# Supplementary Material for
# An Efficient Semismooth Newton based Algorithm for Convex Clustering

**Yancheng Yuan** [1] **Defeng Sun** [2] **Kim-Chuan Toh** [3]

## Proof of Lemma 1

**Lemma 1** *For any $t > 0$, the proximal mapping $\text{Prox}_{t\|\cdot\|_2}$ is strongly semismooth.*

**Proof**. From the definition of $\text{Prox}_{t\|\cdot\|_2}(\cdot)$ and the fact that the projection of any vector onto the second order cone, i.e., the epigraph of the $\ell_2$ norm function, is strongly semismooth (Chen et al., 2003) [Proposition 4.3]. Then, we can obtain the conclusion directly from (Meng et al., 2005) [Theorem 4].

## Proof of Theorem 2

**Proof**. From Lemma 1, we know that $\text{Prox}_{t\|\cdot\|_2}$ is strongly semismooth for any $t > 0$, together with the Moreau identity $\text{Prox}_{tp}(x) + t\text{Prox}_{p^*/t}(x/t) = x$, we know that

$$\nabla\phi(X) = X - A + \mathcal{B}^*(\text{Prox}_{\sigma p^*}(\sigma\mathcal{B}(X) + \tilde{Z}))$$

is strongly semismooth. By (Zhao et al., 2010) [Proposition 3.3], we know that $d^j$ obtained in SSNCG is a descent direction, which guarantees the Algorithm SSNCG is well defined.
From (Zhao et al., 2010) [Theorem 3.4, 3.5], we can get the desired results in this theorem.

## More Numerical Results on Unbalanced Gaussian Dataset

In this section, we show detailed time comparison experiment results between SSNAL and AMA on Unbalanced Gaussian dataset. We summarize our results in Table 1. We can see that AMA almost cannot solve the problem within 100000 iterations.

[1]Department of Mathematics, National University of Singapore [2]Department of Applied Mathematics, Hong Kong Polytechnic University [3]Department of Mathematics, National University of Singapore. Correspondence to: Yancheng Yuan <yuanyancheng@u.nus.edu>.

*Proceedings of the $35^{th}$ International Conference on Machine Learning*, Stockholm, Sweden, PMLR 80, 2018. 

## References

Chen, XD, Sun, D, and Sun, J. Complementarity functions and numerical experiments on some smoothing newton methods for second-order-cone complementarity problems. *Computational Optimization and Applications*, 25 (1-3):39–56, 2003.

Meng, Fanwen, Sun, Defeng, and Zhao, Gongyun. Semismoothness of solutions to generalized equations and the moreau-yosida regularization. *Mathematical programming*, 104(2-3):561–581, 2005.

Zhao, Xin-Yuan, Sun, Defeng, and Toh, Kim-Chuan. A newton-cg augmented lagrangian method for semidefinite programming. *SIAM Journal on Optimization*, 20(4): 1737–1765, 2010.

*Table 1.* Numerical Results on Unbalanced Gaussian Dataset. $k = 10$, $\phi = 0.5$. $Gap = P_{AMA} - P_{Ssnal}$

.

| $\gamma$ | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|---|---|---|---|---|---|
| $t_{AMA}$(S) | 264.54 | 256.21 | 260.06 | 262.16 | 263.27 |
| $t_{Ssnal}$(S) | **1.15** | **0.57** | **0.67** | **0.66** | **0.86** |
| $Iter_{AMA}$ | 100000 | 97560 | 97333 | 100000 | 100000 |
| $Iter_{Ssncg}$ | **23** | **21** | **24** | **24** | **27** |
| $P_{AMA}$ | 0.784611 | 0.990282 | 1.184049 | 1.365560 | 1.535217 |
| $P_{Ssnal}$ | 0.783797 | 0.990281 | 1.184048 | 1.365554 | 1.535209 |
| $Gap$ | 0.000814 | 1E-6 | 1E-6 | 6E-6 | 8E-6 |
| $\gamma$ | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
| $t_{AMA}$(S) | 270.88 | 264.12 | 264.58 | 271.85 | 264.84 |
| $t_{Ssnal}$(S) | **0.97** | **1.03** | **1.03** | **1.05** | **1.04** |
| $Iter_{AMA}$ | 100000 | 100000 | 100000 | 100000 | 100000 |
| $Iter_{Ssncg}$ | **28** | **28** | **27** | **27** | **26** |
| $P_{AMA}$ | 1.693393 | 1.840384 | 1.976499 | 2.101939 | 2.216956 |
| $P_{Ssnal}$ | 1.693375 | 1.840369 | 1.976470 | 2.101920 | 2.216925 |
| $Gap$ | 0.000018 | 0.000015 | 0.000028 | 2E-5 | 3E-5 |