From Local to Global Norm Emergence: Dissolving Self-reinforcing Substructures with Incremental Social Instruments

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Abstract

Norm emergence is a process where agents in a multi-agent system establish self-enforcing conformity through repeated interactions. When such interactions are confined to a social topology, several self-reinforcing substructures (SRS) may emerge within the population. This prevents a formation of a global norm. We propose incremental social instruments (ISI) to dissolve these SRSs by creating ties between agents. Establishing ties requires some effort and cost. Hence, it is worth to design methods that build a small number of ties yet dissolve the SRSs. By using the notion of information entropy, we propose an indicator called the BA-ratio that measures the current SRSs. We find that by building ties with minimal BA-ratio, our ISI is effective in facilitating the global norm emergence. We explain this through our experiments and theoretical results. Furthermore, we propose the small-degree principle in minimising the BA-ratio that helps us to design efficient ISI algorithms for finding the optimal ties. Experiments on both synthetic and real-world network topologies demonstrate that our adaptive ISI is efficient at dissolving SRS.

1. Introduction

From greeting etiquette and driving habits to personal conducts in workplace, social norms shape behaviors that are generally accepted by a society (Morris-Martin et al., 2020; Santos et al., 2018). Online societies such as those facilitated by Facebook, Twitter, and Reddit present large platforms for expressing vastly different opinions and practices. In a decentralized online society, social norms are often organically grown. Take, as examples, the spread of Internet memes (Bauckhage, 2011) and the evolving narratives of emoji (Miller et al., 2016). The decentralized nature of open platforms leaves open the possibility for social bots to manipulate public opinion, disrupt communication, and influence public sentiment (Ferrara et al., 2016). *Norm emergence* is the study of the process in which social norms arise naturally through the interactions between individuals of the society. Understanding norm emergence in diverse societies is important as norms regulate communication, help alleviate conflicts, enhance coordination, and ensure meeting societies' goals (Levin, 2002).

To capture the essence of norm emergence, imagine an individual who faces several possible actions, among which only one is to be taken. The decision, which actions is taken, depends on the interactions between this individual and others in the network. One can naturally model this scenario as a *multi-agent system* (MAS) (Jiang & Jiang, 2013). Here, individuals are *agents* who can internalize sensory input and derive actions autonomously. Through interacting with other agents, a type of self-enforcing behaviors, for instance a social norm, may be adopted by every agent. One represents inter-agent interactions as games that contain multiple equilibria. A *norm* is then defined as one of these equilibria which is accepted by every agent in the MAS (Young, 1996). Norm emergence is then the process that leads to the establishment of such norm.

Sen and Airiau (Sen & Airiau, 2007) proposed the *social learning* mechanism. This is a simple yet powerful paradigm for norm emergence. At every iteration, selected agents participate in a fixed *stage game* through *trial-and-error*. Such games are meant to simulate repeated interactions between agents. After observing the outcomes of their games, these agents accumulate their experiences with a learning mechanism. The experience will, in turn, direct their game plays in subsequent iterations. The process

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continues indefinitely and captures norm emergence as a process of "gradual accretion of precedent" as defined in (Young, 1996).

Despite the general belief that social learning prepares the way for norm emergence, there are situations where a norm fails to emerge (Hu & Leung, 2017; Villatoro et al., 2011; Toivonen et al., 2009). Often these situations occur when one confines agents' interactions in a pre-defined *social topology*. To digest these failures, one may resort to empiricism by relaxing the notion of norm. A common practice is to declare that a norm emerged when 90% of the population (rather than all) have the same behavior (Kittock, 1993). However, even this 90% relaxation might be too strong. It has been observed that communities may converge to different behaviors when the social topology is naturally segregated into communities (Hu & Leung, 2017). These are called *local norms*. In this sense, each community forms a *self-reinforcing substructure* (SRS) that remain stable over time (Villatoro et al., 2011).

In many situations, a desirable outcome is have these SRS dissolved so that the entire population achieves a global norm. This applies when the goal is to, e.g., establish a common frequency of communication among sensors in a WSN, or enforce protocols in a multi-agent system, or unify opinions among multiple online users (Mihaylov, 2012). Given that the social topology plays a vital role in the formation of SRSs, a natural approach to achieve global norm in the presence of SRSs is to modify the social ties. In particular, adding social ties corresponds to establishing interactions between agents, e.g., in a WSN, adding a tie means creating a channel to allow two sensors to communicate. In an online social platform, users can be linked through social recommendation (Yan et al., 2018) (Moskvina & Liu, 2016). Based on this intuition, we introduce *incremental social instruements* (ISI) for dissolving SRSs that guarantee emergence of a global norm (Villatoro et al., 2011). An ISI process adds new social links to the social topology to promote norm emergence in the entire population. Such a process makes sense since in fully connected networks a social norm is expected to be formed (Sen & Airiau, 2007). A natural question arises as to how we can implement such ISIs *most effectively*, i.e., with the small number of new ties.

The contributions of this paper are the following:

1. We use the concept of information entropy, and define two entropies: A-entropy and B-entropy. These entropies, respectively, measure uncertainty of ties and beliefs between the agents. We define their ratio as BA-ratio that, in some ways, measures the interaction diversity among the agents in the population.

2. We propose that minimizing BA-ratio is an efficient approach to dissolve SRSs and promote norm emergence. We further prove the *small-degree principle* in minimizing BA-ratio at the process of creating links. Based on this, we design efficient ISIs for finding the optimal link.

3. We introduce two ways that implements ISIs, one *static* and one *adaptive*. We perform experiments and compare them with several benchmarks and discuss their effect on both synthetic and real-world networks. We demonstrate that our adaptive ISI is effective at dissolving SRSs. We also discuss the impact of various network properties at dissolving SRSs.

2. Related Work

We review key breakthroughs in the study of norm emergence. Axelrod was among the first who studied social norms from an emergent point of view (Axelrod, 1986). His work laid down many key characteristics in subsequent studies, e.g., norms are self-enforcing behaviors of social actors. Shoham and Tennenholtz introduced the paradigm of norm emergence through repeated coordination games (Shoham & Tennenholtz, 1992). In this paradigm, a norm is an equilibrium achieved through repeated interactions between agents. This paradigm has become a standard model of norm emergence; we too adopt this paradigm in this work. Importantly, they also focused on the *efficiency* of norm emergence. Efficiency is critical as agents' incoordination is regarded undesirable and thus faster norm emergence is preferred.

Kittock (Kittock, 1993) was the first who studied the effect of social topology to norm emergence. The author observed that in many cases, even though the majority of agents coordinate their actions after only a few repeated interactions, it is often impossible for the entire population to adopt the same action. Thus it makes sense to accept 90% convergence as a threshold that indicates a norm has emerged. (Young, 1996) studied norm emergence under various social topologies. The author observed a "global diversity effect" where the presence of several sub-populations that do not interact with one another could lead to different, and sometimes conflicting norms.

Sen and Airiau (Sen & Airiau, 2007) proposed the *social learning* framework that greatly extends the varieties of learning mechanism of agents. In particular, Q-learning, also adopted in our paper, is used to guide agents' decisions. Once again, the authors observed that disjoint sub-populations with infrequent inter-community interaction could lead to different local norms. Since then, there have been a lot of studies that explored the roles network topologies play in norm emergence under various learning paradigms (Mukherjee et al., 2008; Yu et al., 2013; Hao et al., 2017; Hu et al., 2019). Notably, Hu and Leung in (Hu & Leung, 2017) studied networks with salient community structure and verified the emergence of stable diverse local norms in such networks.

These works clearly show that network connectivity and SRSs may play a major role in global convergence. Villatoro, *et al.* (Villatoro et al., 2011) introduced *social instruments* aimed at dissolving the SRSs. A social instrument is an agent-level mechanism that exploits the social topology with the purpose of influencing agents' behaviors. The authors conducted experiments on regular and scale-free networks using two social instruments: rewiring the social ties or expanding agents' scopes of observation. These social instruments differ from our proposed social instruments in three aspects: (1) rewiring might destroy the original topological connection between individuals contrasting our desire to preserve the connections; (2) their mechanism tend to produce disconnected components, which even hinder the emergence of global norms; (3) their work focuses on individuals choosing actions that are beneficial to them, while our work focuses on a central organization that helps individuals establish connections to achieve global norms. We thus aim to design *incremental social instruments* that help individuals to establish new connections, and we look for techniques that integrate different local norms in one global norm in an effective way.

Since SRS is related to the community structure of the graph, how to measure the structural information of the graph is of great significance for the study of global convergence. Recently, Li and Pan proposed a notion of structural information entropy to quantify structural information of a graph (Li & Pan, 2016). This notion advances the study of problems related to graph structure, for example, measuring the security of a network (Li et al., 2016), defending networks against virus attack (Li et al., 2017) and hiding the community structure by modifying the original graph (Liu et al., 2019). Inspired by those works, in this paper, we study a metric to measure SRS, and then use this metric to guide the dissolution of SRS and finally reach the global norm.

3. Preliminaries

We present the game-theoretic framework for norm emergence in networks proposed in (Sen & Airiau, 2007). Let $N = \{v_1, \ldots, v_n\}$ be a set of n agents. Assume that these agents are embedded in a *social topology*, i.e., an undirected graph $G = (N, E)$ where every edge $\{u, v\}$ represents a social link between agents u and v. Informally, the edge $\{u, v\}$ provides a venue where the agents u and v interact.

3.1. Stage game

When interacting with others, agents can choose from the set $A = \{a_1, \ldots, a_m\}$ of m actions. Consider the situation when these actions are equally desirable by the agents, meaning that the outcome of the interaction only depends on coordination of the players. By coordination we mean that the two agents align their actions as specified in a (2-player m-action) *coordination game*. The payoff matrix of this game is in Table 1. Intuitively, the agents make independent moves by selecting the actions. If the agents choose the same action, they are both rewarded with a payoff 1; otherwise, they are both punished with a penalty -1. In this way, the game has m Nash equilibria (a_i, a_i) where $a_i \in A$, all of which are equally preferred by the two agents.

Table 1. 2-player-m-action coordination game

	a_1	a_2	...	a_m
a_1	$(1,1)$	$(-1,-1)$	...	$(-1,-1)$
a_2	$(-1,-1)$	$(1,1)$	...	$(-1,-1)$
...	...	...	...	...
a_m	$(-1,-1)$	$(-1,-1)$	...	$(1,1)$

3.2. Social learning

Social learning is used as the action selection mechanism. The process proceeds in iterations. At each iteration, a number of disjoint edges are chosen randomly. The pair of agents in a selected edge then play a coordination game. An agent only receives its own payoff as the outcome of playing the stage game. During the play, each agent v_i maintains a private *belief state* $b_i \in A$ which directs v_i to the next action. Once agent v_i receives a payoff, the belief state b_i is adjusted with the payoff. In the literature, a number of algorithms have been used to implement the social learning paradigm (Morris-Martin et al., 2020), e.g., Q-Learning, WoLF-PHC, and Fictitious play (FP), among which Q-learning has been widely used (Yu et al., 2013; Villatoro et al., 2009; Hu & Leung, 2017). The main reason is that Q-learning tend to lead the majority of agents to adopt the same action with fewer number of iterations.

In Q-learning, every agent v_i maintains a *Q-value* $Q_i(a_j)$ for each action $a \in A$. The action b_i is thus the action a_j that has the maximum Q-value, i.e., $b_i = \arg\max_{a_j} Q_i(a_j)$. At every iteration, the agent v_i chooses the action b_i with ϵ-greedy exploration. After getting a payoff r from playing action a_j, the agent v_i updates $Q_i(a_j)$:

$$Q_i(a_j) := (1 - \lambda)(Q_i(a_j)) + \lambda r,$$

where $\lambda \in [0, 1]$. In this paper, we set $\lambda = 0.5$ and $\epsilon = 0.1$ (exploration parameter) according to (Hu & Leung, 2017).

3.3. Norm emergence

The desired outcome of a social learning process is when agents converge to actions that would maximize their expected utility. This would corresponds to a state when the

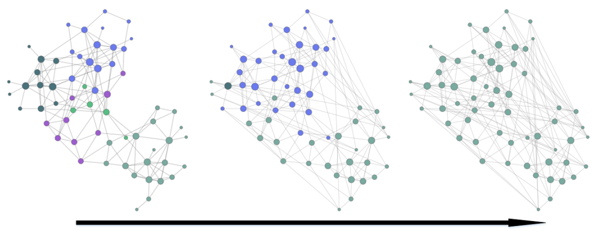


Figure 1. The effect of applying ISI to the dolphin network[1]. Each picture displays stabilized SRS indicated by different colors. The original network contains 62 nodes and 159 edges. Then the configurations after adding 20 and 40 edges respectively.

agents hold the same belief states. This state gives rise to the notion of norms, as defined below. Here, we focus on the belief state b instead of the chosen actions as we adopt ϵ-greedy exploration which forbids the convergence of actions. We therefore regard b as the outcome of the social learning process.

Definition 1. *We say that a* local norm *emerges in a subpopulation* $M \subseteq N$ *if the belief states* b_i *of all agents* $v_i \in M$ *converge to the same* $a_j \in A$. *In this case, if* $M \neq N$, *then* M *is called a* self-reinforcing substructure (SRS) *in* N, *and if* $M = N$, *we say that a* (global) norm *has emerged.*

Remark 1. The notions above are defined for a social learning process which in theory has infinitely many iterations. Empirically, though, we may declare that a (local or global) norm has emerged when convergence is observed within a sufficient number (e.g., 5000) of iterations. This has been adopted by the vast majority of work that study norm emergence (Sen & Airiau, 2007; Hu & Leung, 2017; Yu et al., 2013; Hao et al., 2017; Morris-Martin et al., 2020).

3.4. Incremental social instrument

Villatoro, *et al.* (Villatoro et al., 2011) observed that SRSs form a major bottleneck for the emergence of a global norm. Moreover, SRSs arise mainly due to connectivity issues in the social topology. (Hu & Leung, 2017) have shown that communities often play the roles of SRSs. This is consistent with studies in social psychology (Cialdini & Trost, 1998) where online communities build their own social conventions. Interestingly, a fully connected network does not exhibit any SRSs and global norm has been shown to always emerge.

Let $G = (N, E)$ be a social topology on which SRSs are observed. As we mentioned above, adding edges to G until G becomes fully connected will dissolve SRSs. We thus propose the idea of *incremental social instrument* (ISI) to facilitate norm emergence, for example, applying ISI to the dolphin network[1]. In order to implement this idea we need to design methods that find small number of edges such that a global norm emerges. Let E' be a set of pairs $\{u, v\}$ where $u \neq v$ and $u, v \in N$. By $G \oplus E'$ denote the graph $(N, E \cup E')$ obtained by adding edges E' into G. Our problem thus aims to find a set E' such that a global norm is more likely to emerge. In the rest of the paper we design appropriate implementations of the incremental social instrument.

4. ISI based on minimizing BA-ratio

To better understand how agents interaction in a social topology, we adopt an information-theoretic argument. In the following, we invoke information entropy to interpret various components of the social learning process that we described above. In particular, we focus on (1) the pattern of interactions, and (2) the outcomes of these interactions,

4.1. Interaction

Agent interaction: As explained Section 3.2, each iteration of the social learning process involves the interactions from randomly sampled set S of pairs of agents. These pairs of agents can be viewed as edges in the graph $G = (N, E)$, the graph G represents the patterns of agent interactions. Let us list all agents (vertices) $v_1, \ldots, v_n$ of the graph G. Assume that the interactions between the pairs of agents in S takes place in different discrete times $t = 0, 1, \ldots$ Let $\{x_t, y_t\}$ be the pair of agents that interact at time t. The degree d_i of v_i captures the probability that $v_i \in \{x_t, y_t\}$. This probability is $d_i/2|E|$. By Shannon's definition of information entropy, the *information content* of v_i is $-\log_2(d_i/2|E|)$ that reveals the level of uncertainty if v_i is chosen to participate in an interaction (Cover, 1999). Thus, given the probabilistic distribution on the nodes of the graph G, we have the overall interaction entropy, that we call the *A-entropy*, defined as:

$$\mathcal{H}_a(G) = -\sum_{v_i \in N} \frac{d_i}{2|E|} \log_2 \frac{d_i}{2|E|}. \quad (1)$$

The notion of A-entropy defined above reflects the interaction between different agents. Specifically, a smaller value of A-entropy indicates that the interaction is concentrated in a small set of individuals, while a larger value implies that each individual has a more equal opportunity to participate in the interaction.

Belief in interaction: Suppose every agent v_i has a belief state b_i. In Q-learning, the belief state is an action with the maximum Q-value. The belief states $b_1, \ldots, b_m$ of the agents partition the population N into m disjoint

[1] http://www-personal.umich.edu/~mejn/netdata/

sub-populations $N_1, \ldots, N_m$, where $N_j := \{v_i \in N \mid b_i = a_j\}$. Note that N_j can be empty. When $N_j = N$ for some j, the agents have reached a global norm. For $j \in \{1, \ldots, m\}$, the *volume* of N_j is $\nu_j := \sum_{v_i \in N_j} d_i$. It is clear that $\sum_{j=1}^{m} \nu_j = 2|E|$.

Assume that the players adopt their beliefs as actions (so $\epsilon = 0$) during social learning. An interaction between agents x_t, y_t thus involves agents playing their belief states p_t, q_t, respectively. There are three cases for any action a_j: 1) $a_j \notin \{p_t, q_t\}$, that is, a_j does not appear in the interaction; 2) $a_j \in \{p_t, q_t\}$ and $p_t \neq q_t$, that is, a_j appears once; 3) $a_j = p_t = q_t$, that is, a_j appears twice. Then, for a given graph G and the partition $\mathcal{P} = \{N_1, \ldots, N_m\}$, the probability that action a_j appears in the interaction is

$$\Pr(a_j) = \sum_{b_i = a_j} \frac{d_i}{2|E|} = \sum_{v_i \in N_j} \frac{d_i}{2|E|} = \frac{\nu_j}{2|E|}.$$

The value $\nu_j/2|E|$ measures the reputation of action a_j in the population. We have, in fact, just defined the belief entropy, that we call B-entropy and denote by $\mathcal{H}_b^{\mathcal{P}}(G)$, as follows:

$$\sum_{a_j \in A} \Pr(a_j) \log_2 \Pr(a_j) = - \sum_{a_j \in A} \frac{\nu_j}{2|E|} \log_2 \frac{\nu_j}{2|E|}. \quad (2)$$

The B-entropy captures the average information content of agents' beliefs in their interaction.

Lemma 1. *For any connected graph $G = (N, E)$ and partition $\mathcal{P} = \{N_1, \ldots, N_m\}$, we have $0 \leq \mathcal{H}_b^{\mathcal{P}}(G) \leq \mathcal{H}_a(G)$.*

Proof. Obviously, $\mathcal{H}_b^{\mathcal{P}}(G) \geq 0$. The result is immediate because $-\log_2 \frac{\nu_j}{2|E|} \leq -\log_2 \frac{d_i}{2|E|}$ for $v_i \in N_j$. □

Note that $\mathcal{H}_a(G)$ is the maximal value of $\mathcal{H}_b^{\mathcal{P}}(G)$. The equation $\mathcal{H}_b^{\mathcal{P}}(G) = \mathcal{H}_a(G)$ holds if and only if each agent takes a different beliefs. At this time, we have $m \geq n$.

Remark 2. We point out that, even though in principle, B-entropy is defined on an arbitrary partition of agents, it makes sense in our context only when we apply it to the resulting partition derived from the belief states of agents after a social learning process, i.e., when agents' belief states stabilize. This partition reflects a self-reinforcing substructure of the population due to their patterns of communication $G = (V, E)$, and therefore it can be considered, in this work, as an inherent attribute of G.

4.2. Belief-Agent ratio

We noted above that $\mathcal{H}_a(G)$ is the upper bound for $\mathcal{H}_b^{\mathcal{P}}(G)$. Hence, it makes sense to define their ratio.

Definition 2. *[BA-ratio] The* belief-agent ratio *of a partition $\mathcal{P}$ defined on the population N with respect to their social topology $G = (N, E)$ is $\rho_G(\mathcal{P}) := \mathcal{H}_b^{\mathcal{P}}(G)/\mathcal{H}_a(G)$.*

The BA-ratio reflects the level of diversity of the agents preferred actions (i.e., belief states). $\mathcal{H}_b^{\mathcal{P}}(G) = 0$ if and only if $\frac{\nu_j}{2|E|} = 1$ for some $1 \leq j \leq m$ (Cover, 1999). Combined with Lemma 1, we obtain the following theorem:

Theorem 1. *For any connected graph $G = (N, E)$ on the population N and partition $\mathcal{P} = \{N_1, \ldots, N_m\}$, we have $0 \leq \rho_G(\mathcal{P}) \leq 1$ and $\rho_G(\mathcal{P}) = 0$ if and only if a global norm emerges in N, i.e., for some $1 \leq j \leq m$, $N_j = N$.* □

Clearly, $\rho_G(\mathcal{P})$ is a normalized version of B-entropy. We note that if $\rho_G(\mathcal{P}) = 1$, then each agent holds its own belief; also, if $\rho_G(\mathcal{P}) = 0$, then all agents reach a consensus. Therefore, minimizing $\rho_G(\mathcal{P})$ facilitates the norm emergence. We conclude that in order to design an effective ISI, that is, an edge creation strategy that links members of distinct SRSs to integrate local norms into one global norm, it makes sense to create edges by minimizing $\rho_G(\mathcal{P})$.

4.3. Small-degree principle

When the graph G has SRSs such as $N_1, \ldots, N_m$ described above, it is hard to descrease the BA-ratio. This amounts to either decreasing $\mathcal{H}_b^{\mathcal{P}}(G)$, that is, decreasing beliefs among agents or increasing $\mathcal{H}_a(G)$ interactions between the agents. Which agents should we link in order to decrease the value $\rho_G(\mathcal{P})$? It turns out linking the agents with smallest degrees and distinct beliefs is one such natural tool.

We call agents v_i, v_k *non-interacting-action-inconsistent* (NIAI) agents if $\{v_i, v_k\} \notin E$ and $b_i \neq b_k$. We call a pair $e = \{v_i, v_k\}$ of NIAI agents *BA-ratio minimizing* if $\rho_{G \oplus \{e\}}(\mathcal{P}) \leq \rho_{G \oplus \{e'\}}(\mathcal{P})$ for all NIAI agent pairs e'.

Theorem 2 (Small-Degree Principle). *Consider two NIAI pairs $e = \{v_i, v_k\}, e' = \{v_{i'}, v_{k'}\} \in N_j \otimes N_\ell$.*

(1) If $min\{d_i, d_k\} \leq min\{d_{i'}, d_{k'}\}$ and $max\{d_i, d_k\} \leq max\{d_{i'}, d_{k'}\}$, then $\rho_{G \oplus \{e\}}(\mathcal{P}) \leq \rho_{G \oplus \{e'\}}(\mathcal{P})$.

(2) If $e = \{v_i, v_k\}$ is BA-ratio minimizing, then v_i has the smallest degree among those nodes in N_j that are not interacting with v_k.

Proof. For $h \in \{1, \ldots, m\}$, let Δ_h denote

$$\frac{\nu_h}{2|E| + 2} \log_2 \frac{\nu_h}{2|E| + 2}.$$

By (2), both $\mathcal{H}_b^{\mathcal{P}}(G \oplus \{e\})$ and $\mathcal{H}_b^{\mathcal{P}}(G \oplus \{e'\})$ are equal to

$$- \sum_{h \neq j, \ell} \Delta_h - \sum_{h \in \{j, \ell\}} \left[\frac{\nu_h + 1}{2|E| + 2} \log_2 \frac{\nu_h + 1}{2|E| + 2} \right]$$

Algorithm 1 Incremental Social Instrument (ISI)

Input: $G = (V, E), N = (N_1, N_2, ..., N_m)$
Output: a NIAI pair $\{u, v\}$

1: $\rho_{min} = 1$;
2: **for** $(j, \ell) \in \{1, \dots, m\}^2$ where $\ell > j$ **do**
3: $\quad L_{j\ell} \leftarrow$ sort $N_j \cup N_\ell$ into non-decreasing degree order, $s \leftarrow 1, t \leftarrow |L_{j\ell}|$;
4: $\quad$ **while** $s < t$ **do**
5: $\quad\quad$ **for** $k = s + 1 \to t$ **do**
6: $\quad\quad\quad e \leftarrow \{L_{j\ell}[s], L_{j\ell}[k]\}$;
7: $\quad\quad\quad$ **if** $\delta(L_{j\ell}[s], L_{j\ell}[t]) = 0$ & $\rho_{G\oplus e} < \rho_{min}$ **then**
8: $\quad\quad\quad\quad u \leftarrow L_{j\ell}[s], v \leftarrow L_{j\ell}[k]$;
9: $\quad\quad\quad\quad \rho_{min} = \rho_{G\oplus e}, t = k - 1$; **Break**;
10: $\quad\quad\quad$ **end if**
11: $\quad\quad$ **end for**
12: $\quad\quad s \leftarrow s + 1$;
13: $\quad$ **end while**
14: **end for**
15: **RETURN** $\{u, v\}$

Therefore if we want to prove $\rho_{G\oplus\{e\}}(\mathcal{P}) \le \rho_{G\oplus\{e'\}}(\mathcal{P})$, by the definition of BA-entropy, it remains to prove that $\mathcal{H}_a(G \oplus \{e\}) \ge \mathcal{H}_a(G \oplus \{e'\})$. Define a function $f : \mathbb{R} \to \mathbb{R}$ as $f(x) = (x + 1)\log_2(x + 1) - x \log_2 x$. Denote

$$\eta_h = -\frac{d_h + 1}{2|E| + 2}\log_2\frac{d_h + 1}{2|E| + 2} + \frac{d_h}{2|E| + 2}\log_2\frac{d_h}{2|E| + 2}$$

then,

$$\begin{aligned}\mathcal{H}_a(G \oplus \{e\}) - \mathcal{H}_a(G \oplus \{e'\}) &= \eta_i + \eta_k - \eta_{i'} - \eta_{k'} \\ &= \frac{1}{2|E| + 2}\left(f(d_{i'}) + f(d_{k'}) - f(d_i) - f(d_k)\right) \quad (3)\end{aligned}$$

Without loss of generality, assume $d_i = \min\{d_i, d_k\} \le \min\{d_{i'}, d_{k'}\} = d_{i'}$ and $d_k = \max\{d_i, d_k\} \le \max\{d_{i'}, d_{k'}\} = d_{k'}$, since $f'(x) = \log_2(1 + 1/x) > 0$ when $x > 0$, then the function f is monotonically increasing. Then $f(d_{i'}) \ge (d_i)$ and $f(d_{k'}) \ge f(d_k)$. By equation (3), we have $\mathcal{H}_a(G \oplus \{e\}) \ge \mathcal{H}_a(G \oplus \{e'\})$. Then we complete the proof of (1). Let $v_k = v_{k'}$, the conclusion (2) can be obtained immediately by (1). □

The theorem above suggests that an ISI that puts priority on creating edges between non-interacting agents who have small degrees will result in faster drop of the BA-ratio, hence it facilitates norm emergence in a more effective way.

Remark 2. Denote $\delta(v_i, v_k) = 0$ if v_i and v_k have different beliefs and have no edge between them in Alg.1. The worst-case complexity of Alg.1 is $O(n^2)$. On sparse graph, this complexity is only $O(m^2)$ where m is the number of actions. This is because $\{L_{j\ell}[s], L_{j\ell}[s + 1]\}$ in line 6 is likely to have the minimum degrees of substructures $N_j \otimes N_\ell$ respectively and is non-interacting. In this case, the 'while' loop in line 4 and the 'for' loop in line 5 are executed only once. In addition, it can be easily obtained by Theorem 2 that Alg.1 outputs a NIAI edge e that minimizes $\rho_{G\oplus e}$. Also, this Algorithm help us to design two methods to facilitate global norm, i.e., Static ISI and Adaptive ISI, in Section 5.

4.4. Static and Adaptive ISI

We describe two scenarios for ISI: *static* and *adaptive*. Both scenarios start when agents stabilise on their initial beliefs forming SRS. In the static case, the algorithm only has knowledge about the initial beliefs, while the adaptive case allows the algorithm to take into account changes in agents' beliefs when edges are added iteratively.

Static ISI. Let $G = (N, E)$ be a social topology. Suppose that the agents in N have arrived at local norms, represented by the partition $\mathcal{P} = \{N_1, \dots, N_m\}$. That means the agent $v_i \in N_j$ has its belief $b_i = a_j$. The goal is output a set E' of non-interacting pairs so that adding E' into the social topology G will effectively dissolve the SRS $N_1, \dots, N_m$ such that $N_j = N$ for some j. We call this problem the *static ISI problem* as the central organization only has the have the knowledge of the initial beliefs and the edges in E' can be computed at one go depend on the initial beliefs.

As the description of BA-ratio, our algorithm will aim to reduce the BA-ratio $\rho_G(\mathcal{P})$ of the social topology by creating edges between nodes. Algorithm 2 implements the strategy and outputs an approximate optimal edge set E' of $\rho_{G\oplus E'}(\mathcal{P})$ through the gradient descent method.

Algorithm 2 Static Incremental Social Instrument (SISI)

Input: Social topology $G = (N, E)$, local norm partition $\mathcal{P} = \{N_1, \dots, N_m\}$, budget $h \in \mathbb{N}$
Output: Set E' of h non-interacting pairs.

Create empty sets $E' \leftarrow \varnothing$
for $j = 1, \dots, h$ **do**
$\quad$ Find a NIAI e with the smallest $\rho_{G\oplus(E'\cup e)}(\mathcal{P})$ by Alg.1;
$\quad$ Update $E' = E' \cup e$;
end for
return E'

Adaptive ISI. In static ISI, the knowledge of agents' beliefs are limited in the initial state. This may make a wrong judgment for the subsequent edge addition, since agents may change their beliefs as they have more interaction with other beliefs through the newly added social instrument. In adaptive ISI, suppose that the agents are interacting *while the social instrument* is putting in place. That is, we assume that the actions (i.e. belief states) of the players may change as the social topology changes. When one edge is added between two nodes, the belief state partition $\mathcal{P}$ may also change. Thus the next decision to be made on which edge to add should be made on this updated $\mathcal{P}$ according to Alg. 1.

For this idea to make sense, we need to modify the problem definition above. We propose the *adaptive ISI* prob-

Table 2. The table lists key statistics of the generated graphs using the GRP and LFR models. Parameters in GRP: $k = 10, v = 1$. Parameters in LFR: $\alpha = 2.5, \beta = 1.5, k = 5$.

Type	Parameter	n	$\lvert E\rvert$	m	Modularity
GRP	$\sigma = 0.9$	500	2408	10	0.789
	$\sigma = 0.8$	500	2465	10	0.692
	$\sigma = 0.7$	500	2472	10	0.596
	$\sigma = 0.6$	500	2565	10	0.495
	$\sigma = 0.5$	500	2524	10	0.398
LFR	$\mu = 0.1$	500	1411	9	0.789
	$\mu = 0.2$	500	1378	11	0.670
	$\mu = 0.3$	500	1571	8	0.465
	$\mu = 0.4$	500	1491	7	0.331
	$\mu = 0.5$	500	1362	9	0.300

lem as follows: The input of the problem is still a social topology $G = (N, E)$ with established local norms $\mathcal{P} = \{N_1, \ldots, N_m\}$. A desired algorithm that solves the problem will generate a sequence of new edges $e_1, e_2, e_3, \ldots$ such that after creating every new edge e_t, the agents will continue with their social learning process, until achieving a (meta-)stable state in the form of an updated partition $\mathcal{P}' = \{N'_1, \ldots, N'_m\}$, at which point the algorithm then computes the next edge e_{t+1} with respect to $\mathcal{P}'$. In this sense, the algorithm is adaptive as it is able to adjust its decisions at runtime by observing the belief states of the agents. Moreover, in the adaptive ISI problem there is no need to have a pre-determined number h of new edges to be created; we can simply run the algorithm to iteratively add edges until a global norm emerges.

Algorithm 3 Adaptive Incremental Social Instrument (AISI)

Input: Social topology $G = (N, E)$, local norm partition $\mathcal{P} = \{N_1, \ldots, N_m\}$, budget $h \in \mathbb{N}$
Output: A sequence of h non-interacting pairs $e_1, e_2, \ldots$
  while there exists v_i, v_k such that $b_i \neq b_k$ and $j = 1 \to h$ **do**
    Find an edge e by Alg.1 with respect to the current social topology G and partition $\mathcal{P}$;
    Update $G = G \oplus e$ and output $e_j = e$;
    Run the social learning process until the agents stabilize and record the current partition as the new $\mathcal{P}$;
  end while

5. Experiments

Through experiments, we would like to investigate: 1) The effectiveness of our algorithms w.r.t. integrating local norms and dissolving SRS in both the static and adaptive scenarios; 2) How accurately BA-ratio (as in Def. 2) reveals the diversity of agents' actions; 3) How parameters such as community number, size, and action size affect the algorithms' performance.

5.1. Experiment setup

Network datasets. We adopt both synthetic and real-world networks in our experiments. For synthetic networks, we generate initial social topologies using two well-used random network models: **GRP** (Brandes et al., 2003): This model consists of parameters g, v, ℓ, k and σ: The generated graph has ℓ communities, average degree k, the separation degree σ, the community size g and standard deviation v. More precisely, edges in the graph are added randomly to make sure that the ratio of intra-community edges to the whole edges is σ. **LFR** (Lancichinetti et al., 2008): This model consists of parameters n, γ, β, μ, k, where n is the number of nodes, γ and β are power-law exponents for the degree and community size distribution, μ is the fraction of inter-community edges, and k is the average degree. In fact, the parameter μ in LFR is equivalent to $1 - \sigma$. LFR is different from GRP in that the community size is not uniform but are generated from a power-law distribution. Moreover, the degree distribution within each community also follows a power law.

Table 3. Key statistics of the synthetic and real-world networks.

Type	Graph	n	$\lvert E\rvert$	m	Modularity
GRP	grp_200_0.9	200	468	5	0.726
	grp_500_0.9	500	2408	8	0.789
LFR	lfr_100_0.1	100	302	5	0.638
	lfr_500_0.1	500	1411	8	0.788
RN	enron_email	143	623	7	0.568
	virgili_email	1133	5451	10	0.572
RN	dblp_202	202	387	5	0.510
	dublin_contact	410	2765	7	0.711

For the real-world networks (RN), we choose two email communication network: enron_email[2] and virgili_email[3]; one co-authorship network: dblp[4]; and one human contact network: dublin_contact[5]. We point out that the dblp dataset available online are too large for our experiments, so we only extract several communities from it. See Table 3 for details. In addition, we also list the *modularity* of the initial partition to show the strength of community structure in each graph (Newman, 2006).

Performance metrics. We rely on two indices to reveal global norm emergence: The first is the *proportion of the dominant action*, defined as the maximum proportion of agents who adopt the same belief, i.e., $\max_{1\leq j\leq m} \frac{|\{i|b_i=a_j\}|}{n}$. The second is the *diverseity index* proposed by Hu and Leung in (Hu & Leung, 2017): Given a social topology $G = (N, E)$ and a partition $\mathcal{P} = \{N_1, \ldots, N_m\}$, the *diversity index* $\iota(\mathcal{P})$ is defined as

$$\iota(\mathcal{P}) := \frac{1}{\log_2 n} \sum_{1\leq j\leq m} -\frac{|N_j|}{n} \log_2 \frac{|N_j|}{n}$$

The diversity index is the normalized information entropy

[2] http://networkrepository.com/ia-enron-only.php
[3] https://deim.urv.cat/~alexandre.arenas/data/welcome.htm
[4] https://snap.stanford.edu/data/com-DBLP.html
[5] http://networkrepository.com/ia-infect-dublin.php

for a random variable that selects an action $a \in A$ based on the proportion of the population that adopt it. In this sense, it is a measure on how wide the actions spread within the population $\mathbb{N}$. Normalizing the information entropy by dividing with $\log_2 n$ makes the value of $\iota(\mathcal{P})$ lying within $[0, 1]$. In this experiment, we would like to verify that our notion of BA-ratio $\rho_G(\mathcal{P})$ correlates well with the diversity index $\iota(\mathcal{P})$.

Social learning. As mentioned above, we adopt Q-learning for social learning. On each graph, we run 5000 iterations to convergence for each execution of the social learning process. If a global norm fail to emerge after the social learning process, we observe the SRS that have formed and regard it as the initial partition, i.e., the local norms. After adding one edge by an edge creation strategy, we start another execution social learning, and compute the resulting proportion of dominant action and the new local norm partition. This lasts until the global norm emerges.

Benchmarks. Apart from implementing our static and adaptive strategies (SISI and AISI) to creating edges, we also run four benchmark methods to create edges: **Random**: add a new non-interacting edge randomly; **Min-Degree**: Randomly choose two agent substructures N_j and N_ℓ, add a new edge $\{v_i, v_k\}$ in $N_j \times N_\ell \neq \phi$, where d_i is the smallest degree in N_j and d_k is the smallest degree among non-interacting pair all $\{v_i, v_k\} \in N_j \times N_\ell$; **Max-Degree**: replace smallest by largest in Min-Degree; **MaxMin-Degree**: Randomly choose two agent substructures N_j and N_ℓ, add a new edge $\{v_i, v_k\}$ in $N_j \times N_\ell \neq \phi$, where d_i is the largest degree in N_j and d_k is the smallest degree among non-interacting pair all $\{v_i, v_k\} \in N_j \times N_\ell$.

We point out that all four methods can be implemented using two instantiations: a static and an adaptive one. The static implementation runs on the initial SRS while the adaptive strategy runs on the updated SRS. [6]

5.2. Experimental results

Effectiveness for norm emergence. We use both synthetic and real-world networks listed in Table 3. For every method, we run both the static and adaptive implementations and compare with our SISI and AISI methods. Despite that all methods are shown to produce global norm, they vary greatly in terms of effectiveness. Fig. 2(a) plots the changes of the proportion of the dominant action and of the diversity index on all graphs when applying the static algorithms. Note that only the trend of SISI will suddenly become smooth. This is because SISI only considers the initial partition, and minimizing the BA-ratio will tend to add edges between the two largest initial substructures even though they already have the same belief, which is unnecessary. The other four edge addition algorithms will not have this phenomenon, because they all randomly select two substructures from the initial partition in each edge addition. Apart from this, the Min-Degree has the leading performance on both the synthetic networks and the real-world networks in the static strategy. The good performance of Min-Degree reflects the utility of the small-degree principle. Fig. 2(b) plots the performances when applying the adaptive algorithms. Here it is clear that AISI has superior performance than other methods, verifying the adaptive usage of BA-ratio in eliciting norm emergence.

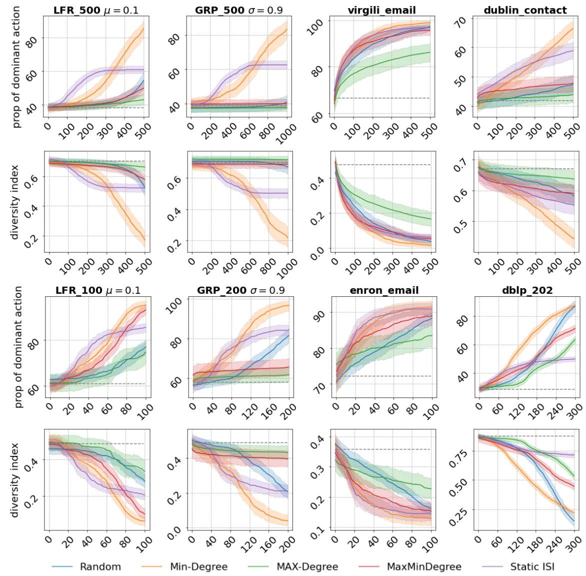
Correlation with diversity index. For this experiment, we generating 10 networks using each of GRP and LFR models (See Table 2 for statistics) and apply AISI to create edges. We then compare the changes on the BA-ratio $\sigma_G(\mathcal{P})$ against changes on the diversity index $\iota(\mathcal{P})$. The experiment is repeated on each graph for 100 times to generate an average value. Figure 3 illustrates the changes on the measured indices on all 10 graphs. Comparing the two plots in the same column, it is clear that the curves of $\rho_G(\mathcal{P})$ closely resemble that of $\iota(\mathcal{P})$ in that they not only show a similar downward trend as more edges are added, but also similar differences between the results of graphs of different parameters. The plots also reflect patterns on the relation between global norm and community structure. A higher σ for GRP or lower μ for LFR leads to a more salient network structure with high modularity, suggesting that it is harder to reach the global norm by adding edges.

Effect of system parameters. For this experiment, we generated 7 GRP graphs and plot the trends of the proportion of the dominant action when we change different graph parameters. Fig. 4 shows that increasing the number of communities ℓ, community size g, and average degree k has a significantly influence in facilitating global norm. Specifically, they make it harder to dissolve SRS. The action number m also has significant impact when it is less than the community number ($m < \ell$), while its impact is much smaller when $m \geq \ell$.

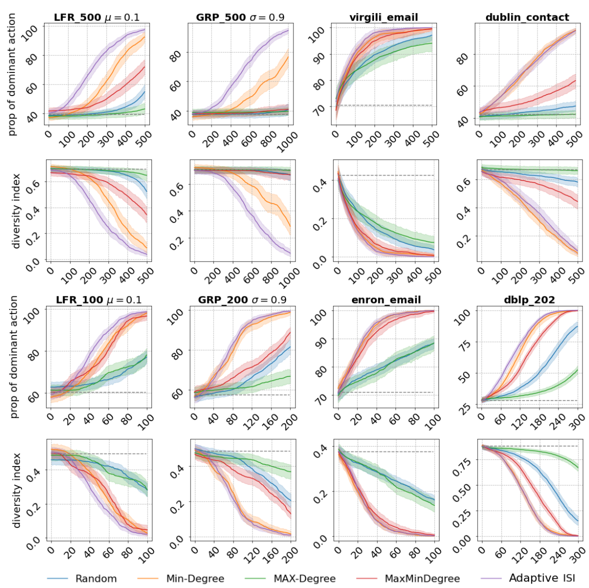
6. Conclusion

This paper addresses the bottleneck to norm emergence in certain social topology that exhibit clear self-reinforcing substructures (SRS), where local norms tend to be stabilized. We demonstrate that in these situations, it is reasonable to implement incremental social instruments to dissolve these SRS and integrate local norms into a single global norm. The significance of this work is three-fold: Firstly, this is one of the few work that address the presence of SRS and integrating local norms through modifying the social topology. Then, this is the first work that aims to

[6] **Reproducibility:** Details of the experiments and the code of our algorithm can be downloaded from https://github.com/CommunityDeception/DissolvingSRS



(a) Static strategy.



(b) Adaptive strategy.

Figure 2. The figure illustrates the performance of the five methods in facilitating global norm emergence in 8 networks. Each graph corresponds to two diagrams, displaying respectively the proportion of the dominant action and the diversity index $\iota(\mathcal{P})$. The horizontal axis indicate the number of new edges added. The horizo ntal dashed line in each graph indicates the value of the original graph. Each line is the result of averaging 100 trials with confidence interval as shown.

bring information-theoretic argument, connecting diversity of interactions with the social topology of the population. Moreover, this is the first work that study small-degree principle. This phenomenon may provide insights in practice when social platform design intervention mechanisms that promote unity and social integration in the online space.

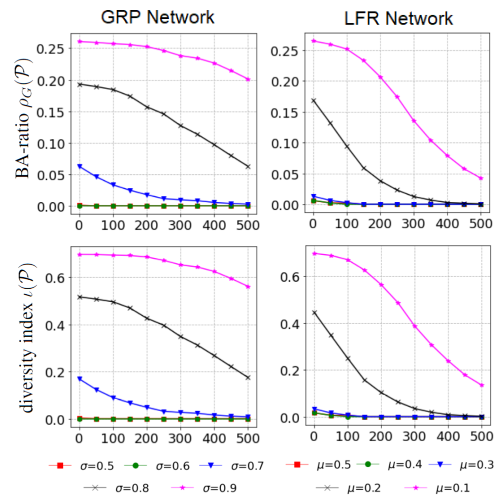


Figure 3. Comparisons of the changes to the BA-ratio $\rho_G(\mathcal{P})$ and the diversity index $\iota(\mathcal{P})$ over ten graphs. The horizontal axis indicates the number of new edges added to each graph. The figure clearly exhibits downwards trend on both types of diversity as more edges are created.

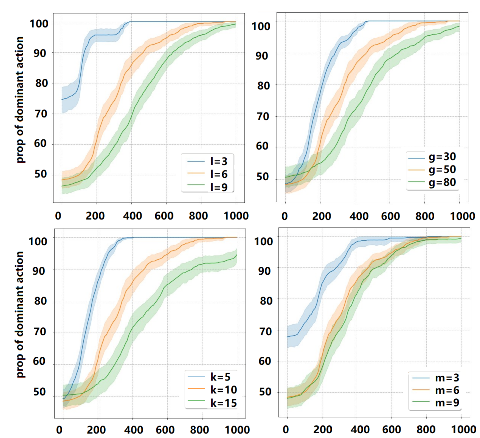


Figure 4. Effects of network properties in facilitating global norm by applying adaptive ISI. The 7 graphs are generated by GPR with different parameters. The reference network are generated by set $l = 6, g = 50, k = 10, m = 6$. We control variables by changing one parameter in each subfigure.

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