Kurt Weichselberger's Contribution to Imprecise Probabilities

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Abstract

Kurt Weichselberger, one of the influential senior members of the imprecise probability community, passed away on February 7, 2016. Almost throughout his entire academic life the major focus of his research interest has been on the foundations of probability and statistics. The present article is a first attempt to trace back chronologically the development of Weichselberger's work on interval probability and his symmetric theory of logical probability based on it. We also try to work out the intellectual background of his different projects together with some close links between them.

Keywords: Weichselberger, Kurt; interval probability; imprecise probabilities; logical probability; symmetric theory; history of probability and statistics.

1. Introduction

Kurt Weichselberger, who passed away last year, has been "a man of the first hour" of the ISIPTA meetings, perceiving them as the natural place to discuss the foundations of probability. He enthusiastically participated in the first six ISIPTAs, from the 1999 Ghent symposium to the Durham meeting in 2009, contributing several papers, a tutorial in 2005 and a special session in 2009. At least from the mid sixties of the last century onwards, the foundations of statistics and probability have always been Weichselberger's great passion – although he had worked on a variety of different topics[1], and had been intensively engaged in academic self-administration and societies.

This paper is a first attempt to trace back fundamental aspects of Weichselberger's ideas as well as their links constituting his challenging research program. Our work is embedded into the *HiStaLMU* project (History of Statistics at LMU Munich). Among other activities, its members interview former leading personalities of the Department of Statistics as oral history and build up an archive around Kurt Weichselberger's office estate.[2] The structure of presentation in this paper is chosen mainly chronologically. After a brief biographic sketch (Section 2), we look at Weichselberger's foundational work and distinguish four main periods: the first intensive research on logical probability (see Section 3), the work on probability intervals in the context of modelling uncertain expert knowledge (Section 4), the axiomatic foundation of an interpretation independent theory of interval probability (Section 5), and eventually the aim to synthesize the previous results towards the *symmetric theory of logical probability* (Section 6). Section 7 concludes.

1. The work on applied statistics includes among others research on survey and census methodology (e.g. Weichselberger (1962)), regional price indices (Weichselberger and Wulsten, 1978), quality control (Weichselberger, 1971), and time series (Weichselberger, 1994).
2. See also the workshop in March 2016 (https://statsoz-neu.userweb.mwn.de/research/ws_historystatistik_2016/index.html).

2. A Short Biographic Sketch

In this section we quickly summarize the main stages of Weichselberger's career.[3] Kurt Weichselberger was born on April 13, 1929, in Vienna. He studied mathematics there, and earned his PhD (*Dr. phil.*) in 1953 for a thesis supervised by Johann Radon (Weichselberger, 1953). Weichselberger started his academic career at the Department of Statistics in Vienna at Wilhelm Winkler's chair, worked at a social research institute in Dortmund, as well as at Johann Pfanzagl's chair in Cologne, where he received his *Habilitation* in 1962 with a thesis on controlling census results (Weichselberger, 1962). From 1963 to 1969 Weichselberger held the chair in statistics at the Technische Universität Berlin. In 1967 he was elected rector of this university and substantially contributed to the then vivid public debate about the role of education and scientists in the modern society.

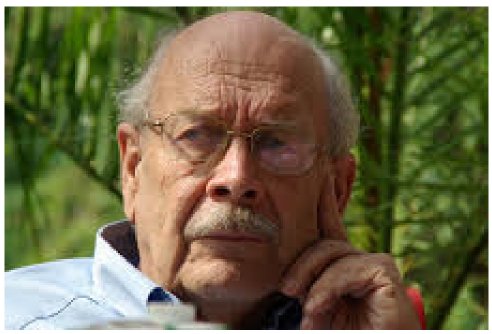

Figure 1: Kurt Weichselberger (photo kindly provided by Weichselberger's family)

From 1969 on, for almost 50 years, Weichselberger has been a member of the Ludwig-Maximilians-Universität München (LMU Munich). During this time he has played a leading role in the sustainable development of statistics as a discipline of its own – far beyond LMU. In particular, he co-founded the Department of Statistics and Philosophy of Science at LMU (see the end of the next section), and substantially contributed, also as Chairman of the Education Committee of the German Statistical Society for more than 10 years, to establishing first study programs for a major in statistics in Germany. From 1997 on, Weichselberger continued his research activities as a professor emeritus. On February 7, 2016, he passed away in his house in Grafing among close family.

3. Logical Probability I

Already at his inaugural speech (Weichselberger, 1968) as rector in Berlin, Weichselberger set out for his great scientific mission and passion: the development of a new theory of statistical inference, putting Fisher's fiducial argument back on its feet and substantially extending it. This theory has to be founded on what Weichselberger called *logical probability*: a non-subjective probability in its literal sense,[4] evaluating, as a two-place function, the reasoning from premises to conclusions and, the other way round,[5] finally allowing to describe the degree of support data give to statistical models. In the last section of his inaugural speech he explicates:

3. For more details see in particular Rüger (1995) and Rüger's obituary (Rüger, 2016). Many students and academic companions until the mid nineties are assembled in the Festschrift edited by Rinne, Rüger, and Strecker (1995).
4. Note the etymological basis of the word probability: *prove-ability*, as well as the constituents of the corresponding German word *Wahr–schein–lich–keit*, i.e. the extent to which something seems to be true.
5. That is the reason, why Weichselberger called his theory *symmetric theory of probability.*

> *[... W]e are challenged with the task to reconceptualise the foundations of probability. The question is whether we can make progress towards a broader concept designation without losing key benefits of the previous – objectivistic – concept. For that matter we have to decide which properties of the objectivistic probability concept we consider to be essential.*
>
> *[...The] two essential properties of Kolmogorov's probability concept that consequently may not be waived [are the following]:*
>
> 1. *The embedding into modern mathematics.*
>   *This needs to remain ensured by determining the mathematical properties of the concept with a consistent system of axioms.*
> 2. *The possibility of the frequency interpretation of probabilities because this presents to date the only known mindset that enables an explanation of the concept and thus guarantees that the ideas of different persons on the meaning of probability can be adapted. Taking these issues into account we are challenged with the task as follows:*
>
> *We have to develop a system of axioms that*
>
> 1. *includes Kolmogorov's system of axioms as a special case;*
> 2. *associates probabilities not with events but with inferences from premises to conclusions;*
> 3. *enables the frequency interpretation of the probability concept;*
> 4. *enables probability propositions in both directions in cases in which the Fisher theory and the Neyman-Pearson theory yield the same results; for example in the case of a sample from a population, it associates a probability with the inference from the population to the sample as well as with the inference from the sample to the population.* (Weichselberger, 1968, p. 46-47) [translation by TA & RS]

Weichselberger is already very clear about the fact that such a theory has to go beyond the restrictions precise probabilities imply, and therefore continues:

> *As in many cases in the history of science it is shown also here that — as a form of compensation for desired benefits — we have to abandon a "habit of thinking" (*Denkgewohnheit*). In the present case this is the habit of thinking that the probability is always a number. We must instead allow sets of numbers – say the interval between 0.2 and 0.3 – to act as the probability of the inference from the proposition B to the proposition A. However, we continue to demand that the set of numbers lies in the interval between 0 and 1.*
>
> *This extension of the probability concept from a number to a set of numbers is encouraged as soon as we try to formalize Fisher's fiducial probability. Therefore a similar approach has already taken the American Henry Kyburg Jr. in his works in the years 1961 to 1964. However, Kyburg's concept is inconsistent at a decisive point, and, to his own statements, it does not lead to useful results in detail. His view is mainly of philosophical and not of mathematical nature.*
>
> *In fact, the definition of probability as a set of numbers – normally an individual number or an interval – leads to mathematical problems. We need a system of*

> *calculation rules for algebraic operations with such sets. This prompts us to similar considerations as the systematization of calculations with inexact or error-prone quantities: one could call it a "theory of tolerance space" (*Spielraum-Theorie*) because we associate tolerance spaces instead of individual numbers. I think that it is possible that this view may give rise to interesting inner mathematical questions.* (Weichselberger, 1968, p. 47) [translation by TA & RS]

During this period Weichselberger had accepted an offer from LMU Munich on a newly installed chair for *Special Topics in Statistics (Spezialgebiete der Statistik)*. In Munich he was strongly engaged in changing the institutional alignment of statistics within the university. In 1974, the Institute for Statistics and Philosophy of Science was founded, as a member of the new Faculty of Philosophy, Philosophy of Science and Statistics. Weichselberger has stayed in intensive intellectual contact with his colleagues from philosophy all the time. Clearly, there have been common scientific interests in particular with Wolfgang Stegmüller, who held the Chair in Philosophy of Science and did research among other topics also on subjective probability and Carnap's concept of logical probability.[6] In the first Munich years Weichselberger worked intensively on a book on logical probability. According to his former assistant Christina Schneider (personal communication), a manuscript of several hundred pages evolved, but, unfortunately, never got published.[7]

4. Probability Intervals, *Uncertainty in Knowledge-Based Systems*

In the mid eighties Weichselberger's research experienced a shift, which gave his interests in imprecise probabilities new impetus, where he had been in-depth engaged in the vivid discussion about modelling uncertain expert knowledge in artificial intelligence. He has understood it as a question of life and death for statistics as a discipline whether statistics can contribute here. Weichselberger agreed with many other researchers mainly from computer science that the problem how to model uncertain knowledge produces a big challenge, where statistics, in its traditional form, reaches its limits. However, he also warned not to throw out the baby with the bath water and end up in a wild arbitrariness of conclusions, by leaving the field to ad hoc calculi. Weichselberger stood for a very clear position: there will be an important contribution of statistics and probability in this area, if, but also radically only if, the concept of probability is ready to overcome the dogma of precision.

Therefore, the book *A Methodology for Uncertainty in Knowledge-Based Systems* (Weichselberger and Pöhlmann, 1990), published by Weichselberger together with his post-doctoral researcher Sigrid Pöhlmann, aims at reconciling probability theory with the objectives of flexible modern uncertainty calculi. In their preface they argue:

> *First of all it must be stated that although the basic ideas prevailing in some considerations about diagnostic systems sound convincing, they violate fundamental requirements for reasonable handling of uncertainty. [. . . We] shall demonstrate that negligence with respect to* [... some basic principles] *may result in the inclusion of information into a diagnostic system which is equivalent to ruining it. (Weichselberger and Pöhlmann, 1990, pp. 1-2)*

6. To which extent a concrete co-operation in research has taken place between Weichselberger and Stegmüller is still an open question, which shall be studied further within the HiStaLMU project.
7. Tragically, that manuscript is not part of Weichselberger's office estate. Up to now it is unclear whether this manuscript still exists.

Indeed, after fundamentally critisising the Dempster-Shafer combination rule and the MYCIN certainty factors, Weichselberger and Pöhlmann develop, in the context of a prototypical special case[8], a neat probabilistic alternative to handle different sources of information in diagnostic systems. It has consistently to be based on a generalisation of probability, synthesising the well-founded concept of probability with the flexibility needed for modelling uncertain knowledge:

> *[...An] argument against a possible application of probability theory [, understood in its traditional, precise form here,] in diagnostic systems is as follows: While probability theory affords statements, using real numbers as measures of uncertainty, the informative background of diagnostic systems is often not strong enough to justify statements of this type. [...] However, it is possible to expand the framework of probability theory in order to meet these requirements without violating its fundamental assumptions. [...W]e believe that the weakness of estimates for measures of uncertainty as used in diagnostic systems represents a stimulus to enrich probability theory and the methodological apparatus derived from it, rather than an excuse for avoiding its theoretical claims. (Weichselberger and Pöhlmann, 1990, p. 2)*

Technically, Weichselberger & Pöhlmann do not yet use interval probability in its full generality, but confine themselves to the case which they call *probability intervals (PRI)*.[9] There an interval-valued probability is assigned to the singletons only, and natural extension is applied to calculate the probabilities of the other events. Moreover, speaking often of "interval *estimates* of probabilities" (italics by TA & RS), Weichselberger & Pöhlmann implicitly rely exclusively on the sensitivity analysis (epistemic) point of view of imprecise probabilities. The book was published one year before Peter Walley's book (Walley, 1991) on general imprecise probability appeared. In Weichselberger and Pöhlmann's book the notions of R- and F-probability ("R" for *reasonable*, corresponding to *avoiding sure loss* to use Walley's terminology, and "F" for *feasible*, corresponding to *coherent*) were developed for the first time. Having been perceived well, mainly in the artificial intelligence community, the book was also criticized strongly as "a little too unfinished" and too example-based in a review in the Journal of the American Statistical Association (Wasserman, 1991). Convenient expressions to work with PRIs were extended in Weichselberger (2001a, Chapter 3.3 and Appendix A.5). The construction of least favourable pairs for testing hypotheses described by PRIs is considered in Martin Gümbel's dissertation (Gümbel, 2009), supervised by Weichselberger.

5. Interval Probability: *Elementare Grundbegriffe ...*

5.1 The Book and the ISIPTA '99 Paper Including its IJAR Extension

Immediately after having finished the book with Pöhlmann, Weichselberger started to develop the theory of interval probability as a "one-place assignment", i.e. assigning probability to events, in its generality.[10] No later than 1992, a first version of a book was ready, which already contained the core of the theory. The material grew and grew in its dimensions, and Weichselberger decided to split the book project into three volumes. Finally, in 2001 the first volume, *Elementare*

8. The general case was later solved in Pöhlmann's Habilitation thesis (Pöhlmann, 1995).
9. See de Campos, Huete, and Moral (1994) for an independent development of almost the same framework.
10. Weichselberger, however, always has been stressing the importance of the "two-place concept" (logical probability with premises and conclusions as functional arguments, see Section 3) as the ideal, calling it still "[...] without doubt the most challenging [...]" concept (Weichselberger, 2001a, p. 33) [translation by TA & RS]. Unless mentioned differently, the term 'probability' is used throughout this section in its one-place meaning as probability of events.

Grundbegriffe einer allgemeineren Wahrscheinlichkeitsrechnung I: Intervallwahrscheinlichkeit als umfassendes Konzept (Elementary Fundamentals of a More General Calculus of Probability I: Interval Probability as a Comprehensive Concept) (Weichselberger, 2001a), appeared.[11] Soon this book became Weichselberger's most influential publication, together with the paper *The theory of interval probability as a unifying concept for uncertainty* (Weichselberger, 2000), which arose from his ISIPTA '99 contribution and serves as an English language reference summarizing some of the main concepts. The title of the book, an immediate allusion to Kolmogorov's *Grundbegriffe* ... (Kolmogorov, 1933), founding traditional probability theory, formulated the research program. Weichselberger develops thoroughly the theory of interval probability by generalizing the Kolmogorovian concept to interval-valued assignments. The book consists of four main chapters:[12]

The first chapter elaborates the background of the theory. It starts with embedding the theory into the historical development of the concept of probability, including other generalizations of probability. Then motives for the paradigmatic shift from traditional probability to interval probability and major objectives of the theory are discussed in-depth.

The second chapter contains the axioms of R- and F-probability. Weichselberger characterises interval-valued assignments $P(\cdot) = [L(\cdot), U(\cdot)]$ on a σ-field $\mathcal{A}$ by their relation to the set $\mathcal{M}$ of classical probabilities (in the sense of Kolomogorov) $p(\cdot)$ they induce. If this set is not empty, then $P(\cdot)$ is an ***R-prob**ability*, and $\mathcal{M}$ is its *structure*. An R-probability is ***F-prob**ability* if $P(\cdot)$ and the structure uniquely correspond to each other by

$$L(A) = \inf_{p(\cdot)\in\mathcal{M}} p(A) \quad \text{and} \quad U(A) = \sup_{p(\cdot)\in\mathcal{M}} p(A)\,, \quad \forall A \in \mathcal{A}\,.$$

In the light of Walley's lower envelope theorems, R-probability and F-probability technically correspond, in essence, to lower and upper probabilities avoiding sure loss and being coherent, respectively (Walley, 1991, Chapters 2 and 3), where, however, Weichselberger, in the spirit of Kolmogorov, demands σ-additivity. In conformity with Walley, Weichselberger stresses that there is no need to require additional restrictive properties (like two- or total monotonicity of the lower bound), but in contrast to him, Weichselberger focuses on interval-valued assignments of *events*, instead of random variables/gambles. For Weichselberger, probability of events is the constitutive entity (of a one-place probability assignment[13]); he sees expectations/previsions as derived entities, explicitly needing an underlying metrical scale. The most important difference for Weichselberger to Walley is that his axiomatisation is, just as the Kolmogorovian approach in traditional probability theory, strictly independent of any interpretation of probability. By this, he emphasises, it provides a sound mathematical basis for expressing all different interpretations of (one-place) generalized probabilities, from subjective to frequentist, which eventually is the key to overcome the methodological antagonisms in statistical inference. Chapter 2.6 reflects on decision criteria based on probabilistic evaluations of events. There Weichselberger also argues that behaviour following Hurwicz-like

11. The book title has the rare addendum "unter Mitarbeit von (with the cooperation of) T. Augustin und (and) A. Wallner", which tributes to the special way the book was written: Augustin entered the project in 1993, Wallner in 1995, both as young PhD students and assistants. They were rather intensively engaged with the book, but rarely as co-authors (Wallner, and to an even smaller extent Augustin, contributed some shorter, clearly marked parts of the book only, listed in Weichselberger (2001a, p. x)), but as critical discussions partners. Weichselberger extended and developed further the theory step by step, and in several meetings per week these steps were immediately and intensively discussed.
12. See also the review by Coolen (2003).
13. See also Footnote 10.

criteria (e.g. Huntley, Hable, and Troffaes (2014, p. 193)) challenges the betting interpretation of imprecise probabilities, which he judges to rely solely on a Γ-maximin point of view.[14]

Chapter 3 generalizes the setting to situations where the limits of an interval probability firstly are only specified on certain subsets of the σ-field $\mathcal{A}$, and then natural extension is applied (*partially determinate probability*). This gives rise to a list of interesting special cases, including PRIs (see Section 4) and a kind of general p-boxes (*cumulative R-/F-probability*[15]). Supplementing natural extension, which already appears in Weichselberger and Pöhlmann (1990) (*derived F-PRI*), Weichselberger also proposes a *cautious standpoint* to proceed from a given R-probability $[L(\cdot), U(\cdot)]$ that is not F-probability to a uniquely defined F-probability $[L^*(\cdot), U^*(\cdot)]$, now such that the original limits $L(\cdot)$ and $U(\cdot)$ are always respected, in the sense that $L^*(\cdot) \leq L(\cdot)$ and $U^*(\cdot) \geq U(\cdot)$.

Specific issues of interval probabilities on finite spaces are in the focus of Chapter 4, see also Weichselberger (1996). In Chapter 4.1 algorithms are developed to check whether assignments constitute R- and F-probability, as well as to calculate the natural extension and its counterpart from the cautious standpoint. Interestingly, linear programming is here not only utilized powerfully for calculations, but also, by duality results, as a mathematical tool for elegant proofs.

5.2 Preceding First Contributions to General Interval Probability; Strongly Related Work and Co-operations

Figure 2: Participants of the Foundations of Statistics Workshop organized by Frank Hampel in 1994: From left to right: Walley, Goldstein, Smets, Coolen, Weichselberger, Morgenthaler, Hampel, Augustin (photo kindly provided by Frank Coolen)

In this section we collect some of Weichselberger's activities when working on his book. His axioms and further core elements of his theory were presented at several workshops, including a workshop in June 1993 honouring Peter J. Huber (Weichselberger, 1996), the Second Gauss Symposium in August 1993 (Weichselberger, 1995a), and a workshop on the foundations of statistics in September 1994 in Zurich organized by Frank Hampel. By that workshop and an associated research retreat to the mountains nearby, Hampel connected researchers interested in the foundations of statistics (see also Figure 2), who only partially knew each other personally. The particpants' ex-

14. See also Coolen (2003, p. 254).

15. Compare also Destercke, Dubois, and Chojnacki (2008) for a related concept.

cited discussions had had a sustainable impact on their further research. Particularly close remained over all the years the relationship of Weichselberger (and Augustin) with Frank Coolen.

In 1995 also a paper on the implications of the rich framework of interval probability on sampling appeared (Weichselberger (1995b), see also Weichselberger (2001a, Chapter 4.3)), which in our eyes by far did not receive the attention it actually deserves. Only with interval probability it becomes possible to express the distinction between different types of symmetry, called *epistemic* versus *physical symmetry* by Weichselberger. Epistemic symmetry relies merely on the lack of knowledge of asymmetry, while for physical symmetry knowledge is available actively supporting symmetry. Only the latter in its purest form justifies the use of precise, traditional probabilities. By these concepts, Weichselberger develops nothing less than a generalization of the principle of insufficient reason, replacing precise uniform probability by a continuum of uniform probabilities, adequately expressing the knowledge on the system under consideration.

Decision theoretic implications of imprecise probabilities are discussed in 1998 in a contribution (Weichselberger and Augustin, 1998) to a Festschrift honouring Weichselberger's Munich long-standing colleague Hans Schneeweiß, working out how interval probability provides an immediate description of the preferences observed in Ellsberg's seminal experiments (Ellsberg, 1961), violating the axioms of traditional subjective utility theory.

As a preparation for the third volume, which was originally devoted to statistical implications of interval probability, the Huber-Strassen theory on robust testing of hypotheses described by neighbourhood models had been intensively discussed by the members of Weichselberger's chair and Helmut Rieder, who spent in 1994 one semester at LMU. Augustin, who originally had started a dissertation about the historical roots of imprecise probability, took over the topic and developed under Weichselberger's supervision a Neyman-Pearson theory under general interval probability, where the hypotheses are described by F-probability instead of two-monotone capacities. In his thesis (Augustin, 1998) it is shown that Weichselberger's condition of continuity of F-fields (Weichselberger, 2001a, p. 152f.) is both necessary and sufficient for the structure to be uniformly dominated. Furthermore, Augustin derives results on different types of least favourable pairs (published in a generalized form for the first ISIPTA and in Augustin (2002a), based on it) and a representation of the optimal test by a single linear program (published later in a decision theoretic context in Augustin (2002b, 2004)), including a Neyman-Person lemma form obtained from duality arguments.

5.3 Further Planned Volumes, Work on Interval Probability After the Book

When the book appeared, a second volume was already in a rather advanced stage. Originally it was devoted to a closer study of types of assignments that lead to two- or totally-monotone capacities (probability intervals, cumulative probabilities, belief-functions), concepts of conditional probabilities and independence, parametric statistical models and a law of large numbers.[16]

In Weichselberger's ISIPTA '01 contribution (Weichselberger, 2001b), *indicator fields* are studied, i.e. interval probabilities that can be understood as basic building blocks for more complex models. In 2002, Lev Utkin visited the Weichselberger chair for almost two years, and a very close relationship with Weichselberger (and Augustin) was established that has endured over all the years. At ISIPTA '03 (cf. Weichselberger and Augustin (2003)), Weichselberger presented his research on conditional probability. He strongly argued in favour of the idea that there cannot be a single concept of conditional probability; several, conceptually different concepts are needed which happen to

16. Some concepts are already briefly sketched in Weichselberger (2000).

coincide in the case of a precise probability. In particular, he elaborates his – rather controversially perceived – *canonical concept of conditional interval probability*, derived from a canon of desirable properties, like a commutative combination of marginals and conditional probabilities.[17]

In autumn of that year, Weichselberger abruptly stopped his research on one-place probability and radically turned all his interest to the foundation of logical probability again.

Anton Wallner, who worked together very closely with Weichselberger at that time (see also Footnote 11), first continued his research on the one-place interval probability and prepared a dissertation under Weichselberger's supervision (Wallner, 2002). There he develops a series of characterisations of interval probabilities in general as well as of uniform interval probabilities, and studies neighbourhood models based on distorted probabilities. Furthermore he presents a rather involved proof that also under general interval probability the structure of an R-probability on a space with cardinality k has, interpreted as a polyhedron in $\mathbb{R}^{k-1}$, at most $k!$ vertices. Related articles, presented at ISIPTA '03 and '05, are Wallner (2003) and Wallner (2007).

6. Logical Probability II

All the development of one-place interval probability described in the previous two sections, as interesting it may be on its own, has been understood by Weichselberger mainly as a preparation for his concept of logical probability, and thus for his general inference theory. Therefore, from 2003 on Weichselberger had devoted all his energy to this topic. Supported by Wallner, Weichselberger started to (re)build a neat framework for logical probabilities, now finding a neat basis in the theory of interval probability, pushing the vision of a closed theory of inference closer to reality. In one of his last public presentations, a special session on the symmetric theory at ISIPTA '09 (Weichselberger, 2009, p. 9), he characterises his major objective in simplified terms as follows:

> *A comprehensive methodology of probabilistic modelling and statistical reasoning, which makes possible hierarchical modelling with information gained from empirical data.*
> *To achieve the goals of Bayesian approach — but without the pre-requisite of an assumed prior probability. (Weichselberger, 2009, p. 3)*

Many of the constituents already mentioned in his inaugural speech as rector in Berlin (Weichselberger, 1968), see also Section 3 above, are revisited in the light of the new foundation. The fundamental idea of logical probability as a two-place function on the reasoning from a premise to the conclusion is formalized in a system of axioms (Weichselberger (2009, p. 8), see Weichselberger (2016, Chapter 4) for more details), while the inference is developed in the context of a duality theory (Weichselberger (2009, p. 8), for the detailed arguments see Weichselberger (2016, Chapter 6)). Also the idea of a frequency interpretation of logical probability could be made rigorous (see Weichselberger (2009, p. 9) and Weichselberger (2016, Chapter 5)). Special aspects have been published in advance at the previous ISIPTAs (Weichselberger, 2005, 2007).

7. Concluding Remarks

We presented a preliminary summary of Kurt Weichselberger's contribution to the theory of imprecise probability. As already emphasized, this paper is a report on current research within the

17. Some aspects are already discussed in Weichselberger (2000, Section 3).

HiStaLMU project, an interdisciplinary project involving statisticians and historians of science to chronicle the history of statistics at LMU Munich. Concerning the research on Weichselberger's scientific biography, the next practical step is to build up the necessary infrastructure by establishing an archive of his office estate, and we hope that we can integrate further material from his family and friends. We also started to prepare a bibliometric network analysis on the spread and influence of Weichselberger's ideas. Far beyond the historical interest, a detailed rework of Weichselberger's unfinished opus and his scattered results will enable a deeper scientific discussion of his scientific inheritance. His results and ideas provide a big challenge, still promising a substantial impact on – nay a paradigmatic shift of – probability and statistics.

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