Supplementary material

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1. Summary of experimental tasks

As mentioned in the main text, we conducted empirical evaluations on a large number of synthetic test problems:

- The tasks mat-2 and mat-6 refer to functions drawn from a Gaussian process(GP) with Matérn-52 kernel in $\mathbb{R}^2$ and $\mathbb{R}^6$ respectively.

- The global optimisation tasks[1] that we considered are the Ackley function defined on $\mathbb{R}^5$ and $\mathbb{R}^{10}$ (ack-5 and ack-10), the Michalewicz function defined on $\mathbb{R}^5$ and $\mathbb{R}^{10}$ (mic-5 and mic-10) and the Eggholder function in $\mathbb{R}^2$ (egg-2).

- We also selected a robot pushing simulation experiment, which was first explored in a BO context by Wang & Jegelka (2017). Here the task is to learn the correct pushing action to minimise the distance of the robot to a goal. The problem has 4 inputs: the robot's location (r_x, r_y), the angle of the pushing force r_θ and the pushing duration t_r. We used the input space suggested by Wang & Jegelka (2017).

2. Asynchronous vs. synchronous parallel BO

Similar to Fig. 4 in the main text, Figs. 1 and 2 show head-to-head comparisons of synchronous and asynchronous methods, here on the ack-10 task.

3. Asynchronous BO

We conducted a large number of experiments testing the different approaches for asynchronous BO. We computed the mean and standard deviation of the log of the simple regret across 30 random initialisations (see Eq. 13 in the main text

for the definition). Tables 1, 2 and 3 show the results after 50, 75 and 100 asynchronous BO steps respectively.

Across all of these experiments, we can see that PLAyBOOK is performing competitively, making it an attractive choice for asynchronous BO problems.

References

Wang, Z. and Jegelka, S. Max-value entropy search for efficient Bayesian optimization. In *International Conference on Machine Learning (ICML)*, 2017.

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[1]Details for these and other challenging global optimisation test functions can be found at https://www.sfu.ca/~ssurjano/optimization.html

k	Task	KB	TS	PLAyBOOK			
				L	LL	H	HL
2	ack-10	-0.30 (0.16)	-0.01 (0.03)	-0.23 (0.15)	**-0.37 (0.25)**	-0.32 (0.18)	-0.27 (0.18)
	ack-5	-0.55 (0.39)	-0.22 (0.20)	**-0.85 (0.52)**	-0.52 (0.40)	-0.64 (0.39)	-0.71 (0.48)
	egg-2	0.28 (0.72)	0.89 (0.92)	-0.12 (1.26)	-0.05 (1.03)	**-0.44 (2.13)**	-0.31 (1.62)
	mat-2	0.81 (0.30)	1.05 (0.26)	**0.78 (0.28)**	0.82 (0.26)	0.81 (0.21)	0.89 (0.20)
	mat-6	1.02 (0.16)	1.13 (0.14)	**0.85 (0.43)**	0.92 (0.33)	0.88 (0.26)	0.86 (0.31)
	mic-10	1.84 (0.08)	1.92 (0.07)	**1.78 (0.11)**	1.80 (0.09)	1.82 (0.11)	1.83 (0.10)
	mic-5	0.71 (0.28)	1.02 (0.13)	0.67 (0.27)	**0.66 (0.26)**	0.69 (0.33)	0.87 (0.16)
	nrobot-4	-0.71 (1.05)	-0.52 (1.21)	-0.88 (0.91)	-0.78 (0.86)	**-0.89 (0.69)**	-0.87 (0.91)
4	ack-10	-0.29 (0.17)	-0.01 (0.03)	-0.26 (0.16)	-0.32 (0.17)	-0.25 (0.14)	**-0.34 (0.18)**
	ack-5	-0.54 (0.36)	-0.21 (0.13)	-0.66 (0.45)	-0.41 (0.27)	-0.53 (0.41)	**-0.71 (0.52)**
	egg-2	0.19 (1.06)	0.79 (0.92)	0.53 (0.91)	0.23 (0.98)	0.29 (0.86)	**-0.06 (1.26)**
	mat-2	**0.77 (0.30)**	0.98 (0.37)	0.94 (0.22)	1.01 (0.28)	0.82 (0.25)	0.87 (0.22)
	mat-6	1.06 (0.17)	1.13 (0.05)	0.99 (0.18)	1.01 (0.26)	0.95 (0.23)	**0.94 (0.20)**
	mic-10	1.82 (0.11)	1.92 (0.07)	1.82 (0.08)	1.83 (0.08)	**1.78 (0.09)**	1.80 (0.08)
	mic-5	**0.66 (0.28)**	1.01 (0.16)	0.78 (0.23)	0.87 (0.19)	0.76 (0.20)	0.82 (0.33)
	nrobot-4	-0.67 (1.00)	-0.39 (1.04)	**-1.01 (0.94)**	-0.94 (0.97)	-0.75 (0.70)	-0.63 (0.79)
6	ack-10	-0.24 (0.19)	-0.01 (0.04)	-0.29 (0.15)	-0.25 (0.14)	-0.20 (0.14)	**-0.31 (0.16)**
	ack-5	-0.51 (0.28)	-0.20 (0.20)	**-0.54 (0.40)**	-0.27 (0.18)	-0.35 (0.24)	-0.49 (0.27)
	egg-2	0.37 (1.05)	0.78 (0.88)	0.71 (0.83)	0.77 (0.82)	**0.07 (1.25)**	0.32 (1.11)
	mat-2	0.85 (0.25)	1.04 (0.19)	0.98 (0.34)	1.06 (0.23)	**0.84 (0.23)**	0.85 (0.25)
	mat-6	1.01 (0.17)	1.07 (0.18)	**0.95 (0.28)**	1.02 (0.28)	1.03 (0.16)	0.99 (0.24)
	mic-10	1.84 (0.08)	1.92 (0.07)	1.84 (0.08)	1.81 (0.08)	1.84 (0.08)	**1.80 (0.09)**
	mic-5	**0.76 (0.21)**	1.04 (0.15)	0.84 (0.23)	0.89 (0.24)	0.88 (0.19)	0.87 (0.20)
	nrobot-4	-0.58 (0.96)	-0.22 (0.95)	**-0.69 (1.04)**	-0.47 (0.85)	-0.63 (1.02)	-0.51 (0.99)
8	ack-10	-0.23 (0.17)	-0.01 (0.03)	-0.26 (0.17)	-0.34 (0.15)	-0.17 (0.11)	**-0.40 (0.14)**
	ack-5	-0.44 (0.27)	-0.19 (0.14)	-0.65 (0.40)	-0.33 (0.21)	-0.38 (0.30)	**-0.68 (0.37)**
	egg-2	0.31 (0.93)	0.65 (0.92)	0.60 (1.20)	0.93 (0.77)	0.42 (0.61)	**0.17 (1.05)**
	mat-2	0.80 (0.22)	0.93 (0.26)	0.81 (0.44)	1.09 (0.20)	0.87 (0.27)	**0.79 (0.30)**
	mat-6	**1.01 (0.19)**	1.13 (0.06)	1.03 (0.17)	1.02 (0.22)	1.03 (0.15)	1.03 (0.20)
	mic-10	1.84 (0.09)	1.91 (0.07)	1.82 (0.08)	1.81 (0.10)	1.80 (0.13)	**1.78 (0.08)**
	mic-5	0.70 (0.33)	1.02 (0.20)	**0.68 (0.32)**	0.79 (0.25)	0.73 (0.32)	0.69 (0.32)
	nrobot-4	-0.61 (0.78)	-0.20 (0.96)	**-0.94 (0.92)**	-0.62 (1.14)	-0.76 (1.11)	-0.39 (0.73)
16	ack-10	-0.13 (0.08)	-0.02 (0.04)	-0.24 (0.13)	-0.24 (0.13)	-0.22 (0.10)	**-0.44 (0.13)**
	ack-5	-0.36 (0.30)	-0.18 (0.16)	-0.49 (0.34)	-0.36 (0.24)	-0.28 (0.19)	**-0.73 (0.26)**
	egg-2	0.58 (0.73)	0.56 (1.10)	1.04 (0.51)	1.22 (0.50)	**0.30 (0.96)**	0.71 (0.53)
	mat-2	**0.82 (0.27)**	0.89 (0.24)	1.07 (0.17)	1.12 (0.20)	0.92 (0.31)	0.90 (0.31)
	mat-6	1.10 (0.05)	1.14 (0.03)	1.05 (0.14)	1.06 (0.21)	1.02 (0.26)	**1.00 (0.26)**
	mic-10	1.86 (0.08)	1.90 (0.07)	1.82 (0.07)	1.82 (0.09)	1.83 (0.08)	**1.79 (0.09)**
	mic-5	**0.81 (0.25)**	1.02 (0.19)	0.85 (0.21)	0.84 (0.23)	0.81 (0.22)	0.82 (0.25)
	nrobot-4	-0.43 (0.94)	-0.22 (0.99)	**-0.78 (0.95)**	-0.14 (0.77)	-0.37 (0.80)	-0.40 (0.81)

Table 1. Mean and standard deviation of the log(regret) after 50 steps of asynchronous BO.

k	Task	KB	TS	PLAyBOOK			
				L	LL	H	HL
2	ack-10	-0.43 (0.18)	-0.01 (0.03)	-0.48 (0.27)	**-0.58 (0.26)**	-0.55 (0.32)	-0.45 (0.28)
	ack-5	-0.91 (0.56)	-0.32 (0.22)	**-1.15 (0.58)**	-0.76 (0.50)	-1.03 (0.52)	-0.92 (0.51)
	egg-2	-0.12 (0.92)	0.87 (0.91)	-0.59 (1.16)	**-1.13 (2.14)**	-0.81 (1.99)	-0.82 (1.68)
	mat-2	0.80 (0.30)	1.05 (0.27)	**0.76 (0.28)**	0.81 (0.26)	0.81 (0.21)	0.87 (0.20)
	mat-6	0.95 (0.21)	1.17 (0.14)	**0.74 (0.53)**	0.87 (0.31)	0.84 (0.34)	0.89 (0.29)
	mic-10	1.79 (0.12)	1.92 (0.07)	**1.75 (0.11)**	1.76 (0.14)	1.79 (0.12)	1.79 (0.13)
	mic-5	**0.52 (0.42)**	0.97 (0.17)	0.61 (0.29)	0.56 (0.24)	0.57 (0.37)	0.73 (0.28)
	nrobot-4	-1.06 (1.08)	-0.83 (1.18)	-1.20 (0.86)	**-1.31 (0.75)**	-1.24 (0.78)	-1.29 (0.80)
4	ack-10	**-0.52 (0.21)**	-0.01 (0.03)	-0.51 (0.27)	-0.42 (0.19)	-0.46 (0.22)	-0.50 (0.22)
	ack-5	-0.83 (0.48)	-0.31 (0.23)	**-1.10 (0.53)**	-0.57 (0.33)	-0.75 (0.62)	-0.90 (0.54)
	egg-2	-0.19 (0.87)	0.69 (1.13)	0.16 (1.70)	-0.81 (2.55)	-0.23 (1.10)	**-0.89 (2.48)**
	mat-2	**0.75 (0.29)**	0.98 (0.37)	0.93 (0.22)	1.01 (0.28)	0.80 (0.25)	0.85 (0.22)
	mat-6	0.97 (0.27)	1.18 (0.06)	0.95 (0.23)	1.02 (0.24)	**0.86 (0.28)**	0.87 (0.30)
	mic-10	1.77 (0.12)	1.92 (0.07)	1.77 (0.09)	1.78 (0.09)	**1.74 (0.09)**	1.77 (0.10)
	mic-5	**0.56 (0.30)**	0.97 (0.14)	0.69 (0.25)	0.76 (0.19)	0.63 (0.22)	0.67 (0.34)
	nrobot-4	-0.92 (0.95)	-1.01 (1.31)	**-1.51 (0.88)**	-1.23 (0.89)	-1.07 (0.68)	-1.04 (0.79)
6	ack-10	-0.41 (0.22)	-0.01 (0.04)	**-0.50 (0.26)**	-0.30 (0.15)	-0.32 (0.18)	-0.38 (0.19)
	ack-5	-0.83 (0.46)	-0.32 (0.23)	**-0.86 (0.54)**	-0.36 (0.23)	-0.52 (0.31)	-0.76 (0.37)
	egg-2	0.19 (1.00)	0.64 (1.14)	0.56 (0.93)	0.75 (0.84)	**-0.34 (1.48)**	-0.21 (1.03)
	mat-2	**0.82 (0.26)**	1.03 (0.20)	0.98 (0.34)	1.06 (0.23)	0.82 (0.22)	0.84 (0.25)
	mat-6	0.92 (0.25)	1.14 (0.17)	**0.86 (0.36)**	1.08 (0.25)	0.95 (0.28)	0.94 (0.26)
	mic-10	1.81 (0.08)	1.92 (0.07)	1.81 (0.08)	1.79 (0.09)	1.80 (0.10)	**1.78 (0.10)**
	mic-5	**0.63 (0.28)**	1.00 (0.14)	0.74 (0.29)	0.82 (0.22)	0.76 (0.26)	0.69 (0.33)
	nrobot-4	-1.02 (0.88)	-0.73 (1.20)	-0.99 (1.04)	-0.78 (0.96)	**-1.04 (1.02)**	-1.02 (0.95)
8	ack-10	-0.44 (0.21)	-0.01 (0.03)	-0.47 (0.21)	-0.40 (0.20)	-0.28 (0.17)	**-0.52 (0.16)**
	ack-5	-0.77 (0.40)	-0.33 (0.20)	**-1.04 (0.45)**	-0.39 (0.22)	-0.52 (0.41)	-0.92 (0.36)
	egg-2	**-0.11 (0.96)**	0.57 (1.00)	0.40 (1.22)	0.85 (0.84)	0.22 (0.51)	-0.05 (0.96)
	mat-2	0.78 (0.23)	0.91 (0.26)	0.81 (0.44)	1.09 (0.20)	0.82 (0.26)	**0.76 (0.30)**
	mat-6	0.95 (0.24)	1.18 (0.10)	0.97 (0.20)	1.10 (0.20)	**0.93 (0.25)**	0.97 (0.28)
	mic-10	1.80 (0.09)	1.91 (0.07)	**1.75 (0.09)**	1.78 (0.09)	1.78 (0.13)	1.76 (0.08)
	mic-5	0.58 (0.38)	0.98 (0.19)	0.59 (0.36)	0.70 (0.26)	0.61 (0.33)	**0.55 (0.53)**
	nrobot-4	-0.92 (0.89)	-0.65 (1.01)	**-1.25 (0.82)**	-0.92 (1.08)	-1.07 (1.10)	-0.92 (0.88)
16	ack-10	-0.27 (0.14)	-0.02 (0.04)	-0.43 (0.21)	-0.31 (0.20)	-0.31 (0.14)	**-0.54 (0.16)**
	ack-5	-0.55 (0.34)	-0.29 (0.19)	-0.77 (0.42)	-0.39 (0.25)	-0.39 (0.28)	**-0.94 (0.30)**
	egg-2	0.26 (0.74)	0.41 (1.18)	0.89 (0.63)	1.21 (0.50)	**-0.39 (1.81)**	0.42 (0.61)
	mat-2	**0.79 (0.27)**	0.88 (0.24)	1.07 (0.17)	1.12 (0.20)	0.87 (0.30)	0.86 (0.31)
	mat-6	1.03 (0.20)	1.20 (0.03)	0.95 (0.21)	1.08 (0.25)	**0.94 (0.26)**	0.96 (0.36)
	mic-10	1.83 (0.09)	1.90 (0.07)	1.79 (0.08)	1.79 (0.09)	1.79 (0.10)	**1.75 (0.13)**
	mic-5	**0.66 (0.27)**	0.95 (0.21)	0.77 (0.25)	0.73 (0.39)	0.75 (0.23)	0.75 (0.25)
	nrobot-4	-0.72 (0.87)	-0.80 (1.24)	**-1.00 (1.01)**	-0.61 (0.95)	-0.72 (0.61)	-0.79 (0.78)

Table 2. Mean and standard deviation of the log(regret) after 75 steps of asynchronous BO.

k	Task	KB	TS	PLAyBOOK			
				L	LL	H	HL
2	ack-10	-0.67 (0.27)	-0.01 (0.03)	-0.72 (0.34)	-0.73 (0.29)	**-0.80 (0.35)**	-0.58 (0.26)
	ack-5	-1.28 (0.70)	-0.35 (0.22)	**-1.49 (0.65)**	-0.95 (0.53)	-1.39 (0.62)	-1.08 (0.47)
	egg-2	-0.42 (1.62)	0.80 (0.92)	-1.10 (1.56)	**-1.58 (2.49)**	-1.54 (2.28)	-1.30 (2.01)
	mat-2	0.79 (0.30)	1.04 (0.25)	**0.76 (0.28)**	0.81 (0.26)	0.81 (0.21)	0.87 (0.19)
	mat-6	1.00 (0.21)	1.27 (0.17)	**0.93 (0.36)**	1.00 (0.31)	0.94 (0.26)	1.01 (0.27)
	mic-10	1.76 (0.13)	1.92 (0.07)	**1.72 (0.10)**	1.73 (0.13)	1.72 (0.13)	1.76 (0.12)
	mic-5	**0.39 (0.51)**	0.94 (0.18)	0.54 (0.31)	0.45 (0.24)	0.45 (0.36)	0.65 (0.28)
	nrobot-4	-1.33 (1.02)	-1.12 (1.21)	-1.54 (0.84)	-1.51 (0.85)	-1.46 (0.74)	**-1.58 (0.78)**
4	ack-10	**-0.72 (0.24)**	-0.01 (0.03)	-0.68 (0.28)	-0.54 (0.24)	-0.67 (0.24)	-0.56 (0.24)
	ack-5	-1.13 (0.59)	-0.41 (0.25)	**-1.46 (0.59)**	-0.68 (0.32)	-0.98 (0.73)	-1.04 (0.52)
	egg-2	-0.27 (0.82)	0.65 (1.12)	-0.16 (1.71)	-1.09 (2.57)	-0.50 (1.03)	**-1.17 (2.53)**
	mat-2	**0.74 (0.29)**	0.98 (0.37)	0.93 (0.22)	1.01 (0.28)	0.79 (0.25)	0.85 (0.22)
	mat-6	1.03 (0.25)	1.30 (0.06)	1.07 (0.19)	1.12 (0.20)	**0.98 (0.25)**	1.01 (0.23)
	mic-10	1.74 (0.11)	1.92 (0.07)	1.75 (0.09)	1.74 (0.10)	**1.70 (0.09)**	1.74 (0.11)
	mic-5	**0.44 (0.32)**	0.96 (0.14)	0.55 (0.27)	0.63 (0.21)	0.50 (0.27)	0.63 (0.34)
	nrobot-4	-1.14 (0.88)	-1.24 (1.23)	**-1.73 (0.88)**	-1.48 (0.72)	-1.32 (0.83)	-1.24 (0.83)
6	ack-10	-0.63 (0.27)	-0.01 (0.04)	**-0.65 (0.29)**	-0.37 (0.20)	-0.44 (0.25)	-0.43 (0.20)
	ack-5	**-1.19 (0.60)**	-0.39 (0.26)	-1.17 (0.65)	-0.39 (0.22)	-0.68 (0.41)	-0.87 (0.40)
	egg-2	-0.18 (1.05)	0.57 (1.15)	0.50 (0.94)	0.65 (0.98)	**-0.53 (1.40)**	-0.45 (0.99)
	mat-2	0.82 (0.25)	1.02 (0.20)	0.98 (0.34)	1.06 (0.23)	**0.81 (0.22)**	0.83 (0.24)
	mat-6	1.02 (0.24)	1.25 (0.16)	**0.99 (0.28)**	1.18 (0.27)	1.05 (0.23)	1.03 (0.23)
	mic-10	1.77 (0.10)	1.92 (0.07)	**1.76 (0.08)**	1.76 (0.08)	1.76 (0.13)	1.77 (0.10)
	mic-5	**0.51 (0.34)**	0.96 (0.15)	0.66 (0.27)	0.72 (0.25)	0.66 (0.37)	0.63 (0.30)
	nrobot-4	-1.27 (0.80)	-1.12 (1.28)	-1.24 (0.89)	-1.03 (0.94)	**-1.35 (0.95)**	-1.27 (1.00)
8	ack-10	-0.57 (0.22)	-0.01 (0.03)	-0.62 (0.26)	-0.44 (0.23)	-0.41 (0.25)	**-0.63 (0.16)**
	ack-5	-1.16 (0.47)	-0.42 (0.24)	**-1.32 (0.46)**	-0.47 (0.23)	-0.70 (0.51)	-1.12 (0.41)
	egg-2	-0.24 (0.91)	0.42 (1.13)	-0.02 (2.20)	0.78 (0.86)	-0.04 (0.54)	**-0.27 (0.85)**
	mat-2	0.78 (0.23)	0.90 (0.25)	0.81 (0.44)	1.09 (0.20)	0.81 (0.26)	**0.75 (0.30)**
	mat-6	**1.01 (0.23)**	1.30 (0.09)	1.07 (0.17)	1.23 (0.17)	1.02 (0.19)	1.04 (0.25)
	mic-10	1.78 (0.08)	1.91 (0.07)	**1.72 (0.10)**	1.75 (0.09)	1.74 (0.13)	1.74 (0.08)
	mic-5	**0.46 (0.37)**	0.93 (0.19)	0.53 (0.35)	0.68 (0.26)	0.50 (0.32)	0.50 (0.52)
	nrobot-4	-1.24 (0.76)	-1.18 (1.23)	**-1.44 (0.84)**	-1.17 (0.98)	-1.35 (1.02)	-1.13 (0.85)
16	ack-10	-0.44 (0.20)	-0.02 (0.04)	**-0.63 (0.24)**	-0.37 (0.18)	-0.40 (0.18)	-0.61 (0.16)
	ack-5	-0.83 (0.43)	-0.41 (0.23)	-1.03 (0.51)	-0.41 (0.24)	-0.43 (0.30)	**-1.11 (0.35)**
	egg-2	-0.41 (2.18)	0.34 (1.17)	0.83 (0.67)	1.21 (0.49)	**-0.86 (2.23)**	-0.53 (2.03)
	mat-2	**0.79 (0.26)**	0.88 (0.24)	1.07 (0.16)	1.12 (0.20)	0.86 (0.29)	0.83 (0.31)
	mat-6	1.09 (0.18)	1.31 (0.04)	1.02 (0.21)	1.21 (0.20)	**1.01 (0.23)**	1.02 (0.42)
	mic-10	1.80 (0.11)	1.90 (0.07)	1.75 (0.09)	1.77 (0.10)	1.76 (0.10)	**1.72 (0.13)**
	mic-5	**0.53 (0.33)**	0.91 (0.21)	0.68 (0.25)	0.67 (0.38)	0.66 (0.28)	0.60 (0.36)
	nrobot-4	-1.10 (0.86)	-1.15 (1.20)	**-1.39 (0.95)**	-0.95 (1.02)	-0.86 (0.60)	-1.07 (0.92)

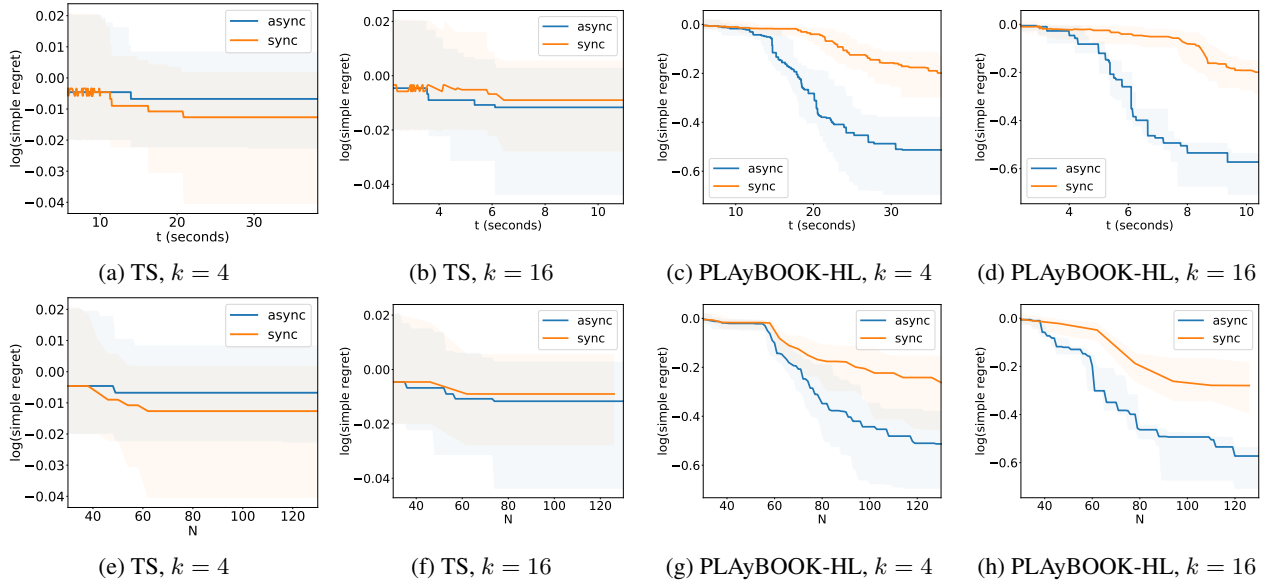*Table 3.* Mean and standard deviation of the log(regret) after 100 steps of asynchronous BO.
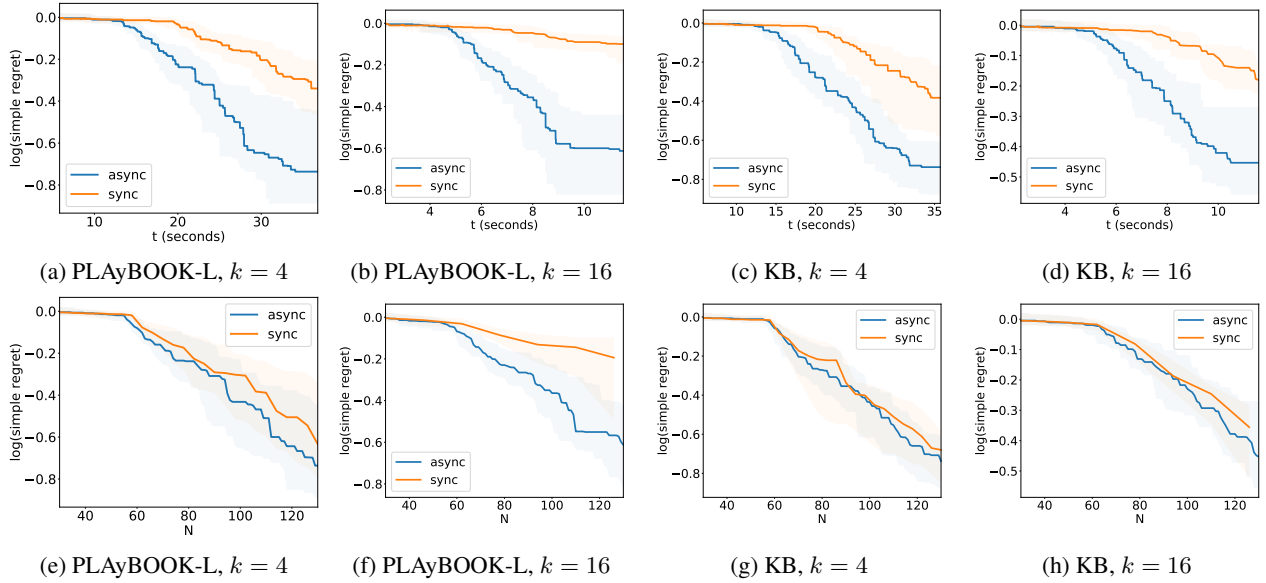
Figure 1. Head-to-head comparison on ack-10

Figure 2. Head-to-head comparison on ack-10