variance	State dim	Action dim	Horizon
CartPole	0.970	0.141	0.225996	0.303	0.077	1.363	3.687	2.324	12.610	4	2	200
Pendulum	0.710	0.242	0.013944	0.034	0.020	4.036	10.901	6.865	23.223	3	1	200
MountainCar	0.491	0.001	0.002337	0.016	0.014	0.047	0.070	0.023	0.054	2	3	200
MountainCarContinuous	0.566	0.305	0.140193	0.250	0.110	6.100	10.331	4.230	8.022	2	1	999
Acrobot	0.759	0.121	0.145020	0.344	0.199	0.670	1.672	1.002	35.891	6	3	500
Ant	0.575	0.316	0.003742	0.567	0.563	3.529	8.730	5.201	37.830	111	8	1000
HalfCheetah	0.523	0.183	0.000402	0.515	0.515	3.195	9.148	5.953	23.468	17	6	1000
Hopper	0.619	0.007	0.000226	0.403	0.403	4.214	7.780	3.565	3.006	11	3	1000
Walker2d	0.583	0.075	5.1e-05	0.480	0.480	4.027	7.445	3.418	0.403	17	6	1000
Humanoid	0.466	0.008	2.9e-05	0.494	0.494	3.810	7.870	4.061	1.917	376	17	1000
cheetah run	0.597	0.030	0.000483	0.473	0.472	2.708	8.335	5.627	1.972	17	6	1000
reacher easy	0.691	0.069	0.010335	0.044	0.034	0.622	3.866	3.244	20.669	6	2	1000
ball_in_cup catch	0.752	0.118	0.036467	0.264	0.227	0.553	1.942	1.389	91.454	8	2	1000
Correlation Coefficient: Score(A)	–	0.139	0.707	-0.251	-0.473	-0.395	-0.344	-0.244	0.372	-0.427	-0.544	-0.452
p-Value: Score(A)	–	0.651	0.007	0.408	0.103	0.182	0.250	0.422	0.211	0.146	0.055	0.121
Correlation Coefficient: Score(R)	0.139	–	0.303	0.050	-0.061	0.421	0.469	0.439	0.314	-0.183	-0.259	-0.016
p-Value: Score(R)	0.651	–	0.315	0.870	0.844	0.152	0.106	0.133	0.297	0.549	0.393	0.959

Table 22. **[Channel Capacity version]**: POIC $\hat{\mathcal{I}}(\mathcal{O};\Theta)$, optimality marginal entropy $\hat{\mathcal{H}}(\mathcal{O})$, optimality conditional entropy $\hat{\mathcal{H}}(\mathcal{O}|\Theta)$, PIC $\hat{\mathcal{I}}(R;\Theta)$, reward marginal entropy $\hat{\mathcal{H}}(R)$, reward conditional entropy $\hat{\mathcal{H}}(R|\Theta)$, normalized variance of return in standard RL benchmark environments, Pearson correlation coefficient to the algorithm-based normalized score (Score(A)), and the random-sampling-based normalized score (Score(R)). We prepare the bags of policy architectures, 56 variants in total; ([0] layers + [1, 2] layers × [4, 32, 64] units) × [Gaussian prior $\mathcal{N}(0, I)$, Uniform prior $Unif(-1, 1)$, Xavier Normal, Xavier Uniform] × [w/ bias, w/o bias]. Since we consider the definition based on channel capacity here, we take the maximum value of each information capacity metrics over these policy architectures. The results suggest that POIC seems to positively correlate with algorithm-based normalized score ($R = 0.707$; statistically significant with $p < 0.01$) better than all other alternatives including variance of returns (Oller et al., 2020).