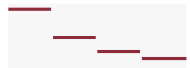
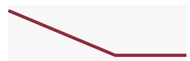
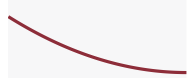
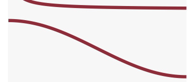
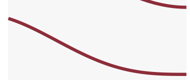
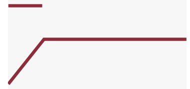
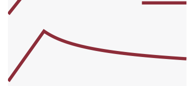
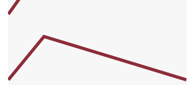
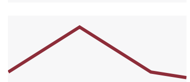
A. List of optimizers and schedules considered

Table 2: List of optimizers considered for our benchmark. This is only a subset of all existing methods for deep learning.

Name	Ref.	Name	Ref.
AcceleGrad	(Levy et al., 2018)	HyperAdam	(Wang et al., 2019b)
ACClip	(Zhang et al., 2020)	K-BFGS/K-BFGS(L)	(Goldfarb et al., 2020)
AdaAlter	(Xie et al., 2019)	KF-QN-CNN	(Ren & Goldfarb, 2021)
AdaBatch	(Devarakonda et al., 2017)	KFAC	(Martens & Grosse, 2015)
AdaBayes/AdaBayes-SS	(Aitchison, 2020)	KFLR/KFRA	(Botev et al., 2017)
AdaBelief	(Zhuang et al., 2020)	L4Adam/L4Momentum	(Rolínek & Martius, 2018)
AdaBlock	(Yun et al., 2019)	LAMB	(You et al., 2020)
AdaBound	(Luo et al., 2019)	LaProp	(Ziyin et al., 2020)
AdaComp	(Chen et al., 2018)	LARS	(You et al., 2017)
Adadelta	(Zeiler, 2012)	LHOPT	(Almeida et al., 2021)
Adafactor	(Shazeer & Stern, 2018)	LookAhead	(Zhang et al., 2019)
AdaFix	(Bae et al., 2019)	M-SVAG	(Balles & Hennig, 2018)
AdaFom	(Chen et al., 2019a)	MADGRAD	(Defazio & Jelassi, 2021)
AdaFTRL	(Orabona & Pál, 2015)	MAS	(Landro et al., 2020)
Adagrad	(Duchi et al., 2011)	MEKA	(Chen et al., 2020b)
ADAHESSIAN	(Yao et al., 2020)	MTAdam	(Malkiel & Wolf, 2020)
Adai	(Xie et al., 2020)	MVRC-1/MVRC-2	(Chen & Zhou, 2020)
AdaLoss	(Teixeira et al., 2019)	Nadam	(Dozat, 2016)
Adam	(Kingma & Ba, 2015)	NAMSB/NAMSG	(Chen et al., 2019b)
Adam$^+$	(Liu et al., 2020b)	ND-Adam	(Zhang et al., 2017a)
AdamAL	(Tao et al., 2019)	Nero	(Liu et al., 2021b)
AdaMax	(Kingma & Ba, 2015)	Nesterov	(Nesterov, 1983)
AdamBS	(Liu et al., 2020c)	Noisy Adam/Noisy K-FAC	(Zhang et al., 2018)
AdamNC	(Reddi et al., 2018)	NosAdam	(Huang et al., 2019)
AdaMod	(Ding et al., 2019)	Novograd	(Ginsburg et al., 2019)
AdamP/SGDP	(Heo et al., 2021)	NT-SGD	(Zhou et al., 2021b)
AdamT	(Zhou et al., 2020)	Padam	(Chen et al., 2020a)
AdamW	(Loshchilov & Hutter, 2019)	PAGE	(Li et al., 2020b)
AdamX	(Tran & Phong, 2019)	PAL	(Mutschler & Zell, 2020)
ADAS	(Eliyahu, 2020)	PolyAdam	(Orvieto et al., 2019)
AdaS	(Hosseini & Plataniotis, 2020)	Polyak	(Polyak, 1964)
AdaScale	(Johnson et al., 2020)	PowerSGD/PowerSGDM	(Vogels et al., 2019)
AdaSGD	(Wang & Wiens, 2020)	Probabilistic Polyak	(de Roos et al., 2021)
AdaShift	(Zhou et al., 2019)	ProbLS	(Mahsereci & Hennig, 2017)
AdaSqrt	(Hu et al., 2019)	PStorm	(Xu, 2020)
Adathm	(Sun et al., 2019)	QHAdam/QHM	(Ma & Yarats, 2019)
AdaX/AdaX-W	(Li et al., 2020a)	RAdam	(Liu et al., 2020a)
AEGD	(Liu & Tian, 2020)	Ranger	(Wright, 2020b)
ALI-G	(Berrada et al., 2020)	RangerLars	(Grankin, 2020)
AMSBound	(Luo et al., 2019)	RMSProp	(Tieleman & Hinton, 2012)
AMSGrad	(Reddi et al., 2018)	RMSterov	(Choi et al., 2019)
AngularGrad	(Roy et al., 2021)	S-SGD	(Sung et al., 2020)
ArmijoLS	(Vaswani et al., 2019)	SAdam	(Wang et al., 2020b)
ARSG	(Chen et al., 2019b)	Sadam/SAMSGrad	(Tong et al., 2019)
ASAM	(Kwon et al., 2021)	SALR	(Yue et al., 2020)
AutoLRS	(Jin et al., 2021)	SAM	(Foret et al., 2021)
AvaGrad	(Savarese et al., 2019)	SC-Adagrad/SC-RMSProp	(Mukkamala & Hein, 2017)
BAdam	(Salas et al., 2018)	SDProp	(Ida et al., 2017)
BGAdam	(Bai & Zhang, 2019)	SGD	(Robbins & Monro, 1951)
BPGrad	(Zhang et al., 2017b)	SGD-BB	(Tan et al., 2016)
BRMSProp	(Aitchison, 2020)	SGD-G2	(Ayadi & Turinici, 2020)
BSGD	(Hu et al., 2020)	SGDEM	(Ramezani-Kebrya et al., 2021)
C-ADAM	(Tutunov et al., 2020)	SGDHess	(Tran & Cutkosky, 2021)
CADA	(Chen et al., 2021)	SGDM	(Liu & Luo, 2020)
Cool Momentum	(Borysenko & Byshkin, 2020)	SGDR	(Loshchilov & Hutter, 2017)
CProp	(Preechakul & Kijsirikul, 2019)	SHAdagrad	(Huang et al., 2020)
Curveball	(Henriques et al., 2019)	Shampoo	(Anil et al., 2020; Gupta et al., 2018)
Dadam	(Nazari et al., 2019)	SignAdam++	(Wang et al., 2019a)
DeepMemory	(Wright, 2020a)	SignSGD	(Bernstein et al., 2018)
DGNOpt	(Liu et al., 2021a)	SKQN/S4QN	(Yang et al., 2020)
DiffGrad	(Dubey et al., 2020)	SM3	(Anil et al., 2019)
EAdam	(Yuan & Gao, 2020)	SMG	(Tran et al., 2020)
EKFAC	(George et al., 2018)	SNGM	(Zhao et al., 2020)
Eve	(Hayashi et al., 2018)	SoftAdam	(Fetterman et al., 2019)
Expectigrad	(Daley & Amato, 2020)	SRSGD	(Wang et al., 2020a)
FastAdaBelief	(Zhou et al., 2021a)	Step-Tuned SGD	(Castera et al., 2021)
FRSGD	(Wang & Ye, 2020)	SWATS	(Keskar & Socher, 2017)
G-AdaGrad	(Chakrabarti & Chopra, 2021)	SWNTS	(Chen et al., 2019c)
GADAM	(Zhang & Gouza, 2018)	TAdam	(Ilboudo et al., 2020)
Gadam	(Granziol et al., 2020)	TEKFAC	(Gao et al., 2020)
GOALS	(Chae et al., 2021)	VAdam	(Khan et al., 2018)
GOLS-I	(Kafka & Wilke, 2019)	VR-SGD	(Shang et al., 2020)
Grad-Avg	(Purkayastha & Purkayastha, 2020)	vSGD-b/vSGD-g/vSGD-l	(Schaul et al., 2013)
GRAPES	(Dellaferrera et al., 2021)	vSGD-fd	(Schaul & LeCun, 2013)
Gravilon	(Kelterborn et al., 2020)	WNGrad	(Wu et al., 2018)
Gravity	(Bahrami & Zadeh, 2021)	YellowFin	(Zhang & Mitliagkas, 2019)
HAdam	(Jiang et al., 2019)	Yogi	(Zaheer et al., 2018)

Table 3: Overview of commonly used parameter schedules. Note, while we list the schedules parameters, it isn't clearly defined what aspects of a schedule are (tunable) parameters and what is a-priori fixed. In this column, α_0 denotes the initial learning rate, α_{lo} and α_{up} the lower and upper bound, Δt indicates an epoch count at which to switch decay styles, k denotes a decaying factor.

Name		Ref.	Illustration	Parameters
Constant				α_0
Step Decay	constant factor			$\alpha_0, \Delta t_1, \ldots, k$
	multi-step			$\alpha_0, \Delta t_1, \ldots, k_1, \ldots$
Smooth Decay	linear decay	e.g. (Goodfellow et al., 2016)		$\alpha_0, (\Delta t, \alpha_{\text{lo}})$
	polynomial decay			$\alpha_0, k, (\alpha_{\text{lo}})$
	exponential decay			$\alpha_0, k, (\alpha_{\text{lo}})$
	inverse time decay	e.g. (Bottou, 2012)		$\alpha_0, k, (\alpha_{\text{lo}})$
	cosine decay	(Loshchilov & Hutter, 2017)		$\alpha_0, (\alpha_{\text{lo}})$
	linear cosine decay	(Bello et al., 2017)		$\alpha_0, (\alpha_{\text{lo}})$
Cyclical	triangular	(Smith, 2017)		$\alpha_{\text{lo}}, \alpha_{\text{up}}, \Delta t$
	triangular + decay	(Smith, 2017)		$\alpha_{\text{lo}}, \alpha_{\text{up}}, \Delta t, k$
	triangular + exponential decay	(Smith, 2017)		$\alpha_{\text{lo}}, \alpha_{\text{up}}, \Delta t$
	cosine + warm restarts	(Loshchilov & Hutter, 2017)		$\alpha_{\text{up}}, \Delta t, (\alpha_{\text{lo}})$
	cosine + warm restarts + decay	(Loshchilov & Hutter, 2017)		$\alpha_{\text{up}}, \Delta t, k, (\alpha_{\text{lo}})$
Warmup	constant warmup	e.g. (He et al., 2016)		$\alpha_{\text{lo}}, \alpha_0, \Delta t$
	gradual warmup	(Goyal et al., 2017)		$\alpha_0, \Delta t, (\alpha_{\text{lo}})$
	gradual warmup + multi-step decay	(Goyal et al., 2017)		$\alpha_0, \Delta t, \Delta t_{\text{steps}}, k_1, \ldots, (\alpha_{\text{lo}})$
	gradual warmup + step number decay	(Vaswani et al., 2017)		$\alpha_0, \Delta t, (\alpha_{\text{lo}})$
	slanted triangular	(Howard & Ruder, 2018)		$\alpha_0, \Delta t, (\alpha_{\text{lo}})$
	long trapezoid	(Xing et al., 2018)		$\alpha_0, \Delta t_{\text{up}}, \Delta t_{\text{down}}, (\alpha_{\text{lo}})$
Super-Convergence	1cycle	(Smith & Topin, 2017)		$\alpha_{\text{up}}, \Delta t, \Delta t_{\text{cutoff}}, (\alpha_{\text{lo}})$

B. List of optimizers selected

Table 4: Selected optimizers for our benchmarking process with their respective color, hyperparameters, default values, tuning distributions and scheduled hyperparameters. Here, $\mathcal{LU}(\cdot,\cdot)$ denotes the log-uniform distribution while $\mathcal{U}\{\cdot,\cdot\}$ denotes the discrete uniform distribution.

Optimizer	Ref.	Parameters	Default	Tuning Distribution	Scheduled
AMSBound	(Luo et al., 2019)	α	10^{-3}	$\mathcal{LU}(10^{-4}, 1)$	✓
		α_l	0.1	$\mathcal{LU}(10^{-3}, 0.5)$	
		β_1	0.9	$\mathcal{LU}(0.5, 0.999)$	
		β_2	0.999	$\mathcal{LU}(0.8, 0.999)$	
		γ	10^{-3}	$\mathcal{LU}(10^{-4}, 10^{-1})$	
		ε	10^{-8}	✗	
AMSGrad	(Reddi et al., 2018)	α	10^{-2}	$\mathcal{LU}(10^{-4}, 1)$	✓
		β_1	0.9	$\mathcal{LU}(0.5, 0.999)$	
		β_2	0.999	$\mathcal{LU}(0.8, 0.999)$	
		ε	10^{-8}	✗	
AdaBelief	(Zhuang et al., 2020)	α	10^{-3}	$\mathcal{LU}(10^{-4}, 1)$	✓
		β_1	0.9	$\mathcal{LU}(0.5, 0.999)$	
		β_2	0.999	$\mathcal{LU}(0.8, 0.999)$	
		ε	10^{-14}	✗	
AdaBound	(Luo et al., 2019)	α	10^{-3}	$\mathcal{LU}(10^{-4}, 1)$	✓
		α_l	0.1	$\mathcal{LU}(10^{-3}, 0.5)$	
		β_1	0.9	$\mathcal{LU}(0.5, 0.999)$	
		β_2	0.999	$\mathcal{LU}(0.8, 0.999)$	
		γ	10^{-3}	$\mathcal{LU}(10^{-4}, 10^{-1})$	
AdaDelta	(Zeiler, 2012)	α	10^{-3}	$\mathcal{LU}(10^{-4}, 1)$	✓
		ε	10^{-8}	✗	
		$1-\rho$	0.95	$\mathcal{LU}(10^{-4}, 1)$	
AdaGrad	(Duchi et al., 2011)	α	10^{-2}	$\mathcal{LU}(10^{-4}, 1)$	✓
		ε	10^{-7}	✗	
Adam	(Kingma & Ba, 2015)	α	10^{-3}	$\mathcal{LU}(10^{-4}, 1)$	✓
		β_1	0.9	$\mathcal{LU}(0.5, 0.999)$	
		β_2	0.999	$\mathcal{LU}(0.8, 0.999)$	
		ε	10^{-8}	✗	
Lookahead Momentum abbr. LA(Mom.)	(Zhang et al., 2019)	α	0.5	$\mathcal{LU}(10^{-4}, 1)$	
		α_f	10^{-2}	$\mathcal{LU}(10^{-4}, 1)$	✓
		k	5	$\mathcal{U}\{1, 20\}$	
		$1-\rho$	0.99	$\mathcal{LU}(10^{-4}, 1)$	
Lookahead RAdam abbr. LA(RAdam)	(Zhang et al., 2019)	α	0.5	$\mathcal{LU}(10^{-4}, 1)$	
		α_f	10^{-3}	$\mathcal{LU}(1e-4, 1)$	✓
		β_1	0.9	$\mathcal{LU}(0.5, 0.999)$	
		β_2	0.999	$\mathcal{LU}(0.8, 0.999)$	
		ε	10^{-7}	✗	
		k	5	$\mathcal{U}\{1, 20\}$	
Momentum	(Polyak, 1964)	α	10^{-2}	$\mathcal{LU}(10^{-4}, 1)$	✓
		$1-\rho$	0.99	$\mathcal{LU}(10^{-4}, 1)$	
NAG	(Nesterov, 1983)	α	10^{-2}	$\mathcal{LU}(10^{-4}, 1)$	✓
		$1-\rho$	0.99	$\mathcal{LU}(10^{-4}, 1)$	
Nadam	(Dozat, 2016)	α	10^{-3}	$\mathcal{LU}(10^{-4}, 1)$	✓
		β_1	0.9	$\mathcal{LU}(0.5, 0.999)$	
		β_2	0.999	$\mathcal{LU}(0.8, 0.999)$	
		ε	10^{-7}	✗	
RAdam	(Liu et al., 2020a)	α	10^{-3}	$\mathcal{LU}(10^{-4}, 1)$	✓
		β_1	0.9	$\mathcal{LU}(0.5, 0.999)$	
		β_2	0.999	$\mathcal{LU}(0.8, 0.999)$	
		ε	10^{-7}	✗	
RMSProp	(Tieleman & Hinton, 2012)	α	10^{-3}	$\mathcal{LU}(10^{-4}, 1)$	✓
		ε	10^{-10}	✗	
		$1-\rho$	0.9	$\mathcal{LU}(10^{-4}, 1)$	
SGD	(Robbins & Monro, 1951)	α	10^{-2}	$\mathcal{LU}(10^{-4}, 1)$	✓

C. Robustness to random seeds

Data subsampling, random weight initialization, dropout and other aspects of deep learning introduce stochasticity to the training process. As such, judging the performance of an optimizer on a single run may be misleading due to random fluctuations. In our benchmark we use 10 different seeds of the final setting for each budget in order to judge the stability of the optimizer and the results. However, to keep the magnitude of this benchmark feasible, we only use a single seed while tuning, analogously to how a single user would progress. This means that our tuning process can sometimes choose hyperparameter settings which might not even converge for seeds other than the one used for tuning.

Figure 5 illustrates this behavior on an example problem where we used 10 seeds throughout a tuning process using grid search. The figure shows that in the beginning performance increases when increasing the learning rate, followed by an area were it sometimes works but other times diverges. Picking hyperparameters from this "danger zone" can lead to unstable results. In this case, where we only consider the learning rate, it is clear that decreasing the learning rate a bit to get away from this "danger zone" would lead to a more stable, but equally well-performing algorithm. In more complicated cases, however, we are unable to use a simple heuristic such as this. This might be the case, for example, when tuning multiple hyperparameters or when the effect of the hyperparameter on the performance is less straight forward. Thus, this is a problem not created by improperly using the tuning method, but by an unstable optimization method.

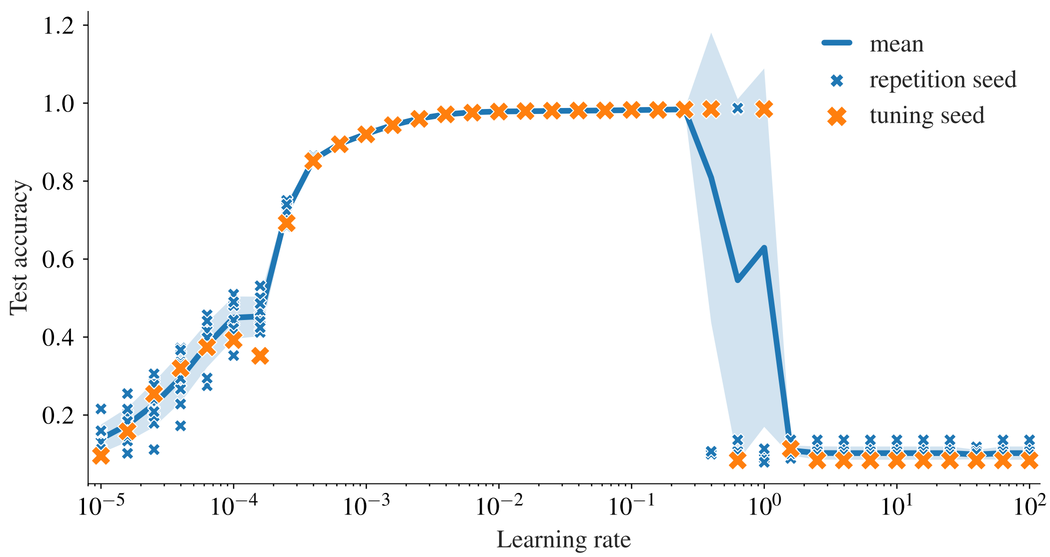


Figure 5: Performance of SGD on a simple multilayer perceptron. For each learning rate, markers in orange (✖) show the initial seed which would be used for tuning, blue markers (✖) illustrate nine additional seeds with otherwise unchanged settings. The mean over all seeds is plotted as a blue line (—), showing one standard deviation as a shaded area ( ).

In our benchmark, we observe a total of 18, 24, and 17 divergent seeds for the small, medium, and large budget respectively. This amounts to roughly 0.5% of the runs in each budget. Most of them occur when using SGD (10, 15, and 7 cases for the small, medium and large budget respectively), ADAGRAD (5, 3, and 5 cases for the small, medium and large budget respectively) or ADADELTA (3, 5, and 3 cases for the small, medium and large budget respectively), which might indicate that modern adaptive methods are less prone to this kind of behavior. None of these cases occur when using a constant schedule, and most of them occur when using the *trapezoidal* schedule (11, 11, and 9 cases for the small, medium and large budget respectively). However, as our data on diverging seeds is very limited, it is not conclusive enough to draw solid conclusions.

D. Re-Tuning experiments

In order to test the stability of our benchmark and especially the tuning method, we selected two optimizers in our benchmark and re-tuned them on all problems a second time. We used completely independent random seeds for both tuning and the 10 repetitions with the final setting. Figure 6 and Figure 7 show the distribution of all 10 random seeds for both the original tuning as well as the re-tuning runs for RMSProp and AdaDelta. It is evident, that re-tuning results in a shift of this distribution, since small (stochastic) changes during tuning can result in a different chosen hyperparameter setting.

These differences also highlight how crucial it is to look at multiple problems. Individually, small changes, such as re-doing the tuning with different seeds can lead to optimization methods changing rankings. However, they tend to average out when looking at an unbiased list of multiple problems. These results also further supports the statement made in Section 3 that there is no optimization method clearly domination the competition, as small performance margins might vanish when re-tuning.

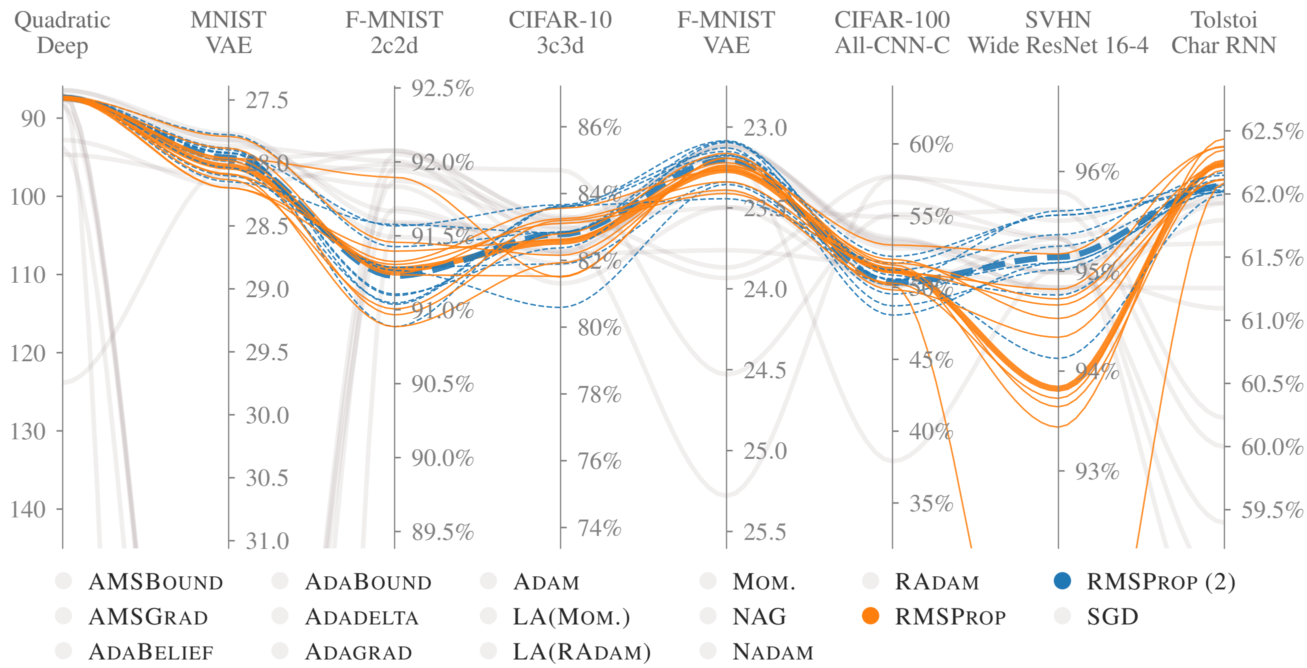


Figure 6: Mean test set performance of all 10 seeds of RMSProp (—) on all eight optimization problems using the *small budget* for tuning and *no learning rate schedule*. The mean is shown with a thicker line. We repeated the full tuning process on all eight problems using different random seeds, which is shown in dashed lines blue (- -). The mean performance of all other optimizers is shown in transparent gray lines.

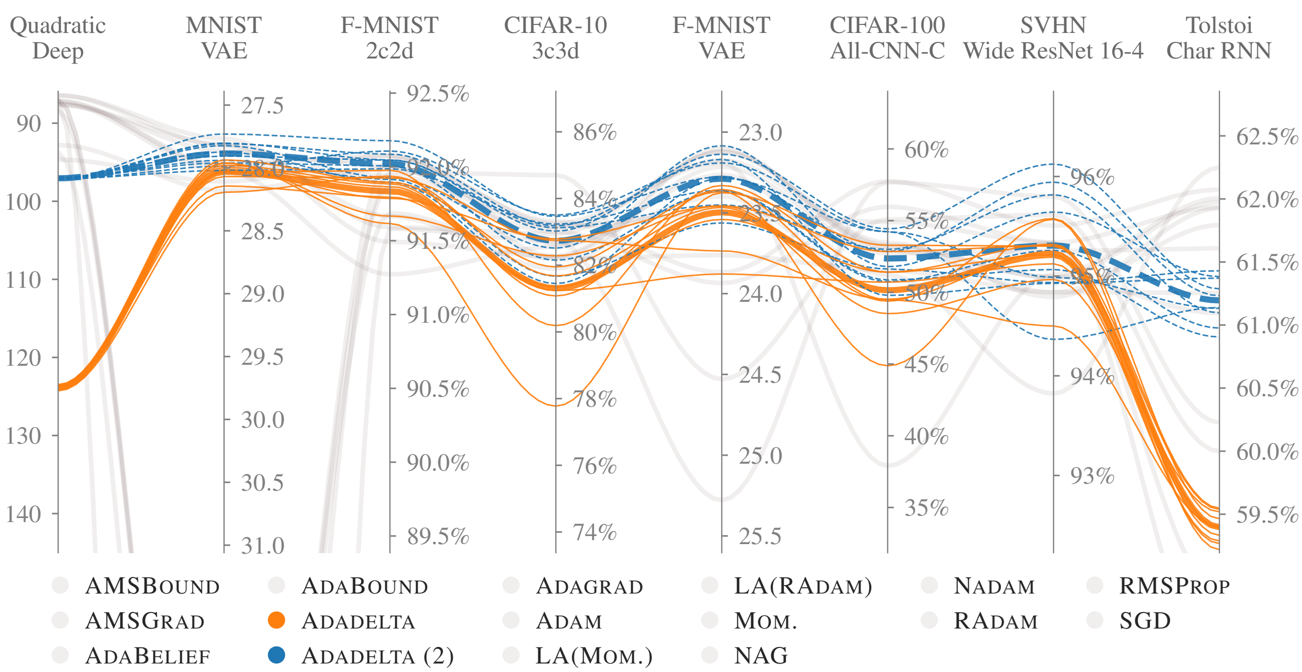


Figure 7: Mean test set performance of all 10 seeds of AdaDelta (—) on all eight optimization problems using the *small budget* for tuning and *no learning rate schedule*. The mean is shown with a thicker line. We repeated the full tuning process on all eight problems using different random seeds, which is shown in dashed lines blue (- -). The mean performance of all other optimizers is shown in transparent gray lines.

E. List of schedules selected

The schedules selected for our benchmark are illustrated in Figure 8. All learning rate schedules are multiplied by the initial learning rate found via tuning or picked as the default choice.

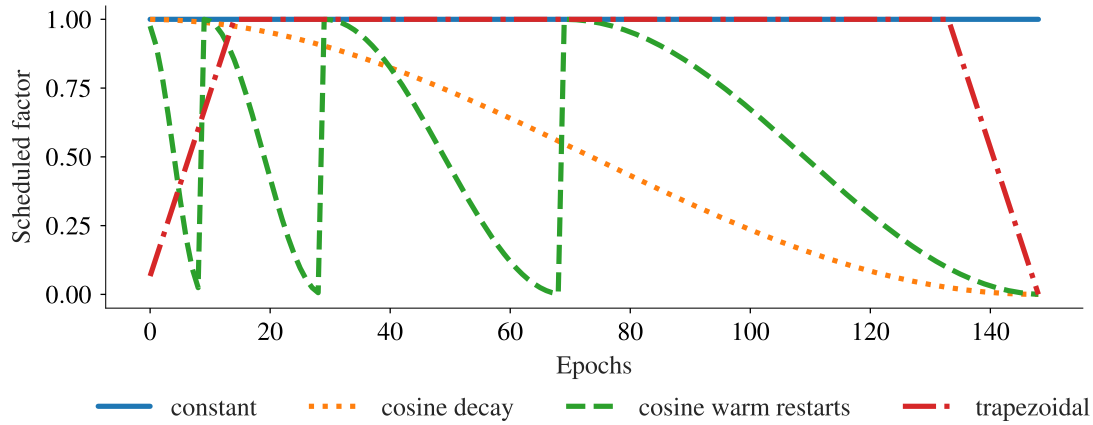


Figure 8: Illustration of the selected learning rate schedules for a training duration of 150 epochs.

We use a *cosine decay* (Loshchilov & Hutter, 2017) that starts at 1 and decays in the form of a half period of a cosine to 0. As an example of a cyclical learning rate schedule, we test a *cosine with warm restarts* schedule with a cycle length $\Delta t = 10$ which increases by a factor of 2 after each cycle without any discount factor. Depending on the number of epochs we train our model, it is possible that training stops shortly after one of those warm restarts. Since performance typically declines shortly after increasing the learning rate, we don't report the final performance for this schedule, but instead the performance achieved after the last complete period (just before the next restart). This approach is suggested by the original work of Loshchilov & Hutter (2017). However, we still use the final performance while tuning.

A representation of a schedule including warm-up is the *trapezoidal* schedule from Xing et al. (2018). For our benchmark we set a warm-up and cool-down period of $1/10$ the training time.

F. ArXiv Mentions

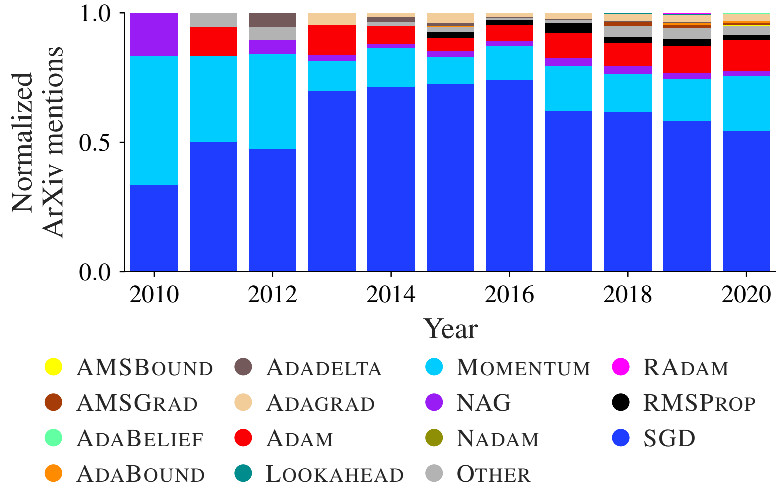


Figure 9: Percentage of times ArXiv titles and abstracts mention specific optimizer per year. This is a normalized version of Figure 1. The data for this figure is shown in Table 5.

Table 5: Mentions of each optimizer in titles and abstracts of papers on ArXiv per year. All non-selected optimizers from Table 2 in the appendix are grouped into *Other*.

Optimizer	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**
AMSBound	0	0	0	0	0	0	0	0	0	1	0
AMSGrad	0	0	0	0	0	0	0	0	7	9	11
AdaBelief	0	0	0	0	0	0	0	0	0	0	3
AdaBound	0	0	0	0	0	0	0	0	0	4	4
AdaDelta	0	0	1	0	1	2	0	1	2	3	3
AdaGrad	0	0	0	2	1	5	3	8	16	22	24
Adam	0	2	0	5	4	7	11	31	47	83	119
Lookahead	0	0	0	0	0	0	0	0	0	2	1
Momentum	3	6	7	5	9	14	23	57	76	124	205
NAG	1	0	1	1	1	3	3	11	17	18	19
Nadam	0	0	0	0	0	0	0	0	1	2	0
Other	0	1	1	0	1	3	2	4	22	34	36
RAdam	0	0	0	0	0	0	0	0	0	2	1
RMSProp	0	0	0	0	0	3	3	13	13	18	18
SGD	2	9	9	30	42	98	129	205	326	451	532

G. Improvement after tuning

When looking at Figure 2, one might realize that few diagonal entries contain negative values. Since diagonal entries reflect the intra-optimizer performance change when tuning on the respective task, this might feel quite counterintuitive at first. *In theory*, this can occur if the respective tuning distributions is chosen poorly, the tuning randomness simply got "unlucky", or we observe significantly worse results for our additional seeds (see Figure 5).

If we compare Figures 10 and 11 to Figures 12 and 13 we can see most negative diagonal entries vanish or at least diminish in magnitude. For the latter two figures we allow for more tuning runs and only consider the seed that has been used for this tuning process. The fact that the effect of negative diagonal entries reduces is an indication that they mostly result from the two latter reasons mentioned.

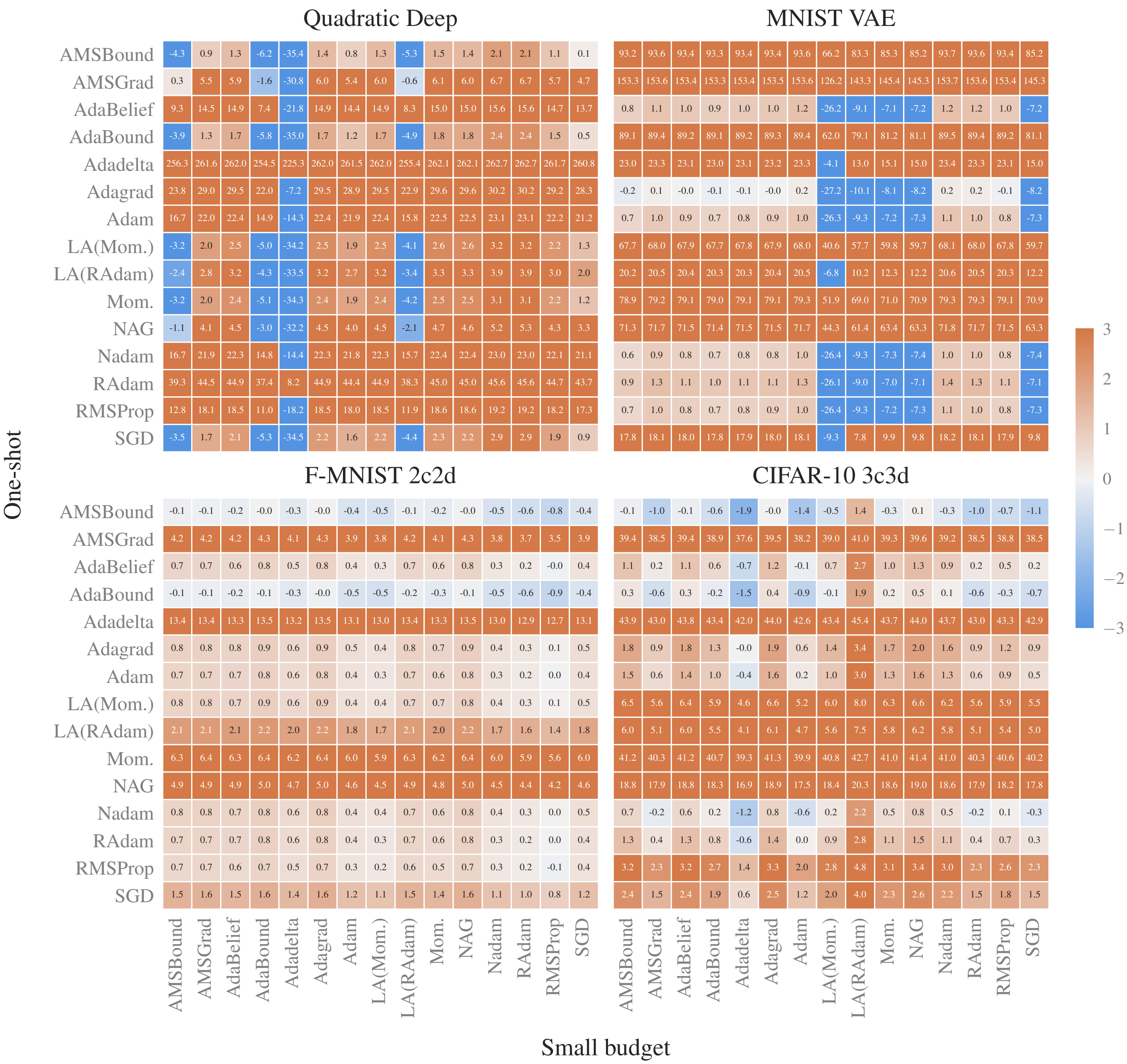


Figure 10: The absolute test set performance improvement after switching from any untuned optimizer (y-axis, *one-shot*) to any tuned optimizer (x-axis, *small budget*) as an average over 10 random seeds for the *constant* schedule. This is a detailed version of Figure 2 in the main text showing the first four problems.

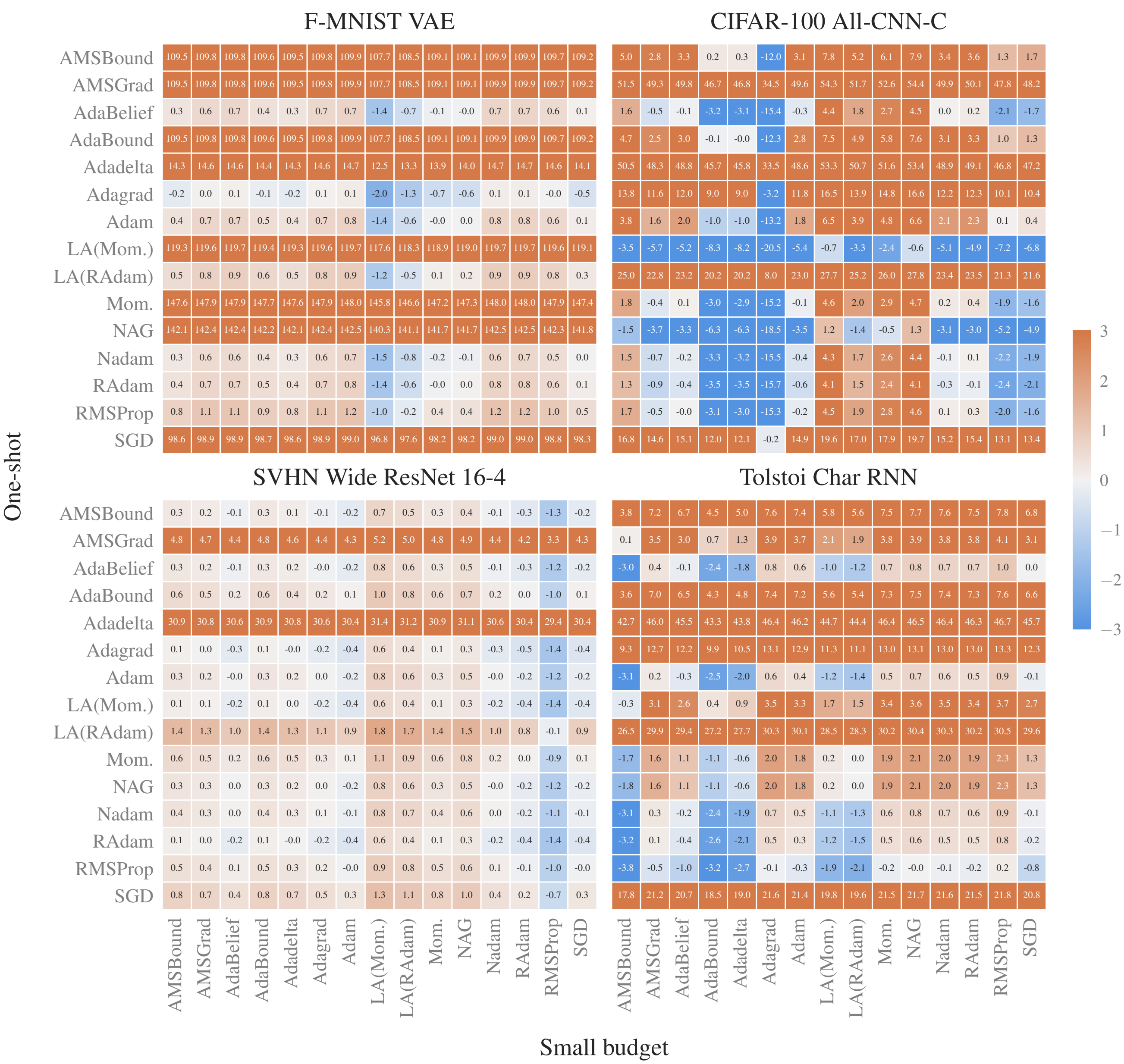


Figure 11: The absolute test set performance improvement after switching from any untuned optimizer (y-axis, *one-shot*) to any tuned optimizer (x-axis, *small budget*) as an average over 10 random seeds for the *constant* schedule. This is a detailed version of Figure 2 in the main text showing the last four problems.

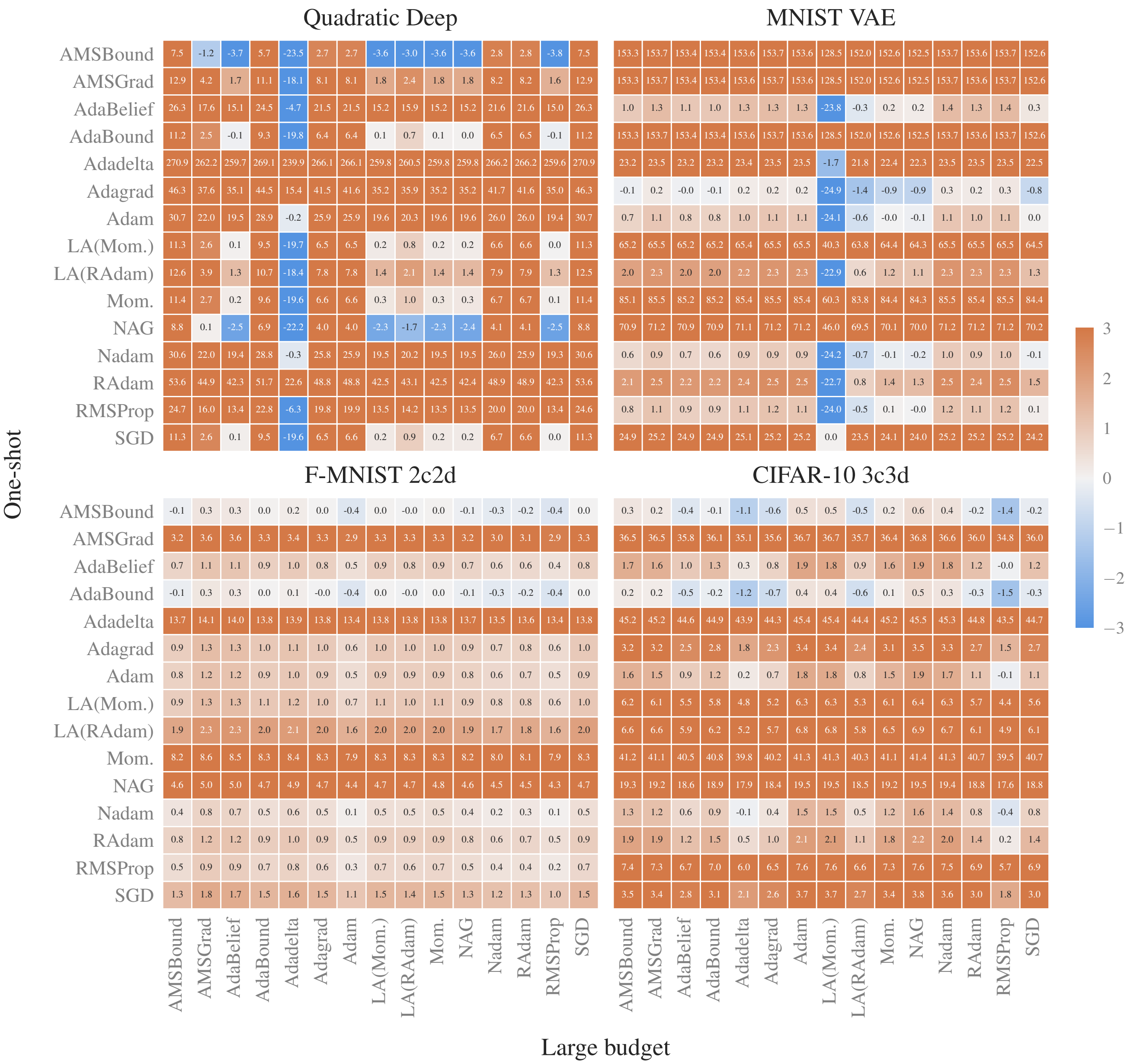


Figure 12: The absolute test set performance improvement after switching from any untuned optimizer (y-axis, *one-shot*) to any tuned optimizer (x-axis, *large budget*) for the *constant* schedule. This is structurally the same plot as Figure 10 but comparing to the *large budget* and only considering the seed that has been used for tuning.

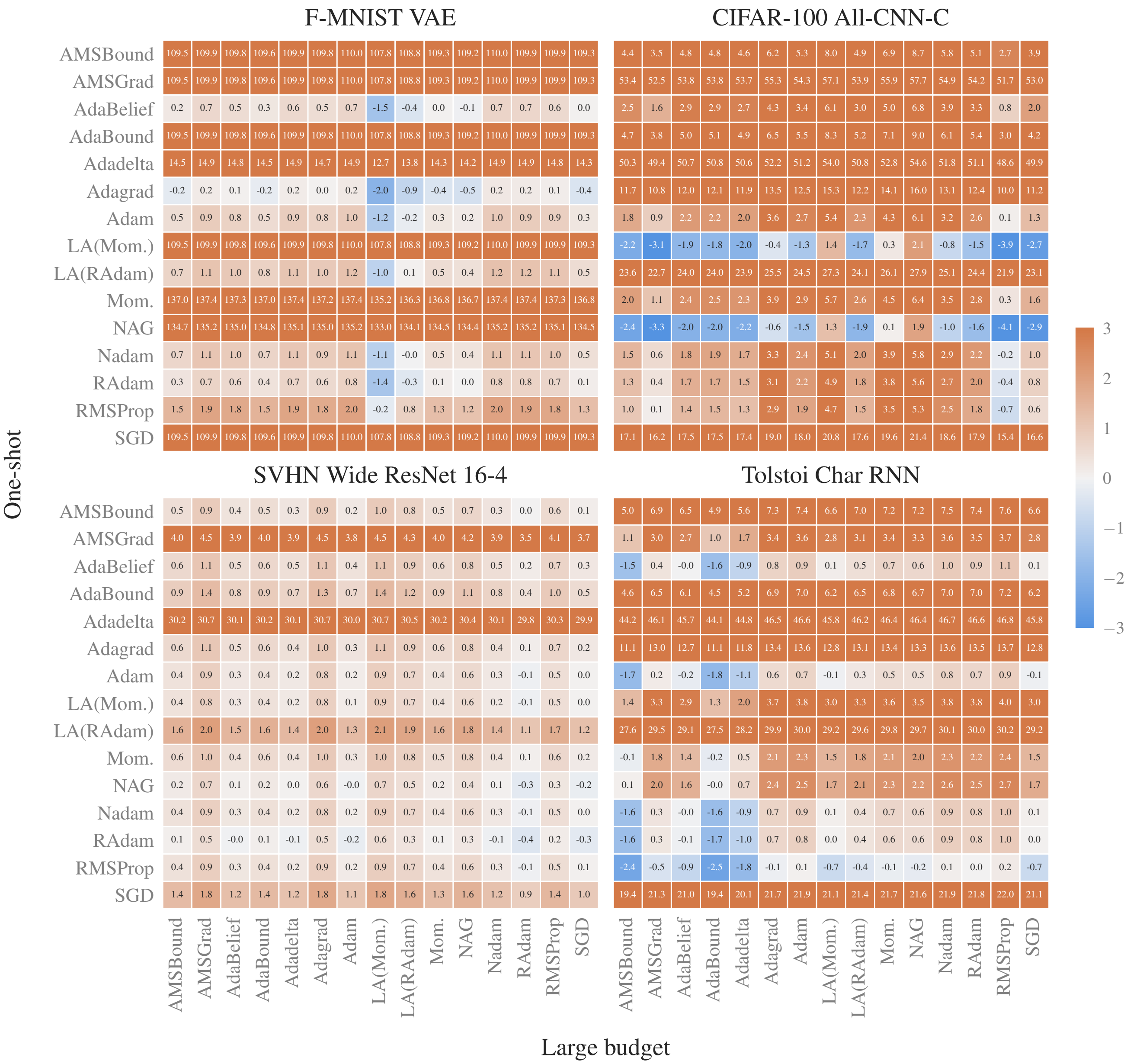


Figure 13: The absolute test set performance improvement after switching from any untuned optimizer (y-axis, *one-shot*) to any tuned optimizer (x-axis, *large budget*) for the *constant* schedule. This is structurally the same plot as Figure 11 but comparing to the *large budget* and only considering the seed that has been used for tuning.

H. Optimizer performance across test problems

Similarly to Figure 4, we show the corresponding plots for the *small budget* with *no learning rate schedule* in Figure 14 and the *medium budget* with the *cosine* and *trapezoidal learning rate schedule* in Figures 15 and 16. Additionally, in Figure 17 we show the same setting as Figure 4 but showing the training loss instead of the test loss/accuracy.

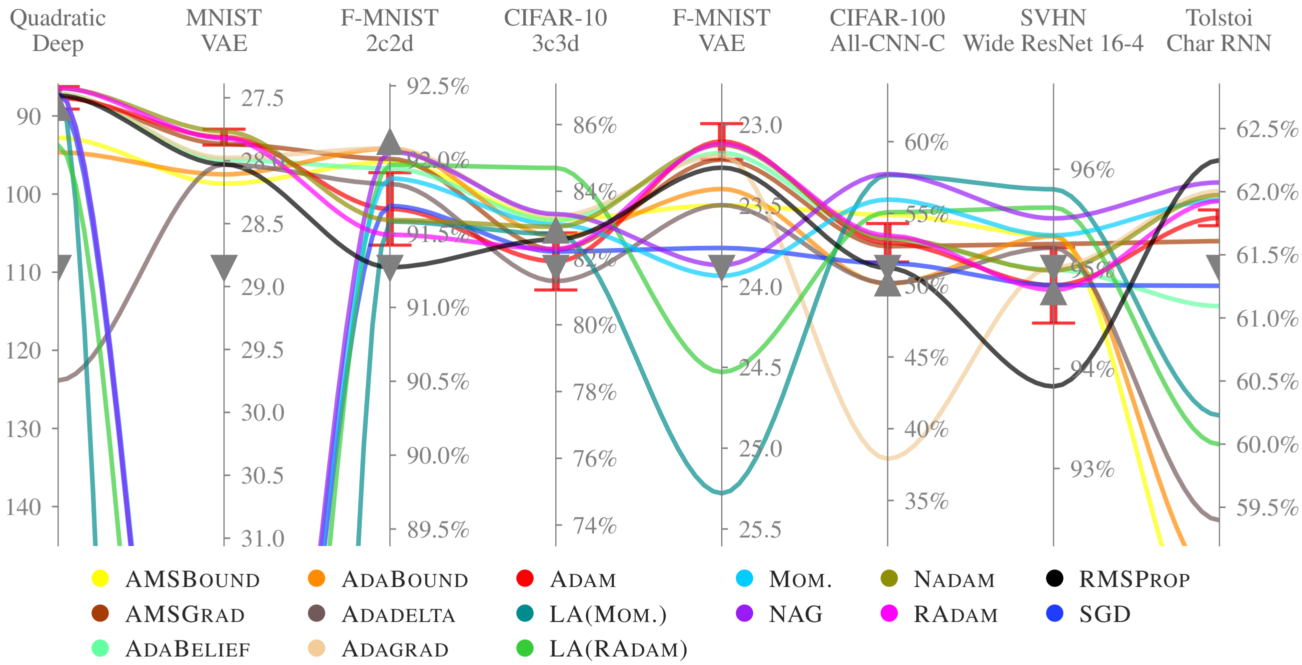


Figure 14: Mean test set performance over 10 random seeds of all tested optimizers on all eight optimization problems using the *small budget* for tuning and *no learning rate schedule*. One standard deviation for the tuned Adam optimizer is shown with a red error bar (I). The performance of the untuned versions of Adam (▼) and AdaBound (▲) are marked for reference. Note, the upper bound of each axis represents the best performance achieved in the benchmark, while the lower bound is chosen in relation to the performance of Adam with default parameters. Tabular version available in the Appendix as Table 7.

The high-level trends mentioned in Section 3 also hold for the smaller tuning budget in Figure 14. Namely, taking the winning optimizer for several untuned algorithms (here marked for Adam and AdaBound) will result in a decent performance in most problems with much less effort. Adding a tuned version Adam (or variants thereof) to this selection would result in a very competitive performance. The absolute top-performance however, is achieved by changing optimizers across different problems.

Note, although the *medium budget* is a true superset of the *small budget* it is not given that it will always perform better. Our tuning procedure guarantees that the *validation* performance on the seed that has been used for tuning is as least as good on the medium budget than on the small budget. But due to averaging over multiple seeds and reporting *test* performance instead of *validation* performance, this hierarchy is no longer guaranteed. We discuss the possible effects of averaging over multiple seeds further in Appendix C.

The same high-level trends also emerge when considering the *cosine* or *trapezoidal learning rate schedule* in Figures 15 and 16. We can also see that the top performance in general increase when adding a schedule (cf. Figure 4 and Figure 16).

Comparing Figure 4 and Figure 17 we can assess the generalization performance of the optimization method not only to an unseen test set, but also to a different performance metric (accuracy instead of loss). Again, the overall picture of varying performance across different problems remains consistent when considering the training loss performance. Similarly to the figures showing test set performance we cannot identify a clear winner, although Adam ands its variants, such as RAdam perform near the top consistently. Note that while Figure 17 shows the training loss, the optimizers have still be tuned to achieve the best validation performance (i.e. accuracy if available, else the loss).

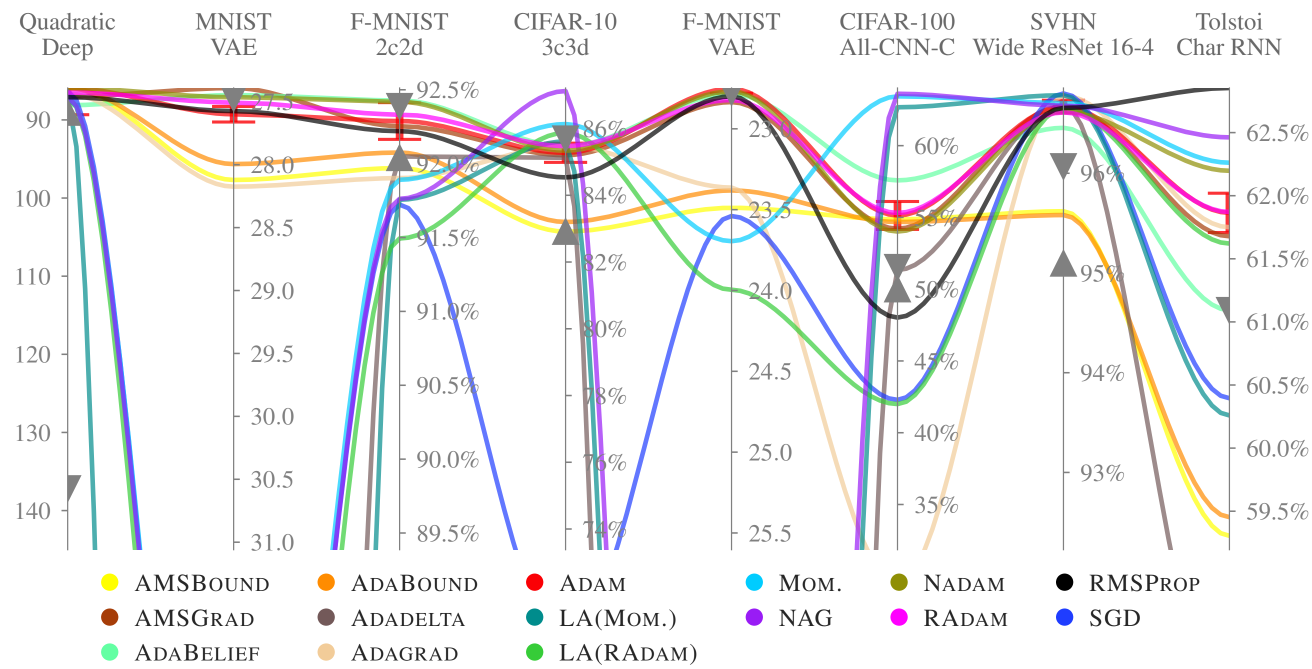


Figure 15: Mean test set performance over 10 random seeds of all tested optimizers on all eight optimization problems using the *medium budget* for tuning and the *cosine learning rate schedule*. One standard deviation for the tuned ADAM optimizer is shown with a red error bar (I). The performance of the untuned versions of ADAM (▼) and ADABOUND (▲) are marked for reference (this time with the *cosine* learning rate schedule). Note, the upper bound of each axis represents the best performance achieved in the benchmark, while the lower bound is chosen in relation to the performance of ADAM with default parameters (and no schedule). Tabular version available in the Appendix as Table 8.

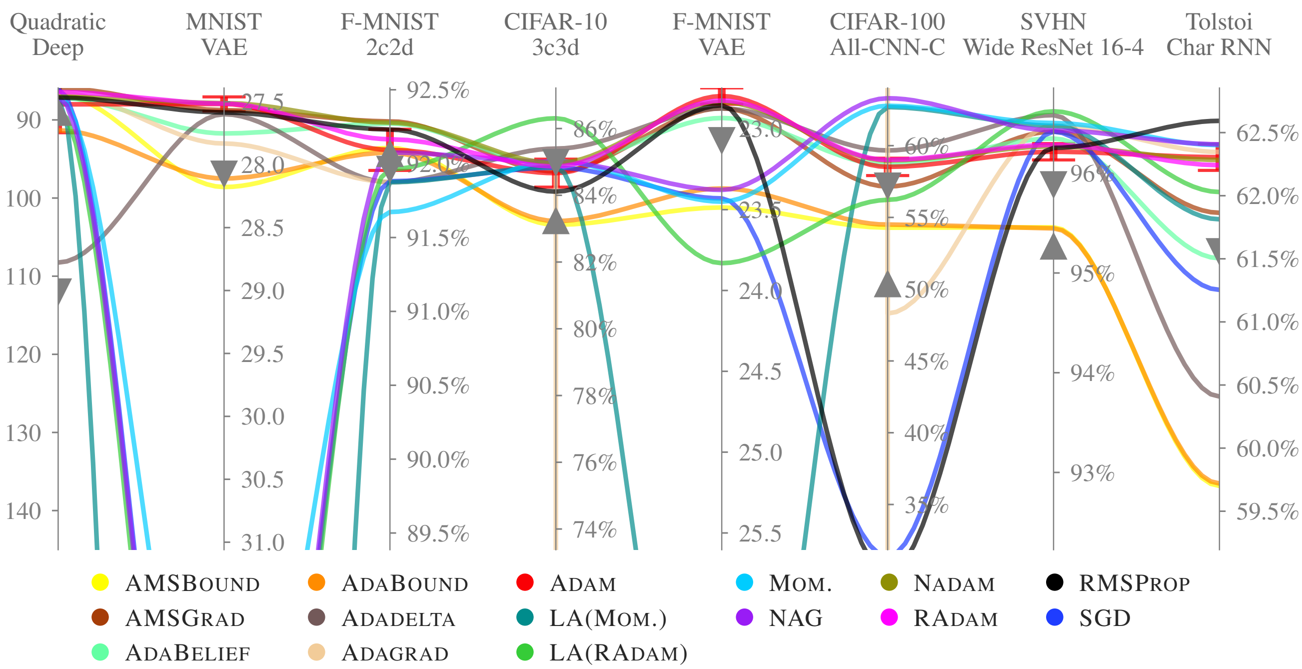


Figure 16: Mean test set performance over 10 random seeds of all tested optimizers on all eight optimization problems using the *large budget* for tuning and the *trapezoidal learning rate schedule*. One standard deviation for the tuned Adam optimizer is shown with a red error bar (I). The performance of the untuned versions of Adam (▼) and AdaBound (▲) are marked for reference (this time with the *trapezoidal* learning rate schedule). Note, the upper bound of each axis represents the best performance achieved in the benchmark, while the lower bound is chosen in relation to the performance of Adam with default parameters (and no schedule). Tabular version available in the Appendix as Table 9.

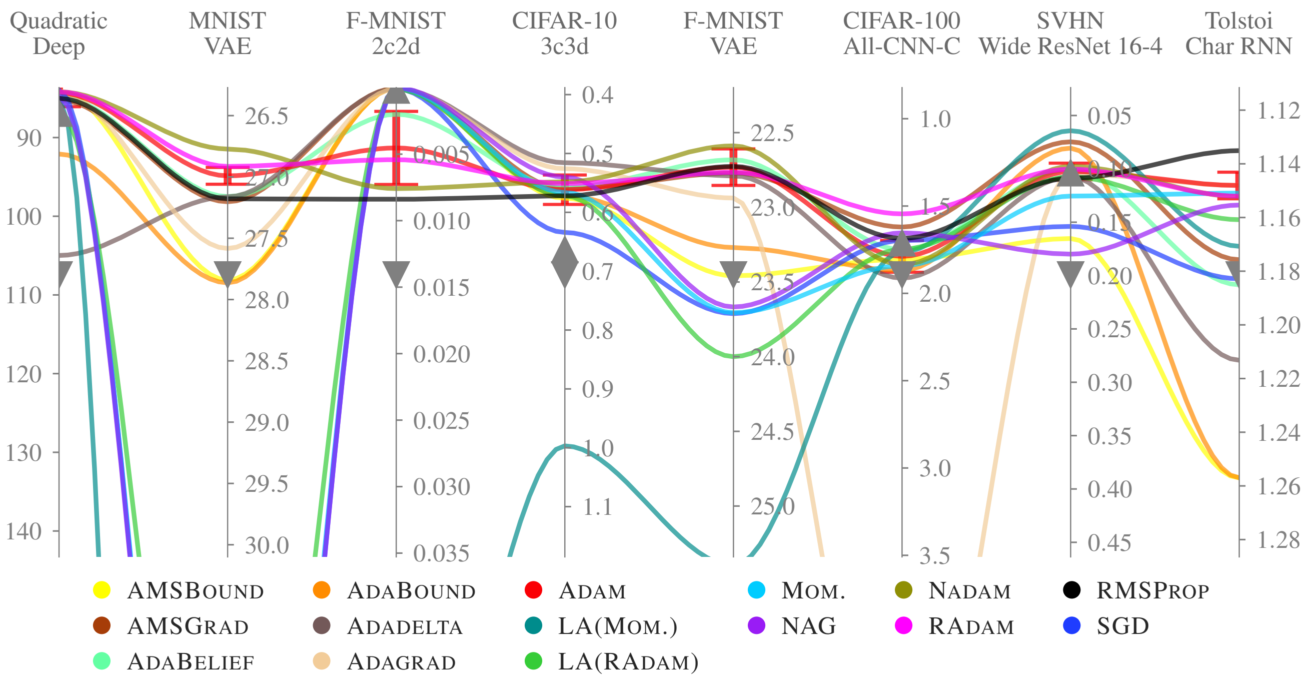


Figure 17: Mean *training* loss performance over 10 random seeds of all tested optimizers on all eight optimization problems using the *large budget* for tuning and *no learning rate schedule*. One standard deviation for the tuned Adam optimizer is shown with a red error bar (I). The performance of the untuned versions of Adam (▼) and AdaBound (▲) are marked for reference. Note, the upper bound of each axis represents the best performance achieved in the benchmark, while the lower bound is chosen in relation to the performance of Adam with default parameters (and no schedule). This figure is very similar to Figure 4, but showing the *training loss* performance instead of the *test accuracy* (or *test loss* if no accuracy is available). Tabular version available in the Appendix as Table 10.

I. Tabular version

Table 6: Tabular version of Figure 4. Mean test set performance and standard deviation over 10 random seeds of all tested optimizers on all eight optimization problems using the *large budget* for tuning and *no learning rate schedule*. For comprehensability, mean and standard deviation are rounded.

Optimizer	Quadratic Deep	MNIST VAE	F-MNIST 2c2d	CIFAR-10 3c3d	F-MNIST VAE	CIFAR-100	SVHN	Tolstoi
AMSBound	86.35 ± 3.47	28.14 ± 0.15	92.15 ± 0.13	82.99 ± 0.78	23.55 ± 0.18	54.64 ± 1.33	95.31 ± 0.31	59.70 ± 0.16
AMSGrad	87.64 ± 1.00	27.85 ± 0.07	**92.26** ± 0.16	83.42 ± 0.65	23.11 ± 0.10	52.34 ± 1.03	95.58 ± 0.31	61.52 ± 0.13
AdaBelief	87.17 ± 0.03	28.01 ± 0.06	92.06 ± 0.24	82.85 ± 0.59	23.22 ± 0.08	53.76 ± 1.35	95.09 ± 0.30	61.26 ± 0.17
AdaBound	94.66 ± 6.25	28.14 ± 0.13	92.03 ± 0.13	83.39 ± 0.53	23.38 ± 0.09	54.77 ± 0.94	95.40 ± 0.29	59.73 ± 0.20
AdaDelta	106.95 ± 0.14	27.87 ± 0.10	92.07 ± 0.11	83.34 ± 0.74	23.18 ± 0.13	53.18 ± 2.48	95.30 ± 0.60	60.54 ± 0.15
AdaGrad	86.70 ± 1.99	28.04 ± 0.29	92.05 ± 0.17	83.08 ± 0.41	23.16 ± 0.04	43.63 ± 21.35	95.34 ± 0.49	62.01 ± 0.10
Adam	86.58 ± 1.95	27.77 ± 0.03	91.69 ± 0.16	82.95 ± 0.70	23.06 ± 0.10	54.84 ± 0.65	94.84 ± 0.30	61.97 ± 0.12
LA(Mom.)	87.17 ± 0.07	52.86 ± 0.84	91.74 ± 0.19	74.01 ± 3.70	25.37 ± 0.35	57.32 ± 0.80	**95.82** ± 0.11	61.44 ± 0.17
LA(RAdam)	89.03 ± 0.87	34.26 ± 9.37	92.05 ± 0.16	83.00 ± 0.64	24.04 ± 0.25	54.92 ± 0.97	95.67 ± 0.11	61.73 ± 0.10
Momentum	87.04 ± 0.02	36.00 ± 11.09	91.87 ± 0.12	83.16 ± 0.56	23.86 ± 0.15	56.21 ± 0.67	95.37 ± 0.27	61.97 ± 0.12
NAG	87.08 ± 0.02	36.16 ± 10.99	91.87 ± 0.12	83.30 ± 0.88	23.85 ± 0.22	**57.85** ± 0.77	95.28 ± 0.23	61.74 ± 0.12
NAdam	86.45 ± 1.94	**27.73** ± 0.09	91.75 ± 0.42	**83.58** ± 0.45	**23.00** ± 0.07	53.44 ± 1.27	95.00 ± 0.25	62.01 ± 0.11
RAdam	86.43 ± 1.93	27.81 ± 0.06	91.63 ± 0.24	82.85 ± 0.52	23.10 ± 0.11	53.98 ± 1.00	94.83 ± 0.38	61.98 ± 0.13
RMSProp	87.38 ± 0.12	27.86 ± 0.08	91.79 ± 0.36	82.16 ± 0.65	23.11 ± 0.08	52.16 ± 0.99	95.25 ± 0.34	**62.24** ± 0.07
SGD	**86.29** ± 3.44	36.17 ± 10.97	91.80 ± 0.23	82.64 ± 0.91	23.83 ± 0.22	50.58 ± 1.49	95.11 ± 0.31	61.29 ± 0.14

Table 7: Tabular version of Figure 14. Mean test set performance and standard deviation over 10 random seeds of all tested optimizers on all eight optimization problems using the *small budget* for tuning and *no learning rate schedule*. For comprehensability, mean and standard deviation are rounded.

Optimizer	Quadratic Deep	MNIST VAE	F-MNIST 2c2d	CIFAR-10 3c3d	F-MNIST VAE	CIFAR-100	SVHN	Tolstoi
AMSBound	92.80 ± 5.99	28.18 ± 0.14	91.99 ± 0.10	83.15 ± 0.65	23.50 ± 0.11	54.91 ± 0.54	95.33 ± 0.17	58.25 ± 0.19
AMSGrad	87.58 ± 0.71	27.87 ± 0.08	92.01 ± 0.09	82.25 ± 0.54	23.21 ± 0.06	52.71 ± 0.97	95.25 ± 0.21	61.61 ± 0.14
AdaBelief	87.18 ± 0.03	27.99 ± 0.06	91.94 ± 0.33	83.13 ± 0.60	23.17 ± 0.07	53.17 ± 1.15	94.99 ± 0.31	61.09 ± 0.09
AdaBound	94.66 ± 6.25	28.11 ± 0.09	**92.08** ± 0.20	82.64 ± 1.03	23.40 ± 0.06	50.10 ± 16.39	95.33 ± 0.16	58.88 ± 0.16
AdaDelta	123.86 ± 0.24	28.03 ± 0.08	91.84 ± 0.11	81.31 ± 1.40	23.50 ± 0.17	50.14 ± 2.29	95.21 ± 0.29	59.40 ± 0.11
AdaGrad	87.14 ± 1.02	27.98 ± 0.16	**92.08** ± 0.23	83.25 ± 0.51	23.19 ± 0.08	37.90 ± 24.22	95.02 ± 0.21	62.01 ± 0.11
Adam	87.68 ± 1.44	27.81 ± 0.06	91.67 ± 0.25	81.90 ± 0.86	**23.10** ± 0.11	52.96 ± 1.34	94.84 ± 0.38	61.79 ± 0.06
LA(Mom.)	87.16 ± 0.06	55.20 ± 0.86	91.58 ± 0.15	82.72 ± 1.24	25.28 ± 0.23	57.68 ± 0.60	**95.80** ± 0.10	60.23 ± 0.26
LA(RAdam)	93.75 ± 3.15	38.11 ± 9.73	91.97 ± 0.22	**84.70** ± 0.30	24.53 ± 0.15	55.09 ± 0.98	95.62 ± 0.19	60.00 ± 0.11
Momentum	87.03 ± 0.02	36.08 ± 11.04	91.87 ± 0.16	83.00 ± 0.71	23.93 ± 0.30	55.96 ± 0.92	95.34 ± 0.23	61.93 ± 0.10
NAG	87.08 ± 0.02	36.18 ± 10.97	92.05 ± 0.13	83.32 ± 0.57	23.87 ± 0.33	**57.75** ± 0.71	95.51 ± 0.21	62.07 ± 0.10
NAdam	86.45 ± 1.94	**27.77** ± 0.06	91.59 ± 0.25	82.94 ± 0.61	23.12 ± 0.06	53.30 ± 0.90	94.99 ± 0.18	61.97 ± 0.08
RAdam	**86.43** ± 1.93	27.82 ± 0.06	91.49 ± 0.40	82.27 ± 0.53	23.12 ± 0.07	53.47 ± 0.86	94.79 ± 0.38	61.93 ± 0.14
RMSProp	87.40 ± 0.14	28.03 ± 0.13	91.27 ± 0.28	82.56 ± 0.71	23.26 ± 0.08	51.20 ± 0.89	93.82 ± 1.64	**62.25** ± 0.12
SGD	88.37 ± 3.55	36.18 ± 10.96	91.69 ± 0.15	82.20 ± 1.32	23.76 ± 0.25	51.53 ± 1.37	94.84 ± 0.56	61.25 ± 0.12

Table 8: Tabular version of Figure 15. Mean test set performance and standard deviation over 10 random seeds of all tested optimizers on all eight optimization problems using the *medium budget* for tuning and the *cosine learning rate schedule*. For comprehensability, mean and standard deviation are rounded.

Optimizer	**Quadratic Deep**	**MNIST VAE**	**F-MNIST 2c2d**	**CIFAR-10 3c3d**	**F-MNIST VAE**	**CIFAR-100**	**SVHN**	**Tolstoi**
AMSBound	85.94 ± 3.41	28.12 ± 0.19	91.97 ± 0.15	82.91 ± 0.83	23.49 ± 0.07	54.87 ± 0.70	95.62 ± 0.15	59.31 ± 0.36
AMSGrad	87.00 ± 0.55	**27.39** ± 0.04	92.25 ± 0.22	85.20 ± 0.34	22.83 ± 0.06	54.21 ± 1.99	96.68 ± 0.07	61.68 ± 0.17
AdaBelief	88.12 ± 0.04	27.45 ± 0.05	**92.43** ± 0.14	85.47 ± 0.26	22.78 ± 0.04	57.58 ± 0.57	96.46 ± 0.08	61.09 ± 0.17
AdaBound	**85.92** ± 3.41	28.00 ± 0.09	92.08 ± 0.17	83.20 ± 0.62	23.38 ± 0.08	54.68 ± 0.81	95.58 ± 0.10	59.45 ± 0.36
AdaDelta	164.58 ± 0.35	58.46 ± 61.52	92.05 ± 0.08	85.12 ± 0.28	60.55 ± 49.27	51.34 ± 0.64	96.68 ± 0.05	57.77 ± 0.19
AdaGrad	86.61 ± 1.94	28.17 ± 0.27	91.90 ± 0.23	85.48 ± 0.35	23.36 ± 0.05	29.40 ± 28.41	96.78 ± 0.07	61.75 ± 0.07
Adam	**85.92** ± 3.41	27.60 ± 0.06	92.29 ± 0.12	85.27 ± 0.29	**22.75** ± 0.03	55.14 ± 0.97	96.67 ± 0.06	61.86 ± 0.16
LA(Mom.)	87.06 ± 0.02	76.78 ± 24.04	91.76 ± 0.20	85.61 ± 0.24	46.09 ± 21.85	62.67 ± 0.81	96.78 ± 0.08	60.26 ± 0.23
LA(RAdam)	87.08 ± 0.42	37.41 ± 10.15	91.49 ± 0.24	85.87 ± 0.18	24.00 ± 0.12	42.00 ± 27.55	96.65 ± 0.09	61.62 ± 0.16
Momentum	87.06 ± 0.02	36.33 ± 10.85	91.89 ± 0.12	86.13 ± 0.19	23.70 ± 0.18	63.43 ± 0.56	96.71 ± 0.05	62.26 ± 0.13
NAG	87.06 ± 0.02	36.53 ± 10.71	91.76 ± 0.13	**87.12** ± 0.19	41.41 ± 21.65	**63.61** ± 0.46	96.68 ± 0.08	62.46 ± 0.10
NAdam	85.93 ± 3.41	27.46 ± 0.10	92.42 ± 0.12	85.34 ± 0.34	22.77 ± 0.07	54.02 ± 0.71	96.62 ± 0.07	62.20 ± 0.12
RAdam	86.49 ± 1.94	27.51 ± 0.05	92.33 ± 0.10	85.47 ± 0.36	22.82 ± 0.08	55.31 ± 0.86	96.61 ± 0.07	61.87 ± 0.19
RMSProp	87.09 ± 0.01	27.57 ± 0.05	92.22 ± 0.18	84.54 ± 0.25	22.80 ± 0.04	48.02 ± 15.69	96.65 ± 0.06	**62.85** ± 0.06
SGD	86.30 ± 3.41	36.47 ± 10.76	91.72 ± 0.21	70.50 ± 30.76	23.54 ± 0.13	42.29 ± 27.05	**96.80** ± 0.08	60.40 ± 0.11

Table 9: Tabular version of Figure 16. Mean test set performance and standard deviation over 10 random seeds of all tested optimizers on all eight optimization problems using the *large budget* for tuning and *trapezoidal learning rate schedule*. For comprehensability, mean and standard deviation are rounded.

Optimizer	**Quadratic Deep**	**MNIST VAE**	**F-MNIST 2c2d**	**CIFAR-10 3c3d**	**F-MNIST VAE**	**CIFAR-100**	**SVHN**	**Tolstoi**
AMSBound	86.78 ± 2.04	28.18 ± 0.19	92.11 ± 0.16	83.11 ± 0.84	23.49 ± 0.11	54.28 ± 1.23	95.46 ± 0.21	59.70 ± 0.14
AMSGrad	**85.94** ± 3.42	27.57 ± 0.06	**92.29** ± 0.12	84.71 ± 0.31	22.87 ± 0.06	57.15 ± 0.89	96.42 ± 0.06	61.86 ± 0.14
AdaBelief	87.19 ± 0.02	27.75 ± 0.05	92.27 ± 0.10	84.90 ± 0.32	22.93 ± 0.07	58.66 ± 0.50	96.35 ± 0.07	61.50 ± 0.15
AdaBound	91.34 ± 5.60	28.11 ± 0.09	92.08 ± 0.14	83.23 ± 0.58	23.37 ± 0.05	54.50 ± 1.23	95.45 ± 0.18	59.72 ± 0.17
AdaDelta	108.26 ± 0.14	27.60 ± 0.08	91.87 ± 0.20	85.40 ± 0.17	22.87 ± 0.08	59.67 ± 0.38	96.58 ± 0.07	60.41 ± 0.11
AdaGrad	86.51 ± 1.95	27.83 ± 0.15	91.88 ± 0.12	84.84 ± 0.23	7e23 ± 2e24	48.31 ± 23.66	96.48 ± 0.10	62.35 ± 0.16
Adam	88.01 ± 3.63	27.52 ± 0.06	92.09 ± 0.14	84.66 ± 0.42	**22.80** ± 0.05	58.52 ± 0.61	96.22 ± 0.08	62.31 ± 0.10
LA(Mom.)	87.12 ± 0.02	52.89 ± 0.00	91.87 ± 0.17	84.85 ± 0.60	28.24 ± 13.23	62.69 ± 0.42	96.48 ± 0.10	61.81 ± 0.17
LA(RAdam)	88.67 ± 1.24	36.14 ± 10.99	91.96 ± 0.14	**86.31** ± 0.25	23.83 ± 0.14	56.22 ± 18.42	**96.62** ± 0.08	62.03 ± 0.14
Momentum	87.06 ± 0.02	33.77 ± 9.62	91.67 ± 0.22	85.02 ± 0.30	23.45 ± 0.22	62.78 ± 0.34	96.50 ± 0.08	62.40 ± 0.08
NAG	87.06 ± 0.02	35.80 ± 11.20	92.08 ± 0.16	85.00 ± 0.44	23.38 ± 0.16	**63.30** ± 0.31	96.43 ± 0.11	62.41 ± 0.09
NAdam	87.03 ± 3.66	**27.51** ± 0.08	92.28 ± 0.11	84.96 ± 0.37	22.83 ± 0.08	58.96 ± 0.77	96.27 ± 0.10	62.28 ± 0.11
RAdam	86.43 ± 1.93	**27.51** ± 0.05	92.17 ± 0.17	84.86 ± 0.32	22.83 ± 0.07	59.01 ± 0.73	96.29 ± 0.09	62.24 ± 0.13
RMSProp	87.14 ± 0.03	27.58 ± 0.07	92.23 ± 0.13	84.11 ± 0.16	22.85 ± 0.05	30.15 ± 29.15	96.25 ± 0.09	**62.59** ± 0.11
SGD	86.05 ± 3.40	35.71 ± 11.26	91.88 ± 0.17	84.83 ± 0.27	23.43 ± 0.19	31.36 ± 30.38	96.42 ± 0.07	61.25 ± 0.11

Table 10: Tabular version of Figure 17. Mean *training* set performance and standard deviation over 10 random seeds of all tested optimizers on all eight optimization problems using the *large budget* for tuning and *no learning rate schedule*. For comprehensability, mean and standard deviation are rounded.

Optimizer	**Quadratic Deep**	**MNIST VAE**	**F-MNIST 2c2d**	**CIFAR-10 3c3d**	**F-MNIST VAE**	**CIFAR-100**	**SVHN**	**Tolstoi**
AMSBound	84.10 ± 3.34	27.84 ± 0.15	**0.00** ± 0.00	0.58 ± 0.02	23.46 ± 0.22	1.80 ± 0.09	0.17 ± 0.00	1.26 ± 0.01
AMSGrad	85.24 ± 1.21	27.20 ± 0.09	**0.00** ± 0.00	0.56 ± 0.01	22.77 ± 0.10	1.62 ± 0.11	0.07 ± 0.00	1.18 ± 0.01
AdaBelief	84.87 ± 0.30	27.16 ± 0.07	**0.00** ± 0.00	0.56 ± 0.02	22.68 ± 0.07	1.75 ± 0.11	0.10 ± 0.00	1.19 ± 0.01
AdaBound	92.10 ± 5.64	27.86 ± 0.17	**0.00** ± 0.00	0.57 ± 0.02	23.27 ± 0.14	1.87 ± 0.09	0.08 ± 0.01	1.26 ± 0.01
Adadelta	104.99 ± 0.30	27.16 ± 0.12	**0.00** ± 0.00	**0.52** ± 0.01	22.79 ± 0.15	1.91 ± 0.17	0.11 ± 0.01	1.21 ± 0.01
Adagrad	84.40 ± 1.73	27.58 ± 0.34	**0.00** ± 0.00	0.53 ± 0.02	22.94 ± 0.06	5.25 ± 6.85	0.10 ± 0.01	1.15 ± 0.01
Adam	84.33 ± 1.76	26.99 ± 0.07	**0.00** ± 0.00	0.56 ± 0.03	22.73 ± 0.12	1.79 ± 0.09	0.10 ± 0.01	1.15 ± 0.00
LA(Mom.)	84.85 ± 0.30	52.85 ± 0.74	0.06 ± 0.02	1.00 ± 0.16	25.40 ± 0.39	1.76 ± 0.06	**0.06** ± 0.00	1.17 ± 0.01
LA(RAdam)	86.68 ± 1.10	34.33 ± 9.29	**0.00** ± 0.00	0.57 ± 0.02	24.00 ± 0.26	1.75 ± 0.08	0.11 ± 0.00	1.16 ± 0.00
Momentum	84.77 ± 0.30	35.98 ± 11.06	**0.00** ± 0.00	0.57 ± 0.02	23.71 ± 0.13	1.84 ± 0.07	0.13 ± 0.00	1.15 ± 0.01
NAG	84.77 ± 0.30	36.15 ± 10.96	**0.00** ± 0.00	0.54 ± 0.02	23.67 ± 0.22	1.65 ± 0.03	0.18 ± 0.00	1.16 ± 0.01
Nadam	84.19 ± 1.74	**26.77** ± 0.12	0.01 ± 0.01	0.55 ± 0.02	**22.59** ± 0.08	1.84 ± 0.08	0.10 ± 0.00	1.15 ± 0.01
RAdam	84.18 ± 1.74	26.91 ± 0.10	0.01 ± 0.00	0.55 ± 0.02	22.77 ± 0.08	**1.54** ± 0.10	0.10 ± 0.00	1.15 ± 0.01
RMSProp	85.02 ± 0.32	27.18 ± 0.13	0.01 ± 0.01	0.57 ± 0.02	22.73 ± 0.08	1.69 ± 0.13	0.11 ± 0.01	**1.14** ± 0.00
SGD	**83.95** ± 3.25	36.15 ± 10.95	**0.00** ± 0.00	0.63 ± 0.02	23.71 ± 0.23	1.70 ± 0.10	0.15 ± 0.01	1.18 ± 0.00

Appendix References

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