An Inference-Based Policy Gradient Method for Learning Options

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Abstract

In the pursuit of increasingly intelligent learning systems, abstraction plays a vital role in enabling sophisticated decisions to be made in complex environments. The options framework provides formalism for such abstraction over sequences of decisions. However most models require that options be given a priori, presumably specified by hand, which is neither efficient, nor scalable. Indeed, it is preferable to learn options directly from interaction with the environment. Despite several efforts, this remains a difficult problem. In this work we develop a novel policy gradient method for the automatic learning of policies with options. This algorithm uses inference methods to simultaneously improve all of the options available to an agent, and thus can be employed in an off-policy manner, without observing option labels. The differentiable inference procedure employed yields options that can be easily interpreted. Empirical results confirm these attributes, and indicate that our algorithm has an improved sample efficiency relative to state-of-the-art in learning options end-to-end.

1. Introduction

Recent developments in reinforcement learning (RL) methods have enabled agents to solve problems in increasingly complicated domains (Mnih et al., 2016; Mousavi et al., 2018). However, as RL agents become increasingly integrated in society, there is an increased need for the behaviour of the agents to be interpretable. Further, in order for such agents to solve difficult and realistic environments–potentially involving long sequences of decisions–more sample efficient techniques are needed. One way to improve on existing agents is to leverage abstraction. By reasoning at various levels of abstraction, it is possible to infer, learn and plan much more efficiently. Further, abstraction makes interpretation easier, by making higher-order properties of data more explicit. Recent developments outside of RL have lead to breakthroughs in terms of learning rich representations for perceptual information (Bengio et al., 2013). In RL domains, however, while efficient methods exist to plan and learn when abstraction over sequences of actions is provided *a priori*, it has proven more difficult to learn this type of abstract behaviour from interaction data.

Many frameworks for formalizing temporal abstraction have been proposed; most recent developments build on the *Options* framework (Sutton et al., 1999; Precup, 2000), which can be parameterized in a flexible manner, potentially amenable to learning. The majority of prior work on learning options has centered around the idea of discovering *subgoals* in state space, and constructing a set of options such that each option represents a policy leading to that subgoal (McGovern & Barto, 2001; Menache et al., 2002; Şimşek & Barto, 2009). These methods can lead to useful abstraction, however, they often require access to a model of the environment dynamics, which is not always available, and can be infeasible to learn. Our contribution instead builds on the work of Bacon et al. (2017), and exploits a careful parameterization of the policy of the agent, as well as a differentiable inference step, in order to simultaneously learn a set of options, while directly optimizing returns. We relax a few key assumptions of this previous work, including the expectation that only options that were actually executed during training can be learned, and the focus on executing options in an on-policy manner, with option labels available. By relaxing these, we can improve sample efficiency and practical applicability, including the possibility to learn control policies from data without knowing which options were executed at each step, as is the case in the context of expert demonstration, or otherwise learn interpretable hierarchical behaviours from behavioural data.

Further to this point, interpretability represents a largely unexplored use-case for options, which have generally been employed for exploration or transfer learning. However, it is often the case that policies learned without options can be difficult to understand, with agents often performing complex sequences of high-dimensional actions. With a discrete set of options, if each represents a distinct, tempo-

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rally extended sub-policy, then it is sufficient to observe that single option in order to understand the general behaviour of the agent. As agents become increasingly complex and increasingly integrated in society, the importance of easily interpreted behaviour is paramount, as it allows us to understand what an agent is doing, and what it plans to do next.

Contributions: We present an algorithm that solves the problem of learning control abstractions by viewing the set of options as *latent variables* that *concisely represent* the agent's behaviour. More precisely, we do not only improve those options that were *actually* executed in a trajectory. Instead, we allow intra-option learning by simultaneously improving all individual options that *could have been* executed, and the policy over options, in an end-to-end manner. The result is an algorithm for learning options that is:

Data Efficient. Learning with options rather than learning flat policies introduces additional complexity. By allowing single experiences to improve all options at the same time, we can nevertheless learn with a sample efficiency that is competitive with that of flat policies.

Applicable with Partial Data. Many state of the art methods for learning options rely on observing the executed options, as well as the actions that led to a behaviour. Our method does not require these option labels, making it applicable in more situations.

Interpretable. Favoring options that were more likely to have generated a learning experience in the policy update leads to increased specialization of individual options. This specialization yields options that are temporally and spatially more coherent, and that can more readily be interpreted, without need for regularization.

We evaluate this algorithm on continuous MDP benchmark domains and compare it to earlier reinforcement learning methods that use flat and hierarchical policies.

2. Related Work

Recent attention in the field of option discovery generally falls into one of two categories. One branch of work focuses on learning options that are able to reach specific subgoals within the environment. Much work in this category has focused on problems with discrete state and action spaces, indentifying salient or bottleneck states as subgoals (McGovern & Barto, 2001; Menache et al., 2002; Şimşek & Barto, 2009; Silver & Ciosek, 2012). Recent work has focused on finding subgoal states in continuous state spaces using clustering (Niekum & Barto, 2011) or spectral methods (Machado et al., 2017). Konidaris & Barto (2009) describe an approach where subgoals of new policies are defined by the initiation conditions of existing options. Specifying options using subgoals generally requires a given or a-priori learned system model, or specific assumptions about the environment. Furthermore, the policies to reach each subgoal have to be trained independently, which can be expensive in terms of data and training time (Bacon et al., 2017).

A second body of work has learned options by directly optimizing over the parameters of function approximations that are structured in a way to yield hierarchical policies. One possibility is to *augment* states or trajectories with the indexes of the chosen options. Option termination, selection, and inter-option behavior then all depend on both the regular system state and the current option index. This approach was suggested by Levy & Shimkin (2011) for learning the parameters of a hierarchical model consisting of pre-structured policies. In the option-critic architecture (Bacon et al., 2017), a similar model is employed, with option-specific value functions to learn more efficiently. Furthermore, neural networks are used instead of a task-specific given structure. Mankowitz et al. (2016) use an explicit partitioning of the state space to ensure policy specialization.

An alternative to state augmentation was proposed by Daniel et al. (2016). In that paper, options were considered latent variables rather than observable variables. That paper employed a policy structure that allowed maximizing the objective in the presence of these latent variables using an expectation-maximization approach. However, the optimization of this structure requires option policies to be linear in state features, which imposes the need to specify good state features a priori. Further, this approach necessitates the use of information from the entire trajectory before policy improvement can be done, eliminating the possibility of an on-line approach. Fox et al. (2017) uses a similar approach in the imitation learning setting with neural network policies instead of a task specific structure.

It should be noted that there are several other related works in hierarchical reinforcement learning outside of the options framework. One possibility is to have a higher-level policy learn to set goals for a learning lower-level policy (Vezhnevets et al., 2017), or to set a sequence of lower-level actions to be followed (Vezhnevets et al., 2016). Another possibility is to have a higher-level policy specify a prior over lower-level policies for different tasks, such that the system can acquire useful learning biases for new tasks (Wingate et al., 2011).

3. Technical Background

We consider an agent interacting with its environment, at several discrete time steps. Generally, the state of the environment at step t, is provided in the form of a vector, $\mathbf{s}_t$, with $\mathbf{s}_0$ determined by an initial state distribution. At every step,

the agent observes $\mathbf{s}_t$, and selects a vector-valued action $\mathbf{a}_t$, according to a stochastic policy $\pi(\mathbf{a}_t|\mathbf{s}_t)$, which gives the probability that an agent executes a particular action from a particular state. The agent then receives a reward r_t and the next state $\mathbf{s}_{t+1}$ from the environment.

We consider episodic setups where, eventually, the agent reaches a terminal state, $\mathbf{s}_T$ upon which the environment is reset, to a state drawn from an initial state distribution. A sequence of states, actions and rewards generated in this manner is referred to as a trajectory τ.

We define the discounted return from step t within a trajectory to be $R_t^{(\tau)} = \sum_{i=t}^{T} \gamma^{(i-t)} r_i$. The objective of the learning agent is to maximize the expected per-trajectory return, given by $\rho = \mathbb{E}_\tau[R_0^{(\tau)}]$.

3.1. Policy Gradient Methods

While several methods exist for learning a policy from interaction with the environment, here, we focus on policy gradient methods, which have benefited from a recent resurgence in popularity. Policy gradient methods directly optimize ρ by performing stochastic gradient ascent on the parameters $\boldsymbol{\theta}$ of a family of policies $\pi_{\boldsymbol{\theta}}$. Policy gradients can be estimated from sample trajectories, or in an online manner. The full return likelihood ratio gradient estimator (Williams, 1992) takes the form:

$$\nabla_{\boldsymbol{\theta}} \rho(\boldsymbol{\theta}) = \mathbb{E}_\tau \left[(R_0^\tau - b) \sum_{t=0}^{T} \nabla_{\boldsymbol{\theta}} \log \pi(\mathbf{a}_t|\mathbf{s}_t) \right], \quad (1)$$

where b is a baseline, used to reduce variance. This is one of the simplest, most general policy gradient estimators, and can be importance sampled if observed trajectories are not generated from the agent's policy. The policy gradient theorem (Sutton et al., 2000) expands on this result in the on-policy case, giving a gradient estimate of the form:

$$\nabla_{\boldsymbol{\theta}} \rho(\boldsymbol{\theta}) = \mathbb{E}_\tau \left[\sum_{t=0}^{T} (R_t^\tau - b) \nabla_{\boldsymbol{\theta}} \log \pi(\mathbf{a}_t|\mathbf{s}_t) \right], \quad (2)$$

which can be shown to yield lower variance gradient estimates.

3.2. Options

The options framework provides the necessary formalism for abstraction over sequences of decisions in RL (Sutton et al., 1999; Precup, 2000). The agent is given access to a set of options, indexed by ω. Each option has its own policy: $\pi_\omega(\mathbf{a}_t|\mathbf{s}_t)$, an initiation set, representing the states in which the option is available, and a termination function $\beta_\omega(\mathbf{s}_t)$, which represents the state-dependent probability of terminating the option. Additionally, the policy over options, $\pi_\Omega(\omega_t|\mathbf{s}_t)$ is employed to select from available options once termination of the previous option occurs.

During execution, option are used as follows: in the initial state, an option is sampled from the policy over options. An action is then taken according to the policy belonging to the currently active option. After selecting this action and observing the next state, the policy then terminates, or does not, according to the termination function. If the option does not terminate, the current option remains active. Otherwise the policy over options can be sampled in the new state in order to determine the next active option.

The policy over options can be combined with the termination function in order to yield the option-to-option policy function:

$$\tilde{\pi}_\Omega(\omega_t|\omega_{t-1}, \mathbf{s}_t) = [1 - \beta_{\omega_{t-1}}(\mathbf{s}_t)]\delta_{\omega_t \omega_{t-1}} + \beta_{\omega_{t-1}}(\mathbf{s}_t)\pi_\Omega(\omega_t|\mathbf{s}_t),$$

where δ is the Kronecker delta.

4. Inferred Option Policy Gradient

To learn options using a policy gradient method we parametrize all aspects of the policy: $\pi_{\Omega,\boldsymbol{\theta}}$ denotes the policy over options, parametrized by $\boldsymbol{\theta}$. $\pi_{\omega,\boldsymbol{\vartheta}}$ then denotes the intra-option policy of option ω, parametrized by $\boldsymbol{\vartheta}$. Finally $\beta_{\omega,\boldsymbol{\xi}}$ is the termination function for ω, parametrized by $\boldsymbol{\xi}$. Rather than explicitly constructing initiation sets, which cannot be done trivially in a differentiable manner, we assume that for all of our options, the initiation set is the entire state space.

We aim to optimize the performance of the agent with respect to a set of policy parameters. The loss function is identical to that employed by traditional policy gradient methods: we optimize the expected return of trajectories in the MDP sampled using the current policy,

$$\rho(\boldsymbol{\theta}, \boldsymbol{\vartheta}, \boldsymbol{\xi}) = \mathbb{E}_\tau \left[R_0^\tau \right] = \int_\tau P(\tau) R_0^\tau \mathrm{d}\tau,$$

where $\mathbb{E}_\tau$ denotes expectation over sampled trajectories.

The expected performance can be maximized by increasing the probability of visiting highly rewarded state-action pairs. To increase this probability, it does not matter which option was originally used in order to generate that state-action pair, rather, we will derive an algorithm that updates all options that *could have* generated that state-action pair. Determining these options is done in a differentiable inference step. As a result the policy can be optimized end-to-end, yielding our *Inferred Option Policy Gradient* algorithm.

In order to compute the gradient of the loss objective, we decompose $P(\tau)$ into the relevant conditional probabilities,

and employ the "likelihood ratio" method, so that it is possible to estimate the gradient from samples:

$$\nabla\rho = \mathbb{E}_\tau\left[R_0^\tau \sum_{i=0}^{T} \nabla \log P(\mathbf{a}_i|\mathbf{s}_{[0:i]}, \mathbf{a}_{[0:i-1]})\right].$$

In this equation, $\mathbf{s}_{[0:i]}$ denotes the sequence of states between step 0 and step i. Gradients are taken with respect to all policy parameters $\boldsymbol{\theta}, \boldsymbol{\vartheta}$, and $\boldsymbol{\xi}$. Note that this is similar to the REINFORCE policy gradient, though here actions are not independent, even when conditioned on states, since information can still pass through the unobserved options. Figure 1 demonstrates this, as the graphical model of trajectories executed under a policy without observed options follows a hidden Markov model-like structure.

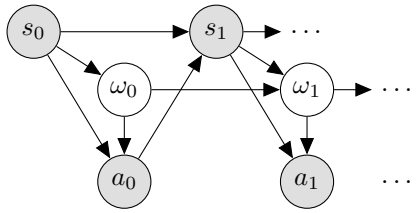


Figure 1. Graphical Model for Option Trajectory

In order to compute the inner gradient, we marginalize over the hidden options at each time step, leading to:

$$\nabla\rho = \mathbb{E}_\tau\left[R_0^\tau \sum_{i=0}^{T} \nabla \log\left(\sum_{\omega_i} \mathbf{m}(i)_{\omega_i} \pi_{\omega_i,\boldsymbol{\vartheta}}(\mathbf{a}_i|\mathbf{s}_i)\right)\right],$$

where $\mathbf{m}(i)$ is the vector of option probabilities at time step i, i.e. $\mathbf{m}(i)_{\omega_i} = P(\omega_i|\mathbf{s}_{[0:i]}, \mathbf{a}_{[0:i-1]})$.

It is this marginalization that explicitly introduces the option structure into our policy. Because we employ marginalization, rather than sampling, it is not necessary to have actually observed the option being executed, as may be the case with data generated by another agent. Further, this introduces a weighted update of all available options, with updates proportional to the likelihood that the agent is in a particular option at that point in the trajectory.

From the graphical model, we observe that the $\mathbf{m}(i)$ term can be expressed in a recursive form, simply as an application of the *forward* algorithm (Baum & Petrie, 1966):

$$\mathbf{m}(i+1)_{\omega_{i+1}} = \sum_{\omega_i} c_i^{-1} \mathbf{m}(i)_{\omega_i} \pi_{\omega_i,\boldsymbol{\vartheta}}(\mathbf{a}_i|\mathbf{s}_i) \tilde{\pi}_{\Omega,\boldsymbol{\theta},\boldsymbol{\xi}}(\omega_{i+1}|\omega_i, \mathbf{s}_{i+1}).$$

In this equation, c_i is a normalization factor, given by:

$$c_i = \sum_{\omega_i} \mathbf{m}(i)_{\omega_i} \pi_{\omega_i,\boldsymbol{\vartheta}}(\mathbf{a}_i|\mathbf{s}_i).$$

The recurrence starts from the initial value $P(\omega_0|\mathbf{s}_0) = \pi_{\Omega,\boldsymbol{\theta}}(\omega_0|\mathbf{s}_0)$. If our policies are differentiable, then this recursive term is differentiable as well, allowing us to perform gradient descent to maximize our objective, using the sampled data to compute the full return Monte Carlo gradient estimate:

$$\nabla\rho \approx R_0^\tau \left[\sum_{i=0}^{T} \nabla \log\left(\sum_{\omega_i} \mathbf{m}(i)_{\omega_i} \pi_{\omega_i,\boldsymbol{\vartheta}}(\mathbf{a}_i|\mathbf{s}_i)\right)\right],$$

where $\tau = (\mathbf{s}_0, \mathbf{a}_0, \ldots, \mathbf{a}_{T-1}, \mathbf{s}_T)$ is a trajectory sampled from the system using the current policy $\pi_{\boldsymbol{\theta}}$. The variance of this estimator can be reduced through inclusion of a constant baseline, through an argument identical to that used for REINFORCE (Williams, 1992).

Here, we notice that actions at any given time step are conditionally independent of rewards received in the past, given the trajectory up to that action. As in other policy gradient methods, we can reduce variance further by removing these terms from our gradient estimator. This is formally expressed as:

$$\forall j < k \quad \mathbb{E}_{\mathbf{s}_{[0:k]},\mathbf{a}_{[0:k]}}\left[r_j \nabla \log P(\mathbf{a}_k|\mathbf{s}_{[0:k]}, \mathbf{a}_{[0:k-1]})\right] = 0.$$

With this realization, we can simplify our estimator to:

$$\nabla\rho \approx \left[\sum_{i=0}^{T}\left(\sum_{j=i}^{T}(r_j) - b(\mathbf{s}_i)\right) \nabla \log\left(\sum_{\omega_i} \mathbf{m}(i)_{\omega_i} \pi_{\omega_i,\boldsymbol{\vartheta}}(\mathbf{a}_i|\mathbf{s}_i)\right)\right], \quad (3)$$

where $b(\mathbf{s}_j)$ is a state-dependent baseline. Note that the estimate is unbiased regardless of the baseline, although good baselines can reduce the variance. In this work, we use a learned parametric approximation of the value function $V_{\boldsymbol{\nu}}$ as baseline. The value function is learned using gradient descent on the mean squared prediction error of Monte-Carlo returns:

$$\sum_{t=0}^{T} \left(V_{\boldsymbol{\nu}}(\mathbf{s}_t) - R_t\right)^2. \quad (4)$$

Estimating the value function can also be done using other standard methods such as LSTD or TD(λ).

The algorithm for learning options to optimize returns through a series of interactions with the environment is given in Algorithm 1. While this algorithm can only be applied in the episodic RL setup, it is also possible to employ the technical insight shown here in an online manner, which is the topic of the next section.

Algorithm 1: Inferred Option Policy Gradient (IOPG)

Initialize parameters randomly
foreach episode **do**
  $\omega_0 \sim \pi_\Omega(\omega|\mathbf{s}_0)$ // sample initial option
  for $t \leftarrow 0, \ldots, T$ **do**
    $\mathbf{a}_t \sim \pi_{\omega_t}(\mathbf{s}_t)$ // sample action from option
    Get $\mathbf{s}_{t+1}$ and r_t from system
    $\omega_{t+1} \sim \tilde{\pi}_\Omega(\omega_{t+1}|\omega_t, \mathbf{s}_{t+1})$ // sample option
  end
  Update $\boldsymbol{\nu}$ according to (4), using sampled data
  $\boldsymbol{\theta}$, $\boldsymbol{\vartheta}$, and $\boldsymbol{\xi}$ according to (3), using sampled data
end

5. Online Gradient Estimates

In addition to the batch gradient estimator described in Section 4, it is also possible to develop an online estimator. While we introduce some bias by updating the parameters in the middle of the trajectory, this increases the number of environments in which our method can be applied. In order to achieve this, we employ a method similar to real-time recurrent learning (Williams & Zipser, 1989), leveraging the recursive structure of the gradient estimate in order to make computation tractable.

For convenience, let $\boldsymbol{\psi}$ denote the concatenation of parameter vectors $\boldsymbol{\theta}$, $\boldsymbol{\vartheta}$, and $\boldsymbol{\xi}$. By noticing that $\mathbf{m}(t)$ is a function of only the data at time t, the current parameters, and the previous value: $\mathbf{m}(t-1)$, alongside a simple application of the chain rule, we observe that:

$$\frac{d\mathbf{m}(t)_\omega}{d\boldsymbol{\psi}} = \frac{\partial \mathbf{m(t)}_\omega}{\partial \mathbf{m}(t-1)} \frac{d\mathbf{m}(t-1)}{d\boldsymbol{\psi}} + \frac{\partial \mathbf{m}(t)_\omega}{\partial \boldsymbol{\psi}}. \quad (5)$$

Thus, in order to efficiently compute this gradient in an online manner, in addition to our parameter vector $\boldsymbol{\psi}$, we maintain an additional set of gradient traces, $g_{\omega\boldsymbol{\psi}}$, for each option, and update them according to equation 5. These values then substitute $\frac{d\mathbf{m}(t-1)}{d\boldsymbol{\psi}}$ when computing the subsequent gradient. This procedure adds an additional memory complexity of $O(|\boldsymbol{\psi}| \times |\Omega|)$, since a gradient trace over all parameters must be maintained for each option.

An inferred option actor-critic (IOAC) algorithm using this gradient estimator is described in Algorithm 2. Note that this algorithm–in addition to learning online–could exhibit lower variance than the IOPG method described above. By using a learned estimator for the returns instead of the Monte Carlo results, updates are more consistent, ideally leading to increased stability, at the cost of some bias.

Algorithm 2: Inferred Option Actor Critic (IOAC)

initialize $\boldsymbol{\psi}$ randomly
foreach episode **do**
  $g_{\omega\boldsymbol{\psi}} \leftarrow \mathbf{0}$
  $\mathbf{s} \leftarrow \mathbf{s}_0$
  $\omega \sim \pi_\Omega(\mathbf{s})$
  for $t \leftarrow 0, \ldots, T$ **do**
    $\mathbf{a} \sim \pi_\omega(\mathbf{a}|\mathbf{s})$
    $\mathbf{s}', r \sim \texttt{step}(\mathbf{a}, \mathbf{s})$
    Update $\boldsymbol{\nu}$ according to TD
    Update $g_{\omega\boldsymbol{\psi}}$ according to (5)
    Substitute $g_{\omega\boldsymbol{\psi}}$ into (3) to update $\boldsymbol{\theta}$ and $\boldsymbol{\vartheta}$
    Draw option termination $b \sim \beta(\mathbf{s}')_\omega$
    if b **then**
      $\omega \sim \pi_\Omega(\mathbf{s}')$
    end
    $\mathbf{s} \leftarrow \mathbf{s}'$
  end
end

6. Experiments

In order to evaluate the effectiveness of our algorithm, as well as the qualitative attributes of the options learned, we examine its performance across several standardized continuous control environments as implemented in the OpenAI Gym (Brockman et al., 2016) in the MuJoCo physics simulator (Todorov et al., 2012). In particular, we examine the following environments:

- Hopper-v1 (observation dimension: 11, action dimension: 3)
- Walker2d-v1 (observation dimension: 17, action dimension: 6)
- HalfCheetah-v1 (observation dimension: 17, action dimension: 6)
- Swimmer-v1 (observation dimension: 8, action dimension: 2).

Generally, they all require the agent to learn to operate joint motors in order to move the agent in a particular direction, with penalties for unnecessary actions. Together, they are considered to be reasonable benchmarks for state-of-the art continuous RL algorithms.

6.1. Comparison of performance

We compared the performance of our algorithm (IOPG) with results from option-critic (OC) and asynchronous actor-critic (A3C) methods, as described in Mnih et al. (2016).

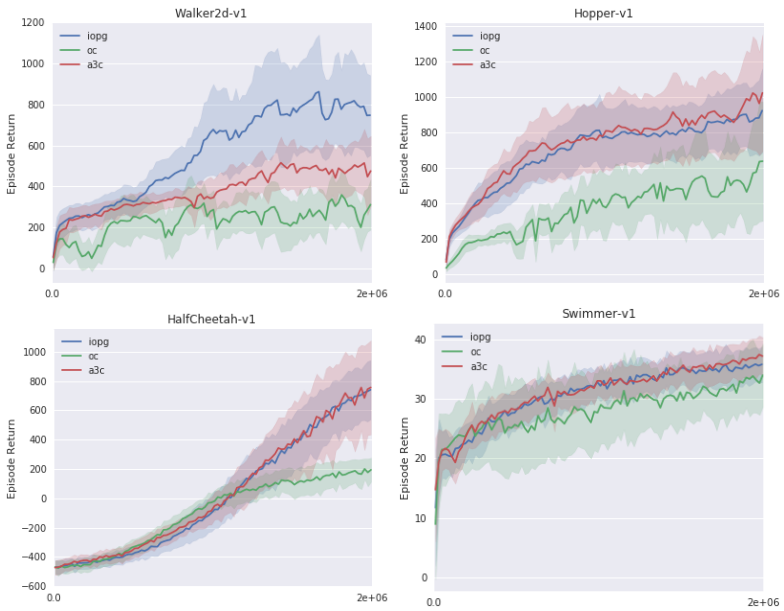


Figure 2. Training curves for 2 million time steps averaged across 10 random seeds for several continuous RL domains. The shaded area represents the 95% confidence interval.

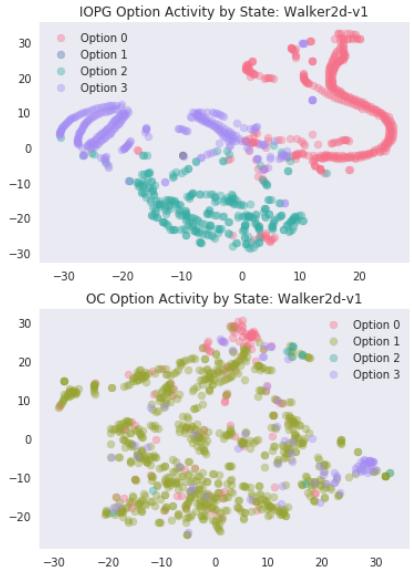


Figure 3. Typical option activity as a function of state. The axes represent T-SNE embeddings of higher dimensional states. Each state is coloured according to the option that was active at that point.

In order to ensure an appropriate comparison, IOPG and OC were also implemented using multiple agents operating in parallel, as is done in A3C. The option-critic algorithm as described in (Bacon et al., 2017) employs greedy option selection according to the learned Q. To ensure a fair comparison, we employed the same parametrized actor as the inter-option policy in our option-critic baseline as was used in IOPG. Since option-critic already learns option-value functions, no SMDP-level value function approximation is needed.

Our model architecture for all three algorithms closely follows that of Schulman et al. (2017). The policies and value functions were represented using separate feed-forward neural networks, with no parameters shared. For each agent, both the value function and the policies used two hidden layers of 64 units with $\tanh$ activation functions. The IOPG and OC methods shared these parameters across all policy and termination networks. The option sub-policies and A3C policies were implemented as linear layers on top of this, representing the mean of a Gaussian distribution. The variance of the policy was parametrized by a linear softplus layer. Option termination was given by a linear sigmoid layer for each option. The policy over options, for OC and IOPG methods, was represented using a final linear softmax layer, of size equal to the number of options available. The value function for IOPG and AC methods was represented using a final linear layer of size 1, and for OC, size $|\Omega|$. All weight matrices were initialized to have normalized rows. RMSProp (Tieleman & Hinton, 2012) was used to optimize parameters for all agents. We employ a single shared set of RMSProp parameters across all asynchronous threads. Additionally, entropy regularization was used during optimization for the AC policies, the option policies and the policies over options. This regularization encourages exploration, and prevents the policies from converging to single repeated actions, as policy gradient methods parametrized by neural networks often suffer from this problem (Mnih et al., 2016).

The results of these experiments are shown in Fig. 2. We see that IOPG, despite having significantly more parameters to optimize, and recovering additional structure, is able to learn as quickly as A3C across all of the domains, and learns significantly faster in the Walker2d environment. This is likely enabled by the fact that all of the options in IOPG can make use of all of the data gathered. OC, on the other hand seems to suffer a reduction in learning speed due to the fact that options are not all learned simultaneously, preventing experience from being shared between them, thus suffering from the additional complexity burden of the additional parameters. It is likely that IOPG performs particularly well in Walker2d due to the relatively high dimensional action space combined with the cyclical nature of good policies being well suited to the option structure.

An additional experiment was performed to investigate the effectiveness of Generalized Advantage Estimation (Schulman et al., 2015) with the IOAC algorithm. Here, gradients were accumulated over the course of a trajectory and applied

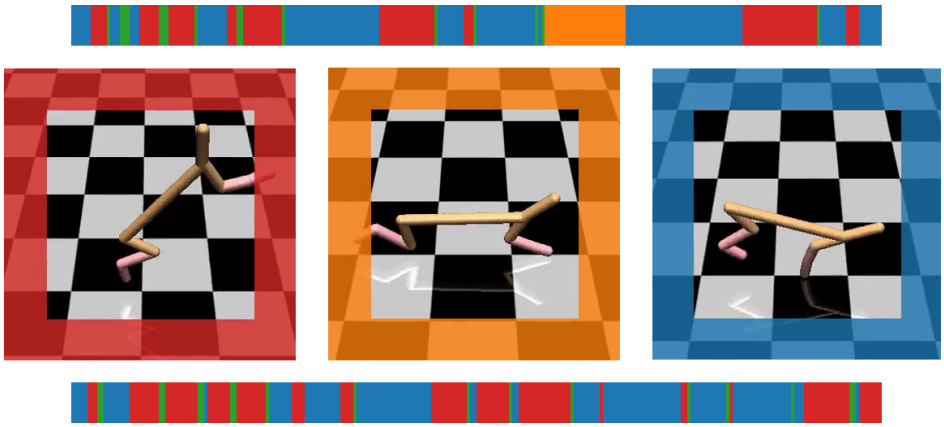


Figure 4. The options discovered by IOPG can be easily interpreted, due to their temporal and dynamic coherence. The coloured bars each represent a trajectory in the HalfCheetah-v1 environment, with each colour representing execution of a different option, and the width of the bar proportional to the number of steps that option was executed. The frames represent the agent behaviour, with the border colour representing the option that was active during that behaviour. Three clear behaviours are observed: a fast "forward leaning" policy (blue), a slower "backward leaning" policy (red), and a slow, but stabilizing "spread" policy (yellow).

only at the end. Results were highly similar to those shown in Fig. 2, with no significant improvement from the use of GAE.

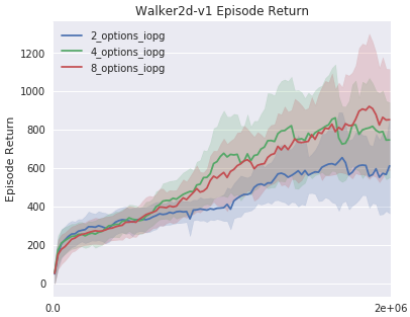
6.2. Option Structure

One important benefit of the extra structure learned by options is that it allows for simplified interpretation of the agent's actions. While the agent's behaviour in the primitive action space may be high dimensional and real-valued, in a spatially and temporally coherent option-space the behaviour can be reduced to a single category. This representation leads to improved interaction and understanding of option behaviour, as well as debugging, monitoring, and potentially verifiable behaviour. In order to further understand if the nature of the options learned by IOPG exhibit these traits, we performed a visualization of them over a random subsample of states in the last 8000 frames. We perform T-SNE (Maaten & Hinton, 2008) on these states in order to represent the high-dimensional state space in two dimensions.

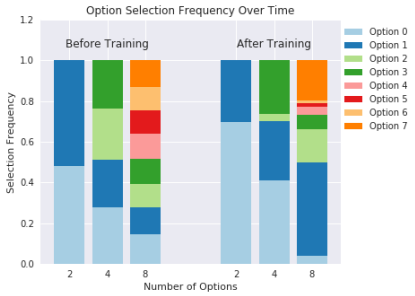
Fig. 3 shows the results of this procedure. We can see that different options are active in different regions of state space. This pattern indicates that the options learned can be interpreted as having some local structure. Options appear to be spatially coherent, as well as having structure in policy space. The relation between state and action abstraction has been observed previously in the RL literature (Andre & Russell, 2002; Provost et al., 2007). This outcome demonstrates the specialization of options learned by IOPG—different options generally do not occupy the same roles in the agent's behaviour.

Further, we find that the options learned represent behaviours that are easily interpreted. We find that options learned by IOPG are both temporally extended—one option often lasts several time steps—and that each option represents a unique policy used by the agent in a coordinated manner to optimize returns. These properties are generally difficult to achieve for automatically learned options, without heuristic structure or regularization. These attributes match the intuitive notion that options represent "skills", or abstract behaviours which can extend over several primitive actions. Fig. 4 clearly demonstrates that learned options possess both temporal coherence and separation in policy-space. We observe that the agent's behavior is represented primarily by three options of the available four: The blue option represents a "forwards leaning" movement, which is the fastest, but leads the agent to become unstable. The red option represents a "backwards leaning" policy, which can slow the agent down, but balance the effects of executing the blue option. Finally, the yellow option stabilizes the agent by lowering its center of mass. This final option is particularly interesting, as it appears to be a transient option learned during training so as to stabilize the agent before it is able to coordinate the other two policies. Once the agent learns to alternate between the red and blue options in a coordinated manner, the yellow option is no longer used.

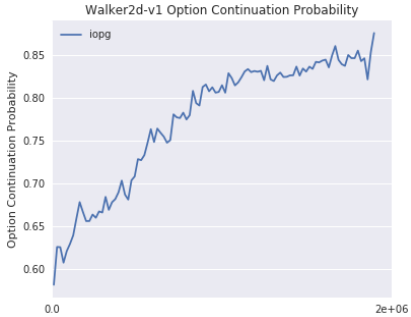
Fig. 5 displays analysis of the options learned in the Walker2d environment. We found that in this particular environment, agents with either four or eight options available perform roughly equally, while having only two options led to sub-optimal performance (Fig. 5a). This effect can be explained by the fact that three options seem to be sufficient, and if more options are given only three of them tend to get frequently selected (Fig. 5b). This finding suggests that only three of the options that IOPG learns are useful here,



(a) Performance in the Walker2d environment as a function of available options. We see that in this environment, having several options available to the agent leads to an improved policy.



(b) In the Walker2d environment, initially option selection is uniform. After training only 3 options tend to be selected, even when more are available. Selection frequencies are averaged over 100 sampled states.



(c) As the options improve during training, the probability of remaining in the active option increases, plateauing at around 0.85. This outcome suggests that the options learned here exhibit temporal extension.

Figure 5. Analysis of the learned options in the Walker2d environment.

perhaps due to the relative simplicity of the environment. In Fig. 5c, we observe further evidence that the options learned by IOPG are temporally extended. A moving average of the continuation probability $(1 - \beta_\omega(s_t))$ during training indicates that early on, when the options are not well optimized, termination occurs quite frequently. As the options improve, termination decreases, until the policy over options is only queried approximately every ten steps on average.

7. Discussion

In this paper, we have introduced a new algorithm for learning hierarchical policies within the options framework, called inferred option policy gradients. This algorithm treats options as latent variables. Gradients are propagated through a differentiable inference step that allows end-to-end learning of option policies, as well as option selection and termination probabilities.

In our algorithms policies take responsibility for state-actions pairs they *could have* generated. In contrast, in learning algorithms for hierarchical policies that use an augmented state space, option policies are updated using only those state-action pairs that were actually generated. As a result, in our algorithm options do not tend to become 'responsible' for unlikely states or actions they generated. This may prevent collapse in the transient option structure, which could be sensitive to rare events. Thus, options are stimulated more strongly to specialize in a part of the state space. We conjecture that this specialization caused the discussed increase in the interpretability of options. Alternatively, it may be the case that fully end-to-end optimization of the policy over options and option policies is what leads to this specialization, as the gradient of the option policies explicitly includes terms concerning the policy over options, which is not the case in previous state-augmented methods.

Furthermore, in our experiments learning with inferred options was significantly faster than learning with an option-augmented state space. In fact, learning with inferred options proved equally fast, or sometimes even faster, than using a comparable non-hierarchical policy gradient method despite IOPG having many more parameters. We conjecture that option inference encourages intra-option learning, thus allowing multiple options to improve as the result of a single learning experience, causing this speed-up.

In future work, we want to quantify the suitability of the learned options for transfer between tasks. Our experiments so far were in the episodic setting. In Section 5, we introduced an on-line actor-critic version of our algorithm, that can learn continuously in infinite-horizon setting. We plan to investigate this variant more closely in future work. Furthermore, while we did not examine such methods here, we want to investigate how techniques for improving learning stability, as used in, e.g., the TRPO and PPO algorithms, can be leveraged in within the context of learning with inferred options.

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